16-51. The wheel is rotating with an angular velocity \( \omega = 8 \text{ rad/s} \). Determine the velocity of the collar \( A \) at the instant \( \theta = 30^\circ \) and \( \phi = 60^\circ \). Also, sketch the location of bar \( AB \) when \( \theta = 0^\circ, 30^\circ, \) and \( 60^\circ \) to show its general plane motion.

\[
v_A = v_B + v_{A/B}
\]

\[
v_A = 1.2 + 0.5 \omega_{A/B} \]
\[
\rightarrow \quad \Delta v = \begin{bmatrix} 0 \\ 0 \\ \frac{\sqrt{3}}{2} \end{bmatrix}
\]

\[
\rightarrow \quad v_A = 1.2 \cos 60^\circ + 0.5 \omega_{A/B} \cos 30^\circ
\]

\[
+ \uparrow \quad 0 = 1.2 \sin 60^\circ - 0.5 \omega_{A/B} \sin 30^\circ
\]

\[
\omega_{A/B} = 4.16 \text{ rad/s}
\]

\[
v_A = 2.40 \text{ m/s} \quad \rightarrow \quad \text{Ans}
\]

Also,

\[
v_A = \omega \times r_B
\]

\[
v_A = v_B + \omega_{A/B} \times r_{A/B}
\]

\[
v_A = (-8k) \times (-0.15 \cos 30^\circ i + 0.15 \sin 30^\circ j) + (0.5 \cos 60^\circ i + 0.5 \sin 60^\circ j)
\]

\[
v_A = 0.60 + 0.433 \omega_{A/B}
\]

\[
0 = 1.039 - 0.25 \omega_{A/B}
\]

\[
\omega_{A/B} = 4.16 \text{ rad/s}
\]

\[
v_A = 2.40 \text{ m/s} \quad \rightarrow \quad \text{Ans}
\]
16-59. The planetary gear A is pinned at B. Link BC rotates clockwise with an angular velocity of 8 rad/s, while the outer gear rack rotates counterclockwise with an angular velocity of 2 rad/s. Determine the angular velocity of gear A.

**Kinematic Diagram:** Since link BC is rotating about fixed point C, then \( v_B \) is always directed perpendicular to link BC and its magnitude is \( v_B = \omega_C r_{BC} \) = \( 8 \times 15 \) = 120 in/s. At the instant shown, \( v_B \) is directed to the left. Also, at the same instant, point E is moving to the right with a speed of \( v_E = \omega_D r_{CE} \) = \( 2 \times 20 \) = 40 in/s.

**Velocity Equation:** Here, \( v_{BE} = \omega_A r_{BE} = 5\omega_A \) which is directed to the left. Applying Eq. 16-15, we have

\[
\begin{align*}
120 &= 40 + 5\omega_A \\
-120 &= -40 - 5\omega_A \\
\omega_A &= 32 \text{ rad/s} \\
\end{align*}
\]

**Ans**
If the angular velocity of link $AB$ is $\omega_{AB} = 3 \text{ rad/s}$, determine the velocity of the block at $C$ and the angular velocity of the connecting link $CB$ at the instant $\theta = 45^\circ$ and $\phi = 30^\circ$. Also, sketch the location of link $BC$ when $\theta = 30^\circ$, $45^\circ$, and $60^\circ$ to show its general plane motion.

$$v_C = v_B + v_{CB}$$

$$\begin{bmatrix} v_C \\ \theta \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} \omega_{CB} (3) \\ \omega_{CB} (3) \cos 45^\circ \end{bmatrix}$$

$$\begin{cases} v_C = 6 \sin 30^\circ - \omega_{CB} (3) \cos 45^\circ \\ 0 = -6 \cos 30^\circ + \omega_{CB} (3) \sin 45^\circ \end{cases}$$

$$\omega_{CB} = 2.45 \text{ rad/s} \quad \text{Ans}$$

$$v_C = 2.20 \text{ ft/s} \quad \text{Ans}$$

Also,

$$v_C = v_B + \omega \times v_{CB}$$

$$-v_C = (6 \sin 30^\circ - 6 \cos 30^\circ) + (\omega_{CB} \times (3 \cos 45^\circ + 3 \sin 45^\circ))$$

$$\begin{cases} v_C = 3 - 2.12 \omega_{CB} \\ 0 = -5.196 + 2.12 \omega_{CB} \end{cases}$$

$$\omega_{CB} = 2.45 \text{ rad/s} \quad \text{Ans}$$

$$v_C = 2.20 \text{ ft/s} \quad \text{Ans}$$
16-74. If the link $AB$ is rotating about the pin at $A$ with an angular velocity of $\omega_{AB} = 5 \text{ rad/s}$, determine the velocity of blocks $C$ and $E$ at the instant shown.

**Kinematic Diagram:** Since link $AB$ is rotating about fixed point $A$, then $v_A$ is always directed perpendicular to link $BC$ and its magnitude is $v_A = \omega_{AB} r_{AB} = 5(1) = 5.00 \text{ ft/s}$. At the instant shown, $v_B = (-5.00\jmath) \text{ ft/s}$. Also, block $C$ and $E$ are moving along the guide and directed toward negative $x$ axis and negative $y$ axis respectively. Then, $v_C = -v_{CL}$ and $v_E = -v_{EL}$. Since the direction of the velocity of point $D$ are unknown, we can assume that its $x$ and $y$ components are directed in the positive direction of their respective axis.

**Velocity Equation:** Here, $v_{CB} = \{ 2 \cos 30^\circ \mathbf{i} + 2 \sin 30^\circ \mathbf{j} \} \text{ ft}$
$= (1.732 \mathbf{i} + 1 \mathbf{j}) \text{ ft}$ and $r_{DB} = \{ -3 \cos 60^\circ \mathbf{i} + 3 \sin 60^\circ \mathbf{j} \} \text{ ft}$
$= (-1.50 \mathbf{i} + 2.598 \mathbf{j}) \text{ ft}$. Applying Eq. 16-16 to link $CBD$, we have

$$v_C = v_s + \omega_{CBD} \times r_{CB}$$
$= \omega_{CBD} \mathbf{L} + \omega_{CBD} \mathbf{F}$
$= (-5.00\jmath + (\omega_{CBD} \mathbf{L}) \times (1.732 \mathbf{i} + 1 \mathbf{j}))$
$= \omega_{CBD} \mathbf{L} + (1.732 \omega_{CBD} \mathbf{L} - 5.00) \jmath$

Equating $i$ and $j$ components gives

$$-v_C = -\omega_{CBD}$$
[1]

$$0 = 1.732 \omega_{CBD} - 5.00$$
[2]

Solving Eqs. [1] and [2] yields

$$\omega_{CBD} = 2.887 \text{ rad/s}$$
$$v_C = 2.89 \text{ ft/s}$$

Ans

The $x$ and $y$ components of velocity of $v_D$ are given by

$$v_D = v_B + \omega_{CBD} \times r_{DB}$$
$= (v_B) + (\omega_{CBD} \mathbf{L}) \times (-5.00\jmath + (2.887\kappa) \times (-1.50\mathbf{i} + 2.598\mathbf{j}))$
$= (v_B) + (\omega_{CBD} \mathbf{L}) \times (-7.50 \mathbf{i} - 9.330 \mathbf{j})$

Equating $i$ and $j$ components gives

$$(v_D)_x = -7.50 \text{ ft/s}$$
$$(v_D)_y = -9.330 \text{ ft/s}$$

Here, $r_{ED} = (-4\jmath) \text{ ft}$. Applying Eq. 16-16 to link $DE$, we have

$$v_E = v_D + \omega_{DE} \times r_{ED}$$
$= (v_D) + (\omega_{DE} \mathbf{L}) \times (-4\jmath)$
$= 4\omega_{DE} - 7.50 \mathbf{i} - 9.330 \mathbf{j}$

Equating $i$ and $j$ components gives

$$0 = 4\omega_{DE} - 7.50$$
$$\omega_{DE} = 1.875 \text{ rad/s}$$
$$v_E = 9.33 \text{ ft/s}$$

Ans
16-06. If bar $AB$ has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block $C$ at the instant shown.

**Kinematic Diagram:** Since link $AB$ is rotating about fixed point $A$, then $v_A$ is always directed perpendicular to link $AB$ and its magnitude is $v_A = \omega_{AB} r_A = 6(0.2) = 1.20$ m/s. At the instant shown, $v_A$ is directed with an angle 45° with the horizontal. Also, block $C$ is moving horizontally due to the constraint of the guide.

**Instantaneous Center:** The instantaneous center of zero velocity of bar $BC$ at the instant shown is located at the intersection point of extended lines drawn perpendicular from $v_A$ and $v_C$. Using law of sine, we have

\[
\frac{r_{BIC}}{\sin 60^\circ} = \frac{0.5}{\sin 45^\circ} \quad r_{BIC} = 0.6124 \text{ m}
\]

\[
\frac{r_{CIC}}{\sin 75^\circ} = \frac{0.5}{\sin 45^\circ} \quad r_{CIC} = 0.6830 \text{ m}
\]

The angular velocity of bar $BC$ is given by

\[
\omega_{BC} = \frac{v_A}{r_{BIC}} = \frac{1.20}{0.6124} = 1.960 \text{ rad/s}
\]

Thus, the velocity of block $C$ is

\[
v_C = \omega_{BC} r_{CIC} = 1.960(0.6830) = 1.34 \text{ m/s} \quad \text{Ans}
\]
16-97. If rod $AB$ is rotating with an angular velocity \( \omega_{AB} = 3 \text{ rad/s} \), determine the angular velocity of rod $CD$ at the instant shown.

**Kinematic Diagram:** From the geometry, \( \theta = \sin^{-1}\left(\frac{4 \sin 60^\circ - 2 \sin 45^\circ}{3}\right) \) = $43.10^\circ$. Since links $AB$ and $CD$ is rotating about fixed points $A$ and $D$, then $v_A$ and $v_C$ are always directed perpendicular to links $AB$ and $CD$ respectively.

The magnitude of $v_A$ and $v_C$ are $v_A = \omega_{AB}r_A = 3 \times 2 = 6.00 \text{ ft/s}$ and $v_C = \omega_{CD}r_C = 4 \omega_{CD}$. At the instant shown, $v_A$ is directed at an angle of $45^\circ$ while $v_C$ is directed at $30^\circ$.

**Instantaneous Center:** The instantaneous center of zero velocity of link $BC$ at the instant shown is located at the intersection point of extended lines drawn perpendicular from $v_A$ and $v_C$. Using law of sines, we have

\[
\frac{r_{BC}}{\sin 103.1^\circ} = \frac{3}{\sin 75^\circ} \quad r_{BC} = 3.025 \text{ ft}
\]

\[
\frac{r_{BC}}{\sin 1.898^\circ} = \frac{3}{\sin 75^\circ} \quad r_{BC} = 0.1029 \text{ ft}
\]

The angular velocity of link $BC$ is given by

\[
\omega_{BC} = \frac{v_B}{r_{BC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s} = 1.98 \text{ rad/s} \quad \text{Ans}
\]

Thus, the angular velocity of link $CD$ is given by

\[
\omega_{CD} = \frac{\omega_{BC}r_{BC}}{4r_{BC}} = 1.983(0.1029) \quad \omega_{CD} = 0.0510 \text{ rad/s} \quad \text{Ans}
\]