Quiz 1

The first letter of your LAST name ________________ First Name ________________ Last Name ________________

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Note: You have 60 min. Be careful about the time allocation. Try not to leave any problems totally blank so that I can give you some partial credit. Good luck!
(1) To solve an engineering problem, you first establish a mathematical model, or a governing equation, by applying a fundamental law of physics. You can then solve the equation by an analytical method or a numerical method. When you choose to use a numerical method, what are two types of errors that you should be aware of? What are the sources of each of the two types of errors?

(2) What is the rate of convergence of the Newton-Raphson method? (Linear, quadratic, cubic, quartic, …?)

(3) In what situation does the modified Newton-Raphson method converge faster than the original Newton-Raphson method?

(4) Which root finding method converges faster to the solution, false-position or secant?
Q1-2 (20pts) Evaluate the function value at $x = 2.14$ with 3-digit arithmetic with chopping.

$$y(x) = 5.81x^2 + 0.628x + 1.08$$

Show the intermediate steps for full credit (or partial credit, if you make a mistake).
Q1-3 (1) Illustrate in the graph below how the false-position method finds a root. Start with an initial bracket, \(2 \leq x \leq 7\), and iterate twice. What is the narrowed bracket (a range of \(x\)) after the two iterations? (Hint: you do NOT need to calculate \(x\) values using the false-position formula—just use a graphical method.)
Illustrate in the graph below how the secant method finds a root. Start with two initial values, $x_0 = 6$ and $x_1 = 8$, and iterate twice. What are $x_2$ and $x_3$? (Hint: you do NOT need to calculate $x$ values using the secant formula—just use a graphical method.)
Q1-4  (1) Complete the following Taylor series formula for a function of one variable. The step size is
\[ h = x_{i+1} - x_i. \]
\[
f(x_{i+1}) = f(x_i) + \frac{f'(x_i)}{1!} h + \frac{f''(x_i)}{2!} h^2 + \cdots + R_n,
\]
where the remainder term is defined as
\[ R_n = \]

(2) Find the second order approximation of the following function using the Taylor series expansion
with a base point at \( x = \frac{\pi}{4} \).
\[
f(x) = x + \tan(x)
\]
This is the last page of Quiz 1, and this page is intentionally left blank so that you can use it if you need more space to write your solution or do some calculations.