

**Definition MunkTop.13.1:**  $\mathcal{B}$  is a *basis* for a topology on  $X$  if and only if  $\mathcal{B} \subseteq \wp(X)$  and for every  $x \in X$ , there exists  $B \in \mathcal{B}$  such that  $x \in B$  and for every  $x \in X$ , for every  $B_1, B_2 \in \mathcal{B}$ , if  $x \in B_1 \cap B_2$  then there exists  $B_3 \in \mathcal{B}$  such that  $x \in B_3$  and  $B_3 \subseteq B_1 \cap B_2$ .

**Definition MunkTop.13.2:** If  $\mathcal{B}$  is a basis for a topology on  $X$  then *the topology on  $X$  generated by  $\mathcal{B}$*  is the unique  $\mathcal{T} \subseteq \wp(X)$  such that for every  $U \subseteq X$ ,  $U \in \mathcal{T}$  if and only if for every  $x \in U$ , there exists  $B \in \mathcal{B}$  such that  $x \in B$  and  $B \subseteq U$ .

**Definition MunkTop.13.3.a.basis:** *The standard basis for a topology on  $\mathbb{R}$*  is the set of  $U \subseteq \mathbb{R}$  such that there exist  $a, b \in \mathbb{R}$  such that  $U = \{x \in \mathbb{R} : a < x < b\}$ .

**Definition MunkTop.13.3.a:** *The standard topology on  $\mathbb{R}$*  is the topology on  $\mathbb{R}$  generated by the standard basis for a topology on  $\mathbb{R}$ .

**Definition MunkTop.13.3.b:** *The lower-limit topology on  $\mathbb{R}$*  is the topology on  $\mathbb{R}$  generated by the set of  $U \subseteq \mathbb{R}$  such that there exist  $a, b \in \mathbb{R}$  such that  $U = \{x \in \mathbb{R} : a \leq x < b\}$ .

**Definition MunkTop.13.3.c:** *The  $K$ -topology on  $\mathbb{R}$*  is the topology on  $\mathbb{R}$  generated by the standard basis for a topology on  $\mathbb{R}$  union the set of  $V \subseteq \mathbb{R}$  such that there exists  $W$  in the standard basis for a topology on  $\mathbb{R}$  such that  $V = W \setminus \{1/n : n \in \mathbb{N}\}$ .

**Definition MunkTop.13.4.a:**  $\mathcal{S}$  is a *subbasis* for a topology on  $X$  if and only if  $\mathcal{S} \subseteq \wp(X)$  and  $\cup \mathcal{S} = X$ .

**Definition MunkTop.13.4.b:** *The topology on  $X$  generated by the subbasis  $\mathcal{S}$*  is the set of  $U \subseteq X$  such that there exists  $\mathcal{A} \subseteq \wp(\mathcal{S})$  such that for every  $A \in \mathcal{A}$ ,  $A$  is finite and  $U = \cup \{\cap S : S \in A\}$ .