**NUMERICAL DIFFERENTIATION FORMULAS**

**First Derivative**

\[ f'(x) = \frac{f(x+h) - f(x)}{h} \]

\[ f'(x) = \frac{-f(x+h) + 4f(x) - 3f(x) + f(x)}{h} \]

**Second Derivative**

\[ f''(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} \]

\[ f''(x) = \frac{-f(x+h) + 4f(x) - 5f(x-h) + 2f(x)}{h^2} \]

**Third Derivative**

\[ f'''(x) = \frac{f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)}{h^3} \]

\[ f'''(x) = \frac{-3f(x) + 16f(x) - 24f(x-h) + 18f(x) - 5f(x-h)}{2h^3} \]

**Fourth Derivative**

\[ f''''(x) = \frac{f(x+4h) - 4f(x+3h) + 6f(x+2h) - 4f(x+h) + f(x)}{h^4} \]

\[ f''''(x) = \frac{-2f(x) + 11f(x) - 24f(x-h) + 26f(x-2h) - 14f(x-3h) + 3f(x)}{h^4} \]

**FIGURE 23.1**

Forward limited-divided-difference formulas. Two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.

**FIGURE 23.2**

Backward limited-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.
First Derivative

\[ f'(x) = \frac{f(x+h) - f(x-h)}{2h} \]

\[ f'(x) = \frac{-f(x+h) + 8f(x) - 8f(x-h) + f(x-2h)}{12h} \]

Second Derivative

\[ f''(x) = \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{h^2} \]

\[ f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2} \]

Third Derivative

\[ f'''(x) = \frac{f(x+3h) - 3f(x+2h) + 3f(x+1h) - f(x)}{2h^3} \]

\[ f'''(x) = \frac{-f(x+2h) + 18f(x+h) - 52f(x) + 18f(x-h) - f(x-2h)}{8h^3} \]

Fourth Derivative

\[ f^{(4)}(x) = \frac{f(x+4h) - 4f(x+3h) + 6f(x+2h) - 4f(x+1h) + f(x)}{2h^4} \]

\[ f^{(4)}(x) = \frac{-f(x+3h) + 30f(x+2h) - 180f(x+1h) + 30f(x) - 180f(x-1h) + 30f(x-2h) - f(x-3h)}{6h^4} \]

**FIGURE 23.3**

Centered finite-divided-difference formulas: two versions are presented for each derivative. The latter version incorporates more terms of the Taylor series expansion and is, consequently, more accurate.