Print Date: 3/5/2010

Inferring Price Sensitivity when Competitor Reactions are Unobserved

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Working Draft, Do Not Distribute

April 2010

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1. Introduction

Distributors can enlarge their sales either by increasing their share of their existing customers or by gaining new customers. Unless the market is growing any increase in sales for an existing customer must come at the expense of the sales of another competing distributor or distributors. One would expect these competing distributors to respond either directly by lowering prices for targeted customers or more broadly by lowering prices for non-targeted customers. The problem is that distributors rarely observe a competitor's price directly, and must infer competitor response indirectly from their own observations about customer purchases. This is the focus of this research, to construct an empirical model of competitive reaction with incomplete information.

The data that we employ in this problem comes from a distributor that operates within a business-to-business context. The customers predominately are small business owners who engage in light manufacturing to sell to their own customer base. The industry is mature and geographically dispersed. The distributor employs its own salesforce and allows it sales representatives to have a high level of autonomy in negotiating prices with customers. This has led to high variability in prices across customers, as well as a high variability in the service quality provided to these customers. Most of the salesforce is on a hybrid compensation scheme, in which some income comes from a salary and the remainder from commissions. Most competitors have a similar structure, although some have only salaried or hourly employees. Additionally some competitors only employ fixed-posted prices, although most allow for negotiated prices.

The goal for this research is to develop a structural model on consumer response and estimate it to a transaction dataset. The model includes several innovations and components that have not been used in this context. First, we link response from individual product sales to an overall measure of loyalty. For example, it may appear that individually customers are insensitive to price changes of individual products, but aggregate price changes may result in switching to competing distributors. Second, we consider strategic behavior on the part of the customer for maintaining multiple suppliers to insure redundancy and secure the lowest prices through competitive process. Finally, our data contains many products that are routinely purchased and anecdotal evidence suggests that customers do not engage in extensive search or comparison shopping. We introduce shopping costs to provide a reason that customers may not routinely switch to new suppliers or engage in extensive comparison shopping.

In this research we describe how could we infer price sensitivity when the prices of the competitors are not observed. We begin in section 2 by introducing an analytical framework in which we assume that customers are rational in making their purchases decisions to derive their demand function. In section 3 we present the model and then we describe a procedure to estimate its parameter using MCMC approach.

2. An economic based model for the multiproduct procurement

Following a long tradition in inventory management, we assume that in each period, each customer minimizes total cost *TC* in *M* product categories. Cost minimization is conducted subject to having a minimum level of utility from optimal purchases (see for example Silver, 1981). If ignoring quality consideration, the constraints translate to minimum levels of purchases for each category.

$$\begin{aligned} &\min \quad TC\left(\{q_{imt}, q_{imt}^{c}\}_{m=1}^{M}\right) \\ &\text{s.t.} \quad q_{imt} + q_{imt}^{c} \geq \tau_{imt} \quad t = 1...T, m = 1...M \end{aligned} \tag{2.1}$$

Where q_{imt} and q_{imt}^{c} are the quantity purchased by customer *i* in category *m* in period *t* from the focal and the competitor companies respectively and τ_{imt} is the minimum quantity required by the customer to cover his consumption in the following period. We postulate that the total cost is compound by fixed and variable components *FC* and *VC*.

$$TC\left(\{q_{imt}, q_{imt}^c\}_{m=1}^M\right) = FC\left(\{q_{imt}, q_{imt}^c\}_{m=1}^M\right) + VC\left(\{q_{imt}, q_{imt}^c\}_{m=1}^M\right) \tag{2.2}$$

The fixed component *FC* depends on whether the consumer buy or not from each company.

$$CF\left(\{q_{imt}, q_{imt}^c\}_{m=1}^M\right) = \left(K_i + \kappa_i r_{it}\right)\delta\left(\sum_m q_{imt}\right) + \left(K_i^c + \kappa_i^c r_{it}^c\right)\delta\left(\sum_m q_{imt}^c\right) \tag{2.3}$$

Where $\delta(\cdot)$ is an index function that takes the value 1 if the argument is positive and 0 otherwise. Then, the fixed value of purchasing in the focal and competitor companies are given by $K_i + \kappa_i r_{it}$ and $K_i^c + \kappa_i^c r_{it}^c$ respectively, being the first component the base cost of transaction while the second component measures how this cost is affected by previous purchase behavior. According to the search cost and learning literature, we expect that transaction might differ with depending on whether the customer has purchased or not from the distributors in the previous periods. The variables r_{it} and r_{it}^c precisely captures the intensity of activities with the focal and competitors firms through exponential smoothing equations:

$$r_{it} = \sum_{u=1}^U \alpha^u \delta\left(q_{imt-u}\right) \qquad\qquad r_{it}^c = \sum_{u=1}^U \alpha^u \delta\left(q_{imt-u}^c\right) \tag{2.4}$$

For identification purposes we fix the parameter α prior to the estimation of all other parameters. The variable component *CV* is a function of the quantities that the customer buys in each firm. We postulate that such cost can be approximated by a quadratic equation as follows.

$$CV\left(\{q_{imt}, q_{imt}^c\}_{m=1}^M\right) = \sum_{m=1}^M p_{imt}q_{imt} + p_{imt}^c q_{imt}^c + \mu_{imt}q_{imt}q_{imt}^c - \frac{1}{2}\eta_{imt}\left({q_{imt}}^2 + {q_{imt}^c}^2\right) \tag{2.5}$$

where parameters η_{imt} capture quantity discounts and parameter μ_{imt} captures transactions costs in procurement. When this parameter is positive the firm prefers to buy from only one firm instead of splitting his demand (all else equal).

3. The simple case without fixed costs

If we assume that the fixed cost of buying from a distributor is zero, the problem is independent between categories.

$$\begin{array}{ll} \min & TC_i\left(q_{imt}, q_{imt}^c\right) \\ \text{s.t.} & q_{imt} + q_{imt}^c \geq \tau_{imt} \quad t = 1 \ldots T \end{array} \tag{2.6}$$

To characterize firm optimal behavior we impose first order conditions on the minimization cost problem defined in(2.6). Let λ_{imt} be the shadow price associated to the minimum quantity constraints. Then, the optimal solution is characterized by the following system of equations.

$$p_{imt} + \mu q_{imt}^c - \eta_{imt} q_{imt} - \lambda_{imt} \leq 0 \ \left(\text{with equality if } q_{imt} > 0\right) \tag{2.7}$$

$$p_{imt}^c + \mu q_{imt} - \eta_{imt} q_{imt}^c - \lambda_{imt} \leq 0 \ \left(\text{with equality if } q_{imt}^c > 0\right) \tag{2.8}$$

$$q_{imt} + q_{imt}^c = \tau_{imt} \tag{2.9}$$

The identification of conditions where interior and boundary solutions occur is a key element to the estimation of a model with imperfectly observed demand. In every period the customer could be in three cases: (a) whole demand is satisfied from the focal retailer ($q_{imt} = \tau_{imt}$), (b) whole demand is satisfied from the competitors ($q_{imt}^c = \tau_{imt}$) and (c) the interior solution where customer buys from the focal and competitor firms in which case demands are given by:

$$q_{imt} = \frac{1}{2}\left(\tau_{imt} + \frac{p_{imt} - p_{imt}^c}{\mu_{imt} + \eta_{imt}}\right) \tag{2.10}$$

$$q_{imt}^{c} = \frac{1}{2}\left(\tau_{imt} + \frac{p_{imt}^{c} - p_{imt}}{\mu_{imt} + \eta_{imt}}\right) \tag{2.11}$$

When applying first order conditions, Kim et al (2002) have a similar structure. However, they are interested in estimated product choice conditional on expenditure and therefore they take differences of first order conditions to take the budget constraint into account. In our application, the value of the demand function is an important component that needs to be estimated and therefore we take a different approach.

The decision of buying from one or other firm depends on the prices, but also on the parameters γ and η. The conditions characterizing the boundaries of each scenario can be derived by intersecting the corresponding first order conditions (see Appendix 1). A graphical depiction of the regions where each alternative in a plane $\delta_{imt} = p_{imt} - p_{imt}^{c}$ vs. $\psi_{imt} = \mu_{imt} + \eta_{imt}$ is chosen is shown in Figure 1. With positive values of ψ_{imt}, the firm has no incentives to buy from more than a firm because it involves higher transaction costs and less volume to negotiate quantity discounts. In this case the customer will always decide to buy from the provider with the lower price. If the total effect of transaction costs and quantity discounts is negative the customer might want to buy from more than one firm. If the difference in prices is small, then a small negative value of ψ_{imt} will suffice to motivate the customer to buy from multiple firms. However if the differences in prices is large, the firm will buy from a single firm unless the benefits of splitting the purchases is very large. We define sets Ω_1 and Ω_2 as the regions in the parameter space where the customer buy from the competitor and focal firm respectively while the set Ω_3 correspond to the case where the customer buys from both.

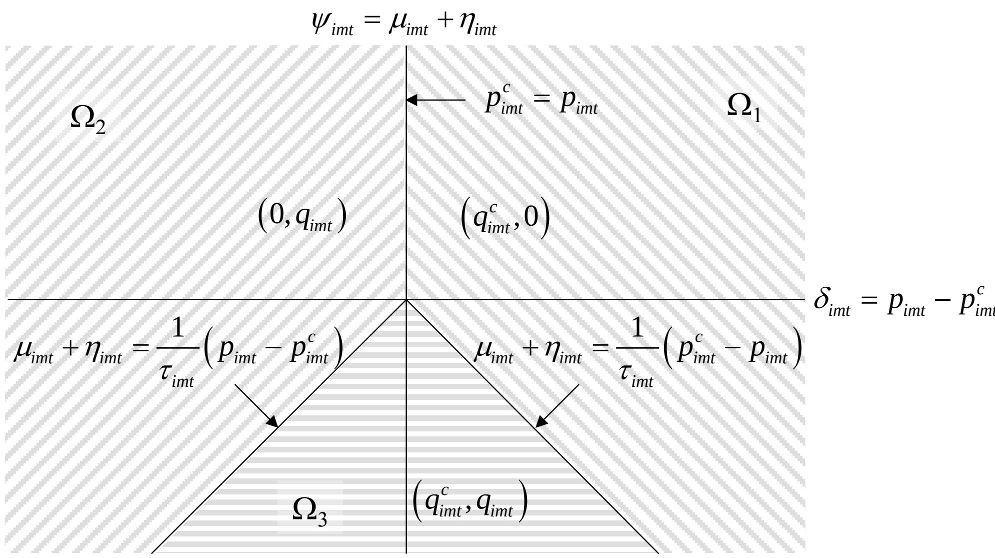

Figure 1: Graphical description of first order condition cases

There are several reasons that might imply negative values for parameters μ_{imt} and η_{imt}. For example a negative value for η_{imt} could signal diseconomies of scale in inventory management while a negative value for μ_{imt} could be triggered by variety seeking[1].

To specify the likelihood function we need to describe the probability behavior of the error term in the model. We start by assuming that all randomness comes from the variations of consumption τ_{imt}. For simplicity we assume that such error is normally distributed. Then, the model can be summarized as follows:

[1] Unlike most literature in variety seeking where the arguments are made at the individual level, here where the decision maker is a firm, variety seeking could be caused by a strategic decision of the customer to purchase from multiple vendors to learn about prices or eliminate the potential for negative prices in the future.

$$q_{imt} = \begin{cases} \tau_{imt} + \varepsilon_{imt} & \text{if } \left(\tau_{imt}, \delta_{imt}, \psi_{imt}\right) \in \Omega_2 \\ \frac{1}{2}\left(\tau_{imt} + \frac{\delta_{imt}}{\psi_{imt}} + \varepsilon_{imt}\right) & \text{if } \left(\tau_{imt}, \delta_{imt}, \psi_{imt}\right) \in \Omega_3 \\ 0 & \text{if } \left(\tau_{imt}, \delta_{imt}, \psi_{imt}\right) \in \Omega_1 \end{cases} \tag{2.12}$$

We further assume that the nonlinear cost parameter $\psi_{imt} = \psi_{im}$ is constant in time. On the other hand, we assume that both the required quantity (τ_{imt}) and the price difference (δ_{imt}) are described by the following linear models[2].

$$\tau_{imt} = \beta_{0im} + \beta_{1im} XT_{it} + \varepsilon_{imt} \tag{2.13}$$

$$\delta_{imt} = \gamma_{0im} + \gamma_{1im} XD_{it} \tag{2.14}$$

Several predictor variables can be included to describe customer level cost parameters. For example, we could postulate that the product requirement τ_{imt} is a function of the average purchase with the company in the recent period (conditionally on that a positive purchase is made: $\sum_{u=1}^{U} q_{imt-u} \Big/ \sum_{u=1}^{U} \delta\left(q_{imt-u}\right)$). Price differences could be described as function of firm size or promotional activity. We introduce customer heterogeneity into the model by specifying that the parameters of the models come from a common population distribution. Let $\theta_{im} = \left(\beta_{0im}, \beta_{1im}, \gamma_{0im}, \gamma_{1im}, \psi_{im}\right)$ be the vector of individual parameters. Then we assume that:

$$\theta_{im} = \Lambda \cdot z_i + \nu_{im} \qquad \nu_{im} \sim N\left(0, V_\theta\right) \tag{2.15}$$

where z_i are customer specific characteristics such as store size, whether it belong to a chain or not among other characteristics.

2 Here we estimate customer product requirement directly. An interesting extension can be derived by modeling customer requirement as the interaction of a consumption and inventory models.

To complete the model, we specify the prior distribution as follows:

$$V_\theta \sim IW(\nu, V) \tag{2.16}$$

$$\text{vec}(\Delta)\big|V_\theta \sim N\left(vec\left(\bar{\Lambda}\right), V_\theta \otimes A^{-1}\right) \tag{2.17}$$

$$\sigma_{im}^2 \sim \nu_\varepsilon ssq \big/ \chi_{\nu_\varepsilon}^2 \tag{2.18}$$

where ν, V, $\bar{\Lambda}$, A, ν_ε and ssq are chosen to have relatively diffuse priors.

4. Data description and estimation

We observe the following vector of information for each product *k* which is a member of category *m* that customer *i* purchases at time *t*: quantity sold (q_{imkt}), negotiated price (p_{imkt}),and promotional activity or direct sales effort (a_{imkt}). Notice that if an item is not sold, this may be due to several reasons: (i) the customer does not need to buy from the category (ii) the customer buys from the competitor. Unfortunately, neither prices (p_{imkt}^c) nor demanded quantities (q_{imkt}^c) from the competitors are observed. We consider convenient to aggregate to a category level through a function $\upsilon_{mk}(\cdot)$ that provides an equivalence mapping of all units within a category. For example, it may translate all weights to a common measure (e.g., gallons to liters) or it could reflect more complex equivalence mappings (i.e., lower quality versus higher quality catalysts).

$$q_{imt} = \sum_{k \in m} \upsilon_{mk}\left(q_{ikmt}\right) \tag{2.19}$$

Demand model specified by equation (2.12) is defined by parts. Then, to compute the likelihood of the model we use law of total probability.

$$p\left(q_{im}\left|\theta_{im}\right.\right) = \sum_{i=1}^{3} p\left(q_{im}\left|\Omega_i\left(\theta_{im}\right)\right.\right) p\left(\Omega_i\left(\theta_{im}\right)\right) \tag{2.20}$$

The probability of being in each region of the parameter space (Ω_i, $i \in \{1,2,3\}$) and the conditional likelihood given the Ω_i are described in Appendix 2. The model derived from imposing first order conditions destroys conjugacy of the hierarchical linear model and therefore we use a Metropolis-Hastings step to update customer level parameters. Population level parameters are computed using the standard conjugate updating. The estimation algorithm proceeds by recursively generating draws as follows:

1. Start with initial values of θ_{im}
2. For each customer
   a. Propose a new value θ_{im} according to a random walk process. And accept it with probability $\min\left\{1, \frac{p\left(\theta_{im}^{\text{new}} | q_{im}\right) p\left(\theta_{im}^{\text{new}}\right)}{p\left(\theta_{im} | q_{im}\right) p\left(\theta_{im}\right)}\right\}$
   b. draw a value of σ_{im} conditional on θ_{im} : $\sigma_{im} \sim \frac{\nu \cdot ssq_i + s_i}{\chi^2_{\nu+T}}$
3. Update upper level regression parameters Λ and V_θ following conjugate multivariate regression model.
4. Repeat as necessary.

Appendix

Appendix 1: Derivation of indifference conditions

- Case 1: customer is indifferent between buying from focal firm only or buying also from the competitor. In the boundary (2.10) hold, but also the $q_{imt} = \tau_{imt}$. Then

$$\left.\begin{aligned} q_{imt} &= \frac{1}{2}\left(\tau_{imt} + \frac{p^c_{imt} - p_{imt}}{\gamma + \eta}\right) \\ q_{imt} &= \tau_{imt} \end{aligned}\right\} \Rightarrow \gamma + \eta = \frac{1}{\tau_{imt}}\left(p^c_{imt} - p_{imt}\right)$$

- Case 2: customer is indifferent between buying from competitor only or buying also from the focal firm.

$$\left.\begin{aligned} q^c_{imt} &= \frac{1}{2}\left(\tau_{imt} + \frac{p_{imt} - p^c_{imt}}{\gamma + \eta}\right) \\ q^c_{imt} &= \tau_{imt} \end{aligned}\right\} \Rightarrow \gamma + \eta = \frac{1}{\tau_{imt}}\left(p_{imt} - p^c_{imt}\right)$$

- Case 3: customer is indifferent between buying from focal firm and competitor. In the boundary there is discontinuity in the demand function and therefore we use first order conditions directly. In the boundary equations (2.7) and (2.8) hold with equality. Then:

$$\left.\begin{aligned} \lambda_{imt} &= p_{imt} + \eta q_{mit} + \gamma q^c_{mit} \\ \lambda_{imt} &= p^c_{imt} + \eta q^c_{mit} + \gamma q_{mit} \end{aligned}\right\} \Rightarrow p_{imt} = p^c_{imt}$$

Appendix 2:

Assuming that ε_{imt}, the error term of the required quantity equation (2.13) is normally distributed with zero mean and variance σ^2_{im}, the probability of being in each region of the parameter space are given by:

$$p\left(\Omega_1\left(\theta_{im}\right)\right)=\begin{cases}1-\Phi\left(\left(\overline{\tau}_{imt}+\delta_{imt}/\psi_{im}\right)/\sigma_{im}^2\right) & \delta_{imt}\geq 0\\ 0 & \text{otherwise}\end{cases} \tag{2.21}$$

$$p\left(\Omega_2\left(\theta_{im}\right)\right)=\begin{cases}1-\Phi\left(\left(\overline{\tau}_{imt}-\delta_{imt}/\psi_{im}\right)/\sigma_{im}^2\right) & \delta_{imt}\leq 0\\ 0 & \text{otherwise}\end{cases} \tag{2.22}$$

$$p\left(\Omega_3\left(\theta_{im}\right)\right)=\Phi\left(\left|\overline{\tau}_{imt}-\delta_{imt}/\psi_{im}\right|/\sigma_{im}^2\right) \tag{2.23}$$

Finally, the conditional likelihood in each region of the parameter space are given by:

$$p\left(q_{imt}\middle|\Omega_1\left(\theta_{im}\right)\right)=\begin{cases}1 & q_{imt}=0\\ 0 & \text{otherwise}\end{cases} \tag{2.24}$$

$$p\left(q_{imt}\middle|\Omega_2\left(\theta_{im}\right)\right)=\begin{cases}0 & q_{imt}=0\\ \varphi\left(\left(q_{imt}-\overline{\tau}_{imt}\right)/\sigma_{im}^2\right) & \text{otherwise}\end{cases} \tag{2.25}$$

$$p\left(q_{imt}\middle|\Omega_3\left(\theta_{im}\right)\right)=\begin{cases}0 & q_{imt}=0\\ \varphi\left(\left(q_{imt}-\left(\overline{\tau}_{imt}+\delta_{imt}/\psi_{im}\right)/2\right)/\sigma_{im}^2\right) & \text{otherwise}\end{cases} \tag{2.26}$$

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