1. Consider a triangular fin as shown in Figure 1. The fin extends infinitely into the page, and loses heat to the surroundings. This heat loss may be modeled as a convective heat transfer to a surroundings temperature of $T_\infty$ through a heat transfer coefficient $h$. Show that the fin heat transfer equation for this geometry is given by:

$$\frac{\partial}{\partial x} \left( x \frac{\partial \theta}{\partial x} \right) - m^2 \theta = 0 \tag{1}$$

where

$$m^2 = \frac{2hL}{kb}$$

and $\theta = T - T_b$.

By using the transformation

$$z = 2mx^{\frac{1}{2}}$$

show that Equation 1 can be transformed into the modified Bessel’s equation

$$z^2 \frac{\partial^2 \theta}{\partial z^2} + z \frac{\partial \theta}{\partial z} - \theta z^2 = 0$$

Though you are not required to find the solution, it may interest you to know that it can be written in terms of modified Bessel’s functions as:

$$\theta = C_1 I_0(z) + C_2 K_0(z)$$

![Figure 1: Fin Geometry for Problem 1](image-url)
2. Let us explore to solution derived in class for steady two-dimensional conduction in the rectangular domain shown in Figure 2. For \( T_1 = 200 \, K \) and \( T_2 = 500 \, K \), \( L = 1 \, m \) and \( W = 2 \, m \)

(a) Find how many terms of the series must be summed to find the temperature at \((0.5, 1.0)\) to an accuracy of 1%.
(b) Plot the temperatures on lines \( x=0.25\,m \), \( x=0.5\,m \), and \( x=0.75\,m \).
(c) Show that the maximum and minimum temperatures lie on the boundaries of the domain.
(d) Find an expression for the heat flux on the top boundary, \( q' (x, 2.0) \).

\[
\begin{align*}
T_1 & \quad y \\
L & \quad x \\
W & \quad T_1
\end{align*}
\]

Figure 2: Conduction in a Rectangular Domain (Problem 2)

3. Let us explore the separation of variables technique which we are using extensively in class to study conduction problems. Consider steady 2-D conduction with constant properties. For each of the cases listed below, determine whether the separation of variables technique will work or not by repeating the derivation done in class.

\[
\begin{align*}
\text{(a)} & \quad T_2 (x) \\
\text{(b)} & \quad T_2 (x) \\
\text{(c)} & \quad h_0 \cdot T_0 \\
\text{(d)} & \quad y = ax + b
\end{align*}
\]

Figure 3: Boundary Conditions for Problem 3