Definition MunkTop.20.1: d is a *metric* on X if and only if d is in the set of maps from $X \times X$ to $\mathbb{R}$ and for every x, $y \in X$, $d(x,y) \geq 0$ and $d(x,y) = 0$ if and only if $x = y$ and for every x, $y \in X$, $d(x,y) = d(y,x)$ and for every x, y, $z \in X$, $d(x,y) + d(y,z) \geq d(x,z)$.

Definition MunkTop.20.2: If d is a metric on X and $x \in X$ and $\varepsilon > 0$ then $B(\varepsilon, x) = \{y \in X : d(x, y) < \varepsilon\}$.

Definition MunkTop.20.3: If d is a metric on X then *the basis for the metric topology on X induced by d* is the set of $B(\varepsilon, x)$ such that $x \in X$ and $\varepsilon > 0$.

Definition MunkTop.20.4: If d is a metric on X then *the metric topology on X induced by d* is the topology on X generated by the basis for the metric topology on X induced by d.

Definition MunkTop.20.5: If (X, T) is a topological space then (X, T) is *metrizable* if and only if there exists d such that T equals the metric topology on X induced by d.

Definition MunkTop.20.6: If (X, T) is a topological space then X is a *metric space* if and only if T equals the metric topology on X induced by d.

Definition MunkTop.20.7: If X is a metric space and $A \subseteq X$ then A is *bounded* in X under d if and only if there exists $M \in \mathbb{R}$ such that for every a_1, $a_2 \in A$, $d(a_1,a_2) \leq M$.

Definition MunkTop.20.8: If X is a metric space and $A \subseteq X$ and $A \neq \varnothing$ and A is bounded in X under d then *the diameter of A* is the unique s such that s is a supremum for $\{d(a_1, a_2) : a_1, a_2 \in A\}$, under $\{(x, y) : x < y\}$.