Definition FS.2.1: A is a *binary relation* if and only if for every $y \in A$, there exist z, w such that $y = (z,w)$.

Definition FS.2.2: A is a *ternary relation* if and only if for every $y \in A$, there exist z, w, u such that $y = (z,w,u)$.

Definition FS.2.3: If R is a binary relation then *the domain of* R is the set of x such that there exists y such that xRy. Otherwise *the domain of* R is undefined.

Definition FS.2.4: If R is a binary relation then *the range of* R is the set of y such that there exists x such that xRy. Otherwise *the range of* R is undefined.

Definition FS.2.5: *The field of* R is the domain of R union the range of R.

Definition FS.2.6: If R is a binary relation then *the converse relation to* R is $\{(x, y) : yRx\}$. Otherwise *the converse relation to* R is undefined.

Definition FS.2.8: If R and S are binary relations then $R \circ S$ is the set of (x,y) such that there exists z such that xRz and zSy. Otherwise $R \circ S$ is undefined. Precedence: 10.

Definition FS.2.9: If R is a binary relation then $R \mid A$ is R intersect the cartesian product of A and the range of R. Otherwise $R \mid A$ is undefined. Precedence: 5.

Definition FS.2.10: If R is a binary relation then *the range of* R *when restricted to* A is the range of $R \mid A$. Otherwise *the range of* R *when restricted to* A is undefined. Precedence: 5.

Definition FS.2.11: R is *reflexive* on A if and only if R is a binary relation and for every $x \in A$, xRx.

Definition FS.2.12: R is *irreflexive* on A if and only if R is a binary relation and for every $x \in A$, it is not the case that xRx.

Definition FS.2.13: R is *symmetric* on A if and only if R is a binary relation and for every x, $y \in A$, xRy if and only if yRx.

Definition FS.2.14: R is *asymmetric* on A if and only if R is a binary relation and for every x, $y \in A$, if xRy then it is not the case that yRx.

Definition FS.2.15: R is *antisymmetric* on A if and only if R is a binary relation and for every x, $y \in A$, xRy and if yRx then $x = y$.

Definition FS.2.16: R is *transitive* on A if and only if R is a binary relation and for every x, y, $z \in A$, xRy and if yRz then xRz.

Definition FS.2.17: R is *connected* on A if and only if R is a binary relation and for every x, $y \in A$, if $x \neq y$ then xRy or yRx.

Definition FS.2.18: R is *simply connected* on A if and only if R is a binary relation and for every x, $y \in A$, xRy or yRx.

Definition FS.2.19: R is *reflexive* if and only if R is a binary relation and R is reflexive on the field of R.

Definition FS.2.20: R is *irreflexive* if and only if R is a binary relation and R is irreflexive on the field of R.

Definition FS.2.21: R is *symmetric* if and only if R is a binary relation and R is symmetric on the domain of R.

Definition FS.2.22: R is *asymmetric* if and only if R is a binary relation and R is asymmetric on the domain of R.

Definition FS.2.23: R is *antisymmetric* if and only if R is a binary relation and R is antisymmetric on the domain of R.

Definition FS.2.24: R is *transitive* if and only if R is a binary relation and R is transitive on the domain of R.

Definition FS.2.25: R is *ϵ-connected* if and only if R is a binary relation and R is connected on the domain of R.

Definition FS.2.26: R is *simply connected* if and only if R is a binary relation and R is simply connected on the domain of R.

Definition FS.2.27: $Id(x) = \{(y, y) : y \in x\}$.

Definition FS.2.28: R is a *quasi order* on A if and only if R is reflexive on A and R is transitive on A.

Definition FS.2.29: R is a *partial order* on A if and only if R is reflexive on A and R is antisymmetric on A and R is transitive on A.

Definition FS.2.30: R is a *simple order* on A if and only if R is antisymmetric on A and R is transitive on A and R is simply connected on A.

Definition FS.2.31: R is a *strict partial order* on A if and only if R is asymmetric on A and R is transitive on A.

Definition FS.2.32: R is a *strict sipmle order* on A if and only if R is asymmetric on A and R is transitive on A and R is connected on A.

Definition FS.2.33: R is a *quasi order* if and only if R is a quasi order on the field of R.

Definition FS.2.34: R is a *partial order* if and only if R is a partial order on the field of R.

Definition FS.2.35: R is a *simple order* if and only if R is a simple order on the field of R.

Definition FS.2.36: R is a *strict partial order* if and only if R is a strict partial order on the field of R.

Definition FS.2.37: R is a *strict sipmle order* if and only if R is a strict sipmle order on the field of R.

Definition FS.2.38: x is a *minimal element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, it is not the case that yRx.

Definition FS.2.39: x is a *first element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, if $x \neq y$ then xRy.

Definition FS.2.40: R is a *well-ordering* on A if and only if R is connected on A and for every $B \subseteq A$, if $B \neq \varnothing$ then there exists x such that x is a minimal element in B, under R.

Definition FS.2.41: y is an *immediate successor* of x, under R if and only if R is a binary relation and xRy and for every z, if xRz then $z = y$ or yRz.

Definition FS.2.42: x is a *last element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, if $x \neq y$ then yRx.

Definition FS.2.43: B is a *section* of A, under R if and only if R is a binary relation and $B \subseteq A$ and the range of A intersect the converse relation to R when restricted to B is contained in B.

Definition FS.2.44: If R is a binary relation then *the initial segment of* A *at* x*, under* R is $\{y \in A : yRx\}$. Otherwise $Seg(R)$ is undefined.

Definition FS.2.45: x is a *lower bound* for A, under R if and only if R is a binary relation and for every $y \in A$, xRy.

Definition FS.2.46: x is an *infimum* for A, under R if and only if x is a lower bound for A, under R and for every $y \in A$, if y is a lower bound for A, under R then yRx.

Definition FS.2.47: x is an *upper bound* for A, under R if and only if R is a binary relation and for every $y \in A$, yRx.

Definition FS.2.48: x is a *supremum* for A, under R if and only if x is an upper bound for A, under R and for every $y \in A$, if y is an upper bound for A, under R then xRy.

Definition FS.2.50: R is an *equivalence relation* if and only if R is reflexive and R is symmetric and R is transitive.

Definition FS.2.51: R is an *equivalence relation* on A if and only if R is an equivalence relation and the field of R equals A.

Definition FS.2.52: If R is an equivalence relation and x is in the field of R then *the coset of x with respect to R* is $\{y : xRy\}$. Otherwise *the coset of x with respect to R* is undefined.

Definition FS.2.53: W is a *partition* of A if and only if $\cup W = A$ and for every B, $C \in W$, if $B \neq C$ then $B \cap C = \varnothing$ and for every $B \in W$, $B \neq \varnothing$.

Definition FS.2.54: W is a *partition* if and only if there exists A such that W is a partition of A.

Definition FS.2.55: If V and W are partitions then V is *finer than* W if and only if $V \neq W$ and for every $A \in V$, there exists $B \in W$ such that $A \subseteq B$.

Definition FS.2.56: If R is an equivalence relation then *the partition induced by R* is the set of the coset of x with respect to R such that x is in the field of R.

Definition FS.2.57: If W is a partition then *the relation induced by W* is the set of (x,y) such that there exists $B \in W$ such that $x \in B$ and $y \in B$.

Definition FS.2.58: f is a *function* if and only if $f = \{(x, y) : f(x) = y\}$.

Definition FS.2.59: f is an *injection* if and only if f and the converse relation to f are functions.

Definition FS.2.60: f is a *function* from A to B if and only if f is a function and the domain of f equals A and the range of f is contained in B.

Definition FS.2.61: f is a *surjection* from A to B if and only if f is a function and the domain of f equals A and the range of f equals B.

Definition FS.2.62: f is an *injection* from A to B if and only if f is an injection and the domain of f equals A and the range of f is contained in B.

Definition FS.2.63: f is a *bijection* from A to B if and only if f is an injection and the domain of f equals A and the range of f equals B.

Definition FS.2.64: *The set of maps from A to B* is the set of f such that f is a function from A to B.