

# DECISION DIAGRAMS FOR DISCRETE OPTIMIZATION

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## TUTORIAL SPEAKER

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## OVERVIEW

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In this tutorial, I will present an overview of the recent successful applications of multivalued decision diagrams (MDDs) to discrete optimization problems, with applications to constraint programming, scheduling, and integer optimization.

In the first part of the tutorial, I will describe how constraint propagation algorithms can be extended to operate on MDDs. By propagating MDDs instead of variable domains, more structural information can be communicated between constraints, which can lead to substantial speedups.

The second part will focus on the use of MDDs in the context of constraint-based scheduling. In particular, we discuss how MDDs can be effectively applied to solve complex disjunctive scheduling problems, and are able to outperform state-of-the-art industrial CP solvers by orders of magnitude in certain cases.

The third part of the tutorial discusses the application of limited-size MDDs to obtain upper and lower bounds for integer optimization problems. The resulting discrete bounds can be much stronger than continuous bounds based on Linear Programming, while being faster to compute.

## BACKGROUND READING

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It is assumed that the audience has basic knowledge about constraint satisfaction problems and/or discrete optimization problems. No detailed knowledge about constraint programming, scheduling, or integer programming is required. Specific key references for the topics that will be discussed in the tutorial are:

- Constraint propagation with relaxed MDDs was first introduced by Andersen et al. [1]. A more systematic overview is provided in [5].
- A detailed study on the use of MDDs for constraint-based scheduling can be found [4].
- The use of BDD *relaxations* for finding optimization bounds is detailed in [2]. The use of BDD *restrictions* is described in [3].

All papers (or preprints) can be downloaded from the following website

<http://www.andrew.cmu.edu/user/vanhoeve/mdd/>

This website also contains prior presentations, source code, and experimental data.

## REFERENCES

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- [1] H.R. Andersen, T Hadzic, J.N. Hooker, and P. Tiedemann. A constraint store based on multivalued decision diagrams. In *Proceedings of the International Conference on Principles and Practices of Constraint Programming (CP)*, volume 4741 of *Lecture Notes in Computer Science*, pages 118–132. Springer, 2007.
- [2] D. Bergman, A. A. Cire, W.-J. van Hoeve, and J. N. Hooker. Optimization Bounds from Binary Decision Diagrams. *INFORMS Journal on Computing*, 2013. To appear.
- [3] D. Bergman, A.A. Cire, W.-J. van Hoeve, and T. Yunes. BDD-Based Heuristics for Binary Optimization, 2013. Under review.
- [4] A. A. Cire and W.-J. van Hoeve. Multivalued Decision Diagrams for Sequencing Problems, 2013. Under review.
- [5] S. Hoda, W.-J. van Hoeve, and J. N. Hooker. A Systematic Approach to MDD-Based Constraint Programming. In *Proceedings of the International Conference on Principles and Practices of Constraint Programming (CP)*, volume 6308 of *Lecture Notes in Computer Science*, pages 266–280. Springer, 2010.