Objectives

• Application of factor model to portfolio optimization
• Explain what the CAPM means and what it doesn’t
• Empirical evidence on CAPM
• Explain what APT is and why…

Estimating covariance matrix

• Lots of parameters in covariance matrix
  – with 10 assets, 55 parameters!
• Don’t want too long data series
  – stationary issues
• Need to invert covariance matrix
• Solution
  – Factor models….
Factor Models and Portfolio Optimization

• Why
  – reduce number of parameters in var-cov matrix to estimate
• Plausible model

Example Single Factor Model

• For each asset,
  \[ \tilde{r}_{i,t} = E[\tilde{r}_{i,t}] + \beta_i \tilde{F}_t + \tilde{\epsilon}_{i,t} \]

• Asset Variance
  \[ \sigma_i^2 = \beta_i^2 \sigma_F^2 + \sigma_{\epsilon_i}^2 \]
Variance-covariance matrix

- Calculate the covariances among assets
  \[ \text{Cov}(\tilde{r}_i, \tilde{r}_j) = \text{Cov}(\beta_i \tilde{F} + \tilde{e}_i, \beta_j \tilde{F} + \tilde{e}_j) \]
  \[ = \beta_i \beta_j \sigma^2 \]

- With 10 assets
  - 10 \( \beta \)'s
  - 10 residual variances and 1 factor variance
  - 21 parameters vs. 55 without factor assumption

Ways to get factors

- Factor Analysis: as in the bond case
- Economic Factors
  - market portfolio
  - interest rates or term structure
  - inflation
  - oil prices, etc.
Combining risky with riskless

- Find efficient portfolio with largest slope, or Sharpe ratio
- Can be done using solver in Excel

$$\max_{\tilde{r}_{port}} \frac{\left( E[\tilde{r}_{port}] - r_f \right)}{\sigma_{port}}$$

Optimal Portfolios
So far...

- Given mean and covariances can find optimal risky position
- Adding riskless
- Practical issues
  - sensitivity
  - where to get inputs

Equilibrium

- Why worry about equilibrium?
  - Benchmark
  - Pricing results
- If we agree on picture
  - all pick same risky position
  - market portfolio!
So What

- easy to figure out optimal strategy: combination of market and risk free
- For all investments, efficient or not calculate beta relative to market

\[ E[\tilde{r}_i] = r_f + \beta_{i, market} \left( E[\tilde{r}_{market}] - r_f \right) \]
Discussion Questions

• You can’t get higher return than market if you don’t hold an efficient portfolio

• Stocks are riskier than bonds: all stocks should have higher expected return than risk free bonds

• You can’t time the market

More Questions...

4/29/99 Equilibrium

mean

σ

A B C

mean

β

4/29/99 Equilibrium
Yet More

- IBM has a beta of 1. Is it as risky as market?

Mispriced Assets in CAPM World

- Run the following regression
- CAPM: intercept (α) is supposed to be zero

\[
\tilde{r}_i - r_f = \alpha_i + \beta_i (\tilde{r}_M - r_f) + \tilde{e}_i
\]
How to take advantage of non-zero $\alpha$...

- Suppose $\alpha$ is positive
  - Asset has higher expected return than CAPM...
- Increase weights in that asset…
- Perhaps hedge with market.

Does $\beta$ explain everything?

- Anything?
- Fama and French (1992) results
  - sort stocks by $\beta$ and price (size), B/M
  - cross-sectional regressions

$$E[r_i] = \gamma_0 + \gamma_{B/M} (B/M_i) + \gamma_{size} \log(ME) + \gamma_\beta \beta_i$$
Results

- The CAPM $\beta$ explains almost nothing
- Size matters
  - small firm effect
- Book to market is important

Issues

- Time varying $\beta$
  - seems to help a lot
- Data snooping
- Is this really CAPM?
Value Investing

- Take advantage of Book to Market Effect
- terrible over the last few years, but...

Momentum and Reversals

- Long and very short horizon reversals
  - 3 year reversals
  - one month reversals
- intermediate term momentum
  - 3 to 12 month holding periods
The APT

- **CAPM**
  - people only care about mean and variance
  - Intuition: only systematic risk should be priced
- **APT**
  - not based on investor preferences but on ‘factor model’ for returns

APT Model

- Common factors that determine co-movements of returns
- systematic and diversifiable risk
- Intuition of model
  - only systematic risk is rewarded...
Factor Model

\[ \tilde{r}_i = E[\tilde{r}_i] + \beta_i^1 \tilde{F}^1 + \beta_i^2 \tilde{F}^2 + \ldots + \beta_i^K \tilde{F}^K + \tilde{\epsilon}_i \]

- F: factor
- \( \beta \): factor loading
- \( \epsilon \): diversifiable risk

Pricing

- Expected returns proportional to factor loadings
- Intuition: only systematic risk should command higher expected returns

\[ E[\tilde{r}_i] = r_{\text{free}} + \lambda_1 \beta_i^1 + \lambda_2 \beta_i^2 + \ldots + \lambda_K \beta_i^K \]
Example

- 3 factors
- Stock i has
  - $\beta_i^1=1.1$, $\beta_i^2=0.5$, $\beta_i^3=2$
  - $\lambda_1=10\%$, $\lambda_2=10\%$, $\lambda_3=1\%$, Riskfree rate =5\%
- Expected return on stock?

Summary

- CAPM
  - what it means in words
  - applications
- APT
  - intuition