Definition builtIn1: $x \cup y$ is the set of z such that $z \in x$ or $z \in y$. Precedence: 40.

Definition builtIn2: $x \cap y$ is the set of z such that $z \in x$ and $z \in y$. Precedence: 30.

Definition builtIn3: $x \setminus y$ is the set of z such that $z \in x$ and it is not the case that $z \in y$. Precedence: 50.

Definition builtIn4: $(a, b) = \{\{a\}, \{a, b\}\}$.

Definition builtIn5: $X \subseteq Y$ if and only if for every x, if $x \in X$ then $x \in Y$.

Definition builtIn6: $\varnothing$ is the unique S such that for every x, it is not the case that $x \in S$.

Definition builtIn7: $\cup X$ is the set of u such that there exists $x \in X$ such that $u \in x$.

Definition builtIn8: $\cap X$ is the set of u such that for every $x \in X$, $u \in x$.

Definition builtIn9: $\wp(X) = \{U : U \subseteq X\}$.

Definition builtIn10: $X \supseteq Y$ if and only if for every y, if $y \in Y$ then $y \in X$.

Definition FS.1.1: $x \Delta_0 y = x \setminus y \cup y \setminus x$. Precedence: 60.

Definition FS.1.2: $x \times y$ is the set of (z,w) such that $z \in x$ and $w \in y$. Precedence: 20.

Definition FS.2.1: A is a *binary relation* if and only if for every $y \in A$, there exist z, w such that $y = (z,w)$.

Definition FS.2.2: A is a *ternary relation* if and only if for every $y \in A$, there exist z, w, u such that $y = (z,w,u)$.

Definition FS.2.3: If R is a binary relation then *the domain of* R is the set of x such that there exists y such that xRy. Otherwise *the domain of* R is undefined.

Definition FS.2.4: If R is a binary relation then *the range of* R is the set of y such that there exists x such that xRy. Otherwise *the range of* R is undefined.

Definition FS.2.5: *The field of* R is the domain of R union the range of R.

Definition FS.2.6: If R is a binary relation then *the converse relation to* R is $\{(x, y) : yRx\}$. Otherwise *the converse relation to* R is undefined.

Definition FS.2.8: If R and S are binary relations then $R \circ S$ is the set of (x,y) such that there exists z such that xRz and zSy. Otherwise $R \circ S$ is undefined. Precedence: 10.

Definition FS.2.9: If R is a binary relation then $R \mid A$ is R intersect the cartesian product of A and the range of R. Otherwise $R \mid A$ is undefined. Precedence: 5.

Definition FS.2.10: If R is a binary relation then *the range of* R *when restricted to* A is the range of $R \mid A$. Otherwise *the range of* R *when restricted to* A is undefined. Precedence: 5.

Definition FS.2.11: R is *reflexive* on A if and only if R is a binary relation and for every $x \in A$, xRx.

Definition FS.2.12: R is *irreflexive* on A if and only if R is a binary relation and for every $x \in A$, it is not the case that xRx.

Definition FS.2.13: R is *symmetric* on A if and only if R is a binary relation and for every x, $y \in A$, xRy if and only if yRx.

Definition FS.2.14: R is *asymmetric* on A if and only if R is a binary relation and for every x, $y \in A$, if xRy then it is not the case that yRx.

Definition FS.2.15: R is *antisymmetric* on A if and only if R is a binary relation and for every x, $y \in A$, xRy and if yRx then $x = y$.

Definition FS.2.16: R is *transitive* on A if and only if R is a binary relation and for every x, y, $z \in A$, xRy and if yRz then xRz.

Definition FS.2.17: R is *connected* on A if and only if R is a binary relation and for every x, $y \in A$, if $x \neq y$ then xRy or yRx.

Definition FS.2.18: R is *simply connected* on A if and only if R is a binary relation and for every x, $y \in A$, xRy or yRx.

Definition FS.2.19: R is *reflexive* if and only if R is a binary relation and R is reflexive on the field of R.

Definition FS.2.20: R is *irreflexive* if and only if R is a binary relation and R is irreflexive on the field of R.

Definition FS.2.21: R is *symmetric* if and only if R is a binary relation and R is symmetric on the domain of R.

Definition FS.2.22: R is *asymmetric* if and only if R is a binary relation and R is asymmetric on the domain of R.

Definition FS.2.23: R is *antisymmetric* if and only if R is a binary relation and R is antisymmetric on the domain of R.

Definition FS.2.24: R is *transitive* if and only if R is a binary relation and R is transitive on the domain of R.

Definition FS.2.25: R is *ϵ-connected* if and only if R is a binary relation and R is connected on the domain of R.

Definition FS.2.26: R is *simply connected* if and only if R is a binary relation and R is simply connected on the domain of R.

Definition FS.2.27: $Id(x) = \{(y, y) : y \in x\}$.

Definition FS.2.28: R is a *quasi order* on A if and only if R is reflexive on A and R is transitive on A.

Definition FS.2.29: R is a *partial order* on A if and only if R is reflexive on A and R is antisymmetric on A and R is transitive on A.

Definition FS.2.30: R is a *simple order* on A if and only if R is antisymmetric on A and R is transitive on A and R is simply connected on A.

Definition FS.2.31: R is a *strict partial order* on A if and only if R is asymmetric on A and R is transitive on A.

Definition FS.2.32: R is a *strict sipmle order* on A if and only if R is asymmetric on A and R is transitive on A and R is connected on A.

Definition FS.2.33: R is a *quasi order* if and only if R is a quasi order on the field of R.

Definition FS.2.34: R is a *partial order* if and only if R is a partial order on the field of R.

Definition FS.2.35: R is a *simple order* if and only if R is a simple order on the field of R.

Definition FS.2.36: R is a *strict partial order* if and only if R is a strict partial order on the field of R.

Definition FS.2.37: R is a *strict sipmle order* if and only if R is a strict sipmle order on the field of R.

Definition FS.2.38: x is a *minimal element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, it is not the case that yRx.

Definition FS.2.39: x is a *first element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, if $x \neq y$ then xRy.

Definition FS.2.40: R is a *well-ordering* on A if and only if R is connected on A and for every $B \subseteq A$, if $B \neq \varnothing$ then there exists x such that x is a minimal element in B, under R.

Definition FS.2.41: y is an *immediate successor* of x, under R if and only if R is a binary relation and xRy and for every z, if xRz then $z = y$ or yRz.

Definition FS.2.42: x is a *last element* in A, under R if and only if R is a binary relation and $x \in A$ and for every $y \in A$, if $x \neq y$ then yRx.

Definition FS.2.43: B is a *section* of A, under R if and only if R is a binary relation and $B \subseteq A$ and the range of A intersect the converse relation to R when restricted to B is contained in B.

Definition FS.2.44: If R is a binary relation then *the initial segment of A at x, under R* is $\{y \in A : yRx\}$. Otherwise $Seg(R)$ is undefined.

Definition FS.2.45: x is a *lower bound* for A, under R if and only if R is a binary relation and for every $y \in A$, xRy.

Definition FS.2.46: x is an *infimum* for A, under R if and only if x is a lower bound for A, under R and for every $y \in A$, if y is a lower bound for A, under R then yRx.

Definition FS.2.47: x is an *upper bound* for A, under R if and only if R is a binary relation and for every $y \in A$, yRx.

Definition FS.2.48: x is a *supremum* for A, under R if and only if x is an upper bound for A, under R and for every $y \in A$, if y is an upper bound for A, under R then xRy.

Definition FS.2.50: R is an *equivalence relation* if and only if R is reflexive and R is symmetric and R is transitive.

Definition FS.2.51: R is an *equivalence relation* on A if and only if R is an equivalence relation and the field of R equals A.

Definition FS.2.52: If R is an equivalence relation and x is in the field of R then *the coset of x with respect to R* is $\{y : xRy\}$. Otherwise *the coset of x with respect to R* is undefined.

Definition FS.2.53: W is a *partition* of A if and only if $\cup W = A$ and for every B, $C \in W$, if $B \neq C$ then $B \cap C = \varnothing$ and for every $B \in W$, $B \neq \varnothing$.

Definition FS.2.54: W is a *partition* if and only if there exists A such that W is a partition of A.

Definition FS.2.55: If V and W are partitions then V is *finer than* W if and only if $V \neq W$ and for every $A \in V$, there exists $B \in W$ such that $A \subseteq B$.

Definition FS.2.56: If R is an equivalence relation then *the partition induced by* R is the set of the coset of x with respect to R such that x is in the field of R.

Definition FS.2.57: If W is a partition then *the relation induced by* W is the set of (x,y) such that there exists $B \in W$ such that $x \in B$ and $y \in B$.

Definition FS.2.58: f is a *function* if and only if $f = \{(x, y) : f(x) = y\}$.

Definition FS.2.59: f is an *injection* if and only if f and the converse relation to f are functions.

Definition FS.2.60: f is a *function* from A to B if and only if f is a function and the domain of f equals A and the range of f is contained in B.

Definition FS.2.61: f is a *surjection* from A to B if and only if f is a function and the domain of f equals A and the range of f equals B.

Definition FS.2.62: f is an *injection* from A to B if and only if f is an injection and the domain of f equals A and the range of f is contained in B.

Definition FS.2.63: f is a *bijection* from A to B if and only if f is an injection and the domain of f equals A and the range of f equals B.

Definition FS.2.64: *The set of maps from A to B* is the set of f such that f is a function from A to B.

Definition FS.3.1: $A \approx B$ if and only if there exists f such that f is a bijection from A to B.

Definition FS.3.2: $x \leq y$ if and only if there exists $z \subseteq y$ such that $x \approx z$.

Definition FS.3.3: $A < B$ if and only if $A \leq B$ and it is not the case that $B \leq A$.

Definition FS.3.4: x is a *minimal element* of A if and only if $x \in A$ and for every $y \in A$, it is not the case that $y \in x$.

Definition FS.3.5: x is a *maximal element* of A if and only if $x \in A$ and for every $y \in A$, it is not the case that $x \in y$.

Definition FS.3.6: x is *finite* if and only if for every $A \neq \varnothing$, if $A \subseteq \wp(x)$ then there exists $y \in A$ such that y is a minimal element of A.

Definition FS.3.7: x is *finite* if and only if for every $y \subseteq x$, if $y \neq x$ then it is not the case that $x \approx y$.

Definition FS.4.1: x is a *transitive set* if and only if for every $y \in x$, for every $z \in y$, $z \in x$.

Definition FS.4.2: x is *ϵ-connected* if and only if for every y, $z \in x$, $y \in z$ or $z \in y$ or $y = z$.

Definition FS.4.3: x is an *ordinal* if and only if x is a transitive set and x is ϵ-connected.

Definition FS.4.4: *The ϵ-connected subset of* x is the set of (y,z) such that $y \in z$ and $z,y \in x$.

Definition FS.4.5: $A < B$ if and only if A and B are ordinals and $A \in B$.

Definition FS.4.6: $A \leq B$ if and only if A and B are ordinals and $A \in B$ or $A = B$.

Definition FS.4.7: $A > B$ if and only if A and B are ordinals and $B \in A$.

Definition FS.4.8: $A \geq B$ if and only if A and B are ordinals and $B \in A$ or $A = B$.

Definition FS.4.9: If x is an ordinal then *the successor of* x is $\{y : y \leq x\}$. Otherwise *the successor of* x is undefined.

Definition FS.4.10: x is a *natural number* if and only if x is an ordinal and the converse relation to the ϵ-connected subset of x is a well-ordering on x.

Definition FS.4.11: ω is the set of x such that x is a natural number.

Definition FS.4.11.a: $\mathbb{N} = \omega$.

Definition FS.4.12: *0* $= \varnothing$.

Definition FS.4.13: *1* $= \{\varnothing\}$.

Definition FS.4.13.2: *2* is the successor of 1.

Definition FS.4.13.3: *3* is the successor of 2.

Definition FS.4.13.4: *4* is the successor of 3.

Definition FS.4.13.5: *5* is the successor of 4.

Definition FS.4.13.6: *6* is the successor of 5.

Definition FS.4.13.7: *7* is the successor of 6.

Definition FS.4.13.8: *8* is the successor of 7.

Definition FS.4.13.9: *9* is the successor of 8.

Definition FS.4.13.10: *10* is the successor of 9.

Definition FS.4.14: *The graph of* $+$ is the unique x such that for every y, $z \in \omega$, $x(y,0) = y$ and x, evaluated at y, the successor of z equals the successor of $x(y,z)$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.15: $x + y$ is the unique z such that (x,y,z) is in the graph of $+$. Precedence: 60.

Definition FS.4.16: *The graph of* $\times$ is the unique x such that for every y, $z \in \omega$, $x(y,0) = 0$ and $x(y,z + 1) = x(y,z) + y$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.17: $x \times y$ is the unique z such that (x,y,z) is in the graph of $\times$. Precedence: 40.

Definition FS.4.18: *The graph of exponentiation* is the unique x such that for every y, $z \in \omega$, $x(y,0) = 1$ and $x(y,z + 1) = x(y,z) \times y$ and for every y, z, $x(y,z)$ is defined if and only if $y,z \in \omega$.

Definition FS.4.19: x^y is the unique z such that (x,y,z) is in the graph of exponentiation. Precedence: 20.

Definition FS.4.20: x is *infinite* if and only if x is not finite.

Definition FS.4.21: x is *denumerable* if and only if $x \approx \omega$.

Definition FS.4.22: x is *infinite* if and only if x is not finite.

Definition FS.4.23: A is *countable* if and only if there exists f such that f is a bijection from ω to A.

Definition FS.4.24: A is *uncountable* if and only if A is not countable.

Definition FS.5.1: If x,$y \in \omega$ and $y \neq 0$ then x / y = (x,y). Otherwise x / y is undefined. Precedence: 5.

Definition FS.5.2: *The set of positive fractions* is the set of x / y such that x / y is defined.

Definition FS.5.3: $x \equiv y$ if and only if there exist a, b, c, d such that x = a / b and y = c / d and $a \times d = b \times c$.

Definition FS.5.4: $x < y$ if and only if there exist a, b, c, d such that x = a / b and y = c / d and $a \times d < b \times c$.

Definition FS.5.5: $x > y$ if and only if $y < x$.

Definition FS.5.6: $x \leq y$ if and only if $x < y$ or $x \equiv y$.

Definition FS.5.7: $x \geq y$ if and only if $x > y$ or $x \equiv y$.

Definition FS.5.8: $x + y$ is the unique z such that there exist a, b, c, d, e, f such that x = a / b and y = c / d and z = e / f and $e = a \times d + b \times c$ and $f = b \times d$. Precedence: 40.

Definition FS.5.9: $x \times y$ is the unique t such that there exist a, b, c, d, e, f such that x = a / b and y = c / d and t = e / f and $e = a \times c$ and $f = b \times d$. Precedence: 20.

Definition FS.5.10: *The set Nra* is the set of the coset of x with respect to $\{(u, v) : u \equiv v\}$ such that x is in the set of positive fractions.

Definition FS.5.10.5: If x is in the set of positive fractions then x is the coset of x with respect to $\{(u, v) : u \equiv v\}$. Otherwise x is undefined.

Definition FS.5.11: $x < y$ if and only if x, y are in the set Nra and there exist u, v such that $u \in x$ and $v \in y$ and $u < v$.

Definition FS.5.12: $x > y$ if and only if x, y are in the set Nra and there exist u, v such that $u \in x$ and $v \in y$ and $u > v$.

Definition FS.5.13: $x \leq y$ if and only if x, y are in the set Nra and there exist u, v such that $u \in x$ and $v \in y$ and $u \leq v$.

Definition FS.5.14: $x \geq y$ if and only if x, y are in the set Nra and there exist u, v such that $u \in x$ and $v \in y$ and $u \geq v$.

Definition FS.5.15: $x + y$ is the unique z such that x, y, z are in the set Nra and there exist u, v, w such that $u \in x$ and $v \in y$ and $w \in z$ and $u + v \equiv w$. Precedence: 40.

Definition FS.5.16: $x \times y$ is the unique z such that x, y, z are in the set Nra and there exist u, v, w such that $u \in x$ and $v \in y$ and $w \in z$ and $u \times v \equiv w$. Precedence: 20.

Definition FS.5.17: 0 is the coset of 0 / 1 with respect to $\{(x, y) : x \equiv y\}$.

Definition FS.5.18: 1 is the coset of 1 / 1 with respect to $\{(x, y) : x \equiv y\}$.

Definition FS.5.19: $x \equiv y$ if and only if there exist a, b, c, d such that x = (a,b) and y = (c,d) and $a + d = b + c$.

Definition FS.5.20: $x < y$ if and only if there exist a, b, c, d such that x = (a,b) and y = (c,d) and $a + d < b + c$.

Definition FS.5.21: $x + y$ is the unique z such that there exist a, b, c, d, e, f such that x = (a,b) and y = (c,d) and z = (e,f) and $a + c + f = b + d + e$. Precedence: 40.

Definition FS.5.22: $x \times y$ is the unique z such that there exist a, b, c, d, e, f such that x = (a,b) and y = (c,d) and z = (e,f) and $a \times c + b \times d + f = a \times d + b \times c + e$. Precedence: 20.

Definition FS.5.23: $\mathbb{Q}$ is the set of the coset of x with respect to $\{(u, v) : u \equiv v\}$ such that x is in the cartesian product of the set Nra and the set Nra.

Definition FS.5.23.5: If x is in the set Nra then x is the coset of (x,0) with respect to $\{(u, v) : u \equiv v\}$.

Definition FS.5.23.8: If x is in the set of positive fractions then $x = x$.

Definition FS.5.23.A: $\mathbb{Q} = \mathbb{Q}$.

Definition FS.5.24: $x < y$ if and only if there exist z, w such that x,$y \in \mathbb{Q}$ and $z \in x$ and $w \in y$ and $z < w$.

Definition FS.5.25: $x + y$ is the unique z such that x,y,$z \in \mathbb{Q}$ and there exist a, b, c such that $a \in x$ and $b \in y$ and $c \in z$ and $a + b = c$. Precedence: 40.

Definition FS.5.26: $x \times y$ is the unique z such that x,y,$z \in \mathbb{Q}$ and there exist a, b, c such that $a \in x$ and $b \in y$ and $c \in z$ and $a \times b = c$. Precedence: 20.

Definition FS.5.27: 0 is the coset of (0,0) with respect to $\{(x, y) : x \equiv y\}$.

Definition FS.5.28: 1 is the coset of (1,0) with respect to $\{(x, y) : x \equiv y\}$.

Definition FS.5.29: $x > y$ if and only if $y < x$.

Definition FS.5.30: $x \leq y$ if and only if $x < y$ or $x = y$.

Definition FS.5.31: $x \geq y$ if and only if $x > y$ or $x = y$.

Definition FS.5.32: $x - y = (!z)x = y + z$. Precedence: 60.

Definition FS.5.33: $|x|$ is the unique $y \in \mathbb{Q}$ such that if $x \geq 0$ then $y = x$ and if $x < 0$ then $y = 0 - x$.

Definition FS.5.35: $\mathbb{N}$ is the unique x such that for every y, $y \in x$ if and only if $y = 0$ or $y > 0$ and $y - 1 \in x$.

Definition FS.5.35.A: $\mathbb{N} = \mathbb{N}$.

Definition FS.5.36: $\mathbb{Z}$ is the unique x such that for every y, $y \in x$ if and only if $y \in \mathbb{N}$ or $0 - y \in \mathbb{N}$.

Definition FS.5.36.A: $\mathbb{Z} = \mathbb{Z}$.

Definition FS.5.37: *The set of all sequences of rational numbers* is the set of maps from ω to $\mathbb{Q}$.

Definition FS.5.38: $x + y$ is the unique z such that x, y, z are in the set of all sequences of rational numbers and for every $n \in \omega$, $z(n) = x(n) + y(n)$. Precedence: 40.

Definition FS.5.39: $x \times y$ is the unique z such that x, y, z are in the set of all sequences of rational numbers and for every $n \in \omega$, $z(n) = x(n) \times y(n)$. Precedence: 20.

Definition FS.5.40: $x < y$ if and only if x,$y \in \omega$ and $x < y$.

Definition FS.5.41: $x > y$ if and only if x,$y \in \omega$ and $x > y$.

Definition FS.5.42: $x \leq y$ if and only if $x < y$ or $x = y$.

Definition FS.5.43: $x \geq y$ if and only if $x > y$ or $x = y$.

Definition FS.5.44: *The set of Cauchy sequences of rational numbers* is the set of x in the set of all sequences of rational numbers such that for every $\varepsilon > 0$, there exists $n \in \omega$ such that for every m, $r > n$, $|x(m) - x(r)| < \varepsilon$.

Definition FS.5.45: $x \equiv y$ if and only if for every $\varepsilon > 0$, there exists $n \in \omega$ such that for every $m > n$, $|x(m) - y(m)| < \varepsilon$.

Definition FS.5.46: $x < y$ if and only if x, y are in the set of Cauchy sequences of rational numbers and there exists $\delta > 0$ such that there exists $n \in \omega$ such that for every $m > n$, $x(m) + \delta < y(m)$.

Definition FS.5.47: $\mathbb{R}$ is the set of the coset of x with respect to $\{(u, v) : u \equiv v\}$ such that x is in the set of Cauchy sequences of rational numbers.

Definition FS.5.48.1: $x < y$ if and only if there exist z, w such that x,y $\in \mathbb{R}$ and $z \in x$ and $w \in y$ and $z < w$.

Definition FS.5.48.2: $x > y$ if and only if $y < x$.

Definition FS.5.48.3: $x \leq y$ if and only if $x < y$ or $x = y$.

Definition FS.5.48.4: $x \geq y$ if and only if $x > y$ or $x = y$.

Definition FS.5.49: $x + y$ is the unique z such that x,y,$z \in \mathbb{R}$ and there exist a, b, c such that $a \in x$ and $b \in y$ and $c \in z$ and $a + b \equiv c$. Precedence: 40.

Definition FS.5.49.A: $x - y = (!z)x = y + z$. Precedence: 60.

Definition FS.5.50: $x \times y$ is the unique z such that x,y,$z \in \mathbb{R}$ and there exist a, b, c such that $a \in x$ and $b \in y$ and $c \in z$ and $a \times b \equiv c$. Precedence: 20.

Definition FS.5.51: 0 is the unique $x \in \mathbb{R}$ such that there exists $w \in x$ such that for every $n \in \omega$, $w(n) = 0$.

Definition FS.5.52: 1 is the unique $x \in \mathbb{R}$ such that there exists $w \in x$ such that for every $n \in \omega$, $w(n) = 1$.

Definition FS.5.53: $|x|$ is the unique $y \in \mathbb{R}$ such that if $x \geq 0$ then $y = x$ and if $x < 0$ then $y = 0 - x$.

Definition FS.5.53.A: *The identity function on* $\mathbb{R}$ is the unique $y \in \mathbb{R}$ such that there exists $w \in y$ such that for every $n \in \omega$, $w(n) = x$.

Definition FS.5.53.B: If x is in the set Nra then x is the identity function on $\mathbb{R}$.

Definition FS.5.53.C: If x is in the set of positive fractions then x is the identity function on $\mathbb{R}$.

Definition FS.5.54: $\mathbb{Q}$ is the set of the identity function on $\mathbb{R}$ such that $x \in \mathbb{Q}$.

Definition FS.5.55: *The set of sequences of real numbers* is the set of maps from ω to $\mathbb{R}$.

Definition FS.5.56: *The set of Cauchy sequences of real numbers* is the set of x in the set of sequences of real numbers such that for every $\varepsilon > 0$, there exists $n \in \omega$ such that for every m, $r > n$, $|x(m) - x(r)| < \varepsilon$.

Definition FS.5.57: If x is in the set of sequences of real numbers then $\lim x$ is the unique $y \in \mathbb{R}$ such that for every $\varepsilon > 0$, there exists $n \in \omega$ such that for every $m > n$, $|x(m) - y| < \varepsilon$.

Definition FS.5.58: x is an *upper bound* on A if and only if $x \in \mathbb{R}$ and $A \subseteq \mathbb{R}$ and for every $y \in A$, $y \leq x$.

Definition FS.5.59: If $A \subseteq \mathbb{R}$ then *the minimal element of* A is the unique $x \in A$ such that for every $y \in A$, it is not the case that $y < x$.

Definition FS.5.59.5: If $A \subseteq \mathbb{R}$ then *the maximal element of* A is the unique $x \in A$ such that for every $y \in A$, it is not the case that $x < y$.

Definition FS.5.60: If $A \subseteq \mathbb{R}$ then *the least upper bound of* A is the minimal element of the set of x such that x is an upper bound on A.

Definition FS.5.61.pre: If there exists $n \in \omega$ such that f is a function from n to $\mathbb{R}$ then *the graph of the finite sum function* is the unique x such that for every $m \in \omega$, if $m < n$ then $x(0) = 0$ and x, evaluated at the successor of m equals $x(m)$ plus f, evaluated at the successor of m and if $m \geq n$ then x, evaluated at the successor of m equals 0 and for every m, $x(m)$ is defined if and only if $m \in \omega$.

Definition FS.5.61: If there exists $n \in \omega$ such that f is a function from n to $\mathbb{R}$ then $\sum_{k \in Dom(f)} f(k)$ is the unique $r \in \mathbb{R}$ such that (the domain of f,r) is in the graph of the finite sum function.

Definition FS.5.62: If $r \in \mathbb{R}$ then $\sqrt{r}$ is the unique $y \in \mathbb{R}$ such that $y \geq 0$ and $y \times y = r$.

Definition FS.5.63: $\sup A$ is the unique s such that s is a supremum for A, under $\{(x, y) : x < y\}$.

Definition FS.5.64: $\inf A$ is the unique g such that g is an infimum for A, under $\{(x, y) : x < y\}$.