**Session 1368**

# Initial Data from a Statics Concept Inventory

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## Introduction

Engineering Statics is a pivotal course in a number of engineering disciplines. Statics lays the foundation for subsequent courses, namely Dynamics and Strength of Materials. Moreover, Statics and these follow-on courses are the basis for engineering design and practice. Instruction in Statics is worthy of significant attention.

While a variety of topics and problems are addressed in typical Statics textbooks, problems focusing on interactions between connected bodies are particularly important. This class of problems, which takes students significantly beyond what they learn in physics, offers experience that is most relevant to applying Statics to practical engineering systems. Recent work by the author[1] has set out to identify and organize the conceptual content of Statics, with particular focus on those concepts that underlie multi-body Statics problems. Four clusters of concepts were proposed, along with a set of skills for implementing those concepts. While mathematical skills, (e.g., resolving and combining forces and computing moments due to forces), are needed for Statics, for our purposes here they are not a part of the conceptual content of Statics. The concepts and skills were then mapped to a set of typical errors which students are observed to make. The relationships to concepts and misconceptions in freshman Newtonian mechanics in physics[2-4] were discussed.

We seek an efficient and accurate means of measuring student understanding of these concepts. Ideally, we would be able to detect which concepts are well in hand, and for concepts that are not understood, whether there is evidence of a consistent misconception. To this end, the development of a concept inventory has been undertaken. There has been some work along this direction by others relevant to Statics[5,6], as well as for a variety of engineering and related subjects[7], although there has not yet been an attempt at a complete inventory in the case of Statics.

## Concepts of Statics

The conceptual content of Statics is subtle and can be viewed on a number of levels. It should be the goal of the mechanics education community ultimately to understand student conceptions of Statics at the most fundamental level, and to determine how those conceptions can explain various observations of student performance. This is not an easy task and will require much time and effort. In one effort to organize the central concepts of Statics, Steif[1] has proposed viewing concepts as forming four clusters as follows:

C1. Forces are always in equal and opposite pairs acting *between* bodies.

C2. Distinctions must be drawn between a force, a moment due to a force about a point, and a couple. Two combinations of forces and couples are statically equivalent to one another if they have the same net force and moment.

C3. The possibilities of forces between bodies that are connected to, or contact, one another can be reduced by virtue of the geometry of the connection and/or assumptions on friction.

C4. Equilibrium conditions always pertain to the external forces acting directly on a chosen body, and a body is in equilibrium if the summation of forces on it is zero and the summation of moments on it is zero.

In addition, there are skills associated with carrying out mathematical operations, such as resolving or combining forces and finding moment moments due to forces. There are also less acknowledged skills such as recognizing the distinct parts making up a mechanical system and discerning how they are connected to one another. Being adept at a skill means the ability to carry it out properly when told to so. By contrast, having a concept in hand is, for example, knowing that, and proceeding as if, forces are always interactions between two bodies, or distinguishing forces, moments and couples and treating each appropriately.

**From Concepts and Errors to Questions in a Concept Inventory**

Each of the questions in the Statics Concept Inventory focuses on a major conceptual task faced in Statics, and the distracters (wrong answers) were devised to single out a distinct error made by students which could have a conceptual basis. These errors, which were organized into categories[1], were arrived at through several means, including the experience of the author as an instructor and those of colleagues at two universities, also long time instructors of Statics. Errors were also based on extensive analysis of written solutions to Statics problems requiring the use of multiple Statics concepts. Some of those solutions were from students just beginning a Statics course (who had some prior experience with Statics in a freshman engineering course). Examples of errors from those solutions are reported along with the above concept organization[1]. Solutions to a second set of problems that were analyzed were from students who had completed a Statics course, and therefore represent conceptual errors which persist after a full semester of instruction in Statics. The analysis of these problems was conducted by the author as part of a comparison between performance on an earlier version of this inventory and other measures of performance in Statics[8].

Again, the inventory seeks to identify errors that are discrete failures that instructors can observe in students' work. (Examples of these errors are given below when Concept Questions are described.) As pointed out[1], however, some of these errors could be the result of errant thinking with respect to more than one concept cluster from above. Thus, while the errors are quite discrete, narrow and conceptual, as will be seen below, there may be a yet more fundamental level of conceptual understanding that could be mined. For example, interviews of students conducted by this author have offered evidence that some students think of forces as associated with one object alone, rather than as an interaction between objects. Unfortunately, the author has not yet been able to devise multiple-choice questions which probe that idea in isolation.

Finally, as pointed out, the inventory is intended to detect errors associated with incorrect concepts, not with other skills (e.g., mathematical) necessary for Statics. Most questions do not involve numbers, although some do. For questions involving some computation, each answer still reflects a different conception. In addition, each wrong answer represents the correct computation based on an incorrect conception, and the computations themselves are trivial. Hence, such questions ought to discriminate between conceptual understanding and computational ability for most students.

**Distribution of Questions on Statics Concept Inventory**

Questions on the Statics Concept Inventory have been separated into 5 groups of questions. The association between questions in each group and the concept clusters above, as well as the conceptual errors which are captured by each group of questions, is as follows:

- *Free body diagrams (5 Questions)*

These questions capture a combination of concept cluster C1 on the inter-body nature of forces and the first half of cluster C4, namely, that equilibrium always pertains to a body. Typical errors include: Leaving a force off; including a force which is exerted between two bodies, both of which are part of the diagram; including a force which is exerted between two bodies, neither of which is part of the diagram; failure to take the forces between two bodies to be equal and opposite.

- *Static equivalence between different combinations of forces and torques (3 Questions)*

These questions capture concept cluster C2 on the distinctions between force, moment and couple and their inter-relations. Typical errors include: presuming a force can be moved perpendicular to its line of action and have no change in its effect; presuming two combinations of loads can be statically equivalent even if the net force is not equal; presuming that a couple at one point is statically equivalent to a different couple at another point

- *Type and direction of loads at connections (3 questions for each of 4 different situations of roller, pin in slot, general pin joint, and pin joint on a two-force member)*

These questions capture one aspect of concept cluster C3, namely reasoning about the forces at various joints or connections when the usual frictionless assumption is made. Typical errors include: incorrectly taking the direction of the force at a connection to be known (assuming it balances some applied load or acts in some direction relative to the member); incorrectly taking the direction of the force at a connection to be unknown (failing to recognize the limits that a slot or roller or presence of a two force member places on the force); incorrectly assuming that a couple can be exerted at a connection (confusing the moment due to a force about some other point with a couple acting at the connection).

- *Limit on the friction force and its trade-off with equilibrium conditions (3 Questions)*

These questions capture a second aspect of concept cluster C3, namely reasoning about the forces between contacting bodies which are modeled using Coulomb friction. Typical errors include: presuming the friction force of the contacting body is equal to $\mu N$ (rather than the force necessary to maintain equilibrium, which could be less than $\mu N$); presuming the friction force of the contacting body to equal the difference between the applied (or balancing) force and $\mu N$; incorrectly setting N (if it is relevant) equal to the weight of the immediate contacting body, rather than taking N to equal the value necessary to maintain equilibrium.

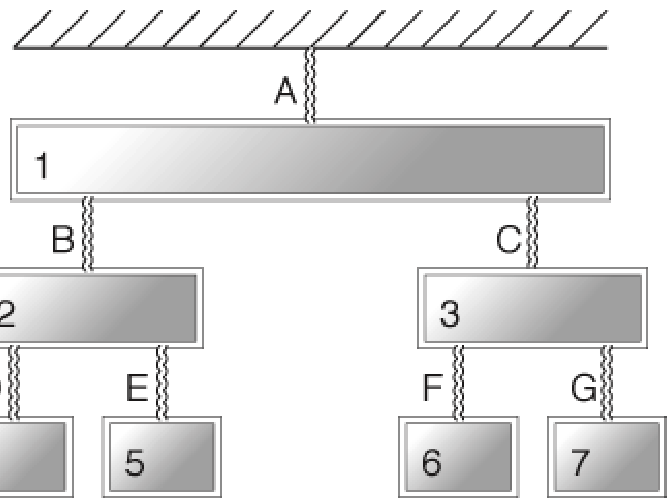
- *Equilibrium conditions (4 Questions)*

These questions capture the second portion of concept cluster C4, namely the necessity for both forces and moments acting on a body to sum to zero.  Typical errors include: not recognizing that the net force is not zero; not recognizing that the net moment is not zero; incorrectly accounting for the contribution of couples to equilibrium.

**Sample Questions from the Statics Concept Inventory**

Below are examples of questions from each group in the inventory as students would see them.  Each is followed by descriptions of the conceptual errors behind each wrong answer.

*Example of question on free body diagrams*
A free body diagram is to be constructed of the assemblage which includes three of the weights ($W_1$, $W_3$ and $W_6$) and the cords connecting them.




Which is the correct free body diagram?

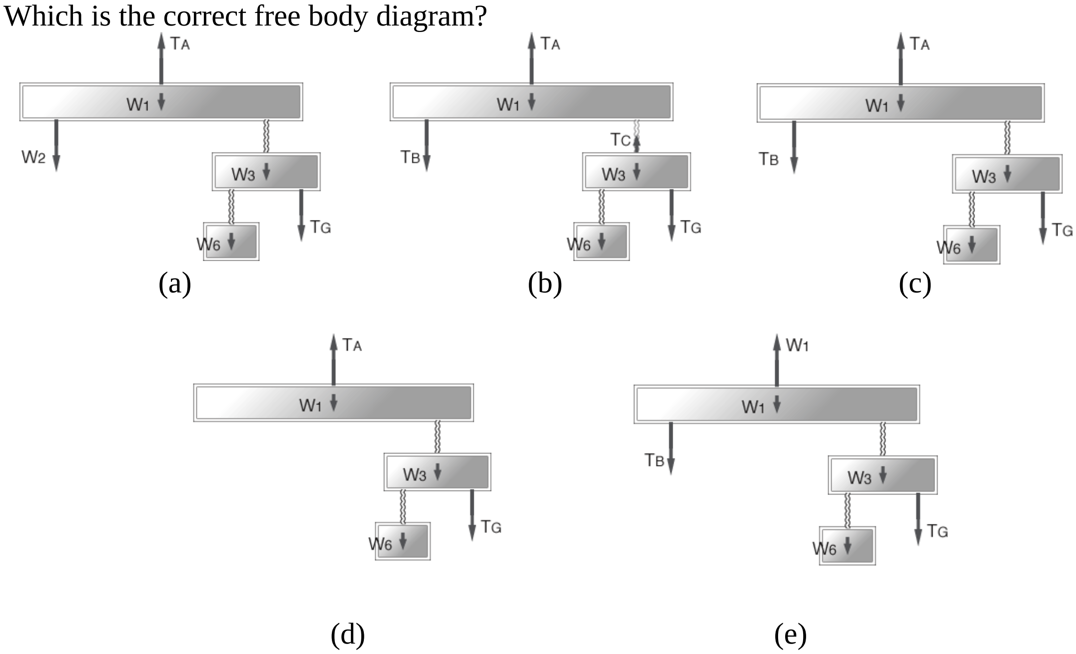
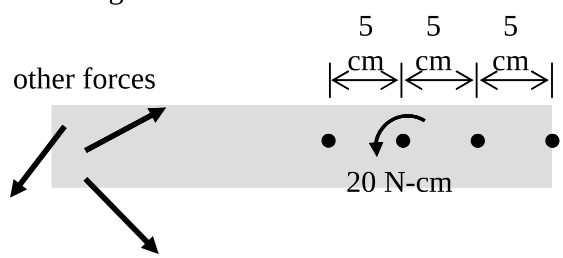



Figure 1. Example of concept question addressing free body diagrams.

Explanations of specific answers: (a) Force between two objects (earth and weight $W_2$), both of which are *not* in the diagram, is falsely presumed to act directly on the body of diagram; (b) Force between two objects (cord C and weight $W_2$), both of which *are* in the diagram, is falsely included in the diagram; (c) Correct answer; (d) Force missing; (e) Premature (and incorrect) application of equilibrium resulted in incorrect value for unknown tension in cord A. Note that the distinct explanations for answers (a) and (e) apply to both.

*Example of question on static equivalency*
One couple of magnitude 20 N-cm keeps the member in equilibrium while it is subjected to other forces acting in the plane at various points (shown at the left). The four dots denote equally spaced points along the member.




Assuming that the same forces are applied at the left, what load(s) could replace the 20 N-cm couple and still maintain equilibrium?

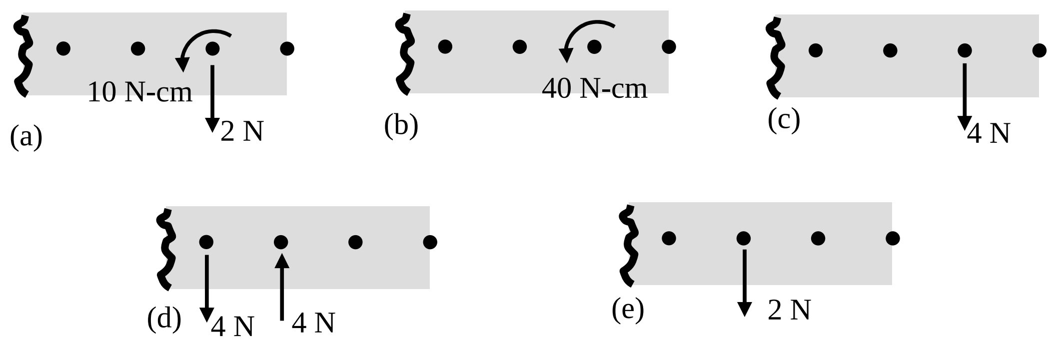



Figure 2. Example of concept question addressing static equivalency.

Although computation is necessary to answer this question, note that each choice of answer could be legitimately arrived at by combining forces, distances and couples appropriately (there are no mathematically improbable answers). This question requires discrimination between forces, moments and couples and understanding the static equivalence of combinations of loads. Explanations of specific answers: (a) A couple alone cannot be equivalent to a force and a couple; (b) Effect of a couple does not change as one moves its position; (c) A couple alone cannot be equivalent to a single force; (d) Correct answer; (e) A couple alone cannot be equivalent to a single force.

*Example of question on loads at connections*
The mechanism (diagram at the left) is acted upon by the downward force shown. A spring acts on the slotted link. Ignore gravity. The pins, which are indicated in black, are well lubricated.

What is the direction of the force exerted by the pin on the horizontal portion of the indicated member? (choices at the right)

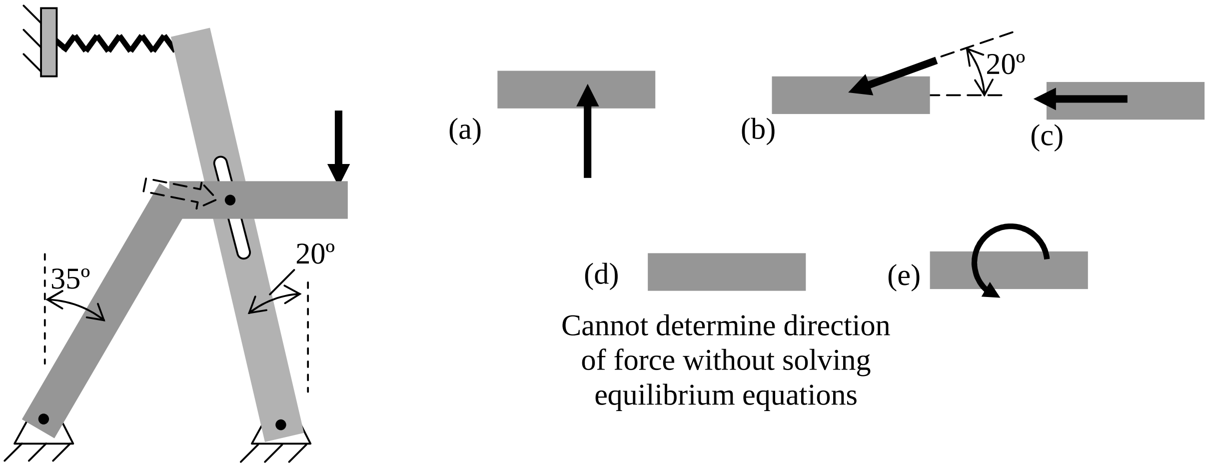
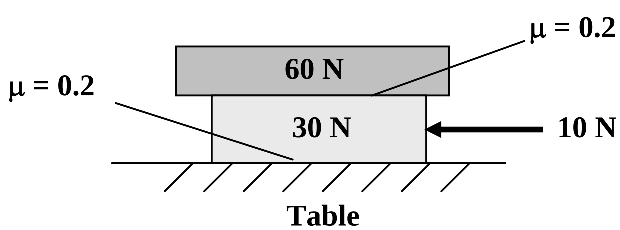



Figure 3. Example of concept question addressing forces at connections.

Explanations of specific answers: (a) Force at connection is chosen to balance the applied force (premature, incorrect application of equilibrium); (b) Correct answer; (c) Force acts along length of member; (d) Pin implies that the force has an unknown direction (say, x and y components), irrespective of other aspects of the connection; (e) The clockwise moment created by the applied force must be balanced by an opposite couple (even though the pin cannot sustain a couple).

*Example of question on limits on friction force*

Two blocks are stacked on a table. The friction coefficient between the blocks and between the lower block and the table is 0.2. (Take this to be both the static and kinetic coefficient of friction). A horizontal 10 N force is applied as shown to the lower block.




What is the horizontal force exerted by the table on the lower block?

(a) 4 N (b) 6 N (c) 8 N (d) 10 N (e) 18 N

Figure 4. Example of concept question addressing friction forces.

Although this problem involves a minor computation, each choice of answer features the correct computation associated with a particular conception. Explanations of specific answers: (a) Force of table is equal to the difference between the balancing force (10) and the friction force μN (with N incorrectly taken to be 30); (b) Force of table is equal to μN (with N incorrectly taken to be 30); (c) Force of table is equal to the difference between the balancing force (10) and the friction force μN (with N correctly found to be 90); (d) Correct answer; (e) Force of table is equal to μN (with N correctly found to be 90).

*Example of question on equilibrium conditions*
The member (shown to the left) is subjected to the force at the lower right corner, and is maintained in equilibrium by a hand gripping the left end (A).

Which of the following (at right) could represent the load(s) exerted by the gripping hand?

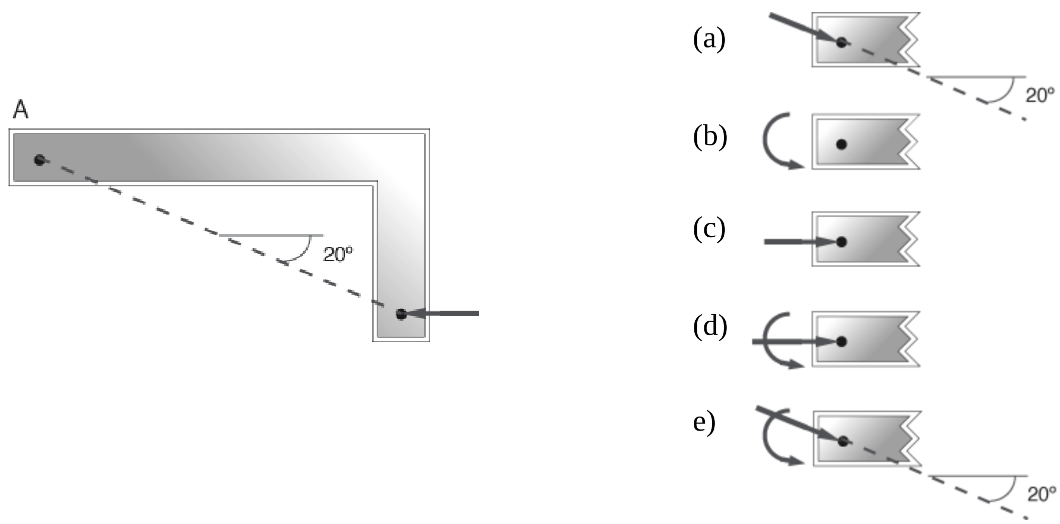



Figure 5. Example of concept question addressing equilibrium conditions.

Explanation of wrong answers: (a) Force balance fails (equilibrium is falsely assumed to hold merely if forces run through a single point); (b) Force balance fails, although moment is apparently balanced by a torque; (c) Moment equilibrium fails (although forces balance); (d) Correct answer; (e) Force balance fails although moment is apparently balanced by a torque.

**Psychometrics based on initial administration of Statics Concept Inventory**

The test described above, having a total of 27 questions, was administered to 125 mechanical engineering students at Carnegie Mellon University on the first day of a sophomore Statics course. Virtually all students had taken the freshman fundamentals of mechanical engineering course that had a 3 week segment on Statics. The psychometrics now presented were based on the results of this administration of this test. From a maximum possible score of 27, the mean score was 10.6, the standard deviation 4.1, the maximum score 22, the minimum 2, and the median 11. The mean and standard deviation, respectively, were for males (102 students) 11.1 and 4.1 for females (23 students) 8.8 and 3.7, and for African-Americans (5 students) 9.4 and 2.4. Interestingly, while there was a continuous distribution in scores among males from the low end to the maximum, except for one female who scored 21, no female scored higher than 13 and no African-American scored higher than 12.

Reliability, as measured by Cronbach's alpha reliability coefficient, was 0.712. This is acceptable evidence of reliability, at least for an initial version, although values nearer to 0.8 would be preferable. On only a single question of the 27 total did more than 70% of the class answer correctly (81% answered that question correctly); on only 8 questions did less than 20% of the class answered correctly. The difficulty of the questions is illustrated in Figure 6.

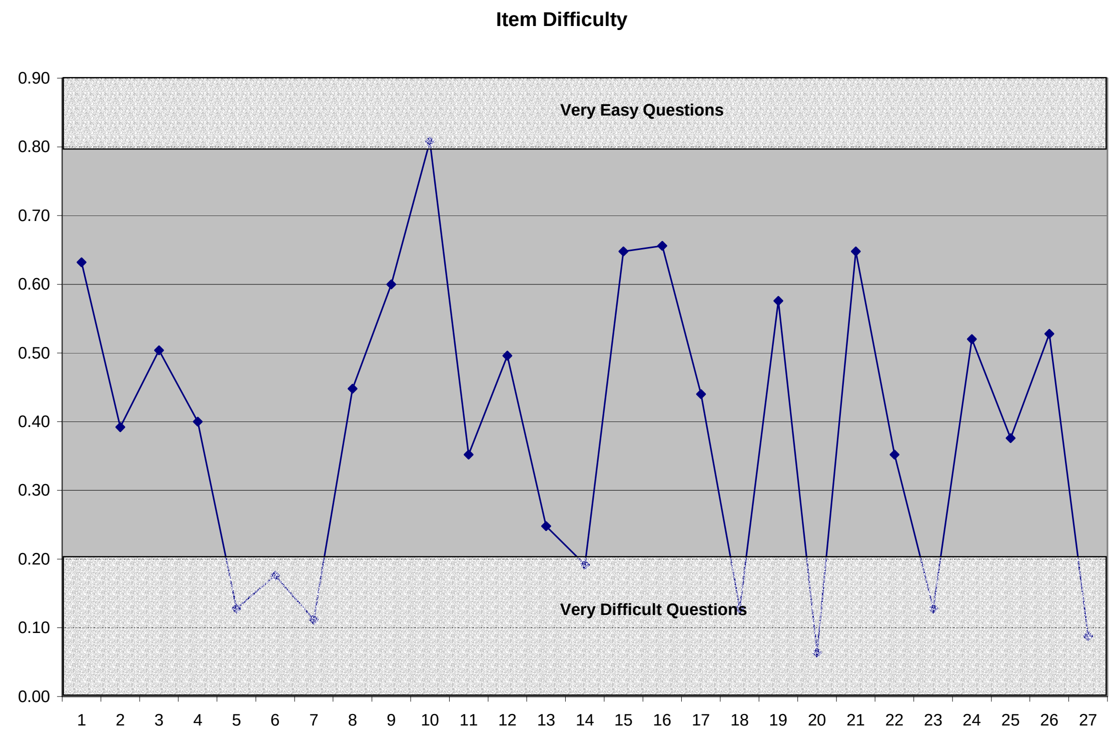



Figure 6. Item difficulty: Fraction of students answering questions correctly.

In another test of quality of the test, it was found that the scores on 9 of 27 questions did not correlate well with the total score (correlation coefficient less than 0.3). Moderate to high item-to-total score correlation coefficients are evidence that the items are good measures of the overall construct of "Statics Conceptual Knowledge". Another test measure, the discrimination scores, is illustrated in Figure 7. (A higher discrimination score, which by definition is in the total range of –1 to 1, signifies that students with overall higher scores are more likely to answer the question correctly than students with lower overall scores.) All but two of the questions with low item-to-total correlation coefficients also had discrimination scores of less than 0.2. The 7 questions with low discrimination scores included three questions testing the understanding of friction and three questions testing the understanding of static equivalence. The remaining question with a low discrimination score, which tests the concept of equilibrium, was found upon review to have wording which was obviously and unnecessarily confusing.

It is no surprise that previous experience would not prepare students for the concept of static equivalence; most students were quite poor at answering these questions, with only 11%, 13% and 9% of students answering them correctly. The poor correlation between scores on friction and overall knowledge of Statics may indicate the prevalence and depth of the misconception over the tradeoff between equilibrium requirements and the upper limit on friction; only 18%, 25% and 35% of students answered these questions correctly. Indeed, from results of an earlier version of the Statics concept inventory[8], this misconception was found to persist in students who had successfully completed a Statics course.

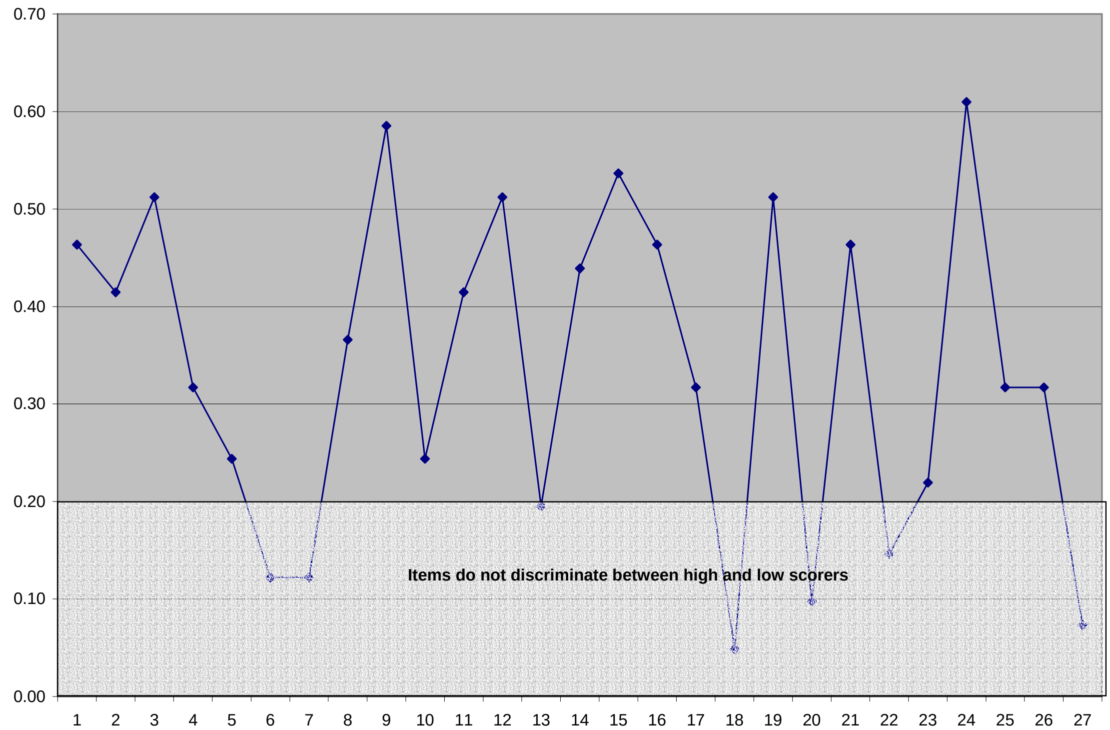



Figure 7. Discrimination scores for questions in concept inventory.

## Potential Uses of Concept Inventories

Concept inventories may be used to improve student learning in many ways, a few of which are pointed out here. When administered at the end of a course, the inventory can provide the instructor with feedback on those concepts that may need more attention in the future. Now, most of the concepts may have already been covered by, say, two-thirds through the course. If the test is administered at that point, and if the results could be analyzed rapidly and provide diagnoses as to conceptual lapses, then remedial exercises might be tailored to address particular lapses. An inventory could be used at the start of a follow-on course (e.g., mechanics of materials), to provide instructors with a picture of the starting knowledge of their students. Finally, the questions themselves might stimulate ideas for instruction that is more conceptually based or suggest ideas for in-class assessment exercises.

## Summary

A multiple-choice Statics Concept Inventory that seeks to measure conceptual knowledge of students in Statics has been developed. Consisting of 27 items, the test examines students in four core areas – free body diagrams, static equivalency of combinations of forces, inferring forces at connections and frictional contact surfaces, and conditions of equilibrium. The alternative (wrong) answers to the questions are intended reflect conceptual errors commonly made by students. This test was administered to 125 mechanical engineering students just entering Statics and the results were analyzed. The analysis showed an acceptable range of difficulty in the questions, and that most questions discriminated effectively between high and low scorers. Further examination of those questions which did not discriminate suggested, in one case, poor question wording and in the other cases perhaps that students have no prior experience with the concept or commonly misunderstand the concept. These uncertainties will be more fully explored when the post-test is administered and analyzed; those results will be presented in the near future.

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