Definition MunkTop.12.1: $\mathscr{T}$ is a *topology* on X if and only if $\mathscr{T} \subseteq \wp(X)$ and $\varnothing \in \mathscr{T}$ and $X \in \mathscr{T}$ and for every $S \subseteq \mathscr{T}$, $\cup S \in \mathscr{T}$ and for every U, $V \in \mathscr{T}$, $U \cap V \in \mathscr{T}$.

Definition MunkTop.12.2: $(X, \mathscr{T})$ is a *topological space* if and only if $\mathscr{T}$ is a topology on X.

Definition MunkTop.12.3: U is *open* in $(X, \mathscr{T})$ if and only if $(X, \mathscr{T})$ is a topological space and $U \in \mathscr{T}$.

Definition MunkTop.12.4.a: If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *finer* than $\mathscr{T}$ on X if and only if $\mathscr{T}' \supseteq \mathscr{T}$.

Definition MunkTop.12.4.b: If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *strictly finer* than $\mathscr{T}$ on X if and only if $\mathscr{T}' \supseteq \mathscr{T}$ and $\mathscr{T}' \neq \mathscr{T}$.

Definition MunkTop.12.4.c: If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *coarser* than $\mathscr{T}$ on X if and only if $\mathscr{T}' \subseteq \mathscr{T}$.

Definition MunkTop.12.4.d: If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *strictly coarser* than $\mathscr{T}$ on X if and only if $\mathscr{T}' \subseteq \mathscr{T}$ and $\mathscr{T}' \neq \mathscr{T}$.

Definition MunkTop.13.1: $\mathscr{B}$ is a *basis* for a topology on X if and only if $\mathscr{B} \subseteq \wp(X)$ and for every $x \in X$, there exists $B \in \mathscr{B}$ such that $x \in B$ and for every $x \in X$, for every B_1, $B_2 \in \mathscr{B}$, if $x \in B_1 \cap B_2$ then there exists $B_3 \in \mathscr{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$.

Definition MunkTop.13.2: If $\mathscr{B}$ is a basis for a topology on X then *the topology on X generated by $\mathscr{B}$* is the unique $\mathscr{T} \subseteq \wp(X)$ such that for every $U \subseteq X$, $U \in \mathscr{T}$ if and only if for every $x \in U$, there exists $B \in \mathscr{B}$ such that $x \in B$ and $B \subseteq U$.

Definition MunkTop.13.3.a.basis: *The standard basis for a topology on* $\mathbb{R}$ is the set of $U \subseteq \mathbb{R}$ such that there exist a, $b \in \mathbb{R}$ such that $U = \{x \in \mathbb{R} : a < x < b\}$.

Definition MunkTop.13.3.a: *The standard topology on* $\mathbb{R}$ is the topology on $\mathbb{R}$ generated by the standard basis for a topology on $\mathbb{R}$.

Definition MunkTop.13.3.b: *The lower-limit topology on* $\mathbb{R}$ is the topology on $\mathbb{R}$ generated by the set of $U \subseteq \mathbb{R}$ such that there exist $a, b \in \mathbb{R}$ such that $U = \{x \in \mathbb{R} : a \leq x < b\}$.

Definition MunkTop.13.3.c: *The K-topology on* $\mathbb{R}$ is the topology on $\mathbb{R}$ generated by the standard basis for a topology on $\mathbb{R}$ union the set of $V \subseteq \mathbb{R}$

such that there exists W in the standard basis for a topology on $\mathbb{R}$ such that $V = W \setminus \{1/n : n \in \mathbb{N}\}$.

Definition MunkTop.13.4.a: $\mathscr{S}$ is a *subbasis* for a topology on X if and only if $\mathscr{S} \subseteq \wp(X)$ and $\cup\mathscr{S} = X$.

Definition MunkTop.13.4.b: *The topology on X generated by the subbasis $\mathscr{S}$* is the set of $U \subseteq X$ such that there exists $\mathscr{A} \subseteq \wp(\mathscr{S})$ such that for every $A \in \mathscr{A}$, A is finite and $U = \cup\{\cap S : S \in A\}$.

Definition MunkTop.14.1: If R is a strict simple order on X and a,$b \in X$ then $(a, b) = \{x \in X : aRxRb\}$.

Definition MunkTop.14.1.A: If R is a strict simple order on X and $U \subseteq X$ then U is an *open interval* in X under the order R if and only if there exist a, $b \in X$ such that $U = (a, b)$.

Definition MunkTop.14.2: If R is a strict simple order on X and a,$b \in X$ then $(a, b]$ is the set of $x \in X$ such that $aRxRb$ or $x = b$.

Definition MunkTop.14.2.A: If R is a strict simple order on X and $W \subseteq X$ then W is an *open-closed interval* in X under the order R if and only if there exist a, $b \in X$ such that $W = (a, b]$.

Definition MunkTop.14.3: If R is a strict simple order on X and a,$b \in X$ then $[a, b)$ is the set of $x \in X$ such that $aRxRb$ or $x = a$.

Definition MunkTop.14.3.A: If R is a strict simple order on X and $W \subseteq X$ then W is a *closed-open interval* in X under the order R if and only if there exist a, $b \in X$ such that $W = [a, b)$.

Definition MunkTop.14.4: If R is a strict simple order on X and a,$b \in X$ then $[a, b]$ is the set of $x \in X$ such that $aRxRb$ or $x = a$ or $x = b$.

Definition MunkTop.14.4.A: If R is a strict simple order on X and $F \subseteq X$ then F is a *closed interval* in X under the order R if and only if there exist a, $b \in X$ such that $F = [a, b]$.

Definition MunkTop.14.5: If R is a strict simple order on X then *the basis for the order topology on* (X, R) is the set of U such that there exist a, $b \in X$ such that $U = (a, b)$ or a is a first element in X, under R and $U = [a, b)$ or b is a last element in X, under R and $U = (a, b]$.

Definition MunkTop.14.6: If R is a strict simple order on X then *the order topology on* (X, R) is the topology on X generated by the basis for the order topology on (X, R).

Definition MunkTop.14.7: If R is a strict simple order on X and $a \in X$ then $(a, +\infty) = \{x \in X : aRx\}$.

Definition MunkTop.14.8: If R is a strict simple order on X and $a \in X$ then $(-\infty, a) = \{x \in X : xRa\}$.

Definition MunkTop.14.9: If R is a strict simple order on X and $a \in X$ then $[a, +\infty)$ is the set of $x \in X$ such that aRx or $x = a$.

Definition MunkTop.14.10: If R is a strict simple order on X and $a \in X$ then $(-\infty, a]$ is the set of $x \in X$ such that xRa or $x = a$.

Definition MunkTop.15.1: If $(X, \mathscr{T})$ and $(Y, \mathscr{T}')$ are topological spaces then *the basis for the product topology on* $(X, \mathscr{T}) \times (Y, \mathscr{T}')$ is the set of $U \times V$ such that $U \in \mathscr{T}$ and $V \in \mathscr{T}'$.

Definition MunkTop.15.2: If $(X, \mathscr{T})$ and $(Y, \mathscr{T}')$ are topological spaces then *the product topology on* $(X, \mathscr{T}) \times (Y, \mathscr{T}')$ is the topology on X generated by the basis for the product topology on $(X, \mathscr{T}) \times (Y, \mathscr{T}')$.

Definition MunkTop.15.3: $\pi_1(x, y) = x$.

Definition MunkTop.15.4: $\pi_2(x, y) = y$.

Definition MunkTop.15.5: If there exist u, v such that $x = (u,v)$ then $\pi_1(x) = u$. Otherwise $\pi_1(x)$ is undefined.

Definition MunkTop.15.6: If there exist u, v such that $x = (u,v)$ then $\pi_2(x) = v$. Otherwise $\pi_2(x)$ is undefined.

Definition MunkTop.16.1: If $(X, \mathscr{T})$ is a topological space and $Y \subseteq X$ then *the subspace topology on Y inherited from* $(X, \mathscr{T})$ is $\{Y \cap U : U \in \mathscr{T}\}$.

Definition MunkTop.16.2: If $(X, \mathscr{T})$ is a topological space and $Y \subseteq X$ then $(Y, \mathscr{T}')$ is a *subspace* of $(X, \mathscr{T})$ if and only if $\mathscr{T}'$ equals the subspace topology on Y inherited from $(X, \mathscr{T})$.

Definition MunkTop.16.3: If then *the dictionary order on* $(R, A) \times (S, B)$ is the unique T such that for every y, $y \in T$ if and only if there exist p, q, r, s such that $y = ((p,q),(r,s))$ and $p,r \in A$ and $q,s \in B$ and pRr or $p = r$ and qSs.

Definition MunkTop.16.4: If $I = [0, 1]$ and R equals the dictionary order on $(\{(x, y) : x < y\}, I) \times (\{(x, y) : x < y\}, I)$ then *the ordered square* I_0^2 is $(I \times I$,the order topology on $(I \times I, R))$.

Definition MunkTop.16.5: If R is a strict simple order on X and $Y \subseteq X$ then Y is a *convex* subset of (X, R) if and only if for every a, $b \in Y$, $(a, b) \subseteq Y$.

Definition MunkTop.17.1: If $A \subseteq X$ then A is a *closed set* in $(X, \mathscr{T})$ if and only if $(X, \mathscr{T})$ is a topological space and $X \setminus A$ is open in $(X, \mathscr{T})$.

Definition MunkTop.17.2: If $(X, \mathscr{T})$ is a topological space and $A \subseteq X$ then $A^\circ = \cup\{U \in \mathscr{T} : U \subseteq A\}$.

Definition MunkTop.17.3: If $(X, \mathscr{T})$ is a topological space and $A \subseteq X$ then $\overline{A}$ is the intersection over the set of F such that $A \subseteq F$ and F is a closed set in $(X, \mathscr{T})$.

Definition MunkTop.17.4: $A \cap B \neq \varnothing$ if and only if $A \cap B \neq \varnothing$.

Definition MunkTop.17.4.5: $A \cap B = \varnothing$ if and only if $A \cap B = \varnothing$.

Definition MunkTop.17.5: If $(X, \mathscr{T})$ is a topological space and $x \in X$ and $U \in \mathscr{T}$ then U is a *neighborhood* of x in $(X, \mathscr{T})$ if and only if $x \in U$.

Definition MunkTop.17.6: If $(X, \mathscr{T})$ is a topological space and $x \in X$ and $A \subseteq X$ then x is a *limit point* of A in $(X, \mathscr{T})$ if and only if for every U, if U is a neighborhood of x in $(X, \mathscr{T})$ then $U \cap A \setminus \{x\} \neq \varnothing$.

Definition MunkTop.17.7: If $(X, \mathscr{T})$ is a topological space and f is in the set of maps from $\mathbb{N}$ to X and $x \in X$ then f *converges to* x under $\mathscr{T}$ if and only if for every $U \in \mathscr{T}$, if U is a neighborhood of x in $(X, \mathscr{T})$ then there exists $N \in \mathbb{N}$ such that for every $n > N$, $f(n) \in U$.

Definition MunkTop.17.8: If $(X, \mathscr{T})$ is a topological space then $(X, \mathscr{T})$ is a *Hausdorff space* if and only if for every x, $y \in X$, if $x \neq y$ then there exist U, $V \in \mathscr{T}$ such that $x \in U$ and $y \in V$ and $U \cap V = \varnothing$.

Definition MunkTop.17.9: If $(X, \mathscr{T})$ is a Hausdorff space and there exists y such that f converges to y under $\mathscr{T}$ then x is a *limit* of f if and only if x equals the unique y such that f converges to y under $\mathscr{T}$.

Definition MunkTop.18.1: If (X, T) and (Y, T') are topological spaces and f is in the set of maps from X to Y then f is *continuous* if and only if for every $V \in T'$, the range of the converse relation to f when restricted to V is in T.

Definition MunkTop.18.2: If (X, T) and (Y, T') are topological spaces and f is in the set of maps from X to Y and $x \in X$ then f is *continuous* at x if and only if for every V, if V is a neighborhood of $f(x)$ in (Y, T') then there exists U such that U is a neighborhood of x in (X, T) and the range of f when restricted to U is contained in V.

Definition MunkTop.18.3: If (X, T) and (Y, T') are topological spaces and f is a bijection from X to Y then f is a *homeomorphism* if and only if f and the converse relation to f are continuous.

Definition MunkTop.18.3.5: If (X, T) and (Y, T') are topological spaces then (X, T) is *homeomorphic to* (Y, T') if and only if there exists f such that f is a homeomorphism.

Definition MunkTop.18.4: If (X, T) and (Y, T') are topological spaces and f is an injection from X to Y and f is continuous then f is an *embedding* if and only if f is a homeomorphism.

Definition MunkTop.19.1: If f is a function and the domain of f equals J then x is a *tuple* relative to f if and only if x is a function and the domain of x equals J and for every $\alpha \in J$, $x(\alpha) \in f(\alpha)$.

Definition MunkTop.19.2: If f is a function and the domain of f equals J then $\prod_{\alpha \in J} f(\alpha)$ is the set of x such that x is a tuple relative to f.

Definition MunkTop.19.2.5: $A^B = \prod_{\alpha \in J} f(\alpha)$.

Definition MunkTop.19.3: If X is a function and the domain of X equals J and T is a function and the domain of T equals J and for every $\alpha \in J$, $(X(\alpha), T(\alpha))$ is a topological space then *the standard basis for the box topology on* $\prod_{\alpha \in J} X(\alpha)$ is the set of $\prod_{\alpha \in J} f(\alpha)$ such that U is a function and the domain of U equals J and for every $\alpha \in J$, $U(\alpha) \in T(\alpha)$.

Definition MunkTop.19.4: If X is a function and the domain of X equals J and T is a function and the domain of T equals J and for every $\alpha \in J$, $(X(\alpha), T(\alpha))$ is a topological space then *the box topology on* $\prod_{\alpha \in J} X(\alpha)$ is ($\prod_{\alpha \in J} f(\alpha)$,the topology on $\prod_{\alpha \in J} f(\alpha)$ generated by the standard basis for the box topology on $\prod_{\alpha \in J} X(\alpha)$).

Definition MunkTop.19.5: If f is a function and the domain of f equals J and $\beta \in J$ then $f(\beta) = (\lambda x \in \prod_{\alpha \in J} f(\alpha))x(\beta)$.

Definition MunkTop.19.6: If X is a function and the domain of X equals J and T is a function and the domain of T equals J and for every $\alpha \in J$, $(X(\alpha), T(\alpha))$ is a topological space then *the standard subbasis for the product topology on* $\prod_{\alpha \in J} X(\alpha)$ is the set of the range of the converse relation to $X(\beta)$ when restricted to U such that $\beta \in J$ and $U \in T(\beta)$.

Definition MunkTop.19.7: If X is a function and the domain of X equals J and T is a function and the domain of T equals J and for every $\alpha \in J$, $(X(\alpha), T(\alpha))$ is a topological space then *the product space on* $\prod_{\alpha \in J} X(\alpha)$ is

($\prod_{\alpha \in J} f(\alpha)$,the topology on $\prod_{\alpha \in J} f(\alpha)$ generated by the subbasis the standard subbasis for the product topology on $\prod_{\alpha \in J} X(\alpha)$).

Definition MunkTop.20.1: d is a *metric* on X if and only if d is in the set of maps from $X \times X$ to $\mathbb{R}$ and for every x, $y \in X$, $d(x,y) \geq 0$ and $d(x,y) = 0$ if and only if $x = y$ and for every x, $y \in X$, $d(x,y) = d(y,x)$ and for every x, y, $z \in X$, $d(x,y) + d(y,z) \geq d(x,z)$.

Definition MunkTop.20.2: If d is a metric on X and $x \in X$ and $\varepsilon > 0$ then $B(\varepsilon, x) = \{y \in X : d(x, y) < \varepsilon\}$.

Definition MunkTop.20.3: If d is a metric on X then *the basis for the metric topology on X induced by d* is the set of $B(\varepsilon, x)$ such that $x \in X$ and $\varepsilon > 0$.

Definition MunkTop.20.4: If d is a metric on X then *the metric topology on X induced by d* is the topology on X generated by the basis for the metric topology on X induced by d.

Definition MunkTop.20.5: If (X, T) is a topological space then (X, T) is *metrizable* if and only if there exists d such that T equals the metric topology on X induced by d.

Definition MunkTop.20.6: If (X, T) is a topological space then X is a *metric space* if and only if T equals the metric topology on X induced by d.

Definition MunkTop.20.7: If X is a metric space and $A \subseteq X$ then A is *bounded* in X under d if and only if there exists $M \in \mathbb{R}$ such that for every a_1, $a_2 \in A$, $d(a_1,a_2) \leq M$.

Definition MunkTop.20.8: If X is a metric space and $A \subseteq X$ and $A \neq \varnothing$ and A is bounded in X under d then *the diameter of A* is the unique s such that s is a supremum for $\{d(a_1, a_2) : a_1, a_2 \in A\}$, under $\{(x, y) : x < y\}$. **Definition MunkTop.21.1:** If (X, T) is a topological space and $x \in X$ then X has a countable basis at x if and only if there exists f such that f is a function from $\mathbb{N}$ to T, and for every U such that U is a neighborhood of x in (X, T), we have that there exists $n \in \mathbb{N}$ such that $f(n) \subseteq U$.

Definition MunkTop.21.2: If (X, T) is a topological space then (X, T) is *first countable* if and only if for every $x \in X$, X has a countable basis at x.

Definition MunkTop.21.3: If (X, T) is a topological space and Y is a metric space and F is a function and the domain of F equals $\mathbb{N}$ and for every $n \in \mathbb{N}$, $F(n)$ is a function from X to Y then $(F(n))_{n=0}^{\infty}$ *uniformly converges* to f if and only if for every $\varepsilon \in \mathbb{R}$, if $\varepsilon > 0$ then there exists $N \in \mathbb{N}$ such that for every $n > N$, for every $x \in X$, $d([F(n)](x), f(x)) < \varepsilon$.

Definition MunkTop.22.1: If (X,T) and (Y,T') are topological spaces and p is a surjection from X to Y then p is a *quotient map* if and only if for every $U \subseteq Y$, $U \in T'$ if and only if the range of the converse relation to p when restricted to U is in T.

Definition MunkTop.22.2: If (X,T) and (Y,T') are topological spaces and p is a surjection from X to Y and $C \subseteq X$ then C is *saturated* if and only if for every $y \in Y$, if C intersects the range of the converse relation to p when restricted to $\{y\}$ then the range of the converse relation to p when restricted to $\{y\}$ is contained in C.

Definition MunkTop.22.3: If (X,T) and (Y,T') are topological spaces and p is a function from X to Y then f is an *open map* if and only if for every $U \in T$, the range of f when restricted to U is in T'.

Definition MunkTop.22.4: If (X,T) and (Y,T') are topological spaces and p is a function from X to Y then f is a *closed map* if and only if for every A such that A is a closed set in (X,T), we have that the range of f when restricted to A is a closed set in (Y,T').

Definition MunkTop.22.5: If (X,T) is a topological space and p is a surjection from X to A then *the quotient topology on A induced by p* is the unique T' such that T' is a topology on A and p is a quotient map.

Definition MunkTop.22.6: If (X,T) is a topological space and X^* is a partition of X and p is a surjection from X to X^* and for every $x \in X$, $p(x) = (!w \in X^*) x \in w$ and T equals the quotient topology on X^* induced by p then (X^*,T) is a *quotient space* if and only if $\top$.