Science Behind Sudoku

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and
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Single Position Technique

- A cell must be assigned a digit if in its row/ column/ block, that is the only position available to the digit.
3 in C6; 4 in R2
Single Candidate Technique

- If there is only one possible candidate for a cell, it must be assigned.
R1C5; then R6C5
Candidate Lines Technique

• If cells accepting a digit in a block are localized to a row (column), remove that digit from rest of the row (column)

• If cells accepting a digit in a row (column) are localized to a block, remove digit from rest of the block.
6 in B7 (localized to line)

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3 in C5 (localized to block)

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3/18/2014
Multiple Lines Technique

- If two blocks at the same level have a digit confined to two lines, the third block must have the digit in the remaining line.
5 in B1 and B3
Naked Pair/Tuples Technique

• If for a group of $k$ inter-dependent cells, the set of all candidates is of size $k$, remove those $k$ digits from everywhere else in the line or block.

• Usually look for $k=2,3$. 
\{1,4\} in C5 as well as B5
\{2, 7, 9\} in R2
Find a naked tuple in B9
Hidden Subsets Technique

• If for a group of k digits, the set of all locations in a block or line is of size k, remove all other candidates in those k locations.

• Duality with naked tuple technique.
C7 – \{7,9\} form hidden subset while \{4,6,8\} form naked tuple
R5 – find a hidden subset

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XWing Technique

- 1 digit : 2 cells/column for 2 columns

- If these 4 cells form vertices of a rectangle, remove the digit from other cells in those rows.
7 in C5 and C9

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**Figure 4.1.** Rule 4.1 at work. In gray, candidates that have been eliminated by other techniques.
Swordfish Technique

- Generalize XWing to k lines.
- Digit occurs in 2 cells/line for k lines.
- Edge-parallel path between the 2k cells.
Digit 1 in rows 4, 5, and 9
References

• Examples from “A to Z of Sudoku” by Narendra Jussien

• “Logical World of Puzzles”, blog by ex-National Champion Rohan Rao (~World Rank 15)