Investment Analysis — Homework # 2

due: Tuesday, April 13, In Class

1. Suppose that the discount function shifts from \( b_t = (1.075)^{-t} \) to \( b_t = (1.076)^{-t} \), where \( b_t \) is the present value of $1 payable at time \( t \), and where \( t \) is time in years.

(a) What do these discount functions tell us about the term structure of interest rates and how it changes?

(b) Calculate the resulting actual percentage price change for a $100 face value bond maturing in 10 years with a coupon rate of 4% per annum. That is, calculate the bond’s price before and after the change in interest rates.

(c) Calculate Macaulay duration for the bond described above. Use duration analysis to calculate the duration approximation for price change for this bond.

(d) Calculate the convexity of this bond (You will likely want to do this on a spreadsheet). Use convexity number to come up with a alternative approximation to the price change.

(e) Now suppose that the only one period spot rate of interest moves. That is, the discount function shifts from \( b_t = (1.075)^{-t} \) to \( b_t = (1.07)^{-t} \), \( b_t = (1.075)^{-t}, t \geq 2 \). Calculate the price change for a $100 face value bond with a ten year maturity and a coupon rate of %4 per year. How does this compare to the expected price change calculated using duration? How about when you include convexity?

2. An investor expects a strong market rally. Assume that the yield curve shifts will be parallel. In order to get the highest return over this rally, which of the following securities should be chosen, and why?

(a) the security with the highest duration

(b) the security with the highest PVBP

3. The next problem should be done using a spreadsheet. Use the Wall St. Journal bond quotes from March 31 and April 7 to solve this problem. Compute the yield spread between the current 5-year T-note and the current 30 year T-bond. Anticipate that this spread will narrow over the next month. Come up with a trade using these two securities to take advantage of your beliefs. Use a repo to for your position, and use a rate of rate of 3% (annualized) per week. Unwind the trade after 5 business days (Feb 3) using the Wall St. Journal prices. How much money did you make/lose. What was your annual rate of return? Note: pages 37-38, and 402-403 of the textbook deal with how to read the Wall St. Journal table to compute prices.

4. In your opinion, two-year zero yields will fall over the next month relative to the yield on a barbell portfolio of 1 and 3 year zeros. The barbell invests an equal amount in each of the zeros. The current term-structure is given by: \( r_1 = 1.05 \), \( r_2 = 1.057 \), and \( r_3 = 1.06 \). If yields are generated by a one factor model, with factor loads of 0.8 on the one year zero, 0.7 on the two year zero, and 0.5 on the 3 year zero.

(a) What position in the barbell makes you factor-adjusted duration neutral for each year you are short the two-year PDB (sometimes called a bullet position).

(b) Interpret the movements of the resulting combined position using the Taylor series argument

\[
\Delta B = \frac{\partial B}{\partial t} \Delta t + \frac{\partial B}{\partial y} \Delta y + 0.5 \frac{\partial^2 B}{\partial y^2} (\Delta y)^2.
\]

When do you win in this position? When do you lose?