Trading and Intro to TS

Goals:

- basic trading terminology
- Explain Law of One Price, and Arbitrage
- Calculate a replicating portfolio of bonds
- Explain relationship: Arbitrage and Replication
- Spot Rates
- Forward Rates

Organized Exchanges

- auction markets
- dealers
- Securities: stocks, futures contracts, options, bonds (somewhat)
- Examples: AMEX, NYSE, CBOE
OTC Market

- dealer market w/out centralized order flow
- NASDAQ: largest
- Scandal?
- Stocks, bonds, and some derivatives

Others

- 3rd Market: trading of listed securities away from exchange
- 4th market: institutions to institutions
Trading Costs

- Commission: paid to broker
- Spread: cost of trading with dealer
  - Bid: dealer buys
  - Ask: dealer sells

Order Types

- Basic idea:
- Market:
- Limit
- Stop loss
Margin Trading

- Maximum margin
  - currently 50%
  - set by Fed
- Maintenance margin
  - minimum equity margin can be
- margin call:

Example–Initial Conditions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yahoo</td>
<td>$70</td>
</tr>
<tr>
<td>50%</td>
<td>Initial Margin</td>
</tr>
<tr>
<td>40%</td>
<td>Maintenance Margin</td>
</tr>
<tr>
<td>1000</td>
<td>Shares purchased</td>
</tr>
</tbody>
</table>

Initial Position

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>$70,000</td>
</tr>
<tr>
<td>Borrowed</td>
<td>$35,000</td>
</tr>
<tr>
<td>Equity</td>
<td>$35,000</td>
</tr>
</tbody>
</table>
**Maintenance Margin**

- Stock Price falls to $60 per share
- New position:
  - Stock $60,000
  - Borrowed $35,000
  - Equity $25,000
- Margin = $25,000/$60,000 = 41.67%
- Margin call: margin must drop to 40%. How much should price drop?

**COUGARS Case**

Questions

- Why have zero coupon bonds been successful?
• Relationship between COUGARS and coupon bonds?

• How much value was created in the COUGARS offering? Where did the value come from?

Arbitrage Opportunity

Strategy that:

• positive cash flow today
• no investment today
• no future obligations

Violates Law of One Price example of violations?
Bonds

Are they really risk-free?

Bond Cash Flows

Pure discount bond, 3 year maturity with 1$ face value:

\[
\begin{array}{cccc}
  t: & 0 & 1 & 2 & 3 \\
  \text{Cash:} & 0 & 0 & 1 \\
\end{array}
\]

\(b_3\) the bond price at \(t = 0\).

\[
\begin{array}{cccc}
  t: & 0 & 1 & 2 & 3 \\
  \text{Cash:} & -b_3 & 0 & 0 & 1 \\
\end{array}
\]

Short sale:

\[
\begin{array}{cccc}
  t: & 0 & 1 & 2 & 3 \\
  \text{Cash:} & b_3 & 0 & 0 & -1 \\
\end{array}
\]
**Coupon** bond with 3 year maturity, Face = 100$, coupon rate = $c%$

<table>
<thead>
<tr>
<th>$t$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash:</td>
<td>$c$</td>
<td>$c$</td>
<td>100 + $c$</td>
<td></td>
</tr>
</tbody>
</table>

If the bond price were $B_3$ and you purchased:

<table>
<thead>
<tr>
<th>$t$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash:</td>
<td>$-B_3$</td>
<td>$c$</td>
<td>$c$</td>
<td>100 + $c$</td>
</tr>
</tbody>
</table>

**Short Sale:**

<table>
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<th>$t$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash:</td>
<td>$B_3$</td>
<td>$-c$</td>
<td>$-c$</td>
<td>-(100 + $c$)</td>
</tr>
</tbody>
</table>

**Real World**

- t–bills
- t–bonds
- long–term bonds
- strips

Created by investment banks
How to Replicate

Can we make a 3 year coupon bond out of PDBs?

<table>
<thead>
<tr>
<th>$t$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>coupon bond</td>
<td>$-B_3$</td>
<td>$c$</td>
<td>$c$</td>
<td>$100 + c$</td>
</tr>
</tbody>
</table>

| $PD_1$ | $-b_1$ | 1   | 0   | 0   |
| $PD_2$ | $-b_2$ | 0   | 1   | 0   |
| $PD_3$ | $-b_3$ | 0   | 0   | 1   |

So if you buy:

- $c$ units of $PD_{B1}$
- $c$ units of $PD_{B2}$
- $100 + c$ units of $PD_{B3}$

<table>
<thead>
<tr>
<th>$t$:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash:</td>
<td>$-ch_1$</td>
<td>$c$</td>
<td>$c$</td>
<td>$100 + c$</td>
</tr>
<tr>
<td></td>
<td>$-ch_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-(100 + c) b_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This gives the same payout

Cost of Replicant:

$$ch_1 + ch_2 + (100 + c) b_3$$
**Arbitrage and Replication**

Suppose $B_3 < cb_1 + cb_2 + (100 + c)b_3$. To take advantage:

<table>
<thead>
<tr>
<th>buy 1 coupon bond</th>
<th>$-B_3$</th>
<th>$c$</th>
<th>$c$</th>
<th>$100 + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sell $c$ $PDB_1$</td>
<td>$cb_1$</td>
<td>$-c$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>sell $c$ $PDB_2$</td>
<td>$cb_2$</td>
<td>0</td>
<td>$-c$</td>
<td>0</td>
</tr>
<tr>
<td>sell $100 + c$ $PDB_3$</td>
<td>$(100 + c)b_3$</td>
<td>0</td>
<td>0</td>
<td>$-(100 + c)$</td>
</tr>
</tbody>
</table>

**Net**

\[
[cb_1 + cb_2 \quad 0 \quad 0 \quad 0] + (100 + c)b_3 - B_3
\]

By assumption

\[
cb_1 + cb_2 + (100 + c)b_3 - B_3 > 0
\]

This meets our definition of an **ARBITRAGE**

Question: what to do if

\[
cb_1 + cb_2 + (100 + c)b_3 < B_3
\]
**Punchline**

- Law of One Price: same payoffs $\implies$ same price
- Replication: $PDB$ gives coupons, etc.
- Replication and Arbitrage

**Question:** No short sales?

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**Bond Prices and Interest Rates**

$b_1$ price of 1 period $PDB$

**Define** $r_1 \equiv$ 1 year interest rate

$$b_1 = \frac{1}{r_1} \implies r_1 = \frac{1}{b_1}$$

$r_i \equiv$ $i$ year interest rate

$$b_i = \frac{1}{r_i^z} \implies r_i = \left(\frac{1}{b_i}\right)^{1/i}$$

Generally, $r_i \neq r_j \implies$ **TERM STRUCTURE**
bond prices $\Rightarrow$ term structure (spot interest rates)

Replication argument:

$$P = \sum_{i=1}^{N} b_i c f_i,$$

or,

$$P = \sum_{i=1}^{N} \frac{c f_i}{r_i^t},$$

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**Forward Rates**

$r_i$ today's rates on $i$ period $PDB$'s. Want to arrange borrowing/lending in the future.

e.g. meet today to arrange for advance borrowing 100$ in 1 yr. to be repaid in 2 yrs. **Forward Contract**

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time:</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Cash:</td>
<td>0</td>
<td>-100</td>
<td>100$</td>
</tr>
</tbody>
</table>

Rate fixed today: NO RISK
Forwards and Spots

Replication again!

1. buy a 2 year PDB. Get $100r_2^2$ in two years.

2. buy a 1 year PDB and forward contract

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 1 yr PDB</td>
<td>-100</td>
<td>100r_1</td>
<td>0</td>
</tr>
<tr>
<td>(2) forward</td>
<td>0</td>
<td>-100r_1</td>
<td>100r_1 f_2</td>
</tr>
<tr>
<td>Net</td>
<td>-100</td>
<td>0</td>
<td>100r_1 f_2</td>
</tr>
<tr>
<td>(1) 2 yr PDB</td>
<td>-100</td>
<td>0</td>
<td>100r_2</td>
</tr>
</tbody>
</table>

NO ARBITRAGE $\implies r_1 f_2 = r_2^2$,

$$1 f_2 = \frac{r_2^2}{r_1}$$

$r_i$ and $b_i$ are related $b_i = \frac{1}{r_i}$

$\implies 1 f_2 = \frac{b_1}{b_2}$

Do this for $i = 1, 2, ...$

$$r_2^2 2 f_3 = r_3^3 \implies 2 f_3 = \frac{r_3^3}{r_2^2} = \frac{b_2}{b_3}$$

$$n f_{n+1} = \frac{b_n}{b_{n+1}} = \frac{r_{n+1}}{r_n}$$
Spots and forwards are just interest rates. Can they be < 1? PDB prices *decrease* as the maturity increases.

\[ b_n \leq b_m \quad n > m \]

**Example**

\[ b_1 = 0.81, \quad b_2 = 0.9 \]

To take advantage:

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) buy ( PDB_1 )</td>
<td>-0.81</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(2) sell ( PDB_2 )</td>
<td>0.90</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>(3) matress from 1–2</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Net</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Arbitrage!* 

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**Example:** Replicating Portfolios, Spots, and Forwards

Coupon bond:

\( M = 5\text{yrs}, \ c = 8\%, \ F = \$1,000. \)

\( P = ? \)

Have the following $10 PDB$ prices:

\[ b_1 = 9.346 \]

\[ b_2 = 8.654 \]

\[ b_3 = 7.939 \]

\[ b_4 = 7.217 \]

\[ b_5 = 6.561 \]
Spot Rates

\[ r_1 = \left( \frac{10}{9.346} \right) = 1.07 \]
\[ r_2 = \left( \frac{10}{8.654} \right)^{\frac{1}{3}} = 1.075 \]
\[ r_3 = \left( \frac{10}{7.939} \right)^{\frac{1}{4}} = 1.08 \]
\[ r_4 = \left( \frac{10}{7.217} \right)^{\frac{1}{7}} = 1.085 \]
\[ r_5 = \left( \frac{10}{6.561} \right)^{\frac{1}{7}} = 1.088 \]
**Forward Rates**

\[ 1f_2 = \frac{(r_2)^2}{r_1} \quad \text{Also,} \quad 1f_2 = \frac{b_1}{b_2} \]

\[ 1f_2 = \frac{(r_2)^2}{r_1} = \frac{(1.075)^2}{1.07} = 1.08 \]

\[ 2f_3 = \frac{(r_2)^3}{(r_2)^2} = \frac{(1.08)^3}{(1.075)^2} = 1.09 \]

\[ 3f_4 = \frac{(r_3)^4}{(r_3)^3} = \frac{(1.085)^4}{(1.08)^3} = 1.10 \]

\[ 4f_5 = \frac{(r_4)^5}{(r_4)^4} = \frac{(1.088)^5}{(1.085)^4} = 1.10 \]

**Conclusions**

- trading terminology
- Replication and arbitrage
- application to bonds and spot rates

**Next Time**

- When is a set of bonds arbitrage free
- Redemption yields