**Definition MunkTop.12.1:** $\mathscr{T}$ is a *topology* on $X$ if and only if $\mathscr{T} \subseteq \wp(X)$ and $\varnothing \in \mathscr{T}$ and $X \in \mathscr{T}$ and for every $S \subseteq \mathscr{T}$, $\cup S \in \mathscr{T}$ and for every $U, V \in \mathscr{T}$, $U \cap V \in \mathscr{T}$.

**Definition MunkTop.12.2:** $(X, \mathscr{T})$ is a *topological space* if and only if $\mathscr{T}$ is a topology on $X$.

**Definition MunkTop.12.3:** $U$ is *open* in $(X, \mathscr{T})$ if and only if $(X, \mathscr{T})$ is a topological space and $U \in \mathscr{T}$.

**Definition MunkTop.12.4.a:** If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *finer* than $\mathscr{T}$ on $X$ if and only if $\mathscr{T}' \supseteq \mathscr{T}$.

**Definition MunkTop.12.4.b:** If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *strictly finer* than $\mathscr{T}$ on $X$ if and only if $\mathscr{T}' \supseteq \mathscr{T}$ and $\mathscr{T}' \neq \mathscr{T}$.

**Definition MunkTop.12.4.c:** If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *coarser* than $\mathscr{T}$ on $X$ if and only if $\mathscr{T}' \subseteq \mathscr{T}$.

**Definition MunkTop.12.4.d:** If $(X, \mathscr{T})$ and $(X, \mathscr{T}')$ are topological spaces then $\mathscr{T}'$ is *strictly coarser* than $\mathscr{T}$ on $X$ if and only if $\mathscr{T}' \subseteq \mathscr{T}$ and $\mathscr{T}' \neq \mathscr{T}$.