24-352 Dynamic Systems and Control: QUIZ 3

Close book and notes. You have 45 minutes to complete the following questions.

NAME: ____________________________ 26 February 2001

1. What is the governing differential equation for the system shown in Figure 1?
2. Assume: $M = 1$, $K = 2$, $B = 3$ and $f(t) = f_0 H(t)$ where $H(t)$ is the Heaviside step function. If $x(0) = 0$ and $x(0) = 0$ find the Laplace transform of $x(t)$, $X(s)$.
3. What are the poles of $X(s)$? Plot their location in the complex s-plane.
4. Use the Laplace transform method to find $x(t)$.

**ANSWER**

\[ M \dddot{x} + B \dot{x} + Kx = f(t) \quad \text{(1)} \]

\[ M = 1, \quad K = 2, \quad B = 3, \quad f = f_0 H(t) \Rightarrow \]

\[ \dddot{x} + 3 \dot{x} + 2x = f_0 H(t) \quad \checkmark \]

**Figure 1**

\[ s^2 X(s) - sx(0) - \dot{x}(0) - s^2 \int_0^t x(t) dt = f_0 \quad \checkmark \]

\[ \Rightarrow \left( s^2 + 3s + 2 \right) X(s) = \frac{f_0}{s} \quad \Rightarrow X(s) = \frac{f_0}{s \left( s^2 + 3s + 2 \right)} \quad \checkmark \]

3. Poles:

\[ s \left( s^2 + 3s + 2 \right) = s \left( s+2 \right) \left( s+1 \right) = 0 \Rightarrow \quad s = 0, \quad s = -2, \quad s = -1 \]

\[ \text{Poles} \quad -2 \quad -1 \quad \checkmark \]

4. Use partial fractions

\[ X(s) = \frac{f_0}{s \left( s^2 + 3s + 2 \right)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} \quad \checkmark \]

null by $s$ and let $s = 0 \Rightarrow A = \frac{f_0}{2 \cdot 1} = \frac{f_0}{2} \quad \checkmark$

null by $s+2$ and let $s = -2 \Rightarrow B = \frac{f_0}{(-2) \cdot (-1)} \quad \checkmark$

null by $s+1$ and let $s = -1 \Rightarrow C = \frac{f_0}{(-1) \cdot (-1)} \quad \checkmark$

\[ x(t) = f_0 \left( \frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} \right) \Rightarrow x(t) = f_0 \left( \frac{1}{2} + \frac{1}{2} e^{-2t} - e^{-t} \right) \quad \checkmark \]