

## Initial Data from a Statics Concept Inventory

Paul S. Steif  
Carnegie Mellon University

### Introduction

Engineering Statics is a pivotal course in a number of engineering disciplines. Statics lays the foundation for subsequent courses, namely Dynamics and Strength of Materials. Moreover, Statics and these follow-on courses are the basis for engineering design and practice. Instruction in Statics is worthy of significant attention.

While there are a number of topics in typical Statics textbooks, the problems that are most relevant to the application of Statics to engineering systems are those addressing the interactions between connected bodies. This class of problems takes students significantly beyond what they learn in physics, extensions that are necessary to addressing practical engineering systems. Recent work by the author<sup>1</sup> has set out to identify and organize the conceptual content of Statics, with particular focus on those concepts that underlie multi-body Statics problems. There are four clusters of concepts and a set of skills for implementing those concepts. While mathematical skills, for example, resolving and adding forces and computing moments due to forces, are needed for Statics, for our purposes here they are not a part of the conceptual content of Statics. These concepts and skills were then mapped to a set of typical errors which students are observed to make. The relationship to concepts and misconceptions in freshman Newtonian mechanics in physics<sup>2-4</sup> were explained.

We seek an efficient and accurate means of measuring student understanding of these concepts. Ideally, we would be able to detect which concepts were well in hand, and for concepts that are not understood, whether there is evidence of a consistent misconception. To this end, the development of a concept inventory has been undertaken. There has been some work along this direction by others relevant to Statics<sup>5,6</sup>, as well as to a variety of engineering and related subjects<sup>7</sup>, although there has not yet been an attempt at a complete inventory in the case of Statics. The individual multiple-choice questions in this inventory are devised to test each concept in isolation, allowing for unambiguous identification of conceptual errors.

### Concepts of Statics

In the framework articulated by Steif<sup>1</sup>, the central concepts of Statics fall into four clusters as follows:

- Forces are always in equal and opposite pairs acting *between* bodies, and equilibrium conditions always pertain to the external forces acting directly on a chosen body.
- Two combinations of forces and couples are statically equivalent to one another if they have the same net force and moment

- The possibilities of forces between bodies that are connected to, or contact, one another can be reduced by virtue of the geometry of the connection and/or assumptions on friction
- A body is in equilibrium if the summation of forces on it is zero and that summation of moments on it is zero.

### **Distribution of Questions on Statics Concept Inventory**

Questions on the Statics Concept Inventory have been separated into 5 groups of questions, which correspond to the clusters of concepts referred to above. The distribution of questions and the conceptual errors which are captured by each cluster of question are as follows:

- Free body diagrams (5 Questions)

Typical errors include: Leaving a force off; including a force which is exerted between two bodies, both of which are part of the diagram; Including a force which is exerted between two bodies, neither of which is part of the diagram; failure to take the forces between two bodies to be equal and opposite

- Static equivalence between different combinations of forces and torques (3 Questions)

Typical errors include: presuming a force can be moved perpendicular to its line of action and have with no change in its effect; presuming two combinations of loads can be statically equivalent even if the net force is not equal; presuming that a couple at one point is statically equivalent to a different couple at another point

- Type and direction of loads at connections (3 questions for each of 4 different situations of roller, pin in slot, general pin joint, and pin joint on two force member)

Typical errors include: incorrectly taking the direction of the force at a connection to be known (assuming it balance some applied load or to acts in some direction relative to the member); incorrectly taking the direction of the force at a connection to be unknown (failing to recognize the limits that slot or roller or presence of a two force member places on the force); incorrectly assuming that a couple can be exerted at a connection (confusing the moment due to a force about some other point with a couple acting at the connection); incorrectly ignoring a couple that could be exerted at a connection (such as a cantilevered)

- Limit on the friction force and its trade-off with equilibrium conditions (3 Questions)

Typical errors include: presuming the friction force of the contacting body is equal to  $\mu N$  (rather than the force necessary to maintain equilibrium, which could be less than  $\mu N$ ); presuming the friction force of the contacting body to equal the difference between the applied (or balancing) force and  $\mu N$ ; incorrectly setting  $N$  (if it is relevant) equal to the weight of the immediate contacting body, rather than taking  $N$  to equal the value necessary to maintain equilibrium.

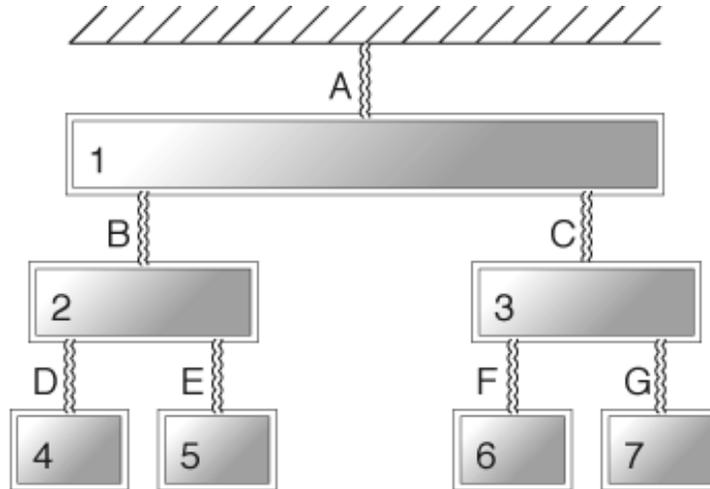
- Equilibrium conditions (4 Questions)

Typical errors include: not recognizing that the net force is not zero; not recognizing that the net moment is not zero; incorrectly accounting for the contribution of couples to equilibrium.

Below are examples of each of the types of questions from the inventory described here.

*Example of question on free body diagrams*

A free body diagram is to be constructed of the assemblage which includes three of the weights ( $W_1$ ,  $W_3$  and  $W_6$ ) and the cords connecting them.



Which is the correct free body diagram?

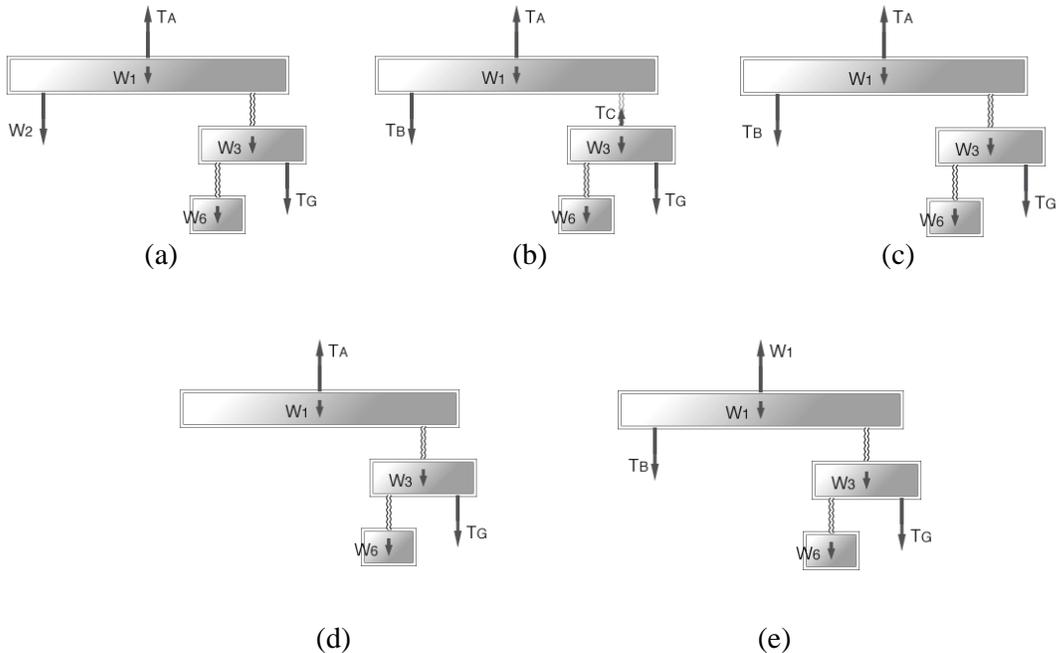
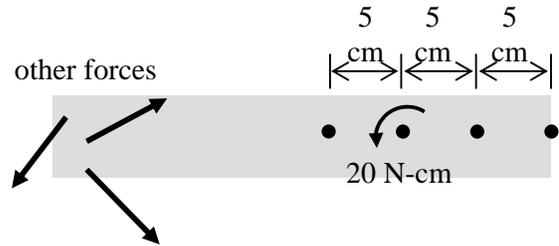


Figure 1. Example of concept question addressing free body diagrams.

Explanation of wrong answers: (a) Force between two objects (earth and weight  $W_2$ ), both of which are *not* in the diagram, is falsely presumed to act directly on the body of diagram; (b) Force between two objects (cord C and weight  $W_2$ ), both of which *are* in the diagram, is falsely included diagram; (c) Correct answer; (d) Force missing; (e) Premature (and incorrect) application of equilibrium resulted in incorrect value for unknown tension in cord A. Note that the distinct explanations for answers (a) and (e) apply to both.

*Example of question on static equivalency*

One torque of magnitude 20 N-cm keeps the member in equilibrium while it is subjected to other forces acting in the plane at various points (shown at the left). The four dots denote equally spaced points along the member.



Assuming that the same forces are applied at the left, what load(s) could replace the 20 N-cm torque and still maintain equilibrium?

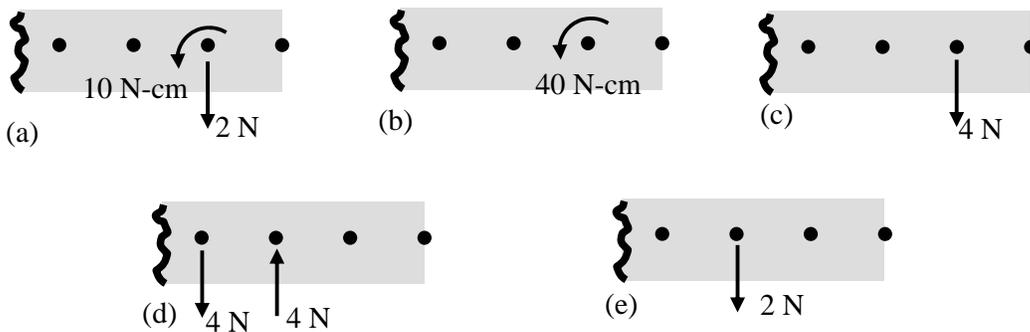


Figure 2. Example of concept question addressing statics equivalency.

Explanation of wrong answers: (a) Torque alone cannot be equivalent to a force, even if there is a couple as well; (b) Effect of torque changes as one moves its position; (c) Torque alone cannot be equivalent to a single force; (d) Correct answer; (e) Torque alone cannot be equivalent to a single force.

*Example of question on loads at connections*

The mechanism (diagram at the left) is acted upon by the downward force shown. A spring acts on the slotted link. Ignore gravity. The pins, which are indicated in black, are well lubricated.

What is the direction of the force exerted by the pin on the horizontal portion of the indicated member? (choices at the right)

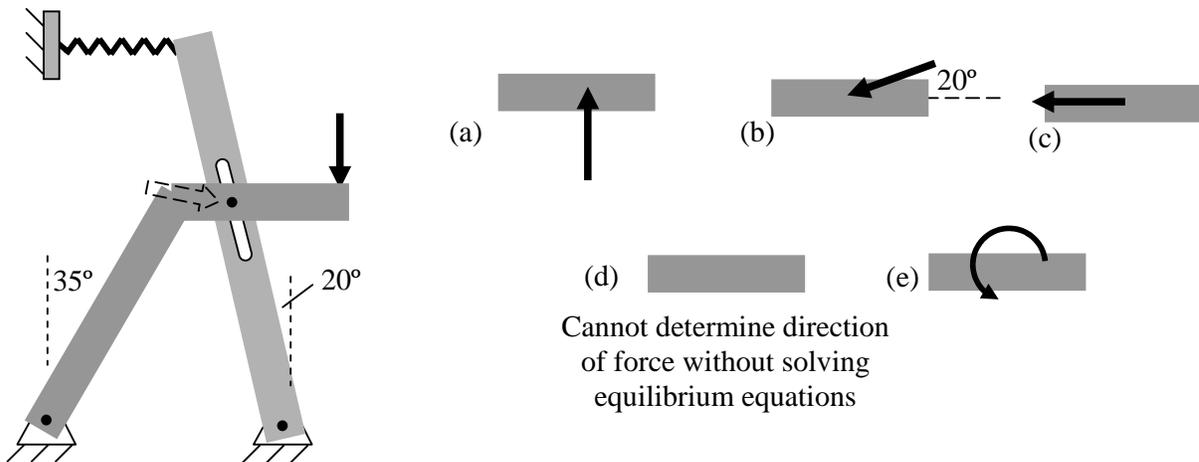
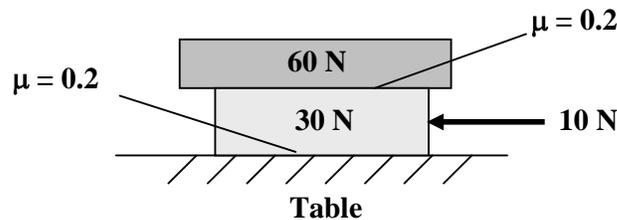


Figure 3. Example of concept question addressing forces at connections.

Explanation of wrong answers: (a) Force at connection is chosen to balance the applied force (premature, incorrect application of equilibrium); (b) Correct answer; (c) Force acts along length of member; (d) Pin implies that the force as an unknown direction (say, x and y components), irrespective of other aspects of the connection; (e) The clockwise moment created by the applied force must be balanced by an opposite couple (even though the pin cannot sustain a couple).

*Example of question on limits on friction force*

Two blocks are stacked on a table. The friction coefficient between the blocks and between the lower block and the table is 0.2. (Take this to be both the static and kinetic coefficient of friction). A horizontal 10 N force is applied as shown to the lower block.



What is the horizontal force exerted by the table on the lower block?

- (a) 4 N      (b) 6 N      (c) 8 N      (d) 10 N      (e) 18 N

Figure 4. Example of concept question addressing friction forces.

Explanation of wrong answers: (a) Force of table is equal to the difference between the balancing force (10) and the friction force  $\mu N$  (with  $N$  incorrectly found to be  $0.2 \cdot 30$ ); (b) Force of table is equal to  $\mu N$  (with  $N$  incorrectly found to be  $0.2 \cdot 30$ ); (c) Force of table is equal to the difference between the balancing force (10) and the friction force  $\mu N$  (with  $N$  correctly found to be  $0.2 \cdot 90$ ); (d) Correct answer; (e) Force of table is equal to  $\mu N$  (with  $N$  correctly found to be  $0.2 \cdot 90$ ).

*Example of question on equilibrium conditions*

The member (shown to the left) is subjected to the force at the lower right corner, and is maintained in equilibrium by a hand gripping the left end (A).

Which of the following could represent the load(s) exerted by the gripping hand? (at right)

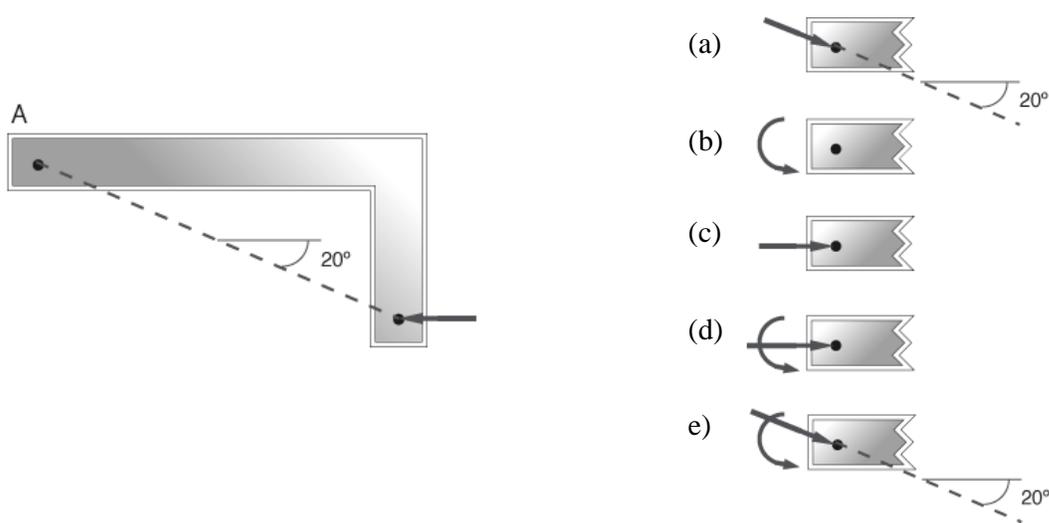


Figure 5. Example of concept question addressing equilibrium conditions.

Explanation of wrong answers: (a) Force balance fails (equilibrium is falsely assumed to be satisfied if forces run through a single point); (b) Force balance fails, although moment is apparently balanced by a torque; (c) Moment equilibrium fails (although forces balance); (d) Correct answer; (e) Force balance fails although moment is apparently balanced by a torque.

**Psychometrics based on initial administration of Statics Concept Inventory**

The test described above, having a total of 27 questions, was administered to 125 mechanical engineering students at Carnegie Mellon University on the first day of a sophomore Statics course. Virtually all students had taken the freshman fundamentals of mechanical engineering course that had a 3 week segment on Statics. The psychometrics now presented were based on the results of this administration of this test. The mean score was 10.6, the standard deviation 4.1, the maximum score 22, the minimum 2, and the median 11. The mean and standard deviation for males (92 students) were 11.1 and 4.1, respectively, for females (23 students) 8.8 and 3.7, and for African-Americans (5 students) 9.4 and 2.4. Interestingly, while there was a continuous distribution in scores among males from the low end to the maximum, except for one female who scored 21, no female scored higher than 13 and no African-American scored higher than 12.

Reliability, as measured by Cronbach's alpha reliability coefficient, was 0.712. This is acceptable evidence of reliability, at least for an initial version, although values nearer to 0.8 would be preferable. On only a single question of the 27 total did more than 70% of the class answer correctly (81% answered that question correctly); on only 8 questions less than 20% of the class answered correctly. The difficulty of the questions is illustrated in Figure 6.

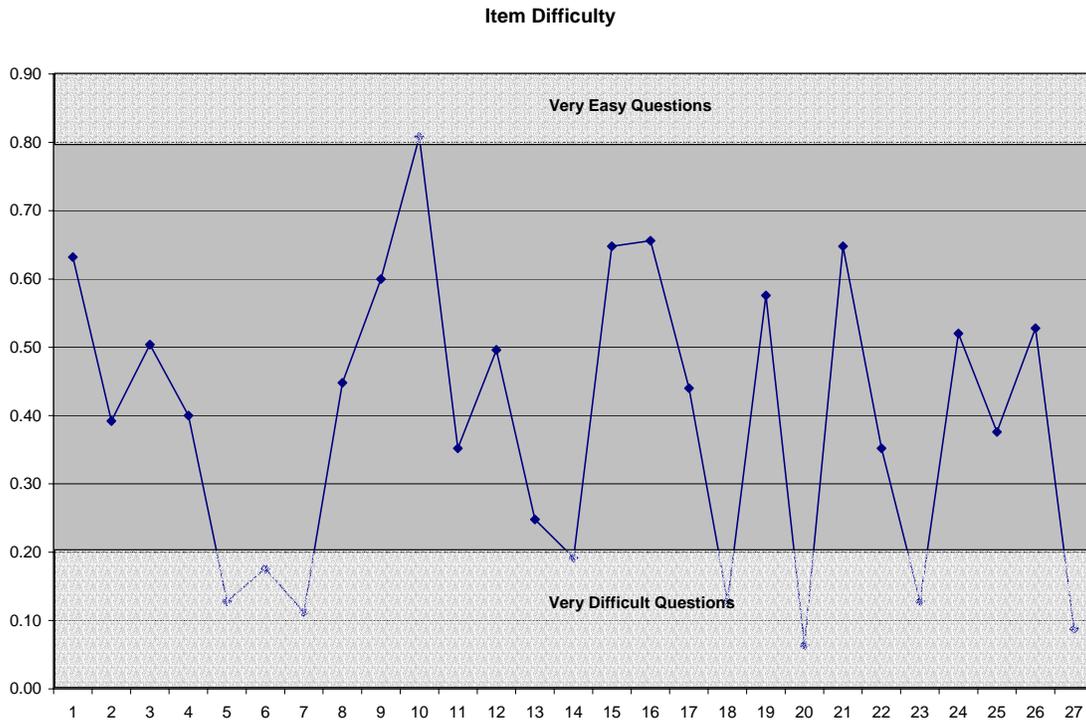


Figure 6. Item difficulty: Fraction of students answering questions correctly.

There was a subset of 9 questions which did not correlate well with the total score (correlation coefficient less than 0.3). Moderate to high item to total score correlation coefficients are evidence that the items are good measures of the overall construct of "Statics Knowledge". All but two of these questions also had a discrimination score of less than 0.2. (A higher discrimination score, which by definition is in the total range of  $-1$  to  $1$ , signifies that students with overall higher scores are more likely to answer the question correctly than students with lower overall scores.) The 7 questions with low discrimination scores included three questions testing the understanding of friction and three questions testing the understanding of static equivalence. The remaining question, which tests the concept of equilibrium, was found upon review to have wording which was obviously and unnecessarily confusing. The difficulty of the questions is illustrated in Figure 7.

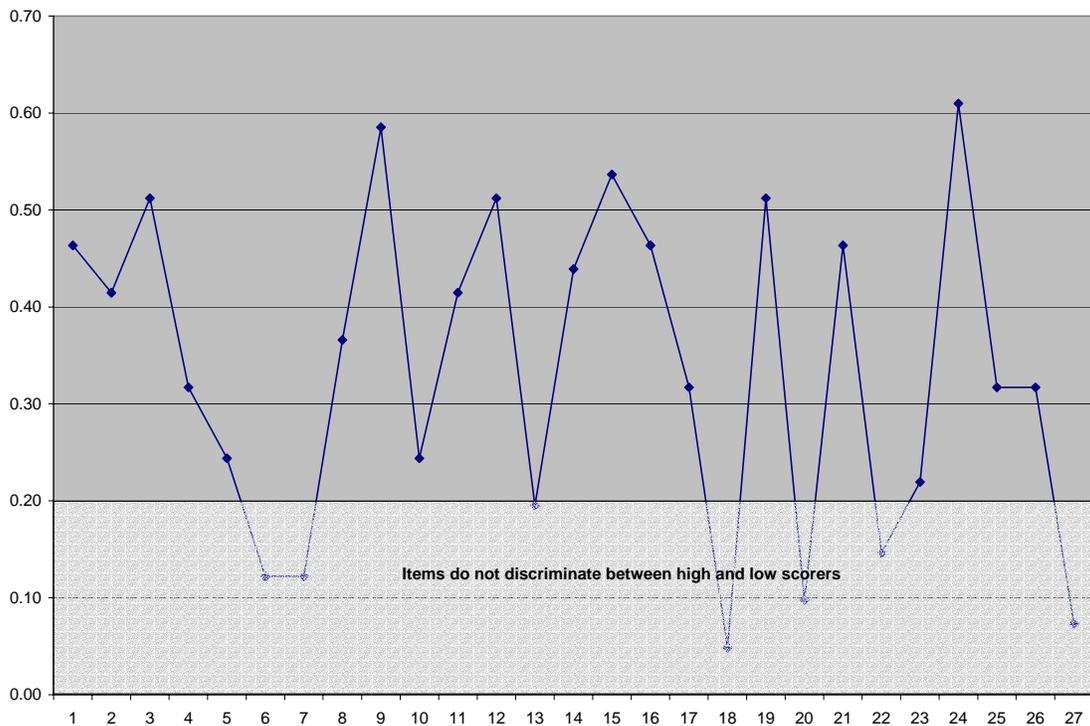


Figure 7. Discrimination scores for questions in concept inventory.

It is no surprise that previous experience would not prepare students for the concept of static equivalence; all students were quite poor at answering these questions, with only 11%, 13% and 9% answering them correctly. The poor correlation between scores on friction and overall knowledge of Statics may be an indication of the prevalence and depth of this misconception - the tradeoff between equilibrium requirements and the upper limit on friction; respectively, only 18%, 25% and 35% of students answered these questions correctly). Indeed, from results of an earlier version of the Statics concept inventory [ref.], this misconception was found to persist in students who had successfully completed a Statics course.

## Summary

A multiple-choice Statics Concept Inventory which seeks to measure conceptual knowledge of students in Statics has been developed. Consisting of 27 items, the test examines students in four core areas – free body diagrams, static equivalency of combinations of forces, inferring forces at connections and contact points, and conditions of equilibrium. The alternative (wrong) answers to the questions are intended reflect conceptual errors commonly made by students. This test was administered to 125 mechanical engineering students just entering Statics and the results were analyzed. The analysis showed an acceptable range of difficulty in the questions, and that most questions discriminated effectively between high and low scorers. Further examination of those questions which did not discriminate suggested, in one case, poor question wording and in the other cases perhaps that students have no prior experience with the concept or commonly misunderstand the concept. These uncertainties will be more fully explored when the post-test is administered and analyzed; those results will be presented in the near future.

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#### **Biographical Information**

##### **PAUL S. STEIF**

Professor, Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, Pa  
Degrees: Sc. B. 1979, Brown University; M.S. 1980, Ph.D. 1982, Harvard University.  
Research area: solid mechanics and engineering education.