PROBLEMS

1. Consider the mass/spring system of Figure 1. Assume \( y(t) = 0 \). Let \( M = 1000 \text{ kg} \) and \( K = 40,000 \text{ N/m} \). Assume \( f(t) \) is from gravity, \( x(0) = 0.1 \) and \( x'(0) = 1 \text{ m/s} \).
   a. What is the natural frequency of the system in radians/second and in Hertz.
   b. Find \( x(t) \). Plot \( x(t) \) for \( 0 < t < 2 \). What is the maximum value of \( x \) over this interval?

2. Consider the system of Figure 1. If \( f(t) = 4000 \cos(\omega t) \)
   a. Find the particular solution to the governing differential equation. Determine the magnitude of the amplitude, \( R \), and the phase of the response, \( \phi \), as functions of the excitation frequency \( \omega \).
   b. Plot \( R(\omega) \) and \( \phi(\omega) \) for \( 0 < \omega < 20 \). Indicate on your plot of \( R \) the regions that are primarily controlled by 1) the spring and 2) the mass.

3. Assume that the force \( f(t) \) is the square wave shown in Figure 2

   \[
   f(t) = \begin{cases} 
   f_0 & \text{for } 0 < t < T/2 \\
   -f_0 & \text{for } T/2 < t < T \\
   0 & \text{otherwise}
   \end{cases}
   \]

   \[
   f_0 = 1000 \text{ N} \quad T = 3 \text{ sec}
   \]
   a. Find the Fourier series for \( f(t) \), i.e. find \( F_n \) and \( G_n \) where
   \[
   f(t) = \frac{F_0}{2} + \sum_{n=1}^{\infty} F_n \cos(n\omega t) + G_n \sin(n\omega t)
   \]
   b. Find the particular solution if the square wave is applied as a force to the system of Figure 1, i.e. assume that \( x(t) \) has the same period as \( f(t) \) and can be represented as a Fourier series of the form
   \[
   x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)
   \]
   Find the \( a_n \) and \( b_n \). Which frequency component of the response is the largest? Why?

4. Suppose that instead of a force being applied to the system in Figure 1 the system is excited by a sinusoidal motion at \( y \). That is \( y(t) = Y_0 e^{\omega t} \). Find the particular solution. Use your result to determine the particular solution if instead \( y(t) = Y_0 \sin(\omega t) \).