Session 1368

REINVENTING THE TEACHING OF STATICS

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Introduction

Learning in the subject of Engineering Statics deserves significant attention. First, Statics lays the foundation for subsequent courses, namely Dynamics and Strength of Materials, both of which rely heavily on free body diagrams and on recognizing the combined effect of forces. While there are new ideas which are emphasized in engineering dynamics, instructors in this subject inevitably find that some of students' difficulties are rooted in ideas from Statics, such as free body diagrams and working with forces. One of the most fundamental concepts in strength of materials is that of internal loads and its relation to external loads. This idea is wholly within the domain of Statics, yet a concept inventory for mechanics of materials currently under development rightfully points to this as one of its core concepts[1]. Second, Statics, together with these courses, forms the basis for much engineering design and practice. Again, instructors in engineering design[2] lament the difficulties students have in using Statics for the purpose of their course. In sum, all is not well with instruction in Statics.

It appears that the ideas of Statics, if learned at all, are often not learned in ways that would enable their extension beyond the course. Many Statics problems are, at least superficially, set in a context of engineering applications and hardware, as evidenced by the problems in many Statics textbooks. Nevertheless, students tend to be largely focused on arriving at a mathematical solution, with little attention paid to how the solution is related to the physical context of the problem. Perhaps this is not surprising given strong emphasis on mathematical manipulation evident in the early chapters of most Statics textbooks. One instance where the practicalities of the actual engineering hardware beg to be considered is that of the mutual forces exerted by bodies connected in various ways. Yet, even here, textbooks largely reduce the treatment to a set of tables which students have no recourse but to memorize.

General Approaches to Improving Learning Which are Relevant to Statics

To improve the situation, instructors of Statics must gather all the tools at their disposal. Several generally accepted approaches to improving learning outcomes are potentially quite relevant to Statics. Students, who are actively engaged in learning, learn more. While it can be more time consuming, ideas that are reached through discovery may be more firmly grasped than those that are acquired through typical lecture or textbook. Students learn through a constant iterative process of assimilating new information and testing out their evolving understanding with feedback from instructors; thus the integration of assessment into the learning process can be of great benefit. This process is aided when new information is placed in the context of knowledge which students have previously acquired; that is, students build on what they already know. Students can learn a great deal from one another; collaboration, if harnessed appropriately, is a powerful tool in learning. Finally, for many subjects in the sciences or technologies, physical referents or manipulatives can serve to enhance learning. Instructional methods which draw together many of these techniques have recently been introduced in Statics[3].

Implications of Conceptual Difficulties Associated with Statics

While the techniques alluded to above can be valuable, how should Statics instruction reflect the conceptual difficulties which are peculiar to Statics? Statics instruction faces the following dilemma. There are a significant number of concepts, which go beyond those addressed in freshman physics, which students must learn and use, in combinations, to solve Statics problems. Moreover, these concepts must be learned in a deep way in the sense of relating the relevant symbols and theory to what they physically represent[4]. Yet, it is known from research into physics education[5-7] that students doubt the existence of forces between unmoving, relatively rigid, inanimate objects. How is one to engage students in learning the concepts of Statics when its core element – the forces between parts of machines and structures – are not viewed as real by significant numbers of students?

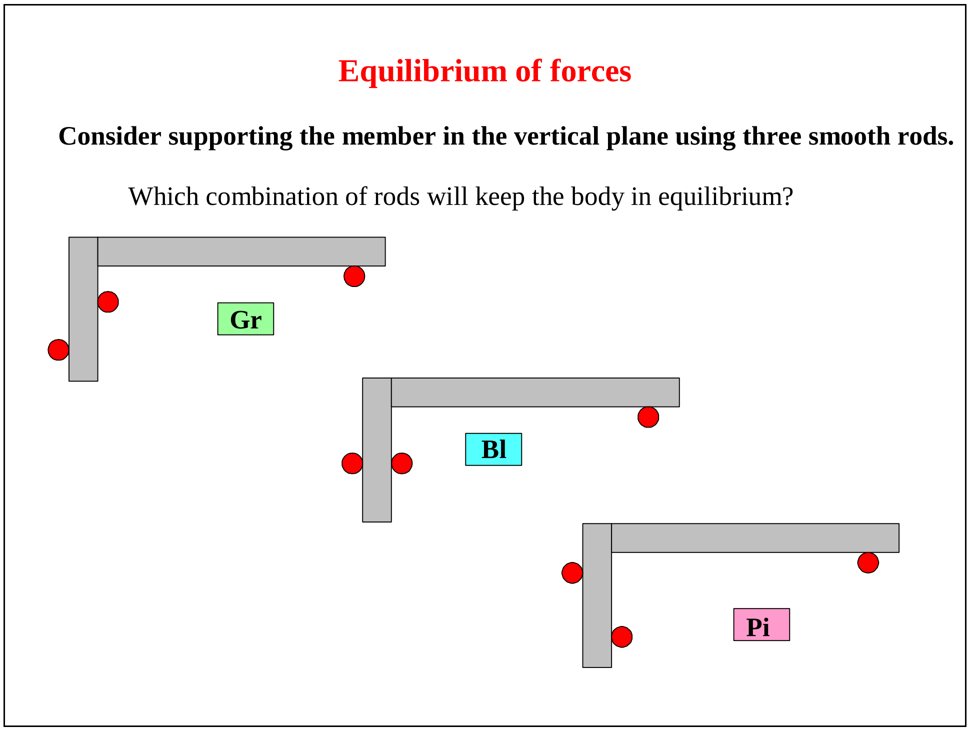
To overcome this dilemma, we have completely reframed our instruction in Statics. The fundamental elements in our instructional approach are: (i) to teach the concepts first entirely in the context of situations in which the forces are indeed real to students, namely ones they can experience by the senses of touch and sight (by sensing deformation or motion); and (ii) to decouple concepts from each other and treat them sequentially, with new concepts building on those which have already been covered.

Thus, we address all the basic concepts of statics (forces, moments, couples, static equivalency, free body diagrams, equilibrium in 2-D and 3-d, friction) in sequence, without recourse to forces between inanimate objects: students feel all the forces. Some elements of this sequence were presented previously[8-9]. Only after that are students gradually introduced to contacts between inanimate objects; each time, the student first exerts the force by hand, prior to witnessing its application by another object (e.g., applying a nonuniform pressure to a member manually, prior to supporting it by another object.) As explained below, this gradual transition from manually exerted forces to contact between inanimate objects prepares students for a far sounder understanding of the loads acting at connections between bodies.

Sequence of Concepts

As pointed out, we have reformulated the concepts of Statics so as to build sequentially on one another. Although, our lecture-by-lecture schedule involves a fine-grained breakdown of concepts, and includes a series of modules to be used in the classroom, for the purposes of this paper we present a higher-level view of the development of concepts. We explain these steps in a concrete way by showing excerpts of PowerPoint presentations through which we address these concepts in the classroom. More about the classroom implementation is given below. While we focus here on *Statics* concepts, it must also be pointed out that the requisite mathematical skills, such as vector algebra, need to be developed as appropriate if the desired level of problem solving ability is to be attained.

1. Equilibrium of bodies requires consideration of both the translational and rotational effects of forces; that is, that the forces and moments must balance. We consider simple situations (Figure 1), some based on an L-shaped object used extensively in class, in which a set of forces maintains the body in equilibrium



Equilibrium of forces

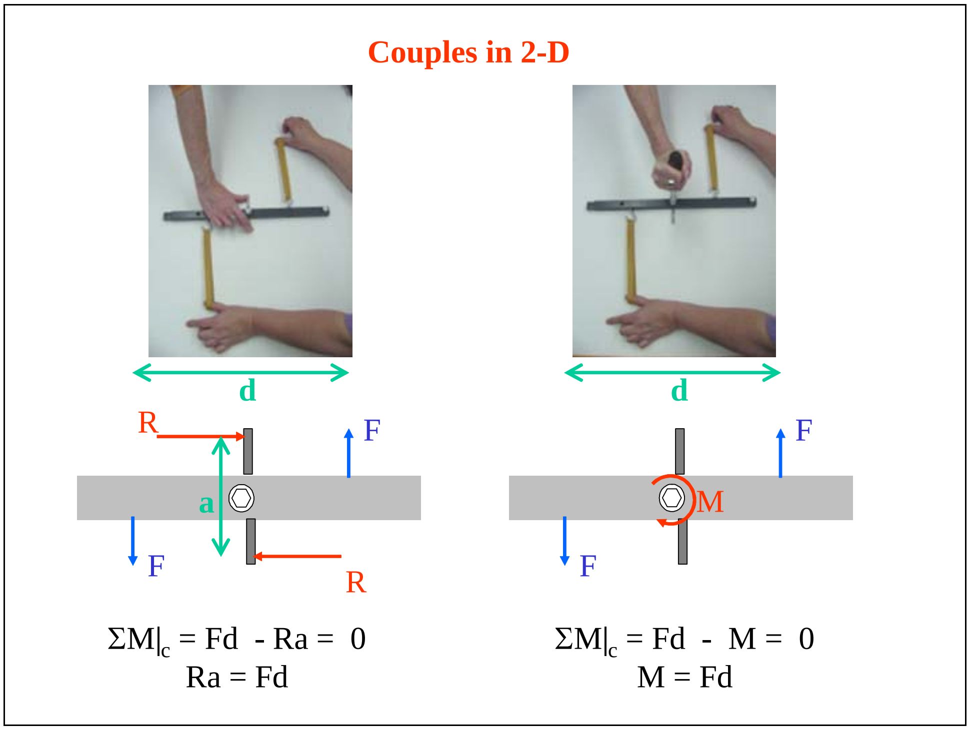
Consider supporting the member in the vertical plane using three smooth rods.

Which combination of rods will keep the body in equilibrium?

Fig. 1

2. Statics involves combinations of forces; one combination of forces can have the equivalent effect as many other combinations. In particular, we focus on combinations of forces which have a tendency only to rotate a body, and the representation of this combination as a couple (Figure 2a). Supporting an elongated object at one end can be seen to occur in many ways (Figure 2b), all of which are declared to be statically equivalent. These concepts are pursued first in 2-D situations, and later in 3-D. For example (Figure 2c), loads are also observed to be statically equivalent by virtue of the deformations they cause. Once students can use both

forces and couples, they consider equilibrium of objects acted upon by such combinations (Figure 2d).

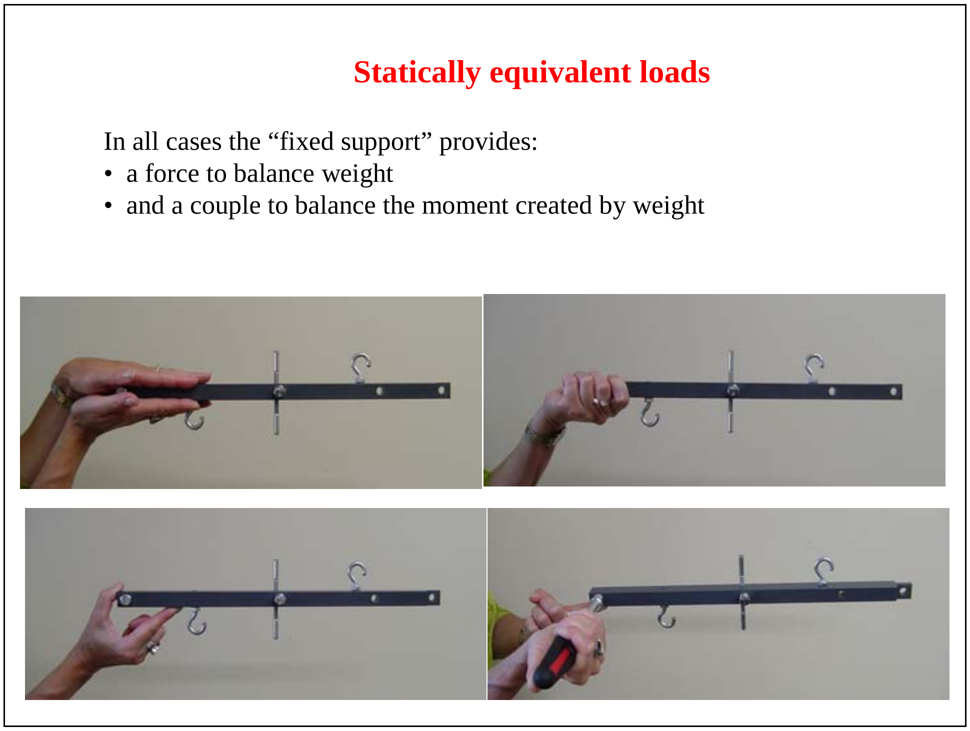


Couples in 2-D

$$\Sigma M|_c = Fd - Ra = 0$$
$$Ra = Fd$$

$$\Sigma M|_c = Fd - M = 0$$
$$M = Fd$$

Fig. 2a

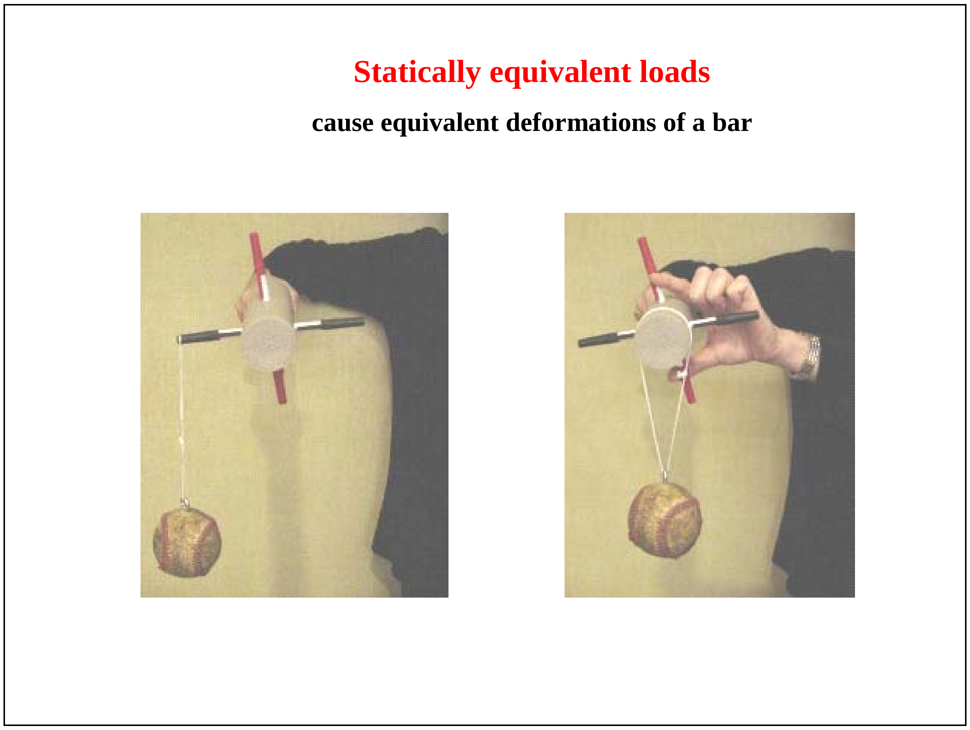


Statically equivalent loads

In all cases the “fixed support” provides:

- a force to balance weight
- and a couple to balance the moment created by weight

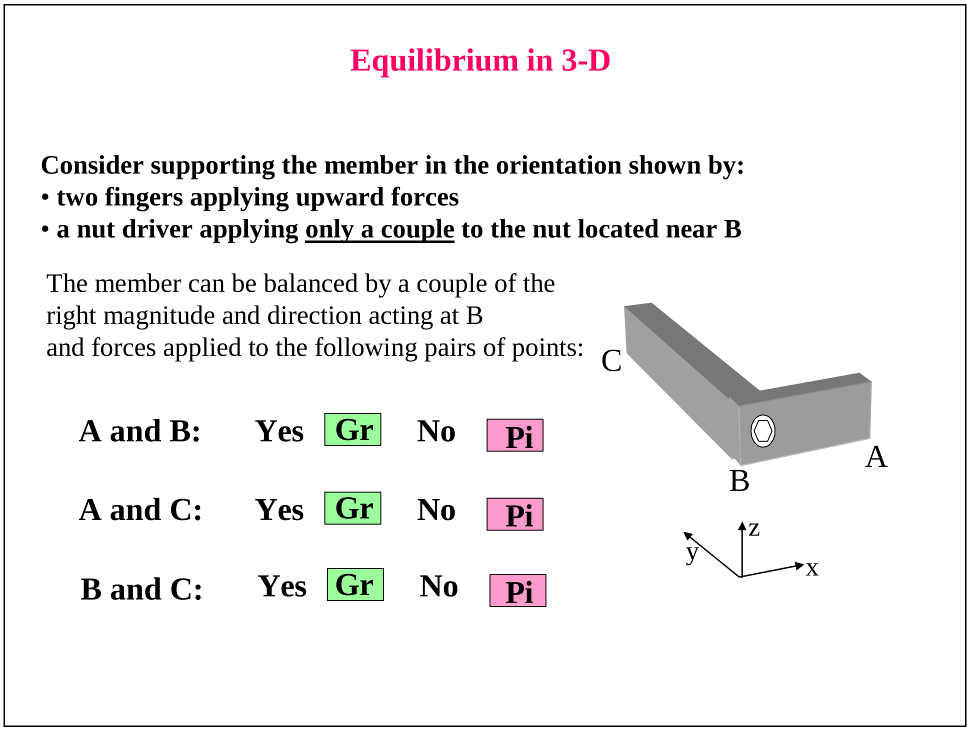
Fig. 2b



Statically equivalent loads

cause equivalent deformations of a bar

Fig. 2c



Equilibrium in 3-D

Consider supporting the member in the orientation shown by:

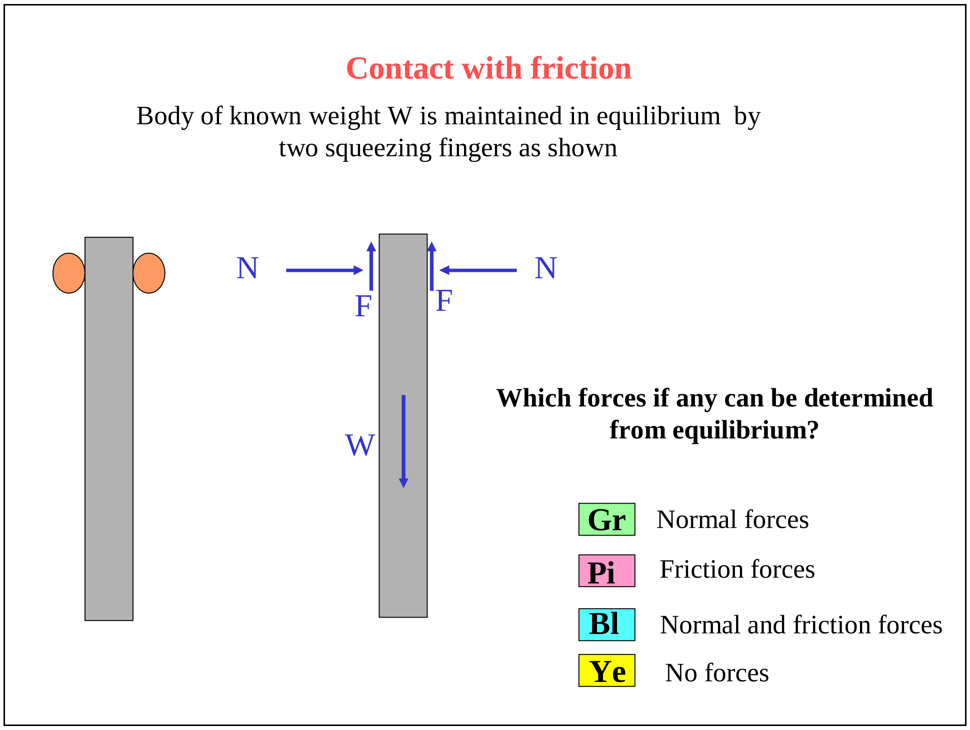
- **two fingers applying upward forces**
- **a nut driver applying <u>only a couple</u> to the nut located near B**

The member can be balanced by a couple of the right magnitude and direction acting at B and forces applied to the following pairs of points:

A and B:	**Yes**	**Gr**	**No**	**Pi**
A and C:	**Yes**	**Gr**	**No**	**Pi**
B and C:	**Yes**	**Gr**	**No**	**Pi**

Fig. 2d

3. To this point, we have focused on situations in which the frictional forces are low. Next, we consider the distinction between forces acting tangential to surfaces (friction) and normal forces. Concepts involving friction are conveyed again using simple objects that can be balanced by forces or couples applied by the hands. One idea is that frictional forces attempt to resist motion and in some instances can maintain a body in equilibrium; the distinction between the frictional force and the *upper limit* on the frictional force (μN) is emphasized (Figure 3a). Also important is the idea that combinations of friction forces can provide couples (Figures 3b and 3c). We view it as critical to set the stage for connections between parts, which we treat later. There we simplify the possible loads at a connection (say no couple at an ideal pin joint). Only if students have had experience observing what friction forces *can* produce are they able to understand the implications of the *neglect* of friction.



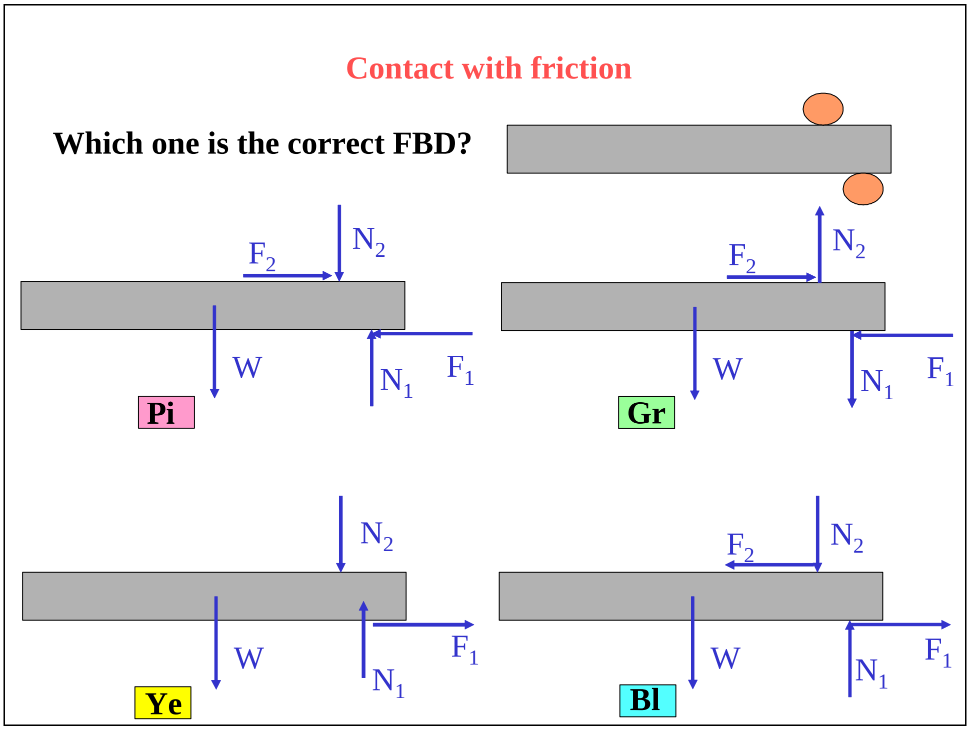
Contact with friction

Body of known weight W is maintained in equilibrium by two squeezing fingers as shown

Which forces if any can be determined from equilibrium?

Gr Normal forces
Pi Friction forces
Bl Normal and friction forces
Ye No forces

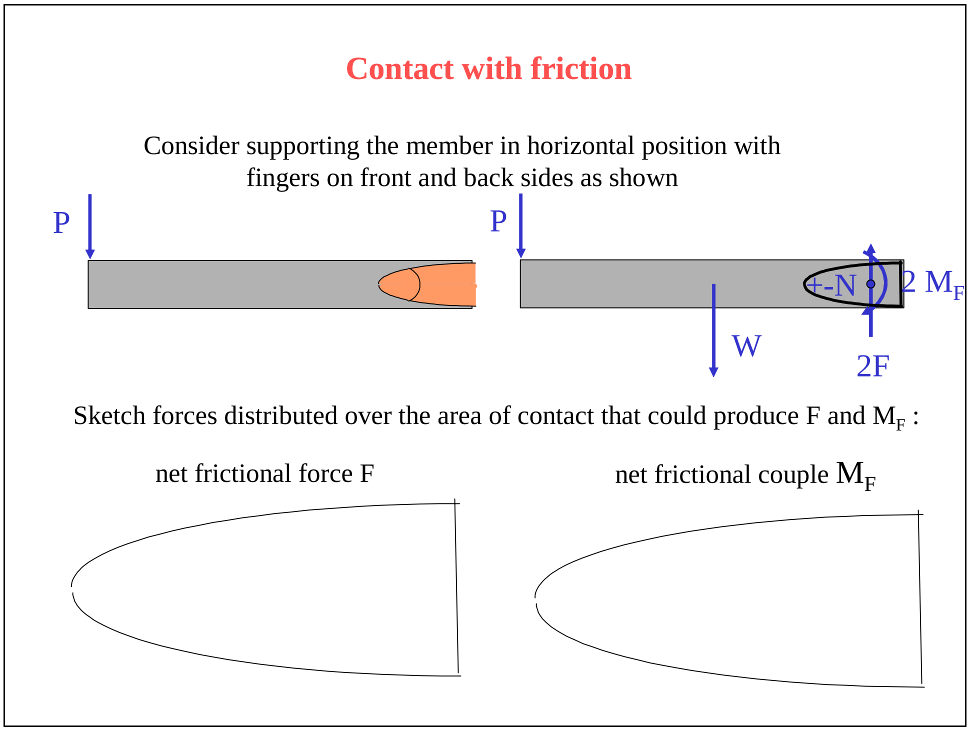
Fig. 3a



Contact with friction

Which one is the correct FBD?

Fig. 3b



Contact with friction

Consider supporting the member in horizontal position with fingers on front and back sides as shown

Sketch forces distributed over the area of contact that could produce F and M_F :

Fig. 3c

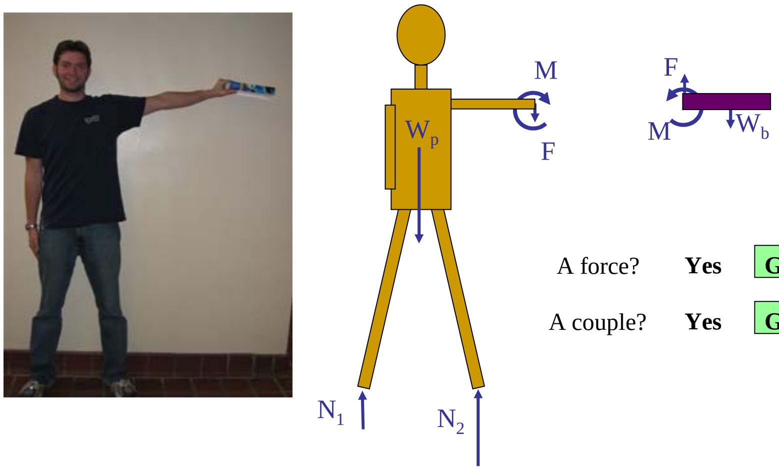
4. Fewer concepts are more pivotal to Statics, and the basis for more errors, than the ideas that every force acts between two bodies (usually in contact), and that a body must be chosen before equations of equilibrium can be derived (equilibrium always pertains to a body). Again, with recourse only to forces that can be *felt*, we have students gain experience in dismembering systems, associating with each force the two acting bodies, and considering equilibrium for various subsets of the system. One example of this is shown in Figure 4, in which a human body which supports the object; these can be considered as single body, or can be separated, when imposing equilibrium.

FBD of interconnected bodies

Draw the FBD of your body only and the FBD of the book.

What do you feel the book is exerting on your hands?

What do you feel you are exerting with your hands to keep the book in equilibrium?

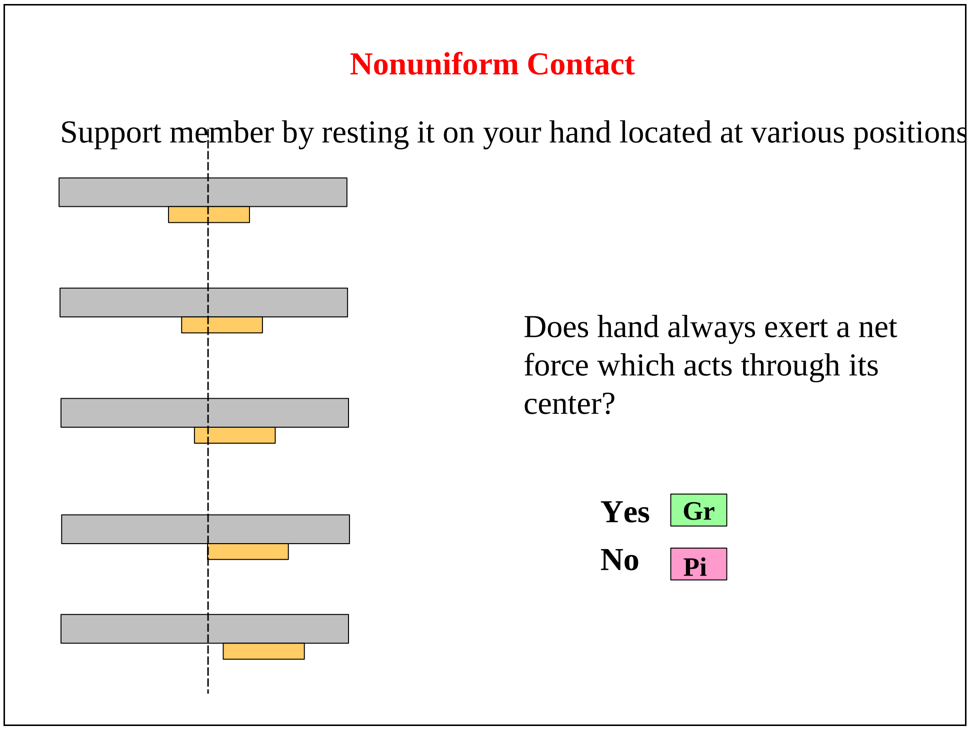


A force? **Yes** **Gr**

A couple? **Yes** **Gr**

Fig. 4

5. When bodies contact one another over an extended surface (rather than at a point), their interaction, can be viewed as a distribution of force. The differing intensities when the same load is acting over different areas can be readily felt by students (e.g., by contrasting the feel of supporting a package either by grasping a wide or narrow strap). Students can also observe how a distribution of contacting force can be uniform or non-uniform, both through supporting a board at different locations by hand (Figure 5a) and through observing deformation of the supporting foam (Figure 5b).



Nonuniform Contact

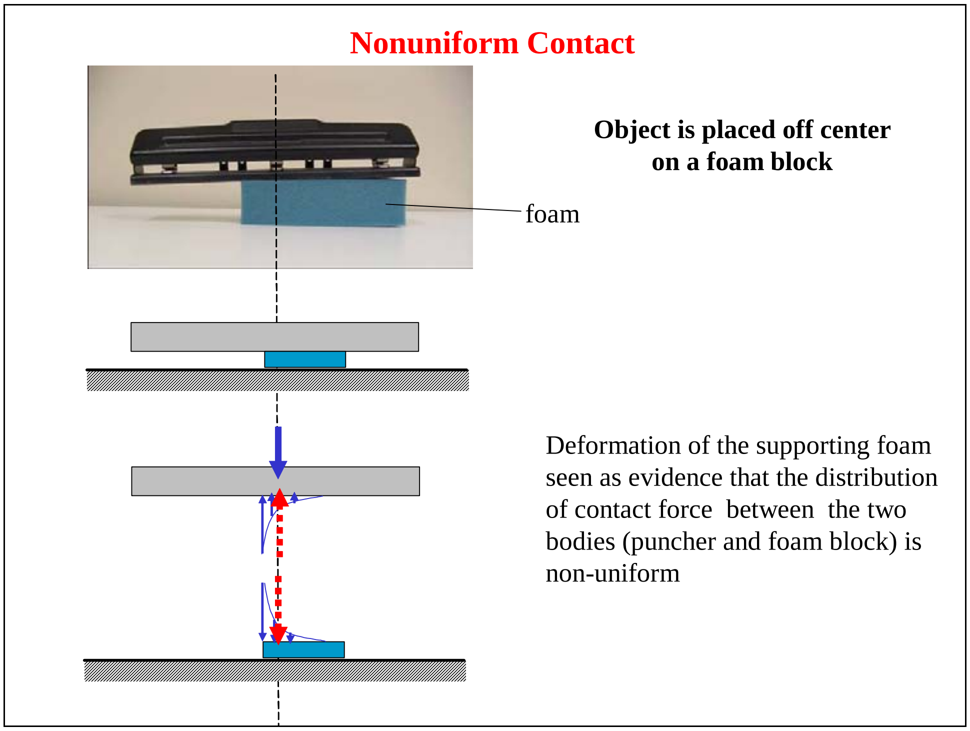
Support member by resting it on your hand located at various positions

Does hand always exert a net force which acts through its center?

Yes **Gr**

No **Pi**

Fig. 5a



Nonuniform Contact

Object is placed off center on a foam block

Deformation of the supporting foam seen as evidence that the distribution of contact force between the two bodies (puncher and foam block) is non-uniform

Fig. 5b

Following the above, our sequence turns to consider interactions between inanimate objects.

6. Equilibrium of contacting bodies for which the position and direction of the net forces are obvious.
   Here students are first solving problems that require separation of contacting *inanimate* bodies (Figure 6). Like in Step 4 above, students must learn to exercise due care in attributing each force to specific pair of bodies and in clearly identifying the body upon which equilibrium is imposed. This exercise also attempts to dispel the common notion of students that the normal force is equal to the weight, rather than merely equaling whatever is necessary to maintain equilibrium.

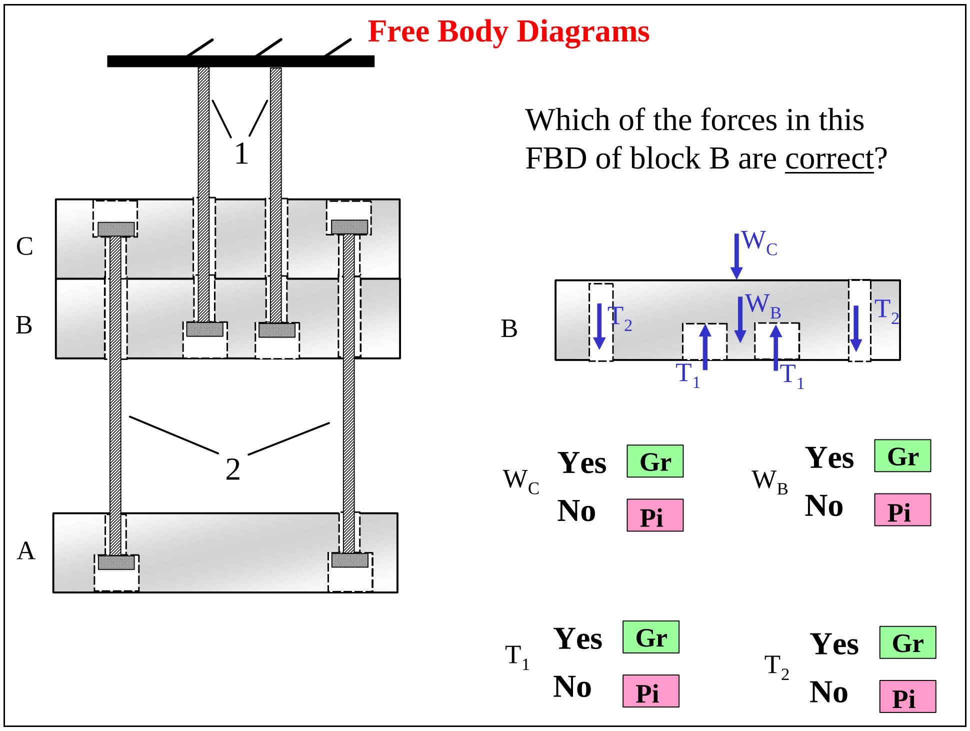
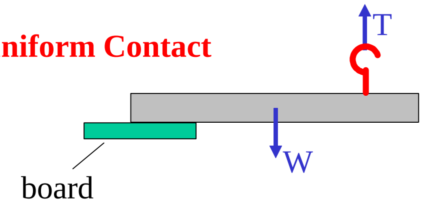


Fig. 6

7. Normal contact between bodies, with distributed forces having a net force not acting at the center of contact.
   Students can now draw together the ideas of Step 5 regarding possible nonuniformity of force distribution with Step 6 regarding separating bodies in various ways to quantify the forces between them. As seen in Figure 7a students learn to use the conditions of equilibrium to infer the *position* of the net load of contact. This reinforces the important idea that the normal force does not necessarily act at the center of the contact area.
   Gripping a body firmly against translation and rotation by a pair of bodies (providing a fixed or cantilevered support, (as seen in Figure 7b) can now be understood: it combines the idea that the net force due to a distribution can act anywhere along the contact length, with the idea that a pair of oppositely directed forces can provide a balancing couple.

Nonuniform Contact

Consider the tension T to maintain equilibrium.



Which of the following is true?

Wh There is only one tension T which will maintain equilibrium

Pi There is a lower limit and an upper limit to the tensions T that can maintain equilibrium

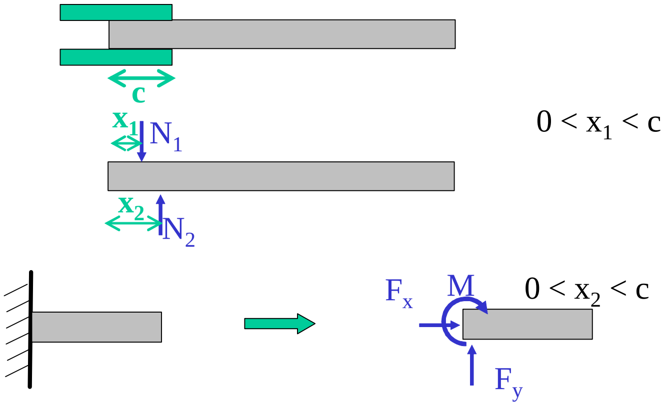
Bl There is a lower limit to the tensions T that can maintain equilibrium, but no upper limit

Ye There is an upper limit to the tensions T that can maintain equilibrium, but no lower limit

Fig. 7a

Nonuniform Contact

What is exerted by a pair of boards?

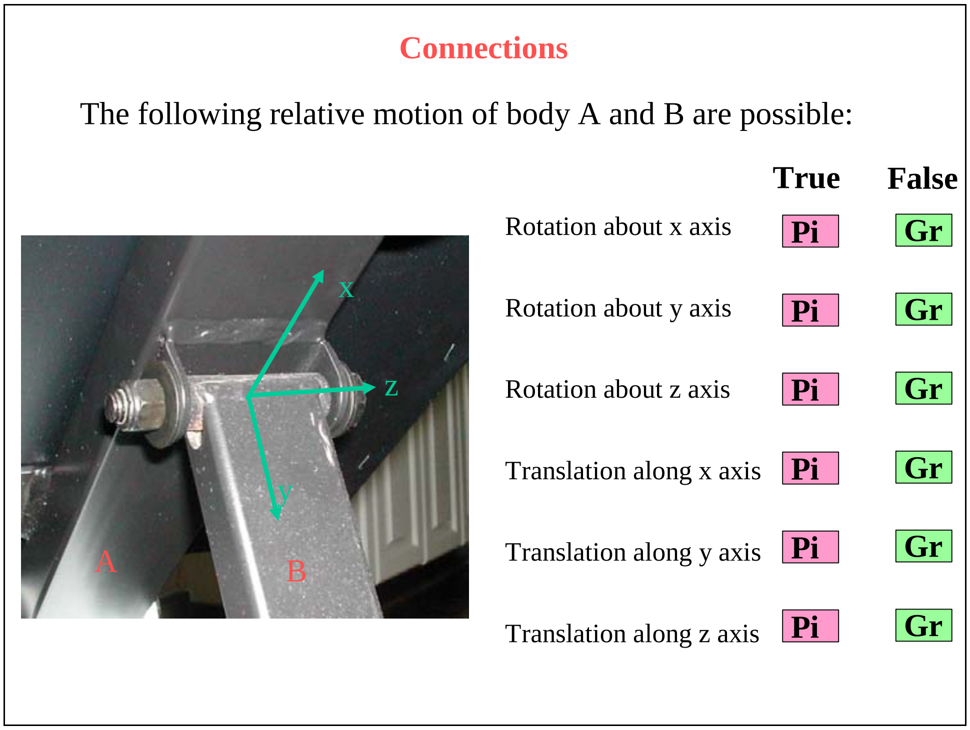


The magnitude and direction of the forces and couple provided by the support depend on the loads applied to the member.

- a horizontal force: F_x - to the left or right
- a vertical force : F_y - up or down
- a couple : M - clockwise or counterclockwise

Fig. 7b

8. Forces exerted by bodies connected by typical joints, with greatest emphasis on pin joints. Much of the conceptual understanding developed above sets the stage for this significant topic. First, we address the very purpose of a joint – to allow for some motions, usually with minimal frictional resistance, and to prevent other motions (Figure 8a). Then, we recognize that this joint is based on contact between a rod and a sleeve. Moreover, through the experience of manually gripping a rod and with previous work on distributed forces and frictional forces, it can be seen that certain loads are only transmitted through friction, whereas others can occur through normal forces (Figure 8b). Next, the loads generated when a single sleeve supports a rod is contrasted with the support provided by a pair of sleeves (Figure 8c). This development paves the way for understanding why symmetric pin joints are preferable, and under what circumstances it is acceptable to model the load at a pin joint with a single transverse force and neglect the transverse couples (Figure 8d). With this approach the stage is set for comprehending how other joints are modeled, based on the shape of the contacting surfaces controlling the motions of the connecting bodies, and on distinguishing between effects of frictional and normal forces. Additionally, this approach can make students more aware of distinctions between circumstances in which slip is to be prevented versus promoted, and of design considerations seeking to minimize forces.

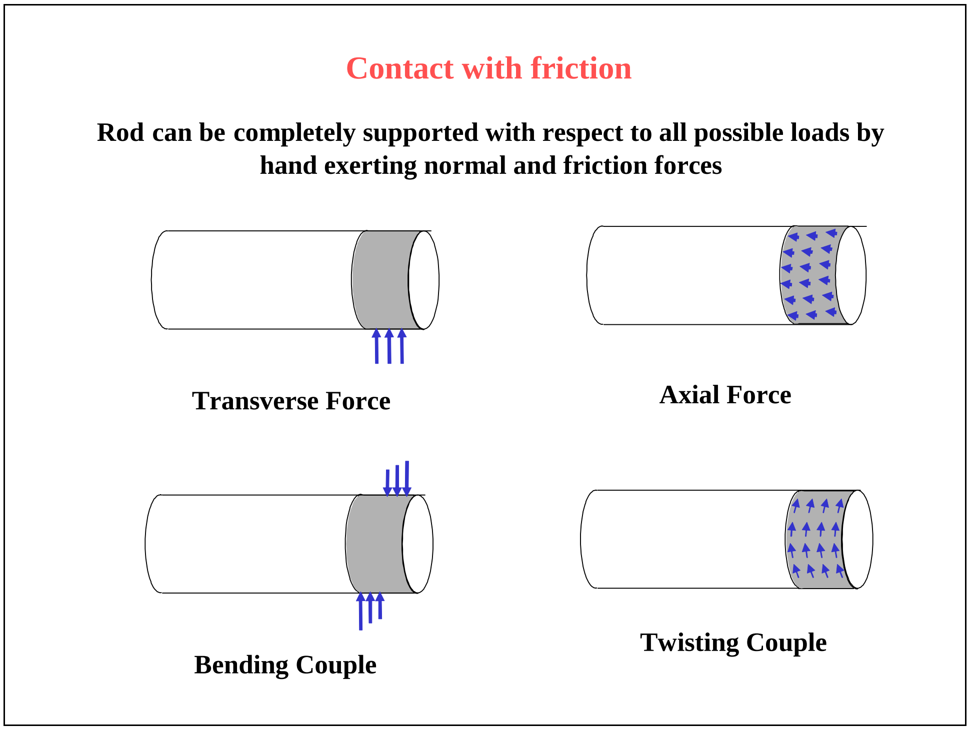


Connections

The following relative motion of body A and B are possible:

	True	**False**
Rotation about x axis	**Pi**	**Gr**
Rotation about y axis	**Pi**	**Gr**
Rotation about z axis	**Pi**	**Gr**
Translation along x axis	**Pi**	**Gr**
Translation along y axis	**Pi**	**Gr**
Translation along z axis	**Pi**	**Gr**

Fig. 8a



Contact with friction

Rod can be completely supported with respect to all possible loads by hand exerting normal and friction forces

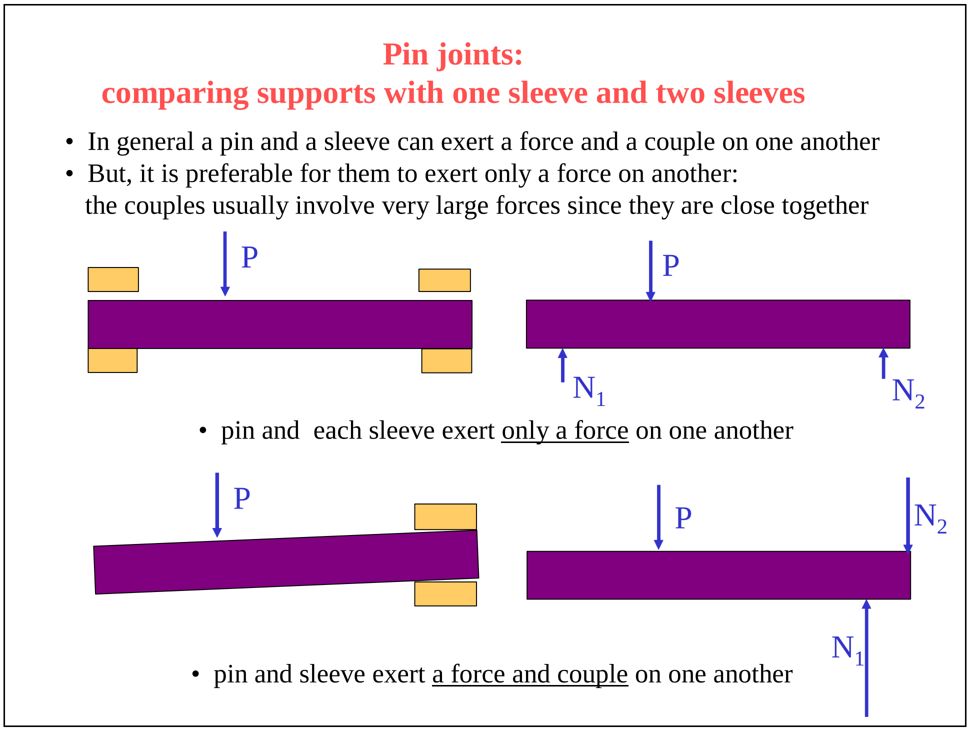
Transverse Force

Axial Force

Bending Couple

Twisting Couple

Fig. 8b



Pin joints: comparing supports with one sleeve and two sleeves

- In general a pin and a sleeve can exert a force and a couple on one another
- But, it is preferable for them to exert only a force on another: the couples usually involve very large forces since they are close together

- pin and each sleeve exert <u>only a force</u> on one another

- pin and sleeve exert <u>a force and couple</u> on one another

Fig. 8c

Connection:
comparison of asymmetric and symmetric pin joints

Motorized Door Opener

Symmetric Pin Joint

Asymmetric Pin Joint

Fig. 8d

Implementation

The succession of concepts just described can be addressed through a combination of activities. In the classroom, we use so-called Learning Modules, which include classroom desktop experiments or demonstrations (featuring the objects shown above), PowerPoint Presentations and, often, Concept Questions. Students manipulate the objects, maintaining them in equilibrium, or creating their motion or deformation, in pursuit of various goals. The instructor controls the PowerPoint Presentations which guide students through the ideas that can be gleaned from manipulating the objects. These presentations typically contain Concept Questions (CQ's), akin to Mazur's ConcepTests[10]: multiple-choice questions that assess student understanding of concepts, and which usually require little or no analysis. Students vote for the different answers, and, depending on the response, they are encouraged to discuss the question further with peers and to manipulate the object.

These modules are used as a part of the lecture period and distributed throughout the semester. Thus, they are interspersed with or replace typical lecture material, such as presenting theory, doing example problems, and discussing practical applications. As a rough guide, there might be such a learning module every other lecture or every third lecture. A given learning module might occupy from as little as 5 or 10 minutes to half of a lecture, to as much as an entire lecture. More details on classroom implementation can be found in other papers by the authors [3, 9].

Based on all the prior research which points to the effectiveness of actively engaged learning, integration of assessment into the learning process, peer collaboration, and hands-on experiences, we expect our approach to contribute significantly to improving learning outcomes. Still, were it feasible, we would be pleased to rigorously compare experimental and control groups of students that differed only in their exposure to our approach. This is difficult to do, however. Our approach involves both altering the entire sequence of topics and the altering the ways in which students encounter this altered sequence in the classroom. Moreover, this new approach pervades most of the semester, rather than being an intervention confined to a single class session. The only possibility, as we see it, would be for one instructor to teach simultaneous sections of Statics. In one section, a traditional approach would be followed; in the second section our new approach would be followed. Then, presumably comparisons could be made on the basis of common assessment instruments, such as tests. However the feedback on our new approach from students has been sufficiently positive, that we have been unwilling to offer a Statics experience which we believe is markedly inferior. Rather, we have chosen to survey students, sometimes regarding the overall approach, and sometimes regarding individual modules. Results of surveys that were conducted in the Fall 2003 are given in Tables 1 and 2.

Table 1 shows results from Miami University. Responses given below are numbered as follows: strongly disagree (0); strongly agree (4).

	4	3	2	1	0	Average
The focus on forces and couples that you can exert (e.g. manipulating objects with fingers and nut drivers; maintaining your own equilibrium) help your understanding of the concepts of Statics.	31	14	1	0	0	3.65
Power Point presentations help your understanding of the concepts of Statics.	28	17	1	0	0	3.59
Asking of concept questions (questions you voted on with colored cards) in lecture helped your understanding.	27	16	3	0	0	3.52
Demonstrations (e.g., friction, moment of inertia) have been helpful.	32	12	2	0	0	3.65

Table 2 shows results from CMU were students were asked how much each of the following contributed to their learning in Statics class. Responses given below are numbered as follows: Nothing (0); Very little (1); Modest amount (2); Quite a bit (3); A lot (4).

	4	3	2	1	0	Average
Voting on questions with colored cards	18	28	33	15	4	2.42
PowerPoint Presentations (aside from voting)	13	29	37	16	3	2.34
Manipulating objects in class.	26	26	35	9	2	2.67

Summary

While many active engagement classroom techniques are available to the instructor, it is up to the instructor to fill in the content which is the substance of these techniques. Moreover, since students learn based on what they know, this substance should be organized in a way that allows knowledge to be built steadily. We have undertaken this reorganization of the conceptual content of Engineering Statics. This reorganization must reflect the significant misconception on the part of many students that rigid, unmoving, inanimate objects do not exert forces. With this in mind, we have rebuilt our instructional approach to Statics by addressing all important concepts sequentially, but, notably, in the context of situations where all relevant forces can be perceived through the senses of touch and sight. Thereafter, we gradually transition students to address the situations of traditional interest in Statics, where forces are exerted by inanimate parts of machines and structures. The sequence of concepts is addressed in class through the use of learning modules, which involve collaboratively manipulating objects, and responding to conceptual questions.

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