A Structural Model of Mental Accounting

Nicholas Pretnar[1], Alan Montgomery[2], and Christopher Olivola[3]

[1] *Economics PhD Student, Carnegie Mellon University*
[2] *Associate Professor of Marketing, Carnegie Mellon University*
[3] *Assistant Professor of Marketing, Carnegie Mellon University*

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Abstract

We develop a formal economic model of mental accounting. Consumers infrequently update their desired expenditure shares in finitely-many consumption categories. Dynamic consumption decisions are affected by consumers' most recent spending habits relative to their pre-determined, implicit mental account budgets. Our model shows that consumers can exhibit loss aversion with respect to over- and underspending relative to a pre-set mental account budget. This is a result of mental accounting inducing lower or higher levels of savings than in the consumption model without mental accounting. Our future work will use the framework presented here to estimate the frequency with which individuals update their mental accounts using transaction-level expenditure data.

1 Introduction

A continuing source of rigorous and healthy debate between neoclassical and behavioral economists involves the precision with which each side's respective models of decision making explain human behavior. The expected utility maximization formulation is supported by decades of analysis on its theoretical and empirical properties. The class of reference dependent utility functions, while not as widely analyzed and tested, can capture certain behavioral properties that have been observed across the social sciences. In this paper we specify an expected utility optimization problem with reference-dependence to test whether individual consumers engage in reference-dependent mental accounting with regards to consumption.

The main contribution of this current draft is to specify a model that captures key elements of the mental accounting process within an expected utility framework. Since much of the research involving expected utility optimization and consumption has taken place on the aggregate level, our modeling specification is informed by both the mental accounting and household consumption literature. We proceed as follows: Section 2 places our ambitions within the context of the behavioral and neoclassical literature, Section 3 outlines the formal theoretical model of mental accounting within an expected utility framework, Section 4 presents different calibrations of the model in 3. In Section 4.2 we show how our model features loss-aversion without having to specify a referential variable directly in the utility function. Section 4.3 features simulations demonstrating the relationship between mental and accounting and price and income responsiveness. Section 5 concludes and outlines additional components of the model to examine moving forward.

2 Background

Research by behavioral economists and psychologists has found that consumers partition their expenditure and savings into mutually exclusive so-called "mental accounts," which correspond to personally relevant categories of consumption [26, 28, 15, 21, 22, 10]. The theory of mental accounting suggests that individuals form an expected budget and try to stick to that budget, though keeping expenditure in line with a pre-set budget is costly [28, 15]. Thaler [27] summarizes the mental accounting process as that of ex-ante and ex-post cost-benefit analysis. Individuals must decide how much to devote to accounts ex-ante. They then engage in consumption and investment where they decide if the cost or benefit of overspending or underspending is worth the additional benefit or cost of consuming or investing in particular products. In this context, mental accounting can be thought of as a self-control mechanism. In an earlier paper, Shefrin and Thaler [24] specify a model that characterizes life-time consumption as a conflict between competing forces within an individual's psyche — those of the "planner" and the "doer." The planner plans for future spending by maximizing total discounted lifetime utility, while the doer earns labor income and consumes in the present. The planner infrequently re-evaluates investments for future consumption. Shefrin and Thaler [24] show that by infrequently re-evaluating consumption and savings allocations, consumption expenditure comes out of labor income more than accrued assets. Our model uses the definition of mental accounting from Thaler [27] and the underlying premise of the model in Shefrin and Thaler [24] as a springboard to describe the short-run consumption process.

In Shefrin and Thaler [24], invested wealth and labor income are not equally fungible from the perspective of the consumer regardless of how liquid the assets are. This is a key facet of mental accounting theory which departs from neo-classical representations of consumer behavior: the marginal utility of wealth varies explicitly across asset classes, and as a corollary, implicitly between different consumption categories for which consumers form different mental budgets. For example, Table 1 shows a consumer's budget for weekly expenditure on groceries, gasoline, food away from home, and entertainment.

Table 1: A Hypothetical Budget

Gasoline	\$40
Groceries	\$75
Food Away From Home	\$30
Entertainment	\$30

Now consider the following expenditure dynamics for the consumer: on day 1 the consumer purchases a tank of gasoline which costs \$35 and groceries which cost \$70. Thus, for the week he has \$5 remaining in each of his mental accounts for gasoline and groceries. Suppose now after day 5 he has run out of food at home and is almost out of gas, but he has not eaten out this week. Instead of purchasing more gas and driving to the grocery store to stock up for the next 5 to 6 days worth of food, the consumer walks down the street to a local restaurant and spends \$20 out to eat on a meal that would have cost him only \$5 if he purchased it at the grocery store. Under the strong assumption that time costs are equal for walking down the street and driving to the grocery store and assuming the consumer gets no additional pleasure from eating out (these are admittedly strong assumptions), the consumer's decision to eat out versus get more groceries can be explained entirely by mental accounting theory. First, the consumer is not treating different consumption categories as equally fungible: under the assumption that the agent is indifferent between driving to the grocery store and eating out, under standard utility maximization subject to a single budget constraint, the consumer is better off driving to the grocery store than eating out. Under mental accounting however, the consumer is loss averse to overspending in any of his mental accounts. Coupled with the fact that the remaining budgeted amounts in different categories are not equally fungible, the consumer will not make the implicit transfer from the "food away from home" account to the "groceries" and "gasoline" accounts in order to achieve a higher utility level. We can model this phenomenon by partitionining the period t budget into different budget constraints for each consumption category, while including an implicit cognitive constraint that prevents the consumer from re-budgeting "on the fly," and thus effectively transferring implicit balances across mental accounts.

Here, the consumption decision is a function of the ex-ante budget the consumer set for himself last period. This budget is his implicit "reference level." In the mental accounting literature, reference levels are often dependent on prices or transactions rather than ex-ante real budgets. Prelec and Loewenstein [21] describe mental accounting within a reference dependent framework under which individuals receive two types of utility upon making a purchase: experiential utility associated with owning a product and transaction utility associated with purchasing the product above or below some pre-expected price. Rabin [23] and Koszegi and Rabin [17] provide more extensive treatments of the behavioral phenomena underscoring reference-dependent preferences, though in most cases they treat the reference level as some price or expenditure level. In a forthcoming paper, Baucells and Hwang [2] extend this convention to a model that treats reference variables as expenditure-based accounting values thus forcing utility to be unitized in terms of prices. We depart from this convention by treating the reference variable as an expenditure budget-share over a given period rather than a specific spot price or absolute budget. Our notion of utility thus takes a conventional, ordinal form that is necessary (as will be discussed later) to facilitate a model that can be estimated. Throughout the remainder of this paper, we use the terms "reference level," "budget," and "budget share" interchangeably when referring to the model we formulate in Section 3.

Mental accounting theory posits that the cognitive capacity required to re-evaluate and switch reference budgets may contribute to loss-aversion in the marketplace [5, 29]. There is evidence that individuals are myopic about switching their reference levels, so they choose these reference levels considering only near-term states of the world but not how such a switch will impact them in the long run [23, 27]. Frequently re-evaluating one's current financial position can also lead to loss-averse investment behavior [5, 29]. In consumption mental accounting, we would expect individuals who re-evaluate their current budgets more frequently to be averse to spending above a desired expenditure level. In this way, a structural model of mental accounting must capture asymmetry around the expenditure budget since individuals are more likely to change their budget when they overspend than when they underspend in a given period.

There is a close relationship between mental accounting and two-stage budgeting from classical consumer-demand theory. In two-stage budgeting, consumers set desired expenditure levels in particular consumption categories prior to engaging in actual consumption [14, 11]. Under two-stage budgeting, individuals first place consumption commodities into aggregated baskets, selecting the quantity to consume in those baskets in the second stage [11]. As long as all goods in the same commodity-class or basket have the same marginal rate of substitution between goods in other commodity baskets, a utility function defined over such a preference structure can be strongly (additively) separated [11]. For example, if a consumer's utility function is additively separated over the four categories described in Table 1 along with an additional outside category that collects all other commodities, then fixing prices and wealth, the rate at which a consumer substitutes consumption of "premium gasoline" with "whole milk" is equal to the rate at which the consumer substitutes "regular gasoline" with "whole milk" or "skim milk" or any other good in the "groceries" category. This assumption is needed to ensure the model can be tractably estimated. In Section 3 we specify a strongly separated utility function where the "goods" are different consumption basket aggregates – gasoline, groceries, restaurant food, etc. The quantity measurements we use are summed to the pay-period level. The agent's reference level corresponds to the ex-ante budget share he forms for expenditure in the corresponding accounts.

On the surface mental accounting with reference consumption may seem like just another version of habit formation; though the two modeling concepts have distinct features that cannot be directly reconciled with each other. First, mental accounting with reference dependence features a dual ex-ante/ex-post decision structure: individuals choose their reference consumption level or budget, realize the present state of the world, and then consume with that budget in mind [27]. Habit formation models feature consumers smoothing consumption over time due to some habit stock of consumption capital [4, 3, 1, 12]. Thus, habit formation models are merely extensions of the expected utility maximization framework designed to capture the smooth, hump-shaped response of real spending to different exogenous shocks [12]. But Campbell and Deaton [9] show that consumption exhibits excess smoothness with respect to exogenous shocks anyway, meaning the habit formation model may not adequately explain real-world behavior. Recently, Gelman et al. [13] found noticeable pay-day effects to the near-term consumption stream, meaning the consumption-smoothing imposed by a smoothly decaying habit stock would inadequately describe near-term behavior. If evidence suggests individuals fail to smooth their consumption over a matter of weeks, how does such evidence conflict with lifetime consumption smoothing as widely posited by the permanent income hypothesis? Incorporating mental accounting into the neo-classical consumer optimization problem may help to explain this near-term cyclical component of the consumption path, while also providing a pivot-point to look at broader, macro phenomena within a tractable, behavioral framework.

In the context of our model, allowing individuals to change their reference level amounts to allowing them to change their consumption capital or habit stock. A traditional habit formation model would thus be inadequate for what we are trying to explain, since it enforces a consumption-smoothing motive. Our model of mental accounting, with its foundations in the behavioral economic literature, constrains individuals to updating their budget shares infrequently, thus imposing stickiness to the consumption path that would otherwise be sub-optimal in a habit formation model. But why should economists even care about mental accounting with respect to consumption? Behavioral researchers employing mental accounting frameworks have found that the marginal propensity to consume from different liquidity sources is not constant [21, 22]. This calls into question an underlying premise of the neo-classical consumer optimization problems where consumption expenditure is constrained by a single budget rather than separate budgets for different goods categories [8, 25, 7, 6].

We introduce this feature of mental accounting into the neo-classical utility maximization problem by constraining the consumer to maximizing his utility subject to separate budget constraints for each separate category. In our model consumers ex-ante choose their budget shares for the different categories. To accomodate the stickiness in changes to reference levels posited in mental accounting theory, we introduce a constraint on the number of budget shares the consumer can re-evaluate and thus update each period. By constraining the consumer to re-evaluate only a few of his mental account budgets each period, we implicitly assume that the consumer is cognitively constrained to process infinitesimal changes to his desired expenditure levels. This is consistent with findings by Lieder, Griffiths, and Goodman [19] that people face limits as to how much numerical information they can cognitively process at once. Imposition of an integer constraint on the number of changes that can be made rather than the absolute value of those changes is consistent with Leslie, Gelmand, and Gallistel [18] who argue that integer representations are innate within individual cognitive processes.

Our primary goal is to integrate mental accounting and cognitive-processing constraints into a rational expectations optimization problem. We choose to incorporate mental accounting and cognition constraints into a standard rational expectations problem, as opposed to a reference-price dependent model, so as to allow future work to exploit the rich literature on structural estimation of dynamic programming problems for "well-behaved" (reflexive, transitive, complete) preferences and "nice" utility functions (strictly quasiconcave, monotone, at least twice differentiable). Our presentation here is testament to the flexibility of the general, expected utility maximization problem, while providing an example of how observed behavioral phenomena can be incorporated into its framework.

3 Model

In the following outline, we suppress the agent-level index i. All variables and functions presented except prices $\boldsymbol{p}_t$, are agent-specific. We assume consumption categories are fixed and known across all individuals. This is a specific departure from previous structural decision models of mental accounting which featured accounts being open and closed for specific transactions (see Prelec and Loewenstein [21]).

Let $u_t(\boldsymbol{q}_t, z_t)$ be a strictly increasing and strictly concave utility function which takes as its arguments a J-dimensional vector of consumption categories, $\boldsymbol{q}_t$, and an outside good that represents cash, or t-period liquid savings, z_t.[1] Assume also that $u_t(\boldsymbol{q}_t, z_t)$ is at least twice-continuously differentiable in all arguments.

Let $\boldsymbol{\theta}_t$ be a $J+1$-dimensional vector of expenditure shares. R_{t-1} is the return on balance holdings B_{t-1}. L_{t-1} is income earned last period, and $M \geq 0$ is a cash borrowing limit. The components of $\boldsymbol{\theta}_t$ are indexed by j and are such that $0 \leq \theta_{jt} \leq (M + R_{t-1} \cdot B_{t-1})/L_{t-1}$, for $1 \leq j \leq J$ and $-(M + R_{t-1} \cdot B_{t-1})/L_{t-1} \leq \theta_{J+1,t} \leq 1$. We require:

$$\sum_{j=1}^{J+1} \theta_{jt} = 1 \tag{1}$$

The model is structured such that period $t-1$ income is available for period t expendi-

[1]Throughout this exposition we use bold face with lower case letters to denote vectors and bold face with upper case letters to denote matrices. Scalars are presented in a regular font style.

ture. We treat income in this fashion since individuals typically consume today using a paycheck they most recently received for wages earned one or two weeks ago. Using a reference level proportional to income also ensures that, all else equal, if income changes and the individual does not choose to make changes to his expenditure shares, the budgets increase proportionally as a response. This assumption is needed to disentangle income effects from mental account balance effects on consumption.

Note that $\boldsymbol{p}_t$ is exogenous to our model, as we assume consumers are price-takers. We denote $\boldsymbol{\zeta}_t$ as a J-dimensional vector of residual exogeneity such that the function $\boldsymbol{\zeta}_t(w_t)$ describes how the state of the world w_t impacts consumption each period, independent of price effects. We thus have $w_t \perp \boldsymbol{p}_t$ each period. w_t can be thought to represent factors in the consumer's immediate environment independent of prices which impact his consumption decision. In mental accounting, consumers form budgets to regulate their consumption. Unexpected changes in the state of the world thus move them off their ex-ante consumption path. For example, a football fan may plan to spend only $20 while out with friends at a bar watching his favorite NFL team, but if his team wins in dramatic fashion, he may exceed his $20 budget to celebrate. On the other hand, if the team loses dramatically, he may exceed his budget drowning his sorrows. Either way, the outcome of the football game — excessively dramatic win or loss — is an exogenous event in the world which acts directly on his consumption. Mental accounting allows him to dynamically update his desired consumption for period $t+1$ as a response to this unexpected event in period t. Since our model is specified with the consumer in mind rather than consumption aggregates and since the extent of exogenous events that affect the consumption path independent of prices is rather enormous, w_t can be thought to represent unmeasured individual heterogeneity that impacts the consumption decision. $\boldsymbol{\zeta}_t$ is described in more detail below.

Each period consumers face J consumption category expenditure constraints:

$$p_{jt}q_{jt} = (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} \quad \forall j \tag{2}$$

The variable $A_{j,t-1}$ in (2) is the over/under in the consumer's mental account for category j from the previous period. We call $A_{j,t-1}$ his "mental account balance." γ is a parameter that governs how going over or under last period's category j budget impacts this period's expenditure. γ does not directly impact the utility a consumer receives from consumption, but the actual amount he consumes. γ can be thought of as the degree to which mental accounting affects a consumer's decisions. When $\gamma = 0$, last period's over- or underexpenditure has no bearing on this period's consumption. The $\gamma = 0$ case directly resembles a multiple-commodity optimization problem without mental accounting but featuring taste shocks for each commodity class – ζ_{jt}.

In (2), ζ_{jt} is an independent and identically distributed multiplicative residual. In the full model, consumers choose θ_{jt} in period $t-1$, prior to realizing ζ_{jt}. Ex-post, a consumer is at par in his mental accounting if his expenditure $p_{jt}q_{jt}$ exactly corresponds to the share of income chosen ex-ante, $\theta_{jt}L_{t-1}$, plus however much he implicitly believes himself to have leftover from last period's expenditure, $\gamma A_{j,t-1}$. Here, γ determines the degree to which mental accounting affects the consumer's expenditure decisions. If $\gamma = 0$, then present consumption is not affected at all by what happened last period. We would typically think $\gamma \in [0,1]$. As $\gamma \to 1$, whether or not the consumer overspent or underspent last period weighs heavily on his current period's decisions, and vice-versa. We can thus think of $\gamma A_{j,t-1}$ as how much richer or poorer the consumer feels with respect to his category j consumption expenditure due to over- or underspending in the previous period. For consumers who overspent last period

$$p_{j,t-1}q_{j,t-1} > \theta_{j,t-1}L_{t-2} + \gamma A_{j,t-2} \tag{3}$$

which implies that their leftover mental budget from last period weighs negatively on this period's expenditure: $A_{j,t-1} < 0$. For consumers who underspent last period, flip the sign in (3) and $A_{j,t-1} > 0$.

Clearly there is a relationship between ζ_{jt} and A_{jt}. $\zeta_{jt}(w_t)$ is a function of some exogenous components orthogonal to prices which impact expenditure temporarily. For example, consider a college student who strictly budgets his beer intake. The week he graduates from college, he engages in excessive celebrations which correspond to heightened beer intake. Here, the impact of graduation on his beer consumption flows through ζ_{jt} which is clearly orthogonal to prices. If $\zeta_{jt} > 1$ then $p_{jt}q_{jt} > \theta_{jt}L_{t-1} + \gamma A_{j,t-1}$ since (2) must hold with equality. Thus, A_{jt} represents the difference between the expenditure budget given a neutral shock, $\zeta_{jt} = 1$, and the actual shock:

$$A_{jt} = \theta_{jt}L_{t-1} + \gamma A_{j,t-1} - p_{jt}q_{jt} \tag{4}$$

$$\Rightarrow \quad A_{jt} = (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})(1 - \zeta_{jt}) \tag{5}$$

(5) can be derived by substituting (2) for $p_{jt}q_{jt}$ in (4). $\zeta_{jt} > 1$ corresponds to $A_{jt} < 0$ and vice-versa.

Note that (5) *does not* require that $\mathbb{E}_t[\zeta_{jt}] = 1$. In fact, if consumers are loss averse and thus averse to going over their budget then $\mathbb{E}_t[\zeta_{jt}] < 1$, so that consumers are more likely to carry forward a positive mental account balance rather than a negative one. One could think of this as the implicit, psychological analog of the precautionary savings motive: consumers prefer to feel they underspent rather than overspent.[2] This implies that consumers expect $A_{jt} > 0$ each period. Further, we impose the natural restriction that $p_{jt}q_{jt} \geq 0$, which requires that $\zeta_{jt} \geq 0$ always. There are periods, however, when consumers may forego expenditure in a category altogether, which implies that $\mathbb{P}(\zeta_{jt} = 0) > 0$. Thus the distribution of ζ_{jt} must incorporate a point-mass at $\zeta_{jt} = 0$. In such periods consumers carry forward a mental account balance exactly equal to that with which they started the period so that when $\zeta_{jt} = 0$, $A_{jt} = \theta_{jt}L_{t-1} + \gamma A_{j,t-1}$.

A_{jt} is one of the mental accounting variables, though it is essentially an expenditure residual (see Equation (4)). θ_{jt}, the expenditure share, is the consumer's primary choice variable corresponding to his ex-ante budgeting process. Each period, the consumer can decide to change all of his expenditure allocations for period $t+1$, change only a few of them, or leave them all the same. The main idea behind mental accounting is just as it sounds: the consumer has a budget in his head with which he seeks to constrain his expenditure. Updating this budget is costly, requiring the consumer to evaluate his recent past expenditure and what he expects prices and the state of the world to be in the near future. Consumers who spend near their budget will not exert cognitive energy to re-evaluate their position unless they expect major changes in prices or the state of the world in upcoming periods. For this reason, we say that the expenditure share allocation decision is subject to a constraint on the number of changes k_t a consumer can make each period. When $k_t = J$, a consumer changes all of his expenditure shares $\theta_{j,t+1}$, $j \in \{1, \ldots, J+1\}$. When $k_t = 1$, a consumer changes only one category $j \in \{1, \ldots, J\}$ plus the share devoted to cash holding $\theta_{J+1,t+1}$. The consumer's cognition constraint, describing the number of changes to the vector $\boldsymbol{\theta}_{t+1}$ he can mentally process in a given period, can be expressed

$$0 \leq \sum_{j=1}^{J} \mathbf{1}\{\theta_{j,t+1} \neq \theta_{jt}\} \leq k_t \leq J \tag{6}$$

(6) says that the number of changes to his shares must be less than or equal to the integer k_t. Here, k_t can be thought of as a resource constraint, or an effort constraint. Changing expenditure shares is mentally costly, so that if the absolute value $|A_{jt}|$ is relatively small, a consumer would not waste precious cognitive resources and time figuring out how to reallocate his budget shares across the components of $\boldsymbol{\theta}_{t+1}$.

We can now fully characterize the consumer's optimization problem under mental

[2]We would like to thank Professor Stephen Spear for cleverly pointing this out.

accounting. Each period, consumers realize $\boldsymbol{p}_t$, $\boldsymbol{\zeta}_t$, and k_t, and must choose $\boldsymbol{q}_t$, z_t, and $\boldsymbol{\theta}_{t+1}$. In the current formulation, we assume that L_t (income) is pre-determined, though this can be relaxed. In the near-term this is not that strong of an assumption if we think of a contracted salary-worker who chooses neither his hours nor his wage rate. Denote the state variables $\boldsymbol{\nu}_t$. Each period the finite-lived consumer seeks to solve

$$\max_{\{\boldsymbol{q}_\tau, z_\tau, \boldsymbol{\theta}_{\tau+1}\}_{\tau=t}^{T}} u_t(\boldsymbol{q}_t, z_t) + \mathbb{E}_t\left[\sum_{\tau=t+1}^{T} \beta^{\tau-t} u_\tau(\boldsymbol{q}_\tau, z_\tau) \mid \boldsymbol{\nu}_t\right] \tag{7}$$

$$\text{subject to} \quad p_{jt}q_{jt} = (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} \quad \forall j, t \tag{8}$$

$$z_t = \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} - A_{J+1,t} \quad \forall t \tag{9}$$

$$-A_{J+1,t} = \sum_{j=1}^{J} A_{jt} \quad \forall t \tag{10}$$

$$\sum_{j=1}^{J+1} \theta_{jt} = 1 \quad \forall t \tag{11}$$

$$\begin{aligned} &0 \le \theta_{jt} \le \frac{M + R_{t-1} \cdot B_{t-1}}{L_{t-1}} \quad \forall j \in \{1, \ldots, J\} \quad \forall t \\ &\qquad\qquad \& \quad -\frac{M + R_{t-1} \cdot B_{t-1}}{L_{t-1}} \le \theta_{J+1,t} \le 1 \qquad \forall t \end{aligned} \tag{12}$$

$$\zeta_{jt} \sim G_j(\cdot) \quad \zeta_{jt} \perp \zeta_{it} \quad \forall j, i, t \tag{13}$$

$$z_t > -(M + R_{t-1} \cdot B_{t-1}) \quad \forall t \tag{14}$$

$$B_t = L_{t-1} + R_{t-1} \cdot B_{t-1} - \sum_{j=1}^{J} p_{jt}q_{jt} \quad \forall t \tag{15}$$

$$0 \le \sum_{j=1}^{J} \mathbf{1}\{\theta_{j,t+1} \neq \theta_{jt}\} \le k_t \le J \quad \forall t \tag{16}$$

3.1 Model Discussion

In Equation (7), notice that utility is a direct function of the demand for money balances z_t. In neo-classical macroeconomic growth models, one way to ensure that consumers wish to hold positive money balances is to provide them with utility for such holdings (see Walsh [30]). In Equation (13) we denote $G(\cdot)$ as any arbitrary probability distribution with positive support and point mass at 0. The set of equations (7) – (16) characterizing the mental accounting optimization problem contains several accounting inequalities and identities not previously discussed. (12) and (14) together ensure that consumers do not spend more than they can borrow plus what they already have in available liquidity, $R_{t-1} \cdot B_{t-1}$. (15) describes how their liquidity balances evolve over time. R_{t-1} is a gross nominal interest rate on balances. Additionally, note that the optimization problem is over choice variables which include $\boldsymbol{\theta}_{t+1}$, next period's expenditure shares. The constraint, (12), ensures that consumers cannot ex-ante choose to set their proportional budget allocations with the desire to spend more than they have available to them in balances B_{t-1} and borrowing M.

Equation (13) imposes independence across consumption categories for ζ_{jt}. Note that z_t is linearly dependent on the choice of $\boldsymbol{q}_t$, so that the consumer chooses z_t as a consequence of choosing $\boldsymbol{q}_t$. Note that $A_{J+1,t}$ can be written as a linear function of a constant and the components of $\boldsymbol{\zeta}_t$. Since each ζ_{jt} is a function of q_{jt}, we can rewrite the system in (8) — (10) as a linearly independent system of J equations with J exogenous unknowns, ζ_{jt}.

Equation (10) is a necessary condition on the mental account balances to ensure that consumers do not ever implicitly think that they have more available liquidity than they actually do. (10) also ensures that summing the expenditure constraints in (8) and (9) over j gives us back a version of the traditional neo-classical budget constraint, illustrated as follows:

$$\sum_{j=1}^{J} p_{jt}q_{jt} + z_t = \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} - A_{J+1,t} \tag{17}$$

Plug in (10) for $-A_{J+1,t}$ and use the identity in (5) to replace every instance of A_{jt}:

$$\begin{aligned}\sum_{j=1}^{J} p_{jt}q_{jt} + z_t &= \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} \\ &\qquad + \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})(1 - \zeta_{jt}) \qquad (18) \\ &= \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1}) + \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} \qquad (19) \\ \Leftrightarrow \quad \sum_{j=1}^{J} p_{jt}q_{jt} + z_t &= \sum_{j=1}^{J+1} \theta_{jt}L_{t-1} = L_{t-1} \qquad (20)\end{aligned}$$

The jump from (19) to (20) comes after replacing $A_{J+1,t-1}$ with $-\sum_{j=1}^{J} A_{j,t-1}$ and canceling. (20) says that if you add up all expenditure on consumption goods $\boldsymbol{q}_t$ and outside cash z_t in any given period, you must get back last period's nominal income. This is because z_t is *not* "savings" in the neo-classical sense, but marginal savings from each period.

Moving z_t to the right side of (20) and substituting into (15) we have

$$B_t = R_{t-1} \cdot B_{t-1} + z_t \tag{21}$$

(21) says that the balances at the end of period t are equal to last period's appreciated balances plus whatever income is leftover after consumption this period, z_t. Note that if $z_t < 0$, then $p_{jt}q_{jt} > L_{t-1}$, so the consumer spent more than he earned. Also note that in the extreme case where $\sum_{j=1}^{J} p_{jt}q_{jt} = M + R_{t-1} \cdot B_{t-1}$, then $z_t = -M - R_{t-1} \cdot B_{t-1}$, which implies that the balance at the end of the period is $B_t = -M$. The consumer has thus hit his credit limit. We do not assume that this credit limit M binds, however. With the wide availability of high-interest financial instruments like payday loans, consumers feasibly always have access to more credit. We simply assume that this credit limit is finite but very large.

While these accounting restrictions may seem obvious and trivial, they are handy for comparing how our mental accounting consumer optimization problem differs from the neo-classical problem with only a single period-specific budget constraint. Given the restrictions we imposed on u_t — strict monotonicity, strict concavity, and at least twice-continuous differentiability — we can use the summed expenditure constraint in (20) to write a version of the single-period neo-classical problem:

$$\begin{aligned} &\max_{\boldsymbol{q}_t, z_t} u_t \\ \text{subject to} \quad &\sum_{j=1}^{J} p_{jt}q_{jt} + z_t \leq L_{t-1} \end{aligned} \tag{22}$$

As long as $p_{jt} >> 0$, (22) has a solution under the assumptions imposed on u_t [20]. Denote this solution $(\overline{\boldsymbol{q}}_t, \overline{z}_t)$. Then $(\overline{\boldsymbol{q}}_t, \overline{z}_t)$ satisfies (20). This leads to a nice theorem.

Theorem 1: Let $(\overline{\boldsymbol{q}}_t, \overline{z}_t)$ solve (22), and let $(\widetilde{\boldsymbol{q}}_t, \widetilde{z}_t)$ be a solution to the mental accounting problem characterized by (7) – (16). Then $(\overline{\boldsymbol{q}}_t, \overline{z}_t) \succeq (\widetilde{\boldsymbol{q}}_t, \widetilde{z}_t)$.

Proof. Fix p_{jt}, $\forall j$. Since u_t is strictly increasing and strictly concave, then u_t represents well-ordered preferences. Suppose for contradiction $(\overline{\boldsymbol{q}}_t, \overline{z}_t) \prec (\widetilde{\boldsymbol{q}}_t, \widetilde{z}_t)$. Given the properties of u_t it must be the case that

$$\sum_{j=1}^{J} p_{jt}\widetilde{q}_{jt} + \widetilde{z}_t > L_{t-1}$$

which implies that

$$\sum_{j=1}^{J} p_{jt}\widetilde{q}_{jt} + \widetilde{z}_t > \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})(1 - \zeta_{jt})$$

Without loss of generality, assume $z_t = \theta_{J+1,t}L_{t-1} + \gamma A_{J+1,t-1} - A_{J+1,t} = 0$. Then $\exists n \in \{1, \ldots, J\}$ such that $\forall i \neq n$:

$$\begin{aligned} p_{nt}\widetilde{q}_{nt} &= \sum_{j=1}^{J} p_{jt}\widetilde{q}_{jt} - \sum_{i \neq n} p_{it}\widetilde{q}_{it} \\ &> \sum_{j=1}^{J} (\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} - \sum_{i \neq n} (\theta_{it}L_{t-1} + \gamma A_{i,t-1})\zeta_{it} \\ \Rightarrow \quad p_{nt}\widetilde{q}_{nt} &> (\theta_{nt}L_{t-1} + \gamma A_{n,t-1})\zeta_{nt} \end{aligned}$$

which contradicts (8). It follows that $(\overline{\boldsymbol{q}}_t, \overline{z}_t) \succeq (\widetilde{\boldsymbol{q}}_t, \widetilde{z}_t)$. □

Theorem 1 says that the neo-classical consumer optimization problem nests the mental accounting problem. That is, consumers cannot achieve higher utility levels by engaging in mental accounting than they otherwise could if they did not constrain their consumption decisions to implicit mental budgets. This is a direct consequence of the fact that u_t is strictly increasing, and $\boldsymbol{q}_t$ are choices over normal goods.

4 Calibrations and Simulations

4.1 Utility Function Assumptions

To demonstrate some stylized results of the model, we need additional assumptions on preferences. First we assume that J, the number of mental accounts, is sufficiently small. The theory behind mental accounting breaks down if consumers have a separate account for each different commodity they purchase, regardless of the substitutability or complimentarity between the different commodities. We must be able to specify a utility function that accomodates grouped commodity baskets. Ideally, we need an additively separable utility function that accomodates the commodity-class separability implicit in two-stage budgeting [14].

For our different commodity classes to be valid we need the following to hold. Suppose there are J categories and suppose the consumer forms an expenditure budget (mental account) in each category. Let j be a category over a certain class of goods and i be another category. Let a and a' be commodities in category j and b and b' be commodities in category i. Denote the marginal rate of substitution between two goods x and y as $MRS(x, y)$. For an additively separable utility function to accurately describe consumer preferences we must have:

$$MRS(a, b) = MRS(a', b) = MRS(a, b') = MRS(a', b') \tag{23}$$

Thus, the marginal rate of substitution for different commodities in different categories must be identical [11]. Note that we require nothing of $MRS(a, a')$, the marginal rate of substitution for goods within categories. This condition implies that the preference ordering for goods between groups j and i must hold for all potential pairs of goods. For example fixing wealth and prices, if $a \succeq b$, then it must be that $a' \succeq b$, $a \succeq b'$, and $a' \succeq b'$. Separability is violated if any of the aforementioned relations fail.

Our model needs two-stage budgeting to hold so that we may treat the expenditure constraints as subset demand functions. Deaton and Muellbauer [11] show that conditional (subset) demand functions will fully characterize cross-sectional demand if and only if the utility function is strongly separable. A subset demand function describes the demand for quantity q_{jt} as a function only of group-specific expenditures x_{jt} and a group price-level p_{jt}:

$$q_{jt} = f(x_{jt}, p_{jt}) \tag{24}$$

The expenditure constraints in (8) fit the definition of a subset demand function pending specification of u_t.

For u_t we choose a flexible, additively separable form that allows for 0 consumption and also incorporates a period-specific utility for holding cash balances, z_t. This is not the same as utility from saving, which could lead to double-counting the utility an individual receives from consumption if he consumes more than he earns in income. In our formulation, "savings" are essentially bank balances B_t, while z_t is the amount of cash from income not spent at the end of the period. Further, we choose a utility function that does not contribute to overall utility when $q_{jt} = 0$ for some t. Thus, we want $u_{jt}(0) = 0$. The category j utility function we specify follows from Kim, Allenby, and Rossi [16]:

$$u_{jt}(q_{jt}) = \alpha_j \ln(q_{jt} + 1) \tag{25}$$

The full, period t utility function is:

$$u_t = \sum_{j=1}^{J} \alpha_j \ln(q_{jt} + 1) + \alpha_{J+1} \ln(z_t + M + R_{t-1} \cdot B_{t-1}) \tag{26}$$

(26) is well-defined for all $q_{jt} \geq 0$ and all $z_t > -(M + R_{t-1} \cdot B_{t-1})$. (26) says that as total consumption expenditure $\sum_{j=1}^{J} p_{jt} q_{jt}$ approaches the borrowing limit plus available balances $M + R_{t-1} \cdot B_{t-1}$, $z_t \to -(M + R_{t-1} \cdot B_{t-1})$ and $u_t \to -\infty$. The latter limit is driven by the log-utility in z_t. This utility function thus ensures that consumers are averse to exhausting all of their potential available credit and account balances for one period's consumption expenditure.

Equation (26) satisfies the two-stage budgeting restrictions. u_t is strictly increasing and strictly concave in each argument, with strictly convex upper contour sets, and therefore the expenditure constraints all hold with equality. Equation (7) with parameterization (26) admits the subset demand specification. Each period, the optimal choice of q_{jt} is

$$q_{jt}^{*} = \frac{(\theta_{jt} L_{t-1} + \gamma A_{j,t-1}) \zeta_{jt}}{p_{jt}} \quad \forall j, t \tag{27}$$

Plugging (27) back into (25) results in the period t, category j indirect utility function

$$\psi_{jt} = \alpha_j \ln \left(\frac{(\theta_{jt} L_{t-1} + \gamma A_{j,t-1}) \zeta_{jt}}{p_{jt}} + 1 \right) \tag{28}$$

For the cash category $J + 1$, we plug in the constraint from (9) and get

$$\psi_{J+1,t} = \alpha_{J+1} \ln \left(\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} - A_{J+1,t} + M + R_{t-1} \cdot B_{t-1} \right) \tag{29}$$

$$= \alpha_{J+1} \ln \left(\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} A_{jt} + M + R_{t-1} \cdot B_{t-1} \right) \tag{30}$$

$$= \alpha_{J+1} \ln \left(\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} (\theta_{jt} L_{t-1} + \gamma A_{j,t-1})(1 - \zeta_{jt}) + M + R_{t-1} \cdot B_{t-1} \right) \tag{31}$$

Equations (30) and (31) follow from substituting in (10) and (5) respectively. Looking one-period ahead, the full period $t+1$, expected conditional indirect utility function is

$$\beta\,\mathbb{E}_t[\psi_{t+1}\mid \boldsymbol{v}_t] = \beta\sum_{j=1}^{J}\alpha_j\mathbb{E}_t\left[\ln\left(\frac{(\theta_{j,t+1}L_t+\gamma A_{jt})\zeta_{j,t+1}}{p_{j,t+1}}+1\right)\,\Big|\,\boldsymbol{\nu}_t\right]$$
$$+\beta\alpha_{J+1}\mathbb{E}_t\left[\ln\left(\theta_{J+1,t+1}L_t+\gamma A_{J+1,t}+\sum_{j=1}^{J}(\theta_{j,t+1}L_t+\gamma A_{j,t})(1-\zeta_{j,t+1})+M+R_t\cdot B_t\right)\,\Big|\,\boldsymbol{\nu}_t\right] \tag{32}$$

By substituting optimal demand into (26), we can characterize the choice of $\boldsymbol{\theta}_{t+1}$ from the full optimization problem in (7) – (16) as an indirect utility maximization problem:

$$\max_{\{\theta_{\tau+1}\}_{\tau=t}^{T}}\ \sum_{\tau=t+1}^{T}\beta^{\tau-t}\mathbb{E}_t[\psi_{t+1}\mid \boldsymbol{v}_t] \tag{33}$$

$$\text{subject to}\quad \sum_{j=1}^{J+1}\theta_{j,t+1}=1\quad\forall t \tag{34}$$

$$0\le\theta_{j,t+1}\le\frac{M+R_t\cdot B_t}{L_t}\quad\forall j\in\{1,\ldots,J\}\quad\forall t$$
$$\&\quad -\frac{M+R_t\cdot B_t}{L_t}\le\theta_{J+1,t+1}\le 1\qquad\forall t \tag{35}$$

$$0\le\sum_{j=1}^{J}\mathbf{1}\{\theta_{j,t+1}\neq\theta_{jt}\}\le k_t\le J\quad\forall t \tag{36}$$

Returning to the definition of ψ_{jt} in (28), note that

$$\frac{\partial\psi_{jt}}{\partial\theta_{jt}}=\frac{\alpha_j L_{t-1}\zeta_{jt}}{(\theta_{jt}L_{t-1}+\gamma A_{j,t-1})\zeta_{jt}+p_{jt}}>0 \tag{37}$$

$$\frac{\partial^2\psi_{jt}}{\partial\theta_{jt}^2}=-\frac{\alpha_j L_{t-1}^2\zeta_{jt}^2}{\left[(\theta_{jt}L_{t-1}+\gamma A_{j,t-1})\zeta_{jt}+p_{jt}\right]^2}<0 \tag{38}$$

Thus ψ_{jt} is increasing and strictly concave in θ_{jt}. Also notice that $\psi_{J+1,t}$ in (31) is a function of θ_{jt} and

$$\frac{\partial \psi_{J+1,t}}{\partial \theta_{jt}} = \frac{\alpha_{J+1} L_{t-1}(1-\zeta_{jt})}{\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} (\theta_{jt} L_{t-1} + \gamma A_{j,t-1})(1-\zeta_{jt}) + M + R_{t-1} \cdot B_{t-1}} \tag{39}$$

$$\frac{\partial^2 \psi_{J+1,t}}{\partial \theta_{jt}^2} = -\frac{\alpha_{J+1} L_{t-1}^2 (1-\zeta_{jt})^2}{\left[\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} (\theta_{jt} L_{t-1} + \gamma A_{j,t-1})(1-\zeta_{jt}) + M + R_{t-1} \cdot B_{t-1}\right]^2} \tag{40}$$

If $\zeta_{jt} < 1$, (39) is greater than 0. (40) is less than 0 for all values of ζ_{jt}, ensuring $\psi_{J+1,t}$ is strictly concave in θ_{jt}. We need to show that for all ζ_{jt}

$$\frac{\partial \psi_{jt}}{\partial \theta_{jt}} + \frac{\partial \psi_{J+1,t}}{\partial \theta_{jt}} > 0 \tag{41}$$

(41) clearly holds for $\zeta_{jt} < 1$. Note that the denominator of (39) is greater than 0:

$$\begin{aligned} &\theta_{J+1,t} L_{t-1} + \gamma A_{J+1,t-1} + \sum_{j=1}^{J} (\theta_{jt} L_{t-1} + \gamma A_{j,t-1})(1-\zeta_{jt}) + M + R_{t-1} \cdot B_{t-1} \\ &= L_{t-1} - \sum_{j=1}^{J} (\theta_{jt} L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + M + R_{t-1} \cdot B_{t-1} \\ &= B_t + M > 0 \end{aligned} \tag{42}$$

Meanwhile, the denominator of (37) is equivalent to

$$p_{jt} q_{jt} + p_{jt} < B_t + M + p_{jt} \tag{43}$$

As long as q_{jt} is not extremely high, i.e. $q_{jt} < (B_t + M - p_{jt})/p_{jt}$, Equation (39) is less in absolute value than Equation (37). Thus for reasonable values of q_{jt} (i.e., individuals do not choose to consume all of their balances and credit limit on one consumption category in one period):

$$\begin{aligned} \frac{\partial \psi_{jt}}{\partial \theta_{jt}} &> \left| \frac{\partial \psi_{J+1,t}}{\partial \theta_{jt}} \right| \quad \forall \zeta_{jt} > 1 \\ \Rightarrow \quad \frac{\partial \psi_{jt}}{\partial \theta_{jt}} &+ \frac{\partial \psi_{J+1,t}}{\partial \theta_{jt}} > 0 \end{aligned} \tag{44}$$

From (44) it follows that the full indirect utility function ψ_t is increasing and concave in θ_{jt}. Therefore, the expected indirect utility optimization problem characterized by (33) – (36) has a unique solution.

When $k_t \geq J$, (36) is non-binding. Note that when $k_t = J$, the consumer can make exactly $J + 1$ changes to his expenditure shares since we always allow him to change his share devoted to the cash good z_t without cognitive effort. This ensures that if $k_t = 1$, (34) will hold after he changes one consumption category. When $k_t \geq J$, the consumer solves (33) subject to Equations (34) and (35). The Lagrangian for this problem is thus a smooth function under the parameterization in (32). Since $\mathbb{E}_t[\psi_{t+1} \mid \boldsymbol{\nu}_t]$ is increasing and concave in all arguments, it follows that the first-order conditions fully characterize the equilibrium choice of $\boldsymbol{\theta}_{t+1}$ when $k_t = J$.

We do not present the first-order conditions here. The effects of mental accounting on the consumer expenditure path are not all that interesting when $k_t = J$. If consumers re-evaluate and update their budget shares each period, then all variation in the consumption path is driven solely by exogenous changes in prices and consumption shocks. While the $k_t = J$ case closely resembles the neo-classical utility optimization problem, the stochasticity in the model still operates separately on each consumption category, so the marginal utility of wealth in each consumption category varies exogenously, and thus consumption categories themselves are still not equally fungible. In this case, Theorem 1 would still apply.

For the $k_t = J$ case, the choice of $\boldsymbol{\theta}_{t+1}$ is characterized by analytically tractable first-order conditions that follow directly from (37) and (39) and solving for the optimal values of $\theta_{j,t+1}$ using (34). In most periods mental accounting imposes a stickiness in desired expenditure allocations which means that consumers make $k_t < J$ changes. The Lagrangian for the problem where (36) binds is not differentiable since (36) is not continuous. The equilibrium choice $\boldsymbol{\theta}^*_{t+1}$ must yield higher expected indirect utility than all other choices of $\boldsymbol{\theta}_{t+1}$:

$$\mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^*_{t+1}) \mid \boldsymbol{\nu}_t\Big] \geq \mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \quad \forall \boldsymbol{\theta}_{t+1} \neq \boldsymbol{\theta}^*_{t+1} \tag{45}$$

When $k_t < J$, we solve for $\boldsymbol{\theta}^*_{t+1}$ algorithmically. Let $\theta^*_{j,t+1}$ be the j^{th} component of $\boldsymbol{\theta}^*_{t+1}$. Consumers choose $\theta^*_{j,t+1} \neq \theta_{jt}$ if and only if

$$\mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^*_{t+1}) \mid \boldsymbol{\nu}_t\Big] \geq \mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}_t) \mid \boldsymbol{\nu}_t\Big] \tag{46}$$

If $k_t > 1$, consumers must decide whether to make 0 changes, 1 change, 2 changes, etc., up to k_t changes. For example, if $k_t = 2$, it may yield higher utility for consumers to make only one change. Let $\boldsymbol{\theta}^{2*}_{t+1}$ be the choice of expenditure shares that yields the highest indirect utility level making exactly 2 changes, so that $\exists i, j \in \{1, \ldots, J\}$ with $i \neq j$ such that $\theta^{2*}_{i,t+1} \neq \theta_{it}$ and $\theta^{2*}_{j,t+1} \neq \theta_{jt}$ and $\forall n \neq i, j$, $\theta^{2*}_{n,t+1} = \theta_{nt}$. Then

$$\mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^{2*}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \geq \mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^{2}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \quad \forall \boldsymbol{\theta}^{2}_{t+1} \neq \boldsymbol{\theta}^{2*}_{t+1} \tag{47}$$

(47) says that $\boldsymbol{\theta}^{2*}_{t+1}$ is the best choice of expenditure shares a consumer can make if he makes *exactly* 2 changes. This choice may still be suboptimal, however. Let $\boldsymbol{\theta}^{1*}_{t+1}$ be the choice of expenditure shares that yields the highest indirect utility level making exactly 1 change. When $k_t = 2$, the consumer chooses $\boldsymbol{\theta}^{2*}_{t+1}$ if and only if

$$\begin{aligned} &\mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^{2*}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \geq \mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^{1^*}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \\ \text{and} \quad &\mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}^{2*}_{t+1}) \mid \boldsymbol{\nu}_t\Big] \geq \mathbb{E}_t\Big[\psi_{t+1}(\boldsymbol{\theta}_t) \mid \boldsymbol{\nu}_t\Big] \end{aligned} \tag{48}$$

We merely extend the solution algorithm when $2 < k_t < J$, and the logic characterizing the equilibrium choice of $\boldsymbol{\theta}^{*}_{t+1}$ is the same.

4.2 Loss Aversion in the Model

We simulate the model for $T = 35$ periods of consumption over $J = 3$ categories plus the oustide cash good to examine how choices of $\boldsymbol{q}_t$ and $\boldsymbol{\theta}_{t+1}$ are affected by the mental account balances on hand $A_{j,t-1}$, $\forall j$, exogenous shocks $\boldsymbol{\zeta}_t$, and price changes $\boldsymbol{p}_t$. We also examine how the presence of the mental accounting variables affects price responsiveness. In the simulations that follow, we allow prices $\boldsymbol{p}_t$ and exogenous consumption shocks $\boldsymbol{\zeta}_t$ to vary randomly. We show how the consumption path varies under different parameterizations with different frequencies of budget share changes λ_k. We also demonstrate how price- and income-responsiveness are impacted by mental accounting.

We make the following parametric specifications. Denote $\ln \boldsymbol{p}_t$ as the vector of logged-prices. We allow the natural logarithm of prices to follow a VAR(1) process:

$$\begin{aligned} \ln \boldsymbol{p}_t &= \boldsymbol{\mu}_{\boldsymbol{p}} + \boldsymbol{\Gamma}_{\boldsymbol{p}} \cdot \ln \boldsymbol{p}_{t-1} + \boldsymbol{\eta}_t \\ \boldsymbol{\eta}_t &\sim MN\Big(\boldsymbol{0}, \boldsymbol{\Sigma}_{\boldsymbol{\eta}}\Big) \end{aligned} \tag{49}$$

(49) says that $\boldsymbol{\eta}_t$ follows a multivariate normal distribution. In all simulations, we treat

income $L = 100$ as fixed and known.[3] The borrowing limit is $M = 1000$. We use the utility function in (26) imposing constant returns to utility from goods consumption ($\sum_{j=1}^{J} \alpha_j = 1$) and setting the parameter on cash holdings $\alpha_{J+1} >> 0$. It turns out that α_{J+1} is an extremely important parameter describing the relationship between mental accounting and instantaneous utility from savings. For fixed income and balances, higher α_{J+1} induce higher savings levels, but this may be suboptimal if the consumer frequently updates his budget shares in the consumption categories. α_{J+1} thus implicitly governs the degree of loss aversion exhibited by the consumer, which is discussed in more detail later in this section.

First though, note that MU_z, the marginal utility from cash holdings, is such that if $M >> 0$ and $B_{t-1} >> 0$:

$$MU_z = \frac{\alpha_{J+1}}{z_t + M + R_{t-1} \cdot B_{t-1}} \approx 0 \tag{50}$$

which implies that α_{J+1} is proportional to MU_z multiplied by end-of-period cash balances. Since MU_z is diminishing due to the concavity of $\ln(\cdot)$, α_{J+1} is an increasing function of the borrowing limit M, available balances B_{t-1}, and the interest rate R_{t-1}. The model thus says that α_{J+1} is increasing in an individual's wealth. Thus, wealthy individuals will have extremely high α_{J+1} while less wealthy individuals who hold less cash will have a low α_{J+1}.

In our simulations we give ζ_{jt} a log-normal mixture distribution to accomodate the potential for zero-expenditure in the categories:

$$\begin{aligned} \zeta_{jt} &\sim \left[\pi_{\omega_j}\omega_{jt} + (1-\pi_{\omega_j})(1-\omega_{jt}) \cdot \xi_{jt}\right] \\ \omega_{jt} &\in \{0,1\} \\ \xi_{jt} &\sim LN\left(\overline{\mu}_{\xi_j}, \overline{\sigma}_{\xi_j}\right) \end{aligned} \tag{51}$$

ζ_{jt} is thus a mixture between 0 and a log-normal *iid* random variable ξ_{jt} with location parameter μ_{ξ_j} and scale parameter σ_{ξ_j}. π_{ω_j} is the probability of zero expenditure in category j and ω_{jt} is a zero-expenditure binary variable. We assume $\omega_{jt} \perp \omega_{i,t+\tau}$, $\forall i, j$ and $\forall \tau \in \{0, \ldots, T-t\}$. To facilitate the simulations, we set starting values $\boldsymbol{\theta}_0$ by setting initial prices to $\boldsymbol{p}_0 = e^{\mu_p}$. Table 2 lists the full parameterization we use throughout the simulations:

[3] Future work should examine the mechanics of the model with stochastic income variation, though this would likely add considerably to computation time.

Table 2: Full Simulation Parameters

Parameter	Value	Description
β	0.95	Utility discounting
γ	0.5	Mental acct. balance discounting
α_1	0.3	Utility parm. $j = 1$
α_2	0.4	Utility parm. $j = 2$
α_3	0.3	Utility parm. $j = 3$
α_4	14 or 9	Utility parm. z_t
$\overline{\mu}_{\xi_j}$	-0.14	Location parm. $\forall \xi_j$
$\overline{\sigma}_{\xi_j}$	0.1	Scale parm. $\forall \xi_j$
π_{ω_1}	0.1	Prob. $\zeta_{1t} = 0$
π_{ω_2}	0.02	Prob. $\zeta_{2t} = 0$
π_{ω_3}	0.05	Prob. $\zeta_{3t} = 0$
μ_{p_1}	1	Location parm. η_1
μ_{p_2}	1.01	Location parm. η_2
μ_{p_3}	1	Location parm. η_3
$\boldsymbol{\Gamma_p}$	(see below)	Coef. matrix for price VAR(1)
$\boldsymbol{\Sigma_\eta}$	(see below)	Var/Cov for $\boldsymbol{\eta}_t$
L	100	Income (fixed)
M	1000	Borrowing limit

For the log-price VAR(1) we set $\boldsymbol{\Gamma_p}$ and $\boldsymbol{\Sigma_\eta}$ to allow for positive co-movement of prices across time:

$$\boldsymbol{\Gamma_p} = \begin{pmatrix} 0.2 & 0 & 0.02 \\ 0.2 & 0.03 & 0.1 \\ 0.2 & 0.03 & 0.1 \end{pmatrix}$$

$$\boldsymbol{\Sigma_\eta} = \begin{pmatrix} 0.001 & 0 & 0.001 \\ 0 & 0.004 & 0.001 \\ 0.001 & 0.001 & 0.02 \end{pmatrix}$$

For illustration, it helps to first understand how the dynamic variables evolve when the consumer updates all of his budget shares every period, $k_t = J$, $\forall t$, verses what happens when he is allowed to make just one change each period, $k_t \leq 1$, $\forall t$. Note that $j \in \{1, 2, 3\}$ are consumption categories and $j = 4$ is the cash holdings category. In Figures 1 and 2, the bottom-most trend line is the cash category which changes each time the consumer makes a change to one of his consumption category budgets. In Figure 2 notice that we give the consumer the option of making a change $k_t = 1$ each period, but the consumer only makes the change if a higher utility level can be achieved by doing so. For periods $t = 14 - 18$ in Figure 2, the consumer is better off by sticking to his current budget in all categories than he would be if he made just one change to any of the categories.

Figure 1: Budget share dynamics for $k_t = J = 3, \forall t$.

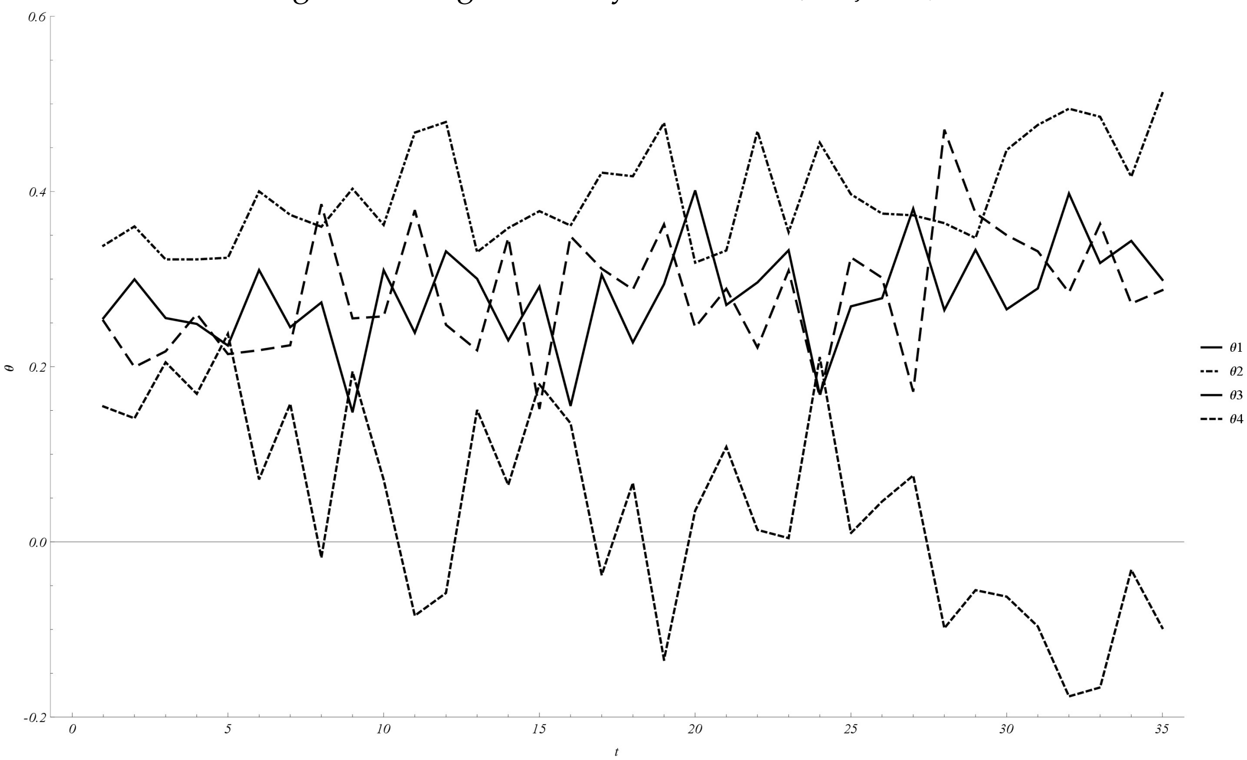
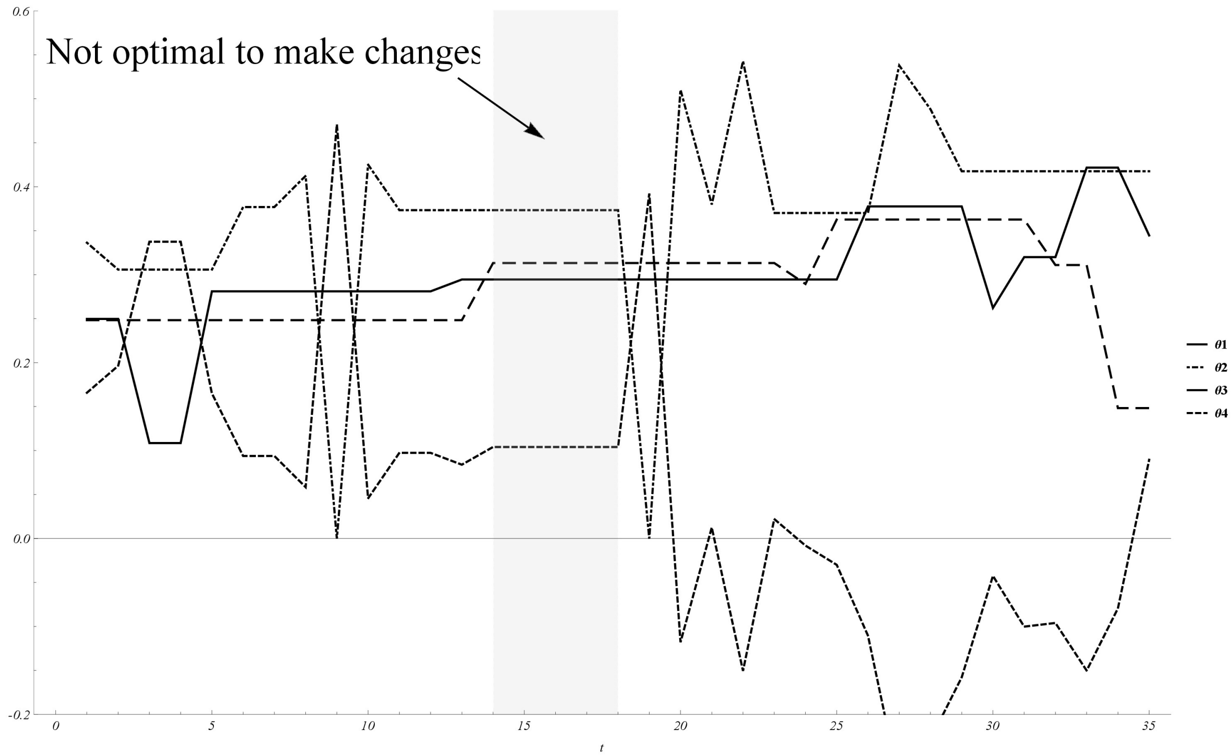


Figure 2: Budget share dynamics for $k_t = 1, \forall t$.



We want the quantity consumed q_t and the mental account balance carried forward from last period $A_{j,t-1}$ to co-move negatively. For example, suppose a consumer budgets \$30 for himself in gasoline in period t, but he spends \$50. Then he carries forward a balance of $A_t = -20$. Next period, he either must consume less or make up the difference by allowing himself to consume more out of income, adjusting $\theta_{j,t+1}$ upward. Figure 3 illustrates this case over periods $t = 17 - 20$, showing the co-movement of consumption and the mental account balance when $k_t = 0$. Figure 4 shows consumption and mental account balances under the same shock process, but when the consumer is allowed to make $k_t = J = 3$ changes each period. The consumer is updating his consumption shares dynamically, so that q_{jt} in Figure 4 is a dynamic function of θ_{jt}, chosen ex-ante and $A_{j,t-1}$. In Figure 3, since L_t is fixed $\forall t$ and θ_{jt} is the same $\forall t$, consumption dynamics act solely through $A_{j,t-1}$. After overspending, the mental account balance only recovers when next period's consumption expenditure is buffered. When the consumer is allowed to update his expenditure shares frequently — $k_t = J = 3$, for example — $\theta_{j,t+1}$ moves negatively with A_t as demonstrated in Figure 5.

Figure 3: Co-movement of q_{jt} and A_{jt} when $k_t = 0$.

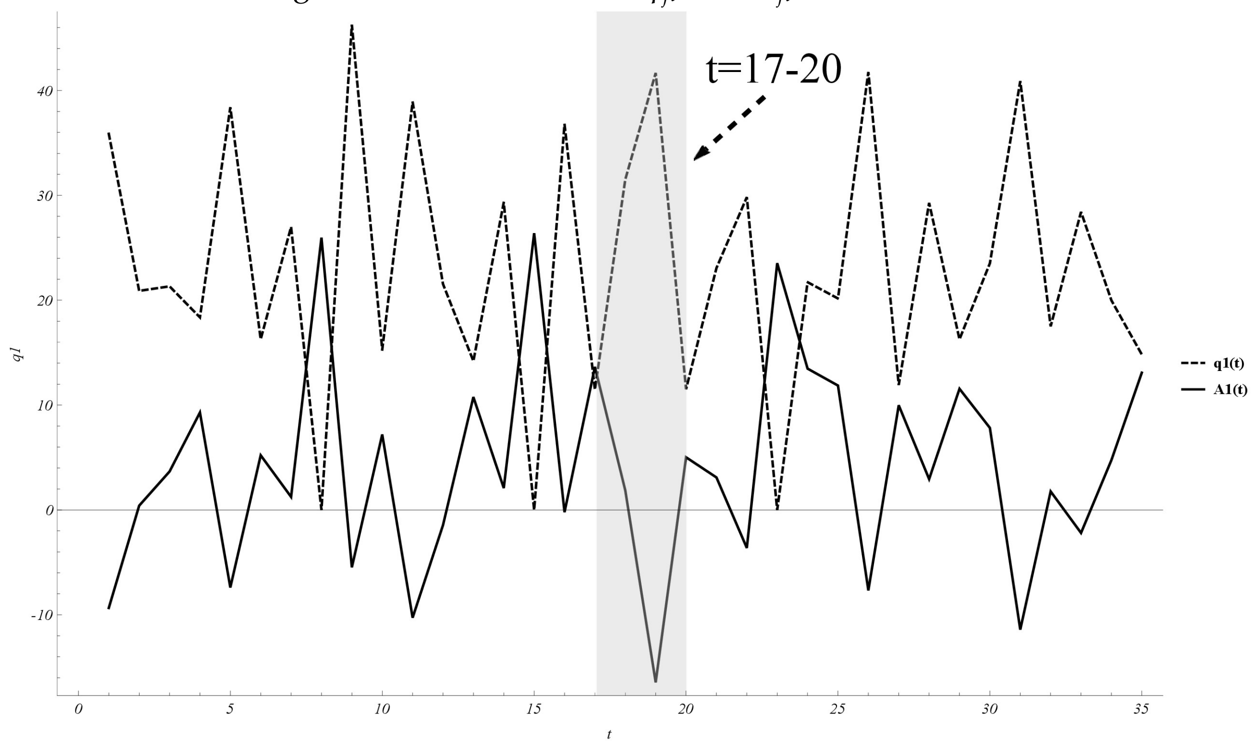


Figure 4: Co-movement of q_{jt} and A_{jt} when $k_t = J = 3$.

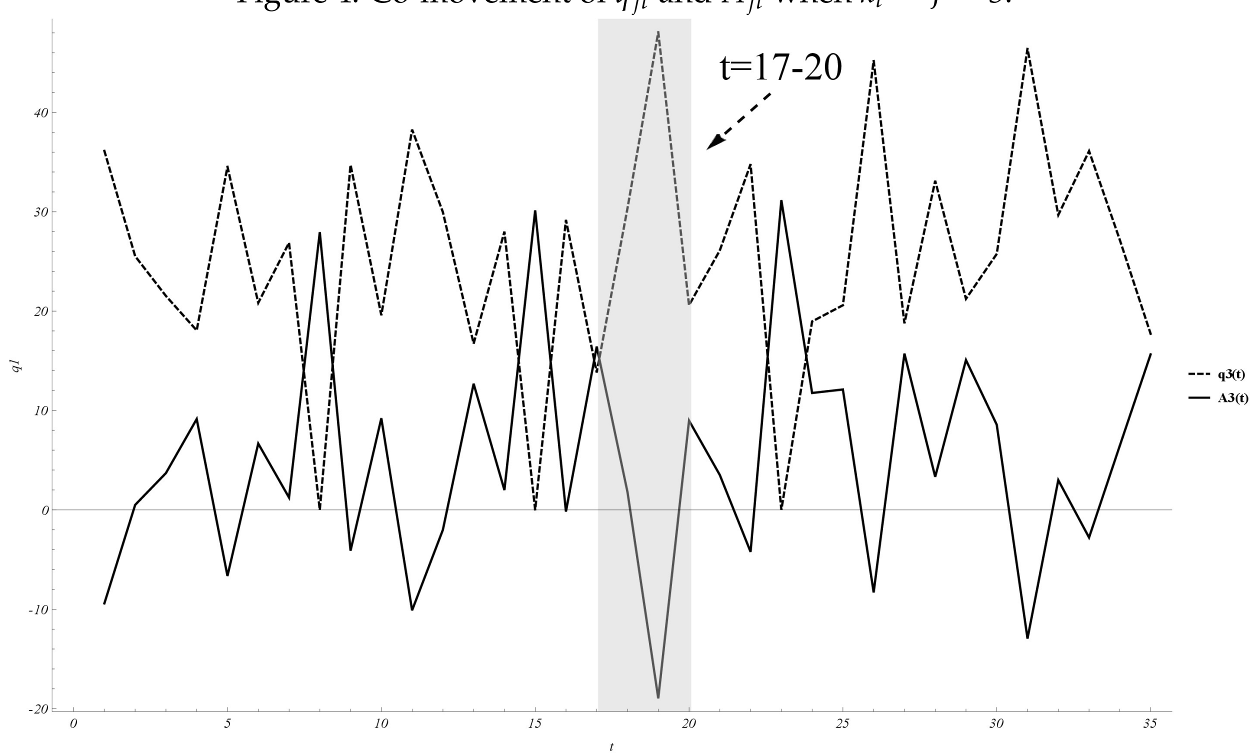
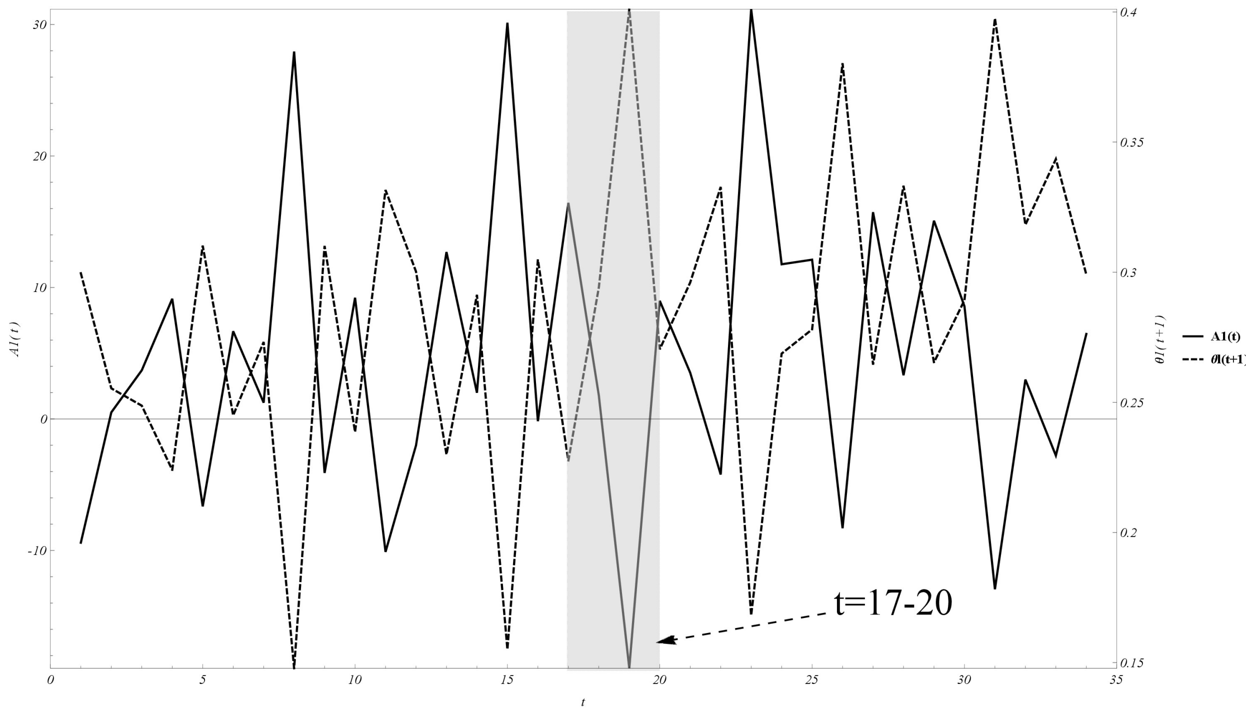


Figure 5: Co-movement of θ_{jt} and A_{jt} when $k_t = J = 3$.



Turn now to Figure 6. Here, we plot the consumption paths for a single good under three different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J$. Notice the consumption paths are relatively similar under each regime. In our model, mental accounting impacts the overall welfare of the individual through his savings process, or demand for balance holdings, z_t. Shefrin and Thaler [24] discuss how the process of mental accounting can act as a self-control mechanism that people use to ensure they invest a sufficient portion of their income for future use. In our model, this is the equivalent of keeping $\theta_{J+1,t}$ relatively high to ensure that balances accrue rather than dissipate over time. Whether or not individuals prefer to accumulate balances or consume out of pre-existing balances depends on the value of α_{J+1}, the parameter governing the degree to which utility from cash holdings contributes to the overall welfare of the consumer. Figures 7 and 8 show that when $k_t = 3 = J$ and $\alpha_4 = 14$, the consumer will consume more out of his balance holdings if given the opportunity to frequently change his expenditure shares. In Figure 9, we show how period-specific utility u_t evolves over time as the consumer is allowed to make $k_t = 0$, $k_t = 1$, and $k_t = J$ changes for each t. The solid line on the bottom corresponds to the $k_t = J$ case, so consumers are always updating their expenditure shares each period which adversely affects their overall utility since they are explicitly transfering funds from their balances to consumption. Figure 10 plots the $u_{J+1,t}(z_t)$ additively-separated component of the utility function, while Figure 11 plots the sum of the consumption-good utilities. Clearly, $u_{J+1,t}(z_t)$ is adversely weighing on the overall welfare of the individual, not $\sum_{j=1}^{J} u_{jt}(q_{jt})$.

Figure 6: Consumption quantities for a single good under different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J = 3$.

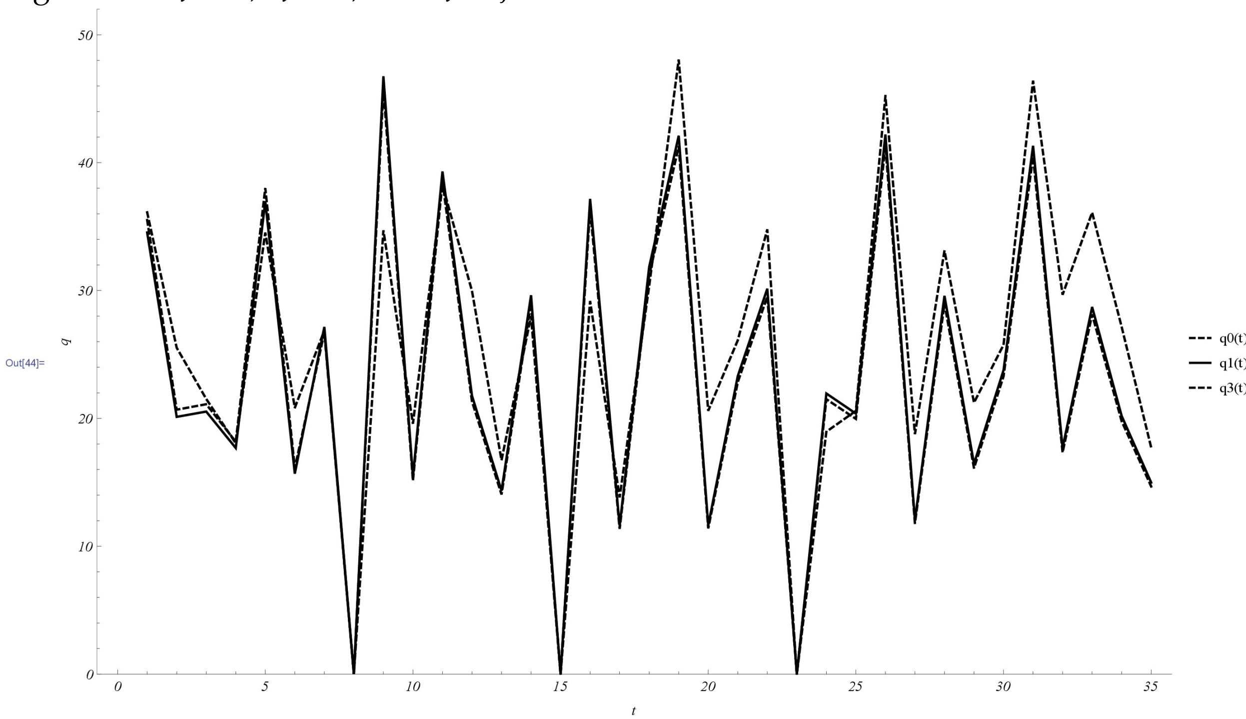


Figure 7: Demand for cash z_t under different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J = 3$.

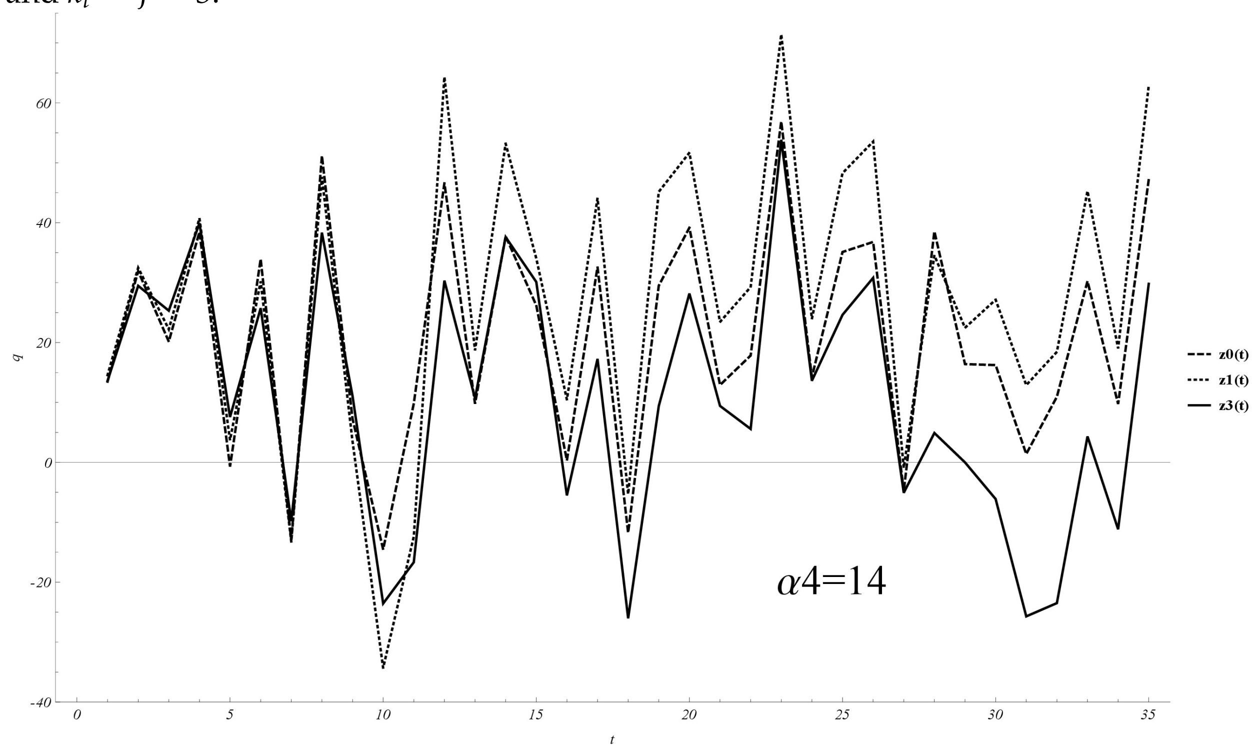


Figure 8: Balance evolution under different frequencies of budget share updating.

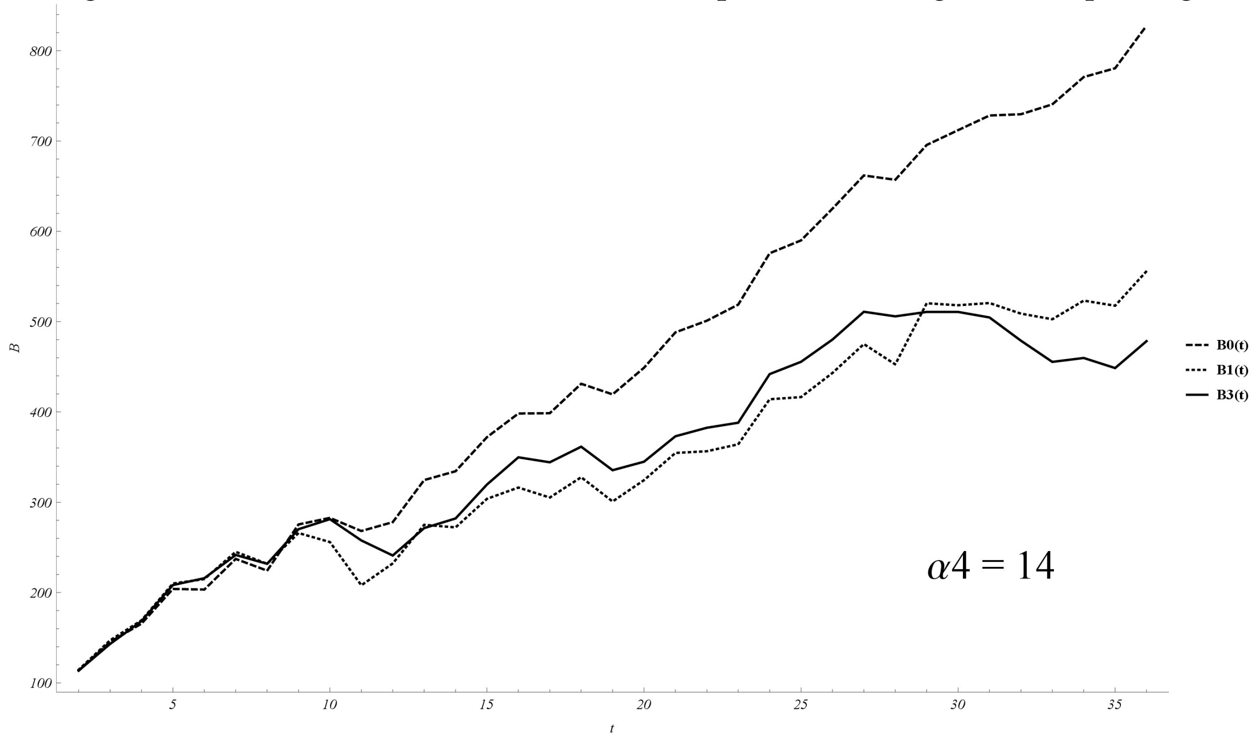


Figure 9: $u_0(t)$ is period t utility under consumption with $k_t = 0$, $u_1(t)$ with $k_t = 1$, and $u_3(t)$ with $k_t = 3 = J$.

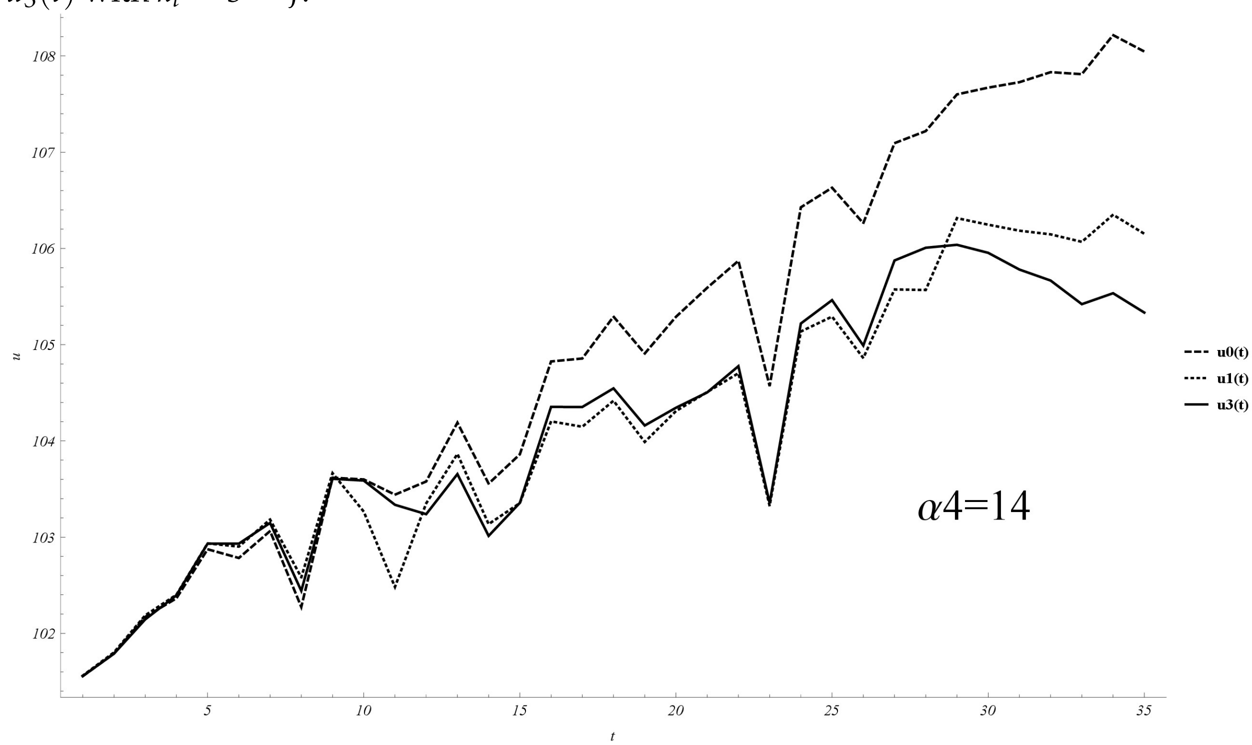


Figure 10: Utility from cash balances under different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J = 3$.

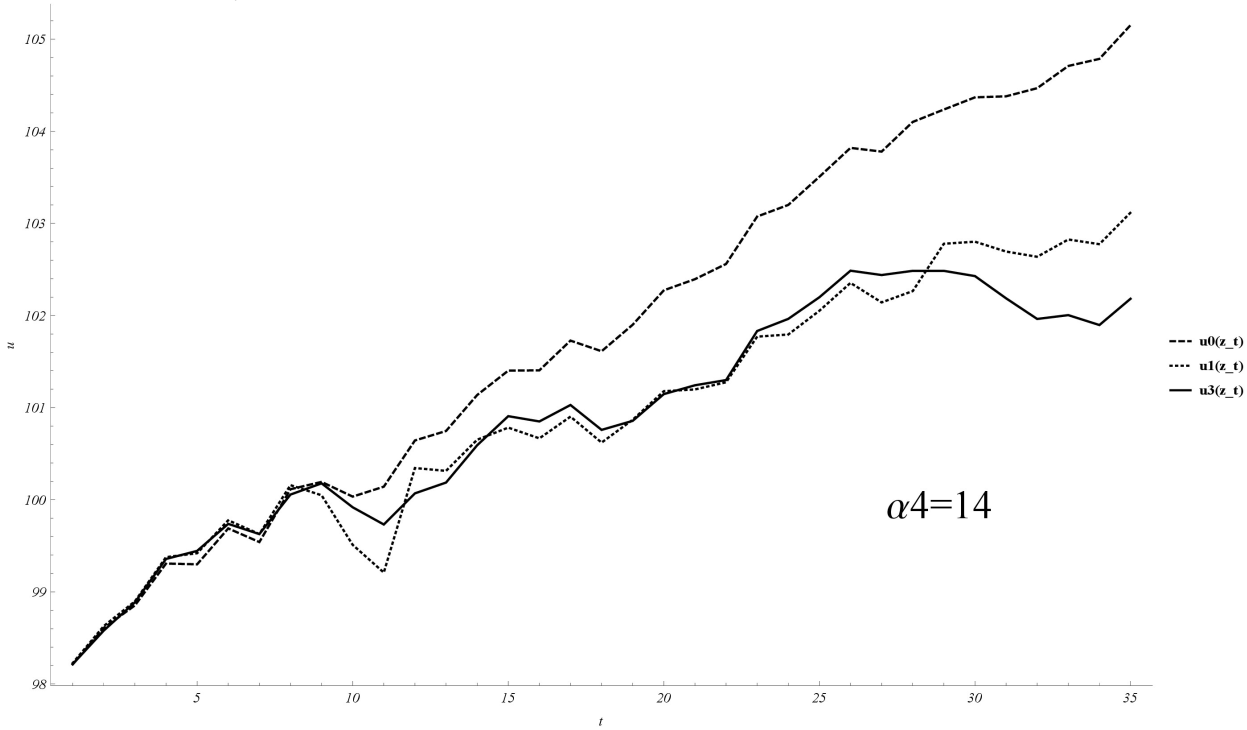
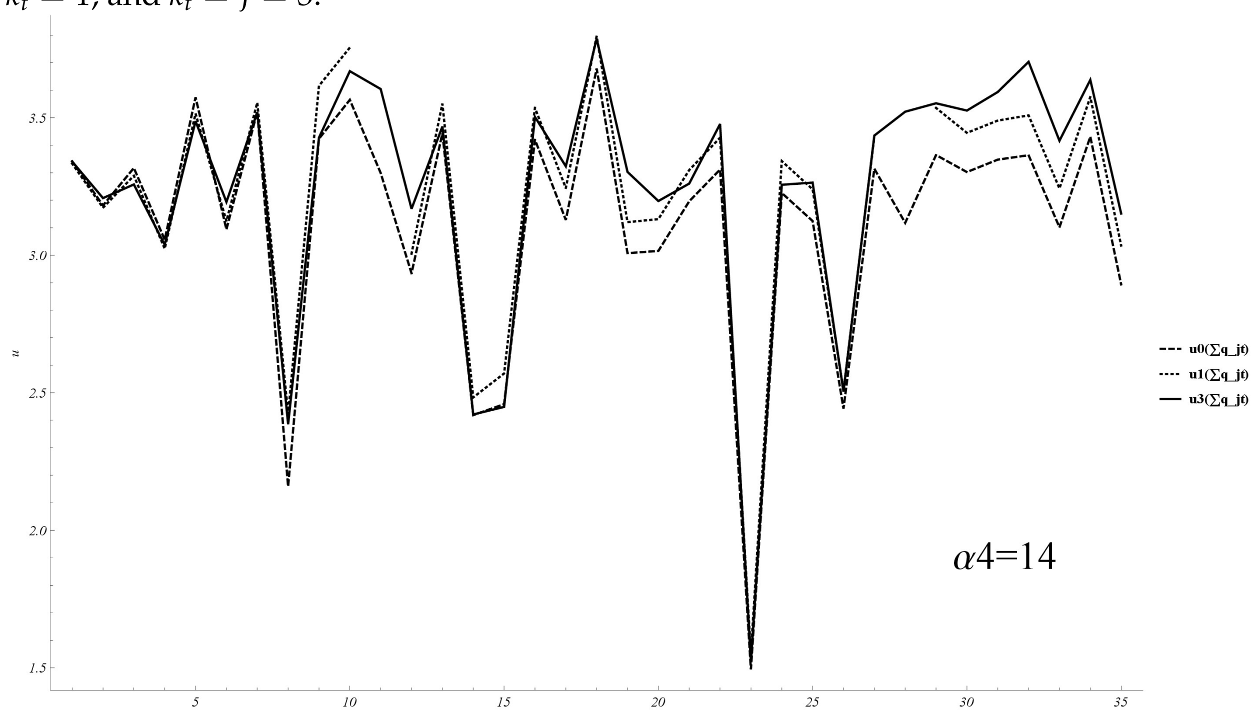


Figure 11: Utility from consumption under different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J = 3$.



We used the same shock process to run a simulated example with $\alpha_{J+1} = \alpha_4 = 9$ and all other parameters the same as before to show that the value of α_{J+1} is critical to the balance holding dynamics, B_t. With $\alpha_4 = 9$, the consumer is a perpetual borrower with negative balances after 35 periods. In this case, mental accounting helps the consumer regulate his borrowing and debt spending. Figure 12 shows that when $k_t = 3$ (the solid line on top) the consumer borrows less than he does if he cannot update his budget shares frequently. Essentially, as the consumer engages in excess consumption, he can update his expenditure shares to make sure that he does not engage in *too much* excess consumption in subsequent periods. Figure 13 shows how the consumer adjusts the proportion of his income he plans to devote to cash holdings upward whenever balances are negative. In contrast with the $\alpha_4 = 14$ case, utility for $k_t = J = 3$ is higher overall than when the consumer can make no changes. As we see in Figures 14 and 15, overall utility and utility from cash holdings is higher when the consumer updates budget shares more frequently, under this parameterization.

In this context, consumer loss-aversion is two-pronged: first, consumers who frequently re-evaluate their mental account balances thus updating their budgets are engaging in loss-averse behavior if they have high propensities to save; second, consumers with lower propensities for saving can achieve higher utility by updating their mental account balances more frequently. Loss-aversion in this context is an implicit function of the consumer's marginal utility of cash balances MU_z, which is directly dependent on the specific parameterization of the utility function. Here, we have incorporated loss-aversion *implicitly* into the standard, concave utility maximization problem without having to specify an explicit reference-level variable inside the static utility function. Future exploration should explore how the model outlined here exhibits loss-aversion under different parameterization of u_t.

Figure 12: Balance evolution under different frequencies of budget share updating.

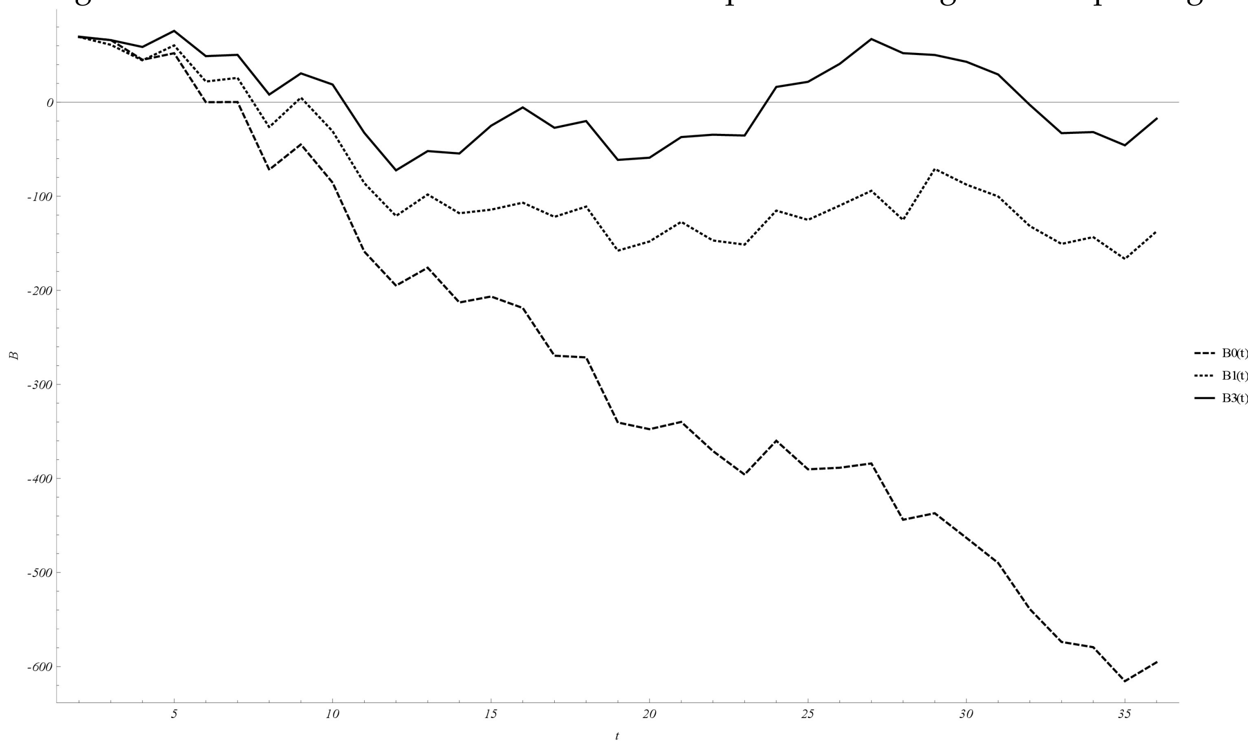


Figure 13: When $B_t < 0$ the consumer adjusts $\theta_{4,t+1}$ upward.

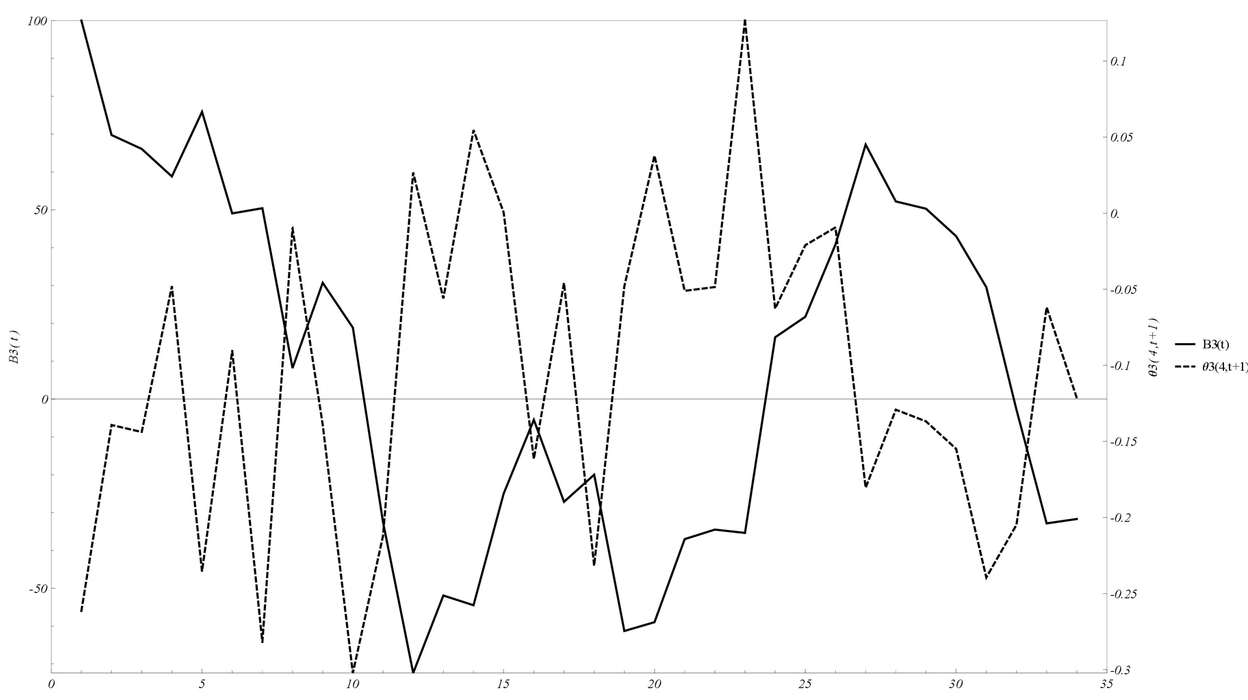


Figure 14: $u_0(t)$ is period t utility under consumption with $k_t = 0$, $u_1(t)$ with $k_t = 1$, and $u_3(t)$ with $k_t = 3 = J$.

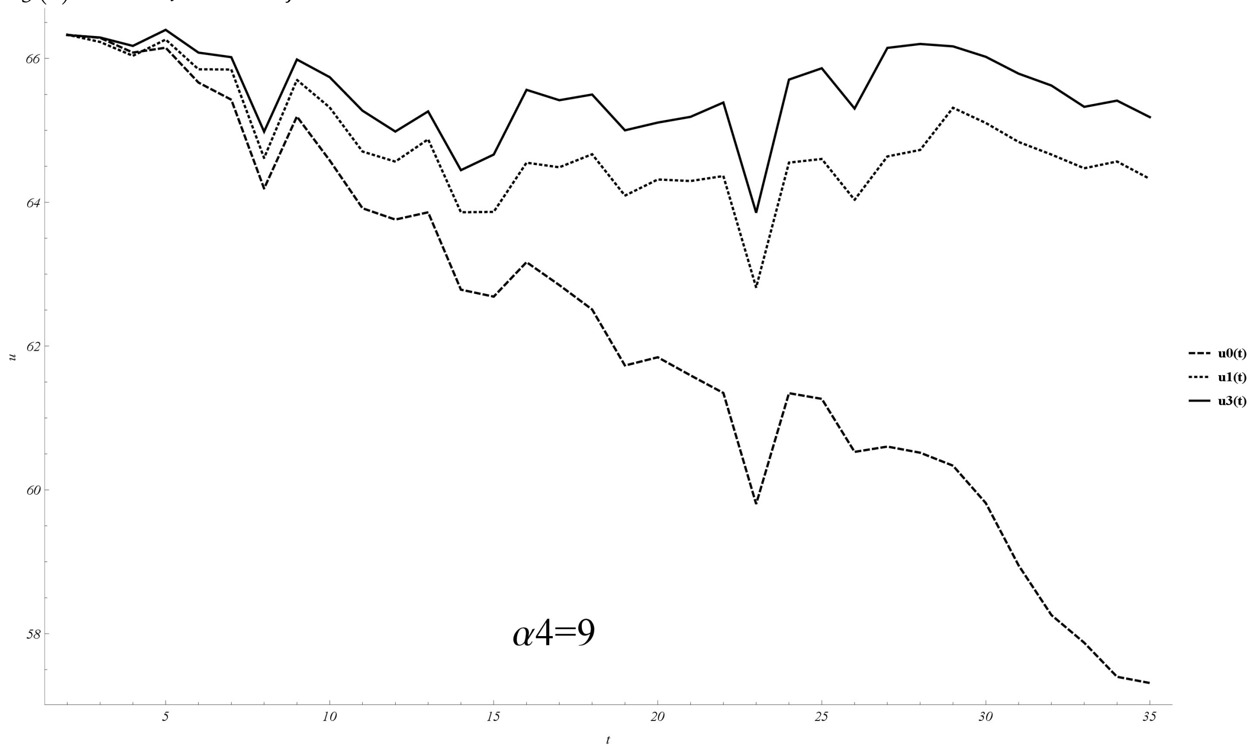
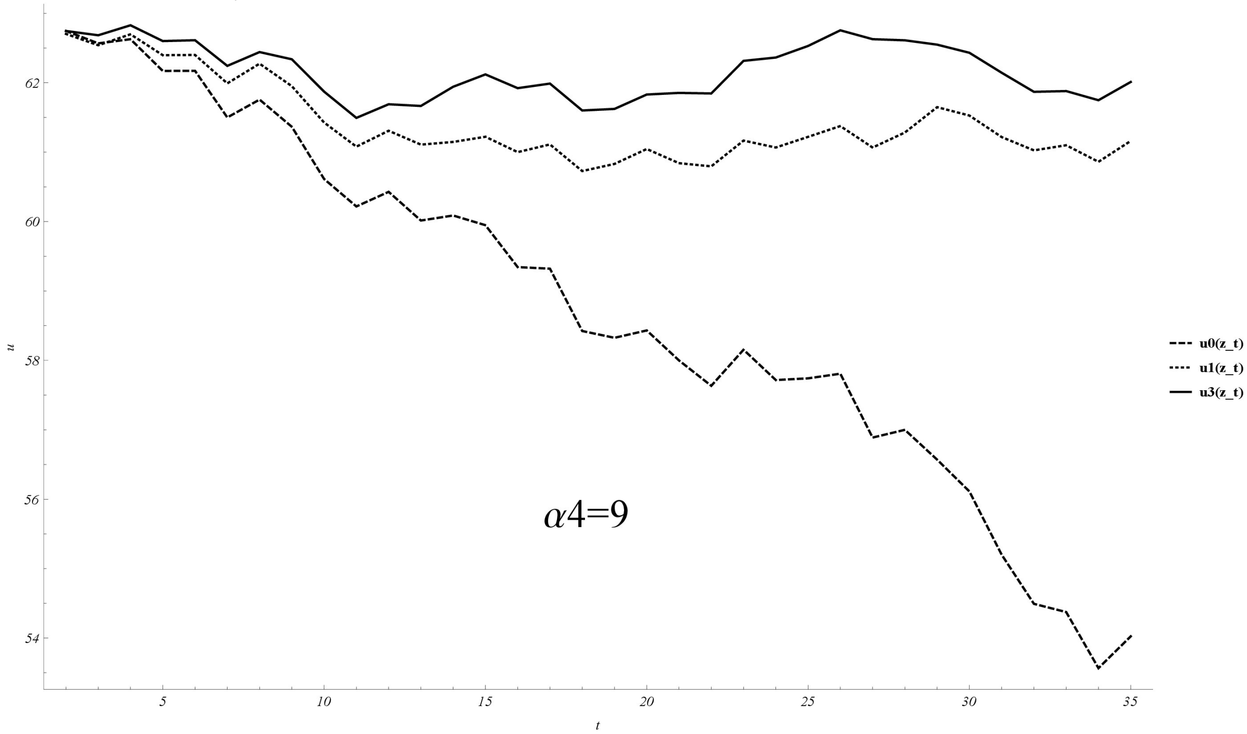


Figure 15: Utility from cash balances under different mental accounting regimes — $k_t = 0$, $k_t = 1$, and $k_t = J = 3$.



4.3 Price and Income Responsiveness

We also seek to measure how the mental accounting framework deals with price and income responsiveness. To do this, we summed the expenditure constraints for the goods categories $j \in \{1, \ldots, J\}$ and differentiated (26) subject to this constraint to derive the mental accounting period-t Marshallian demand function for good j:

$$g_{jt}(\boldsymbol{p}_t, \boldsymbol{\theta}_t, \boldsymbol{\zeta}_t, A_{1,t-1}, \ldots, A_{J,t-1}, L_{t-1}) = \frac{\sum_{j=1}^{J}(\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + \sum_{i \neq j}(p_{it} - p_{jt} \cdot \alpha_i/\alpha_j)}{p_{jt}\left[1 + (1/\alpha_j)\sum_{i \neq j}\alpha_i\right]} \tag{52}$$

From this, we get expressions for the own-price elasticity of demand $\epsilon_{jt}(p_{jt})$ for good j in period t:

$$\epsilon_{jt}(p_{jt}) = \frac{\partial g_{jt}}{\partial p_{jt}} \cdot \frac{p_{jt}}{q_{jt}} = -\frac{\sum_{j=1}^{J}(\theta_{jt}L_{t-1} + \gamma A_{j,t-1})\zeta_{jt} + \sum_{i \neq j} p_{it}}{p_{jt}q_{jt}\left[1 + (1/\alpha_j)\sum_{i \neq j}\alpha_i\right]} \tag{53}$$

In the price-responsiveness simulations Figures 16 and 17 show how the choice of next period's budget share $\theta_{1,t+1}$ moves negatively with own-price responsiveness and income responsiveness. This is intuitive: inelastic demand is associated with higher wealth levels, thus inelastic demand is correlated with how implicitly (mentally) wealthy a consumer feels with respect to a particular consumption category. In such cases, mental accounting predicts that consumers will reallocate their budgets away from those categories in which they feel implicitly wealthy, hence the co-movement in Figures 16 and 17.

Figure 16: Next period's expenditure share $\theta_{1,t+1}$ and the absolute value of this period's own price elasticity of consumption $|\epsilon_{1t}(p_{1t})|$.

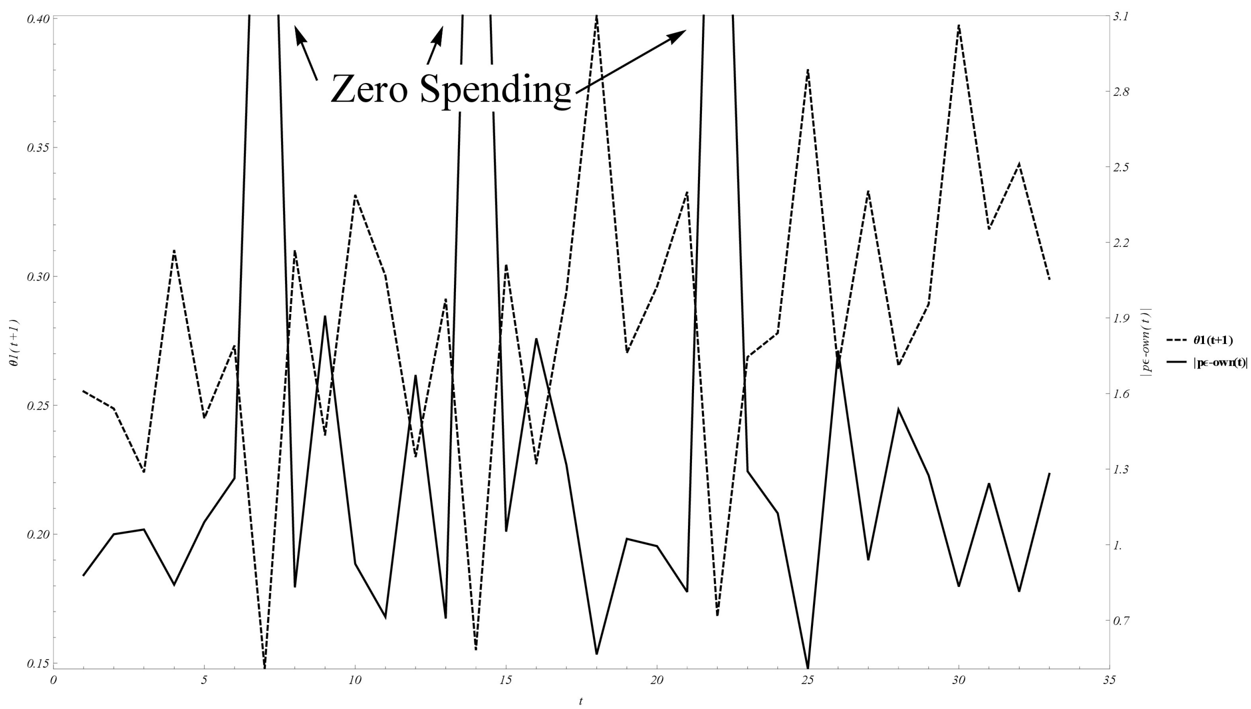
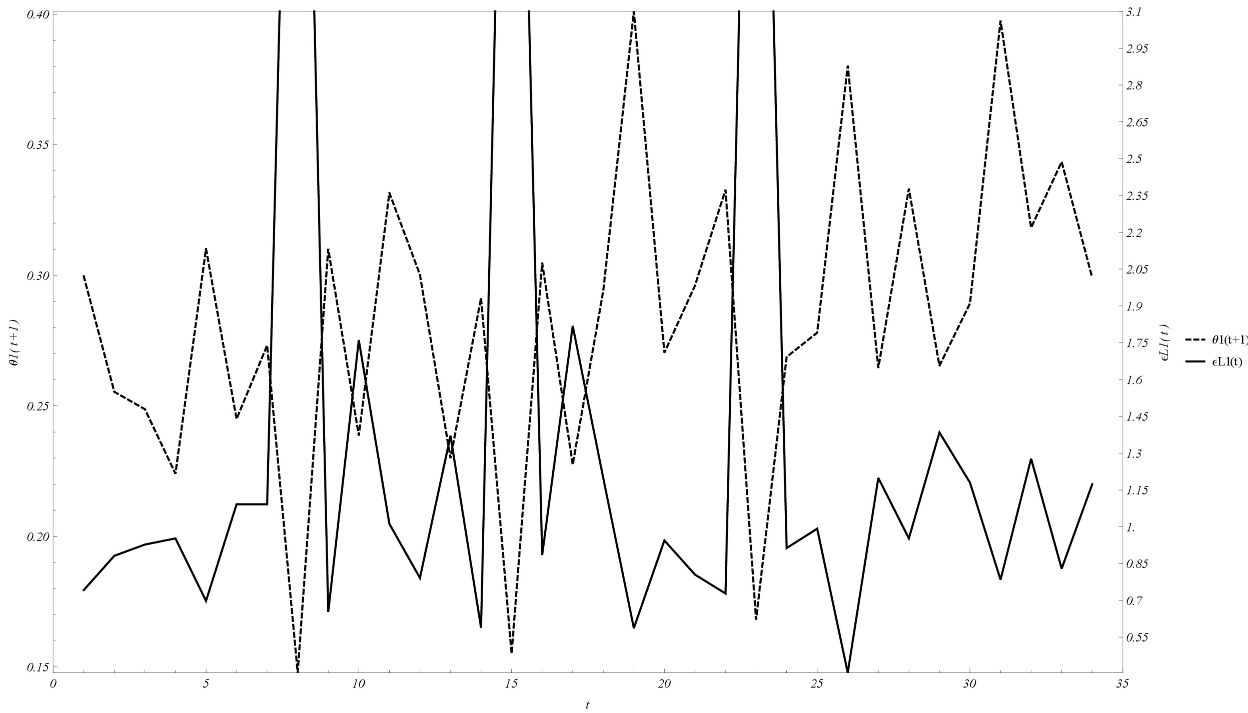


Figure 17: Higher ex-ante expenditure shares are associated with lower period-t income elasticity of demand.



5 Conclusion & Moving Forward

We have developed a structural model of mental accounting that features both reference-dependence and loss aversion using a standard, quasiconcave, monotone, and continuously-differentiable utility function. In this paper we show that under certain assumptions mental accounting can be modelled as a behavioral control mechanism that provides additional constraints on consumption and savings decisions within the neo-classical economic framework. We show that consumers who engage in mental accounting cannot achieve higher utility levels than if they treated their consumption and investment classes as equally fungible in all periods. As a consequence, the model we present here is nested in the neo-classical economic consumer optimization problem if we were to allow all exogenous stochasticity to act through prices. We show that mental accounting can partially explain the short term dynamics around individual and income-responsiveness to consumption. We also demonstrate that the degree to which consumers' mental account reference levels are sticky either adversely or positively affects their overall welfare depending on their present demand for cash balances. Consumers who frequently update their mental accounts and demand high cash balances will consume more and save less than if their budgets were stickier. On the other hand, consumers who are prone to high consumption expenditure will be forced to save more if they re-evaluate their mental accounts more frequently. Future work will examine how these specific phenomena may relate to different discounting regimes, such as hyperbolic discounting verses exponential discounting. There is also potential work to relate a structural, economic model of mental accounting to literature on consumer risk aversion and the macroeconomic literature on consumption smoothing.

Appendix A. List of Parameter and Variable Names

Table 3: List of Indices

Indices	Description
i	Indexes consumers
j	Indexes consumption categories
t	Indexes time
n	Indexes Monte Carlo simulations

NOTE: capital letters — I, J, T, N etc. — correspond to total elements in index.

Table 4: List of Variables

Variable	Description
$\boldsymbol{p}_t$	J-dimensional vector of prices
$\boldsymbol{q}_t$	J-dimensional vector of consumption
z_t	Scalar, liquid cash demand in period t
u_t	Utility function with additively separable components u_{jt}
$\boldsymbol{\theta}_t$	$J+1$-dimensional vector of expenditure shares
$\boldsymbol{A}_t$	$J+1$-dimensional vector of mental account balances after consumption in period t
$\boldsymbol{\zeta}_t$	J-dimensional vector of *iid* exogenous shocks to consumption
M	Borrowing limit
B_t	Balances after consumption in period t
L_t	Income earned from labor in period t
R_t	Gross interest rate on balances
k_t	Integer number of changes to period $t+1$ expenditure shares from period t
$\boldsymbol{\nu}_t$	The vector of state variables (all of the above)
$\widetilde{\boldsymbol{\nu}}_t$	The vector of *latent* state variables, $\boldsymbol{\theta}_t$, $\boldsymbol{A}_t$, and k_t

NOTE: all variables except $\boldsymbol{p}_t$ and R_t are indexed by agent, i.

Table 5: List of Agent-Level Parameters

Parameter	Description
β	Utility discount parameter
γ	Mental account balance discount parameter
$\boldsymbol{\alpha}$	J-dimensional Cobb-Douglas utility parameters
α_{J+1}	Controls contribution of cash demand to utility

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