Interest Rate Risk Management

Risk management tools: duration etc.

Objectives

- Apply duration, PVBP, etc. to trading situations
- Application to Orange Country Bankruptcy
- Explain how repo works
Duration

• % Change in bond price for a % Change in Yield
• Formula:

\[ D = -\frac{\partial B}{\partial Y} \times \frac{Y}{B} \]

\[ D = \frac{CF_1}{1+Y/B} \times 1 + \frac{CF_2}{(1+Y)^2/B} \times 2 + \cdots + \frac{CF_T}{(1+Y)^T/B} \times T \]

Interpretation

• parallel shifts in term structure, return proportional to duration
Example

- Yield = 8%
- 30 year PDB and 1 year PDB
- Durations?

\[ b_1 = \frac{1}{1.08} = 0.93 \]
\[ b_{30} = \frac{1}{1.08^{30}} = 0.0994 \]

Yield rise

- New Yield: 1.082
- % Price changes:

\[ b_1 = \frac{1}{1.082} = 0.93, \text{ or } 0.185\% \text{ drop} \]
\[ b_{30} = \frac{1}{1.082^{30}} = 0.0940, \text{ or } 5.4\% \text{ drop} \]
Example

- Bond with 10% semi-annual, 8 payments left, annualized yield of 9%.
- Aside: in practice semi-annual compounding

\[
P = 5 \times \left( \frac{1}{0.045} \right) \times (1 - (1.045/2)^{-8}) + 100 \times (1.045)^{-8}
\]

\[
= 103.2979
\]
Duration

\[ D = -\frac{\partial P}{\partial Y} \times \frac{Y}{P} \]

\[ = \frac{5/1.045}{103.30} \times 1 + \frac{5/1.045^2}{103.30} \times 2 + \cdots + \frac{105/1.045^8}{103.30} \]

\[ = 6.81 \]

3/30/99

Bond Returns or So What?

• Suppose yield drops to 8.8%, next day
• New bond price: 103.97, Change: 0.67
• Duration approximation:

\[ \Delta P = -D \times \frac{\Delta Y}{Y} \times P \]

\[ = -6.81 \times \frac{0.044 - 0.045}{1.045} \times 103.30 \]

\[ = 0.67791 \]
Holding Period Returns

- Buy bond at 9% and sell after yield drop
- Return = \( \frac{(103.97 - 103.30)}{103.30} = 0.648\% \)

\[
\frac{\Delta P}{P} = -D \frac{\Delta Y}{Y} \\
= 6.81 \frac{0.044 - 0.045}{1.045} \\
= 0.6526\% 
\]

Portfolio duration

- Invest \( w_i \% \) of your $ in bond I
- \( D_i \) duration of bond I
- Portfolio duration:
  - \( D_p = w_1 D_1 + w_2 D_2 + \ldots w_I D_I \)
Example

- Invest half your $ in a 2 year PDB and the rest in a 30 year PDB
- Portfolio duration?

Duration Application: Immunization

- Match duration of liabilities and assets
- Why?
Immunization

- You have a liability of 100 in 2 years
- Duration?
- Suppose you use 1 year PDB and previous coupon bond to form immunized portfolio?
- Current yield is 9%

Strategy

- Invest x% in one year PDB and (1-x)% in coupon bond

\[
4 = xD_{1\text{year}} + (1-x)D_{\text{coupon}}
\]
\[
= x2 + (1-x)6.81
\]
\[
\rightarrow x = \frac{2.81}{4.81} = 0.5842
\]
Position

\[ V_{\text{liability}} = \frac{100}{1.045^4} = 83.85613 \]

\[ P_{\text{pdb}} = \frac{100}{1.045^2} = 91.573 \]

\[ \# \text{ PDB} = \frac{0.5842 \times 83.86}{91.573} = 0.535 \]

\[ \# \text{ Coupon} = \frac{0.4156 \times 83.86}{103.30} = 0.338 \]
Modified Duration

- % Change of bond price for given yield change
- Units of percentage returns
- same info as duration

\[ MD = -\frac{\partial P}{\partial Y} \frac{1}{P} = D \times \frac{1}{Y} = 6.81 \times \frac{1}{1.045} \]

PVBP

- Change in price for change in basis point
- Units: $ per bond

\[ PVBP = -\frac{\partial P}{\partial Y} / 100 = D \frac{P}{Y} / 100 = 6.81 \times \frac{103.30}{1.045} / 100 \]
Using PVBP to hedge

- 2 bonds: 2 year and 20 year
- currently long 1 20 year bond, and want to hedge with 2 year

\[ n_{20}PVBP_{20} + n_2PVBP_2 = 0 \]

\[ n_2 = -n_{20} \frac{PVBP_{20}}{PVBP_2} \]

Speculation

- Suppose 20 year yield > 2 year yield
- Expect spread to widen
- What should you do?
Orange Country

- Dec 1994, $1.6 billion loss
- largest municipality to declare bankruptcy
- Treasurer: Bob Citron, $7.5 investor pool
- Pool: Orange County Investment Pool
- Customers
  - country, city, schools

What was going on?

- Duration!
- Citron previously made 2% higher returns than State pool
- local schools issued ST notes to invest in pool
- 1994 election, opponent pointed out problems
What happened

• 1994 Fed increased rates
  – big paper losses in pool
  – bankruptcy declared
  – losses realized

Citron’s strategy

• Upward sloping TS
• Dec 1993
  – short term yields < 3%
  – 5 year yields roughly 5.2%
• after liquidation, 2.5% interest rate drop
• borrowed short and invested long
• levered too!
Returns

Before Bankruptcy
How to increase duration

- Increase duration in 2 ways:
  - invest in longer bonds
  - leverage

Example

- $100 5 year note
- use as collateral for cash in reverse repo
- use cash to buy new note,
- continue...
- If duration of notes is 4 and you do this 3 times?
Reverse Repo Transaction

- Short term borrowing or lending using treasuries as collateral
- very common
- give your bonds to dealer for $ today
- commit to buy back bond in future
- Haircut

Reverse Repo

Initial Date

Deliver Bond

Get price-haircut

Repo Dealer

Final Date

Get bond back

Pay price + interest
Position (simplified)

- $7.8 B in contributed funds
- $12.8 B borrowed funds: reverse repos
- $20.6 B invested in 5 year T-notes
- Duration: 7.4
- Yields up by 2.6%
Potential Losses

Summary

• Applying duration
• convexity: intuition and formula
• Next Time
  – Swaps and convexity
    • Chapter 16, text
  – Factor Models for Bonds
    • Litterman and Scheinkman article
    • Kritzman on factor models