Definition FS.4.1: x is a *transitive set* if and only if for every $y \in x$, for every $z \in y$, $z \in x$.

Definition FS.4.2: x is *ϵ-connected* if and only if for every y, $z \in x$, $y \in z$ or $z \in y$ or $y = z$.

Definition FS.4.3: x is an *ordinal* if and only if x is a transitive set and x is ϵ-connected.

Definition FS.4.4: *The ϵ-connected subset of* x is the set of (y,z) such that $y \in z$ and z,$y \in x$.

Definition FS.4.5: $A < B$ if and only if A and B are ordinals and $A \in B$.

Definition FS.4.6: $A \leq B$ if and only if A and B are ordinals and $A \in B$ or $A = B$.

Definition FS.4.7: $A > B$ if and only if A and B are ordinals and $B \in A$.

Definition FS.4.8: $A \geq B$ if and only if A and B are ordinals and $B \in A$ or $A = B$.

Definition FS.4.9: If x is an ordinal then *the successor of* x is $\{y : y \leq x\}$. Otherwise *the successor of* x is undefined.

Definition FS.4.10: x is a *natural number* if and only if x is an ordinal and the converse relation to the ϵ-connected subset of x is a well-ordering on x.

Definition FS.4.11: ω is the set of x such that x is a natural number.

Definition FS.4.11.a: $\mathbb{N} = \omega$.

Definition FS.4.12: *0* $= \varnothing$.

Definition FS.4.13: *1* $= \{\varnothing\}$.

Definition FS.4.13.2: *2* is the successor of 1.

Definition FS.4.13.3: *3* is the successor of 2.

Definition FS.4.13.4: *4* is the successor of 3.

Definition FS.4.13.5: *5* is the successor of 4.

Definition FS.4.13.6: *6* is the successor of 5.

Definition FS.4.13.7: *7* is the successor of 6.

Definition FS.4.13.8: *8* is the successor of 7.

Definition FS.4.13.9: *9* is the successor of 8.

Definition FS.4.13.10: *10* is the successor of 9.

Definition FS.4.14: *The graph of* $+$ is the unique x such that for every y, $z \in \omega$, $x(y,0) = y$ and x, evaluated at y, the successor of z equals the successor of $x(y,z)$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.15: $x + y$ is the unique z such that (x,y,z) is in the graph of $+$. Precedence: 60.

Definition FS.4.16: *The graph of* $\times$ is the unique x such that for every y, $z \in \omega$, $x(y,0) = 0$ and $x(y,z + 1) = x(y,z) + y$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.17: $x \times y$ is the unique z such that (x,y,z) is in the graph of $\times$. Precedence: 40.

Definition FS.4.18: *The graph of exponentiation* is the unique x such that for every y, $z \in \omega$, $x(y,0) = 1$ and $x(y,z + 1) = x(y,z) \times y$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.19: x^y is the unique z such that (x,y,z) is in the graph of exponentiation. Precedence: 20.

Definition FS.4.20: x is *infinite* if and only if x is not finite.

Definition FS.4.21: x is *denumerable* if and only if $x \approx \omega$.

Definition FS.4.22: x is *infinite* if and only if x is not finite.

Definition FS.4.23: A is *countable* if and only if there exists f such that f is a bijection from ω to A.

Definition FS.4.24: A is *uncountable* if and only if A is not countable.