Probability Basics (Lecture 20)

Analysis of Software Artifacts
Decision Making

- Tradeoff between reliability and time to market
- Software now "buggy"
  - Should I test more or release a possibly buggy
- Tradeoffs between cost and reliability
- Develop it in-house
  - Should I go with an off-the-shelf component or
- Applications of Probability and Statistics
Applications

Bayesian statistics used in cost models as well – any more? – such as COCOMO

Linear regression used in software cost models

Cost models •

Applications
Applications

- Markov chains also used to generate test cases
- Random sampling used in testing
  - Testing
    - Expected time between two failure events
      - Mean time between failures or MTBF
    - Entirely based on statistics
    - Metrics for software reliability

Applications
Role of Probability Theory

- the cost of producing it
- composition of the team
- amount of testing already done
  \( \mathcal{H} \) denoted by
- known quantities about the software
- reference time will be denoted by \( T \)
- probability is dependent on time
- uncertainties
  probability used as a means for quantifying

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Rule of Probability Theory
Random quantities

- Reliability
- Number of bugs remaining
- MTBF

Quantities that are unknown we will call random

Random quantities
Types of Random Quantities

- Random Variables
- $\mathcal{X}$
- $\mathcal{X}'$
- $\mathcal{X}^*$
- $x$ and $\mathcal{X}$
- $\mathcal{X}'$
- realization denoted by smaller case letters
- realization of these variables are numbers
- could real numbers or integers
- for random variables $\mathcal{X}$, $\mathcal{X}'$, realizations denoted by $\mathcal{X}'$
– any more?

– \( Z \) is greater than a specified time \( M \).

– e.g., true or false

– random events will be generally propositions,

– only takes two values

– distinguishing feature is that a random event

Random Events •
(\mathcal{H}|\mathcal{F})_{\perp} \text{ is not same as } (\mathcal{H}|\mathcal{F})_{\perp+} \\
\mathcal{H} \text{ given the past history} \\
\rightarrow \text{ probability at time } \tau \text{ that event } \mathcal{F} \text{ happens} \\
(\mathcal{H}|\mathcal{F})_{\perp} \\

\underline{Notation}
How should we interpret probability?

- Frequency of a repeatable event
- Probability interpreted as the
  \[ \frac{N}{y} \]
  probability of a head is
- Let us say coin comes up head \( y \) number of times
- Flip a coin \( N \) times (large, say a million)
- When a coin is flipped
- Probability of a head occurring

How should we interpret probability?
Subjective View

Which view is better for software engineering?

We call this view subjective probability.

Will happen in exchange of one dollar? $E$

How much a person is willing to bet that event $E$ will occur? $P(E|H)$

Interpreted as the belief of a person given that he/she knows the history $H$ that will occur $E$. $(H|E)P$
Let us start

(\mathcal{H}|x)^X \text{ and denoted by } (\mathcal{H}|x)^X_{\mathcal{H}} \text{ at } x \text{ is called the probability density function of } (\mathcal{H}|x)^X \text{ denoted by } (\mathcal{H}|x)^X_{\mathcal{D}} \text{ is called the distribution function of } X \text{ if it is said to have a point mass at } x \text{ then } 0 < (\mathcal{H}|x)^X_{\mathcal{D}} \text{ abbreviated as } (\mathcal{H}|x = X)_{\mathcal{D}} \text{ that realizes the value } x \text{ and denote the event } x = X.
Some questions

- If $x^1$ jumps at $x$, what does it mean?
- Is $(x)^X_H$ always smooth?
- What is $\dot{x}^X_f$?
- What is $\dot{x}^X_H$?

Let $X$ be uniformly distributed between $[0, 1]$
\[(\exists x \, p + \exists x \geq \exists X \geq \exists x \text{ and } \exists x \geq \exists X \geq \exists x) d - \]

\[
\text{approximates } \exists x p \exists x p (\mathcal{H}|\exists x, \exists x) \exists X \exists x \exists x f
\]

\[
(\mathcal{H}|\exists x \geq \exists X \text{ and } \exists x \geq \exists X) d
\]

\[
\text{as } (\mathcal{H}|\exists x, \exists x) \exists X \exists x \exists x f
\]

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Multiple Random Variables
Example

- consider an unit square and $X$ and $Y$ be the two coordinates
- let $F_{X,Y}(x, y)$ be $x \cdot y$
- what is $f_{X,Y}(x, y)$?
this knowledge affects your judgment about $X_1$ •

suppose you know the value of $X_2$ •

consider two random variables $X_1$ and $X_2$ •

Conditional Probabilities and Independence
\( \mathcal{H} \) is known as the conditional distribution of \( X \) given \( x \)

\( (\mathcal{H}_x, x^X|_X x^X) X^X \)

is abbreviated by

\( (\mathcal{H}_x, x^X|_X x^X) X^X \)

this is called the conditional probability of \( X \) given \( x \)

\( \mathcal{H} \) is the probability that \( X \) has the value \( x \)

\( \mathcal{H}_x, x^X|_X x^X \) is the probability that \( X \) realizes

\[ \text{Conditional Probabilities} \]
Example:

- $X$ and $\mathcal{A}$ are independent in our unit square.
- $X$ is said to be independent of $X^\perp$.
- $X^\perp$ distribution is not affected by realization of $X$.
- What does this mean?

\[
(\mathcal{H}|^\perp x = ^\perp X) \mathcal{P} = (\mathcal{H}|^\perp x = ^\perp X| ^\perp x = ^\perp X) \mathcal{P}
\]

Suppose the following equation is true.

Independence
Independence

Similarly for $X^B$.

First time software developed by $A$ fails $X^A$.

And $B$.

Suppose a software is developed by two teams $A$.

$$\mathcal{H}|^x = ^x \forall X \text{ and } ^x = ^1 X \text{ if } \exists X \text{ and } ^1 X \text{ are independent, then what is}$$

Independence
Independence

What does it mean to say that $X$ and $Y$ are independent? •

Definition: $X$ and $Y$ are independent if and only if

$$\forall d \in \mathcal{D}, \mathcal{H} \models \langle X \rangle_d \text{ and } \mathcal{H} \models \langle Y \rangle_d$$

\[ \Rightarrow \] 

Independence
Why independence?

- Generally a very idealistic assumption
- Independence assumption makes calculations easier
- Independence assumption
- Many experiments performed which refute the code developed from same specification
- Generally the independence assumption is not true
\[ (\mathcal{H} | \mathcal{E})_{P} (\mathcal{H}, \mathcal{E} | \mathcal{F}_{1})_{P} = (\mathcal{H} | \mathcal{E} \text{ and } \mathcal{F}_{1})_{P} \]

**Multiplicativity**: 

\[ (\mathcal{H} | \mathcal{E})_{P} + (\mathcal{H} | \mathcal{F}_{1})_{P} = (\mathcal{H} | \mathcal{E} \text{ or } \mathcal{F}_{1})_{P} \]

Simultaneously (they are mutually exclusive), then

**Additivity**: If both \( \mathcal{E} \) and \( \mathcal{F}_{1} \) cannot occur

\[ 1 \geq (\mathcal{H} | \mathcal{E})_{P} \geq 0 \]

**Convexity**: For any event \( \mathcal{E} \)

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Laws of Probability
\[
(\mathcal{H}|^u\mathcal{E})_d \times \\
\cdots \times (\mathcal{H},^u\mathcal{E}, \cdots,^u\mathcal{E}|^{\mathcal{E}})^d \\
\times (\mathcal{H}|^u\mathcal{E}, \cdots,^u\mathcal{E}|^{\mathcal{E}})^d = (\mathcal{H}|^u\mathcal{E} \cdots \text{and } ^u\mathcal{E} \text{ and } ^{\mathcal{E}})^d
\]

multiplicative law takes the following

\[
(\mathcal{H}|^{\mathcal{E}})^d \bigodot_{u=1}^{l=1} = (\mathcal{H}|^u\mathcal{E} \cdots \text{or } ^u\mathcal{E} \text{ or } ^{\mathcal{E}})^d
\]

exclusive

consider u events \( ^u\mathcal{E}, \cdots,^u\mathcal{E} \),\( ^{\mathcal{E}} \) that are mutually

Generalizations
(\mathcal{H}|\mathcal{E})d(\mathcal{H}|\mathcal{I}\mathcal{E})d-(\mathcal{H}|\mathcal{E})d+(\mathcal{H}|\mathcal{I}\mathcal{E})d = (\mathcal{H}|\mathcal{E} \text{ or } \mathcal{I}\mathcal{E})d

\text{if } \mathcal{I}\mathcal{E} \text{ and } \mathcal{E} \text{ are independent} \bullet

(\mathcal{H}|\mathcal{E} \text{ and } \mathcal{I}\mathcal{E})d-(\mathcal{H}|\mathcal{E})d+(\mathcal{H}|\mathcal{I}\mathcal{E})d = (\mathcal{H}|\mathcal{E} \text{ or } \mathcal{I}\mathcal{E})d

\text{suppose } \mathcal{I}\mathcal{E} \text{ and } \mathcal{E} \text{ are not mutually exclusive} \bullet

More equations
An Example

- The system fails if either hardware or software fail
- $E_s$ denote the event that the software experiences a fault within the next day
- $E_H$ denote the event that the hardware experiences a fault within the next day
- Consider a system made up of a hardware and software component
\[(S\Phi) d (H\Phi) d - (S\Phi) d + (H\Phi) d\]

simplifies to

if and are independent, then the expression \(H\Phi\) and \(S\Phi\) are independent

\[(S\Phi \text{ and } H\Phi) d - (S\Phi) d + (H\Phi) d\]

(\(H\Phi\) history)

Given by the following expression (suppressing the \(H\Phi\) on the right)

\[(H|S\Phi \text{ or } H\Phi) d\]

the system reliability is given by

An Example
$(S \lor (B \land H))P$

the probability that the system will fail is  

$(B \lor H)P(B|E)H(E)$

the probability given above evaluates to  

$(B \lor H)P(B|$  

the probability that the hardware component fails is  

Suppose the hardware has a backup system  

Example Extended
The Law of Total Probability

\[
(\mathbb{P}x = \mathbb{P}X \mid \mathbb{P}x = \mathbb{P}X) \bigotimes \sum = (\mathbb{P}x = \mathbb{P}X) \bigotimes \\
\text{the marginal of } \mathbb{P}X \text{ alone is } \bullet \\
\text{is the joint probability of } (\mathbb{P}x = \mathbb{P}X, \mathbb{P}x = \mathbb{P}X) \bullet \\
\text{variables } \mathbb{P}X \text{ and } \mathbb{P}X \text{ are two discrete random variables } \bullet \\
\text{suppose } \mathbb{P}X \text{ and } \mathbb{P}X \text{ are two discrete random variables } \bullet \\
\text{The Law of Total Probability}
\]
Bayes Law

\[
\frac{(\overline{x} = \overline{X} | \overline{z}x = \overline{z}X)}{(\overline{x} = \overline{X}) \cdot (\overline{x} = \overline{X} | \overline{z}x = \overline{z}X)}
\]

Applying the multiplicative rule we get

\[
\frac{(\overline{z}x = \overline{z}X, \overline{x} = \overline{X})}{(\overline{z}x = \overline{z}X, \overline{x} = \overline{X})} = (\overline{z}x = \overline{z}X | \overline{x} = \overline{X})
\]

Compute

\[
\text{Bayes Law}
\]