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UNCERTAINTY IN MEASUREMENTS OF FLUID TEMPERATURE IN TUBES

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Summary: Temperature measurements of fluid in tubing is common practice in many applications of biology and medicine for the purpose of temperature control, analysis, and modeling. Thermocouples are commonly applied for those measurements and, ignoring the thermal interaction between the thermocouple and the fluid, it is widely assumed that the thermocouple's tip reads the undisturbed fluid temperature. The current report provides a simple and explicit formulation for the estimation of the effect of this thermal interaction on the measured temperature value. Results show that this thermal effect, between the sensor and the sensed phenomenon, may result in an uncertainty in measurements which overwhelms uncertainties from other sources.

Keywords: Temperature Measurements, Fluid Flow, Uncertainty, Mathematical Analysis

Introduction

Thermocouples are perhaps the most commonly used instruments for temperature measurements, with the possible exception of the glass thermometer, for medical applications. Temperature measurements of fluid in tubing is common practice in many applications of biology and medicine for the purpose of temperature control, analysis, and modeling. Examples are numerous from both bench-top experiments and medical applications such as measurements during perfusion, or transfusion, temperature control with thermal blankets, temperature control by means of water bath, etc. The fluid temperature is commonly measured with thermocouples, as is schematically presented in Fig. 1: (a) using a T connector and standard thermocouple wires, or (b) using a thermocouple needle which is inserted through the tube wall.

It is commonly assumed that the thermocouple's tip reads the undisturbed fluid temperature, once the tip is immersed in the fluid. In practice, however, the thermocouple's tip reads a fluid temperature which is affected by the presence of the thermocouple. More specifically, the thermocouple conducts heat to, or from, the point of measurement, which heats up, or cools down, the fluid at the same location, respectively. Thus, the thermocouple as a sensor interacts with the sensed phenomenon, as has been discussed recently in (4) for cryosurgical applications. The current study arises from efforts to provide a simple and explicit formulation for estimating this thermal effect on the measured temperature value. While this study deals with one source of uncertainty in temperature measurements, there are other sources which contribute to the uncertainty interval, such as electrical amplifiers, analog to digital converters, surroundings temperature compensation, the presence of electrical/ magnetic fields, random ground currents, etc. (1,5).

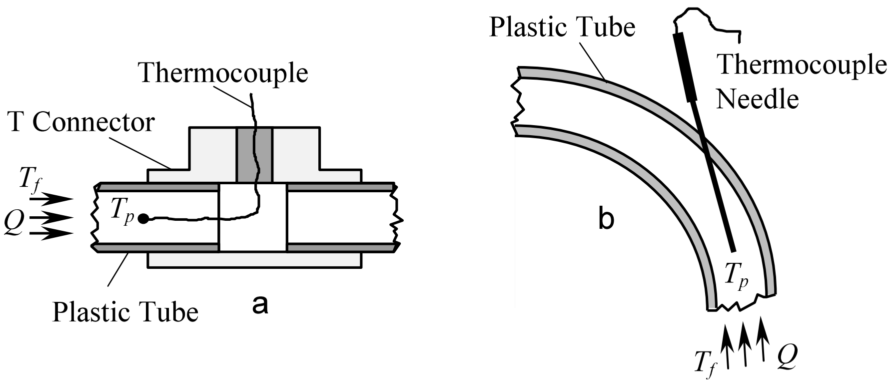

Figure 1: Thermocouple localization setup in plastic tubing using (a) a T connector and (b) a thermocouple needle.

Mathematical Analysis

The thermocouples shown schematically in Fig. 1 can be modeled as two fins, from heat transfer considerations, having a common base (Fig. 2). The fins' common base is the point at which the thermocouple penetrates the tube wall. The thermal model of fin is applicable for one-dimensional heat flow in thin plates or wires. The basis for the fins analysis is the assumption of a uniform temperature distribution in each cross-section of the fin, due to low thermal resistance to heat flow by conduction within the fin's cross-section, with respect to the thermal resistance to heat flow by convection of the surrounding fluid. The criterion for the validity of the thermal analysis of fins is given at the end of this section. Mathematical analyses of heat conduction by fins and heat convection around them are widely available in literature (2,3), where the relevant formulations are presented hereon.

The portion of the thermocouple within the tube can be modeled as a fin with a finite length L, having a temperature distribution:

$$\frac{T - T_f}{T_0 - T_f} = \frac{\cosh[m_f (L - x_f)]}{\cosh(m_f L)} \tag{1}$$

where T is the variable temperature along the fin, T_f is the undisturbed fluid temperature, T_0 is the temperature at the base of the fin, x_f is a coordinate measured from the base of the fin, *cosh* is the hyperbolic cosine function, and the constant m_f is defined below. Equation (1) is the solution of the ordinary heat conduction equation in the fin with a boundary condition of convection.

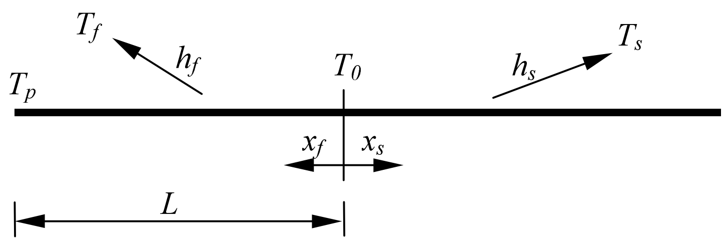

Figure 2: Schematic illustration of modeling the thermocouple as two fins having a common base. The indexes f and s denote the portions of the thermocouple exposed to the fluid inside the tube, and to the surroundings, respectively.

The portion of the thermocouple outside of the tube can be modeled as an infinite fin having the temperature distribution:

$$\frac{T - T_s}{T_0 - T_s} = \exp(-m_s x_s) \tag{2}$$

where T_s is the surrounding temperature (room temperature), and x_s is a coordinate measured from the base of the fin. The analysis of an infinite fin is applied in the special case at which the fin is so long that the tip experiences the surrounding temperature (Eq. (2) is the limit of Eq. (1) when L goes to infinity). The constants m_f and m_s of Eqs. (1) and (2) are defined as:

$$m_n = \frac{4U_n}{d_p k_p} \qquad n = s, f \tag{3}$$

where U_n is an overall coefficient of heat transfer, and k_p is the thermal conductivity of the thermocouple. The diameter d_p is an effective value of the thermocouple wires, which creates the same cross section area as both wires of the thermocouple. Thus, d_p equals approximately 1.4 times the diameter of each wire.

The continuity of heat fluxes at the fins' bases can be written as:

$$\left.\frac{dT}{dx_s}\right|_{x_s=0} = -\left.\frac{dT}{dx_f}\right|_{x_f=0} \tag{4}$$

which means that the heat conducted from the fluid is the heat transferred to the surrounding. A negative sign has to be given to one of the derivatives in Eq. (4) to indicate that the coordinates x_s and x_f have opposite directions. From Eqs. (1), (2), and (4), one finds the base temperature to be:

$$T_0 = \frac{m_s T_s + m_f T_f \tanh(m_f L)}{m_s + m_f \tanh(m_f L)} \tag{5}$$

where *tanh* is the hyperbolic tangent function.

The temperature difference between the sensed temperature (thermocouple's tip) and the undisturbed fluid temperature, which is defined as ΔT_p, can now be calculated from Eq. (1) by setting x_f to L:

$$\Delta T_p \equiv T_p - T_f = \frac{T_0 - T_f}{\cosh(m_f L)} \tag{6}$$

The coefficient of heat transfer by convection is addressed next. For the outer fin case, heat transfer is expected to be solely governed by free convection. For the case of air and small temperature differences, say, less than 30°C, one may assume this coefficient to be constant and equal to h_s=15 W/m^2-°C (2). In the fluid case, inside the tube, the coefficient of heat transfer by convection, h_f, is calculated from the empirical correlation for Nusselt number, *Nu*:

$$Nu \equiv \frac{h_f(D-d_i)}{k_f} = \begin{cases} 4 & Re \leq 2300 \\ 0.023Re^{0.8}Pr^{0.35} & Re > 2300 \end{cases} \tag{7}$$

where D is the inner diameter of the tube, d_i is the diameter of the electrical insulator of the thermocouple, and Pr is known as the Prandtl number of the fluid (given as a thermophysical property in tables). Reynolds number, Re, of Eq. (7) is calculated by:

$$Re = \frac{4Q}{\pi \nu (D + d_i)} \tag{8}$$

where Q is the volumetric flow rate, and ν is the dynamic viscosity of the fluid. Reynolds number indicates the flow regime, where the value 2300 is typically taken as a threshold above which the flow transforms from laminar to turbulent.

Finally, the overall coefficient of heat transfer, U_n, is found from:

$$\frac{1}{\pi\, d_p\, U_n} = \frac{1}{\pi\, d_i\, h_n} + \frac{\ln(d_i / d_p)}{2\pi\, k_i} \qquad n = f, s \tag{9}$$

where k_i is the thermal conductivity of the electrical insulation of the thermocouple. The terms of Eq. (9), from left to right, represent the total resistance to heat flow from the thermocouple, the thermal resistance to heat transfer by convection from the electrical insulator, and the thermal resistance to heat conduction across the electrical insulator, respectively. Rearranging Eq. (9), one finds the overall coefficient of heat transfer to be:

$$U_n = \frac{d_i}{d_p} \cdot \frac{2k_i h_n}{d_i h_n \ln(d_i / d_p) + 2k_i} \qquad n = f, s \tag{10}$$

The basis for the fins analysis is the assumption of a uniform temperature distribution in each cross-section of the fin, due to low thermal resistance to heat flow by conduction within the fin's cross-section, with respect to the thermal resistance to heat flow by convection of the surrounding fluid. Biot number, Bi, is a dimensionless number which represents this thermal resistance ratio:

$$Bi \equiv \frac{U_n d_p}{k_p} \tag{11}$$

As a rule of thumb, the fin analysis is typically applied for a Biot number less than 0.1.

Results and Discussion

The physical properties of most biological solutions do not differ much from those of water. Hence, for biological applications, the discussion is focused on temperature measurements of water. Two general cases are considered here: temperature measurements of water at 0 and at 37.8°C, at a standard room temperature of 20°C. The physical properties of water at 0°C are: $\nu = 17.9 \cdot 10^{-7}$ m^2/s, $Pr = 13.25$, $k_f = 0.566$ W/m-°C; and at 37.8°C are: $\nu = 6.87 \cdot 10^{-7}$ m^2/s, $Pr = 4.53$, $k_f = 0.630$ W/m-°C. Teflon-coated copper-constantan thermocouple is assumed, which has a thermal conductivity of 386 W/m-°C. Typical thermal conductivity of plastic electrical insulators, such as Teflon, is 0.1 W/m-°C.

Figures 3-5 present the difference between the sensed temperature by the thermocouple tip and the known fluid temperature, ΔT_p, Eq. (6), for an effective thermocouple diameter of 0.5 mm, and an electrical insulation thickness of 0.15 mm. It can be seen that for a given tube and thermocouple diameters, ΔT_p is a weak function of the flow rate for a given flow regime. This temperature difference has a constant value in laminar flow, and an almost constant value in turbulent flow. It can also be seen that ΔT_p decreases dramatically with the increase in the immersed length of the thermocouple, L. The temperature difference ΔT_p is much higher at 0°C than at 37.8°C due to the significant elevation of viscosity with the decrease in temperature. The higher viscosity value also causes the flow regime to stay laminar up to higher flow rates.

From Figs. 3-5 one can see that the uncertainty in measurements increases with the increase of the inner diameter of the tube. This can be explained by the fact that the Nusselt number, Eq. (7), has a constant value in laminar flow. It follows that the heat transfer coefficient h_f is an inverse function of the space between the tube inner diameter and the thermocouple diameter. The heat transfer decreases with the decrease in the heat transfer coefficient value, and, hence, a larger temperature difference appears at the thermocouple tip for larger tube diameter. Similar dependency of uncertainty in the tube inner diameter is observed in turbulent flow.

Tables 1 and 2 list the maximal ΔT_p for inner tube diameters of 2.5, 5 and 10 mm, and for thermocouple length of up to 30 mm, where ΔT_p in turbulent flow is calculated at the extremely high flow rate of 10 liters per minute. These tables also list the flow rates at which the flow regime transforms from laminar to turbulent. Table 1 lists values for a thermocouple diameter of 0.5 mm while Table 2 lists values for a thermocouple diameter of 0.25 mm. The electrical insulation thickness is 0.15 mm in both cases. Comparing the results listed in Tables 1 and 2 one can see that ΔT_p decreases with the diameter of the thermocouple.

Finally, it is noted that this study deals with one source of uncertainty in temperature measurements, which is due to heat interaction by the temperature sensor. There are other sources which contribute to the uncertainty in measurements, such as electrical amplifiers, analog to digital converters, surrounding temperature compensation, the presence of electrical/magnetic fields, etc. (1,4). For example, the uncertainty interval due to the quality of the thermocouple materials, values which are provided by the manufacturer, are typically in the range of ±0.5 to ±1.0°C. Uncertainty intervals due to the quality of peripheral equipment are typically smaller than ±0.2°C. Uncertainty interval due to surroundings temperature compensation is typically smaller than ±0.5°C. However, the high values of temperature differences presented in this paper indicate that heat conduction by the thermocouple may be the most severe source of uncertainty in measurements, which can easily overwhelm commonly accepted uncertainties from other sources. Thus, it is highly recommended to use the above formulation for uncertainty analysis as a routine for experimental planning and analyses.

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Table 1: Temperature differences between the thermocouple tip and the fluid far from the tip, for thermocouple diameter of 0.5 mm and electrical insulation thickness of 0.15 mm.

D, mm	L, mm	Laminar Flow $\Delta T_p = T_p - T_f$, °C		Turbulent Flow $\Delta T_p = T_p - T_f$ °C		Flow Rate, liter/min at which $Re = 2300$	
		T_f=0°C	T_f=37°C	T_f=0°C	T_f=37°C	T_f=0°C	T_f=37°C
2.5	1.25	11.6	-10.2	9.9	-8.8	0.64	0.25
	5	4.8	-4.2	3.6	-3.2		
	10	2.3	-2.0	1.6	-1.4		
	20	0.7	-0.6	0.4	-0.4		
5	2.5	9.8	-8.5	6.6	-5.8	1.16	0.43
	5	6.2	-5.3	3.7	-3.3		
	10	3.3	-2.8	1.6	-1.4		
	20	1.2	-1.0	0.4	-0.4		
	30	0.4	-0.4	0.1	-0.1		
10	5	8.4	-7.2	3.9	-3.4	2.10	0.80
	10	4.9	-4.1	1.7	-1.5		
	20	2.1	-1.7	0.5	-0.4		
	30	1.0	-0.8	0.1	-0.1		

Table 2: Temperature differences between the thermocouple tip and the fluid far from the tip, for thermocouple diameter of 0.25 mm and electrical insulation thickness of 0.15 mm.

D, mm	L, mm	Laminar Flow $\Delta T_p = T_p - T_f$, °C		Turbulent Flow $\Delta T_p = T_p - T_f$ °C		Flow Rate, liter/min at which $Re = 2300$	
		T_f=0°C	T_f=37°C	T_f=0°C	T_f=37°C	T_f=0°C	T_f=37°C
2.5	1.25	9.7	-8.5	8.1	-7.2	0.59	0.23
	5	3.3	-2.9	2.4	-2.1		
	10	1.2	-1.1	0.8	-0.7		
	20	0.2	-0.2	0.1	-0.1		
5	2.5	7.6	-6.6	4.9	-4.3	1.08	0.41
	5	4.3	-3.7	2.4	-2.1		
	10	1.8	-1.5	0.8	-0.7		
	20	0.4	-0.3	0.1	-0.1		
	30	0.1	-0.1	0.0	-0.0		
10	5	6.1	-5.2	2.6	-2.2	2.05	0.79
	10	3.0	-2.4	0.9	-0.7		
	20	0.9	-0.7	0.1	-0.1		
	30	0.3	-0.2	0.0	-0.0		

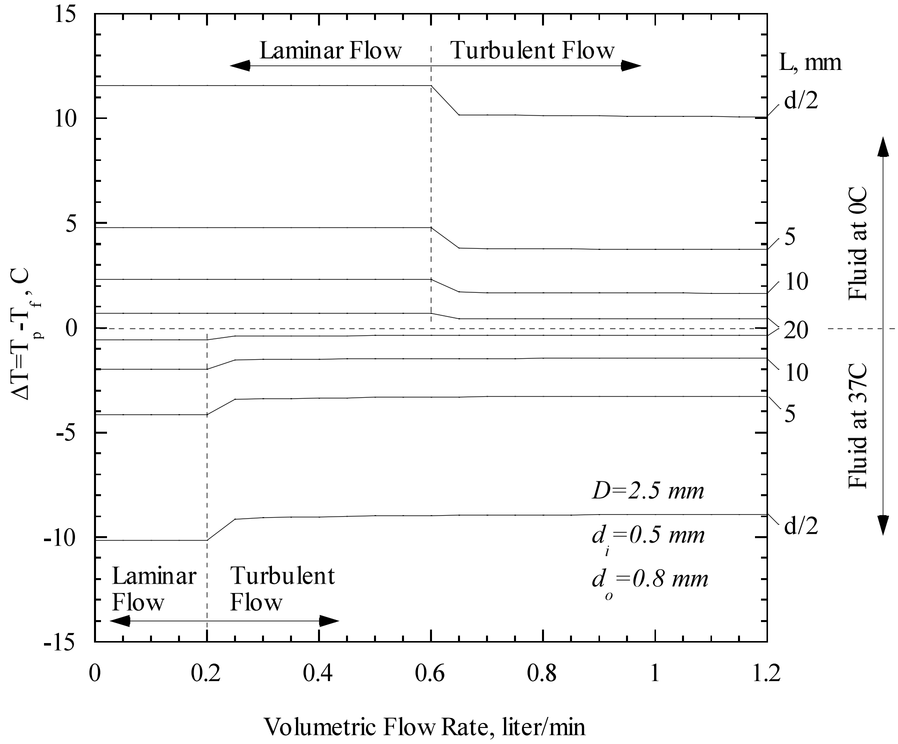

Figure 3: Temperature difference between the thermocouple tip and the fluid temperature, ΔT_p, for an inner tube diameter of 2.5 mm, thermocouple wire diameter of 0.5 mm, and electrical insulation thickness of 0.15 mm; where *L* is the immersed length of the thermocouple.

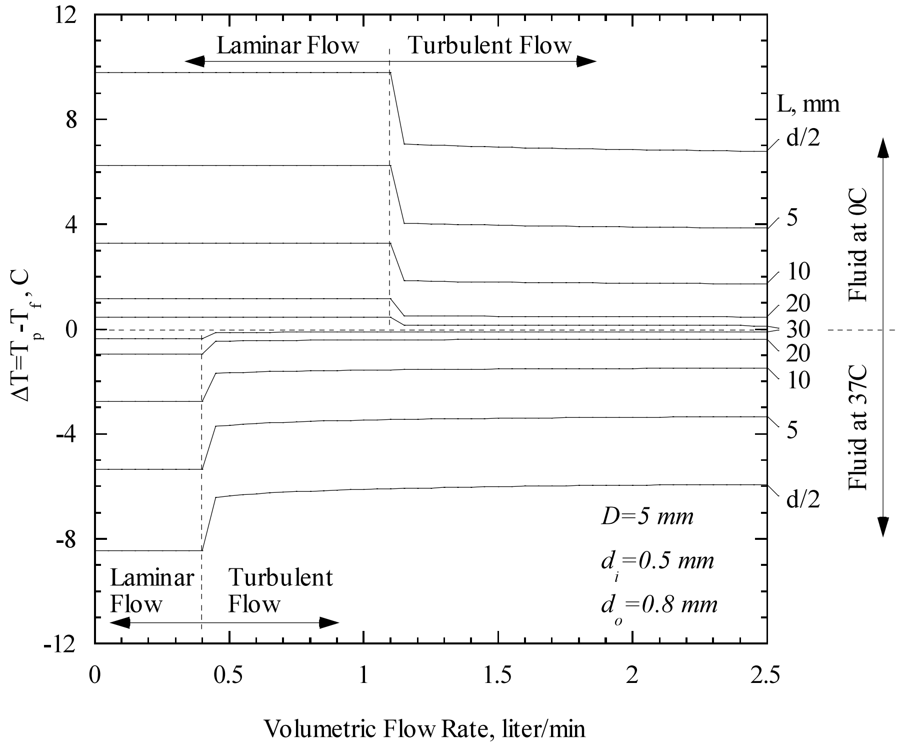

Figure 4: Temperature difference between the thermocouple tip and the fluid temperature, ΔT_p, for an inner tube diameter of 5 mm, thermocouple wire diameter of 0.5 mm, and electrical insulation thickness of 0.15 mm; where *L* is the immersed length of the thermocouple.

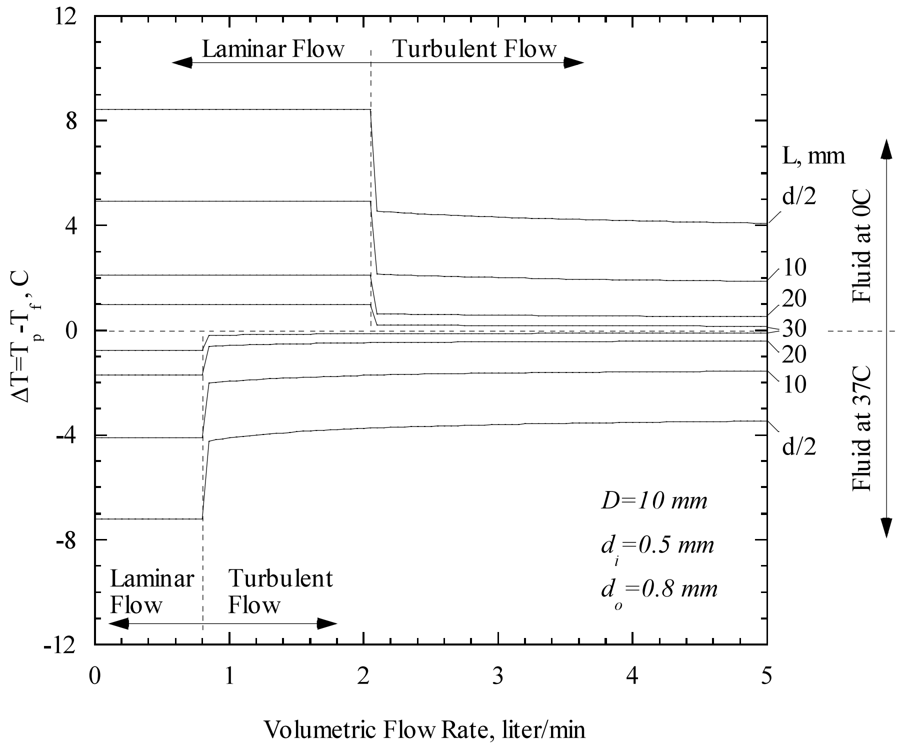

Figure 5: Temperature difference between the thermocouple tip and the fluid temperature, ΔT_p, for an inner tube diameter of 10 mm, thermocouple wire diameter of 0.5 mm, and electrical insulation thickness of 0.15 mm; where L is the immersed length of the thermocouple.