Objectives

• Finish fixed-income
• Options
  – what
  – simple strategies
  – introduction to pricing

Including Time Value or Putting it all together

• Up to now, ignored time value
• Change in position over small unit of time

\[
\frac{\Delta P}{P} \approx y \Delta t - D_p \beta_p \Delta F + 0.5C_p \beta_p^2 (\Delta F)^2
\]

\[
= A_p \Delta t - D_p \beta_p \Delta F + 0.5C_p \beta_p^2 (\Delta F)^2
\]
**Portfolio Movements**

- Invest $x$ in 1 year PDB and $(1-x)$ in 2 year PDB
- 50% in each of the bonds
  - with sensitivities 1.1, and 1.2
- intercepts of 0.001 and 0.002
- set $\Delta t=1$

---

**Computations**

\[
\frac{\Delta P}{P} \approx (xA_1 + (1-x)A_2) - (xD_1\beta_1 + (1-x)D_2\beta_2) \Delta F \\
+ 0.5(xC_1\beta_1^2 + (1-x)C_2\beta_2^2)(\Delta F)^2
\]

\[= (0.5(0.001) + 0.5(0.002)) - (0.5(1.1) + 0.5(2)(1.2)) \Delta F \\
+ 0.5(0.5(1.1)^2 + 0.5(2)^2(1.2)^2)(\Delta F)^2
\]
Adjusted Convexity vs. Yield

- spread position: short bullet and long barbell
- Modified duration = 0
- Positive convexity
- Intercept?
Fixed Income Summary

• PDB prices
• forward rates and no-arbitrage restrictions
• Risk Management
  – duration and convexity for parallel shifts
    • PVBP, modified duration
    • price of convexity
  – adjusting for non-parallel shifts: factor model

Introduction to options

• Options are everywhere
  – traded options: basic and exotic
  – interest rate options
  – many securities have embedded options
    • callable debt, convertible debt, rights issues, warrants, mortgages, etc.
  – Many real investments have option features
Terminology

- Call
- Put

More terminology

- American style
- European style
Terminology

• Exercising:

• Strike (exercise) price:

• expiration date

More terminology

• In the money

• At the money

• Out of the money
Stock Options

- 3 month cycle
- expire on 3rd Saturday of month, trading stocks Friday afternoon
- protected for stock splits, stock dividends, NOT for regular dividends...

Exotics

- Asian options
- lookback
- barrier
- binary
Notation

- $C_t$: price of call, $P_t$: put price, $S_t$: stock price
- $r$: risk free rate

Sequence of events for American call

- Now: purchase call for $C_0$
- Exercise time, $T$
  - $S_T > X$: exercise and pay $X$ for stock
  - $S_T < X$: don’t exercise

$$C_T = \begin{cases} S_T - X, & \text{if } S_T > X \\ 0, & \text{otherwise} \end{cases}$$

$$= \max(0, S_T - X)$$
Puts

- Similar timing
- Buying put not the same as writing call

\[ P_T = \begin{cases} 
0, & \text{if } S_T > X \\
X - S_T, & \text{if } S_T \leq X 
\end{cases} \]

\[ = \max(0, X - S_T) \]
**European call**

Payoffs

Long call

X

Stock price

**European Put**

Options Payoff

Long put

Stock Price

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Some strategies

• Naked
• Protective put
• Covered call
• Straddle
• Strangle

Protective put

• Purchase underlying security
• Purchase put option, exercise price X
Algebraically

<table>
<thead>
<tr>
<th>Position</th>
<th>$S_T &lt; X$</th>
<th>$S_T &gt; X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying</td>
<td>$S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Put</td>
<td>$X - S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Net</td>
<td>$X$</td>
<td>$S_T$</td>
</tr>
</tbody>
</table>

Covered call

- Purchase underlying
- Write call option against it
**Spread**

- Buy put and call, both at the same strike

**Strangle**

- Buy call and put, call exercise < put exercise
Spreads

- combination of 2+ calls or puts, same asset with differing exercise prices or times to expiration
  - Vertical or money spread
    - Same maturity and different exercise price
  - Horizontal or time spread
    - different maturities

Main Points

- Lots of strategies possible
- Options allow you to customize cash flows across states in the future
Put call parity

• Relationship between price of European call and put
• Independent of assumptions about randomness in underlying
  – stocks
  – indexes
  – bonds, currencies, etc.

Final Payoffs: Long call and short put

Position Value

Underlying
Arbitrage relationship

Summary

- Adding passing of time time to bond models
- Options
  - what
  - basic strategies
  - put/call parity
Next Time

- More basic properties of option values
- Basic option pricing and hedging model
  – binomial model