**Unit 4 – Equilibrium of Forces and Moments (Student Sheet)**

Engineering Statics in Physics Project

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**4.1 A Board rests on scales, and people stand on the board**

*Experiment 1: A board resting on two scales*

 Scale 1 board weight: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 Scale 2 board weight: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Mean board weight: \_\_\_\_\_\_\_\_

Scale 2

Scale 1

Scale 1 Reading: \_\_\_\_\_\_\_\_\_\_\_ Scale 2 Reading: \_\_\_\_\_\_\_\_

FBD of the board

Sum of Vertical Forces Sum of Moments about left strip

Where do you think the weight of the board acts and why?

Why is that consistent with equilibrium of the board and the scale readings?

*Experiment 2: A student standing on a board that rests on two scales*

Scale 1 student weight: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Scale 2 student weight: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Mean student weight: \_\_\_\_\_\_\_\_

 dist1 = \_\_\_\_\_\_\_ dist2 = \_\_\_\_\_\_\_\_

Scale 2

Scale 1

Scale 1 Reading: \_\_\_\_\_\_\_\_\_\_ Scale 2 Reading: \_\_\_\_\_\_\_\_

FBD of the board

Sum of Vertical Forces:

Sum of Moments about left strip Sum of Moments about right strip

*Experiment 3: Two students standing on a board that rests on two scales*

Student 1 Weight: \_\_\_\_\_\_\_\_ Student 2 Weight: \_\_\_\_\_\_\_\_ Board Weight: \_\_\_\_\_\_\_\_

Board Length: \_\_\_\_\_\_\_\_

 dist1 = \_\_\_\_\_\_\_ dist2 = \_\_\_\_\_\_\_\_ dist3 = \_\_\_\_\_\_\_\_

Scale 2

Scale 1

Scale 1 Reading: \_\_\_\_\_\_\_\_ Scale 2 Reading: \_\_\_\_\_\_\_\_

FBD of the board (label unknown forces from the scales as N1 and N2).

Sum of Vertical Forces Sum of Moments about left strip

Solve the equations for the unknowns, N1 and N2, and compare with the measured values.

|  |  |  |
| --- | --- | --- |
|  | N1 | N2 |
| Predicted |   |  |
| Measured |  |  |

**4.2 Two fingers support a long object at one end – what are the forces on the fingers?**

FBD of case A FBD of case B

Sum of moments for case A Sum of moments for case B

 (Specify which point you are (Specify which point you are

 taking moments about) are moments about)

Which arrangement (A or B) is easier to support? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Explain how your drawings (above) and work support this observation.

FBD of case A FBD of case C

Sum of moments for case A Sum of moments for case C

Describe how the forces you apply change as the upper rod is moved to the left?

Explain how the moment summations are consistent with what you feel.

Describe how your forces change as you move both rods towards the center of the meter stick (keeping them at a fixed separation with respect to each other). Explain how the moment summations are consistent with what you feel.

**4.3 Finding the location of the center of gravity**

*Method 1:**Balancing the bar in two orientations on a single rod*

 d1 = \_\_\_\_\_\_\_\_\_ d2 = \_\_\_\_\_\_\_\_\_

 ( \_\_\_\_\_\_ , \_\_\_\_\_\_ )

*Method 2: Hanging the bar in two orientations from spring scales*

scale1 \_\_\_\_\_ scale2 \_\_\_\_\_ scale1 \_\_\_\_\_ scale2 \_\_\_\_\_

dist1 \_\_\_\_\_ dist2 \_\_\_\_\_ dist1 \_\_\_\_\_ dist2 \_\_\_\_\_





dist2

dist2

dist1

dist1

d1  d2

Sum of Forces Sum of Forces

Sum of moments about left edge Sum of moments about left edge

d1 = d2  =

(\_\_\_\_\_ , \_\_\_\_\_ )

*Method 3:**Using the central position and the mass of each rectangle composing the bar to generate a predicted center of mass position for the L-shaped bar*

area1 \_\_\_\_\_ area2 \_\_\_\_\_ area1 \_\_\_\_\_ area2 \_\_\_\_\_

dist1 \_\_\_\_\_ dist2 \_\_\_\_\_ dist1 \_\_\_\_\_ dist2 \_\_\_\_\_

dist2

dist2

dist1

dist1

d1

d2

Sum of Forces Sum of Forces

Sum of moments about left edge Sum of moments about left edge

 ( \_\_\_\_ , \_\_\_\_ )

**4.4 Combinations of horizontal and vertical forces**

*Experiment 1:**Holding a rectangular-shaped body with two rods*

 FBD of plate:

Sum of Forces x

Sum of Forces y

Sum of Moments

Can you maintain the plate in this position? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why or why not?

*Experiment 2:**Holding a rectangular-shaped body with three rods*

Case A Case B

FBD (case A) FBD (case B)

Sum of Forces x Sum of Forces x

Sum of Forces y Sum of Forces y

Sum of Moments (about bottom rod) Sum of Moments (about bottom rod)

Use the sum of forces in x and y to equate some of the forces and reduce the number of unknown forces in each case.

In which case do the rods succeed in keeping the plate balanced and level? \_\_\_\_

Explain how the moment summation and the reduced number of unknown forces support this observation.

Use your work (FBD and sum of moments) to rationalize the variation in forces on the side rods when you increased the vertical separation between the side rods.

*Experiment 3:**holding an L-shaped body with three rods*

Case A Case B

FBD (case A) FBD (case B)

Use the sum of forces in x and y to equate some of the forces and reduce the number of unknown forces in each case.

In which case do the rods maintain equilibrium? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Explain how a moment summation and the reduced number of unknown forces support this observation.

Repeat the experiment, but now locate the bottom rod at various points along the whole bottom edge. Again try to balance with the side rods in arrangement case A or B. For one location of the bottom rod, you will find that the arrangement which balances switches from A to B or vise versa.

Draw the FBD when the bottom rod is at the point where the arrangement switches.

Explain how a moment summation and the reduced number of unknown forces support the observation that the balancing arrangement switches for this location of the bottom rod.

**4.5 Horizontal and vertical forces: potential for tipping**

*Experiment 1: Shifting weights to one side of a tipping stand that rests on two bathroom scales*

 Case A Case B

Scale 2

Scale 1

Scale 1

Scale 2

 FBA (Case A) FBD (Case B)

Sum of Forces:

Sum of Moments:

Using your sums of forces and moments, explain how the scale readings shifted when the books were shifted to one side.

*Experiment 2: Applying a horizontally directed force to a person standing on two bathroom scales*

Scale 1

Scale 2

Scale 1

Scale 2

Fpush

FBD



Sum of Forces:

Sum of Moments:

Using your sums of forces and moments, explain the observed change in scale readings when the student is pushed.

*Experiment 3: Applying a horizontally directed force to a tipping stand that rests on two bathroom scales*

Fscale

high friction fabric between stand and scales

Scale 2

Scale 1

 FBD

Sum of Forces:

Sum of Moments:

Using your sums of forces and moments, predict the scale readings for a given value of the horizontally applied force.

*Experiment 4: Altering the vertical position of the horizontally applied force on the tipping scale*

Fpush

fabric between stand and table

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|  |  |
| --- | --- |
| Fabric Type | Tip-slip transition height\*  |
| high friction |  |
| low friction |  |

\*the vertical location of Fpush where the tipping changes to slipping (or slipping to tipping)

Why does the tip-slip transition height change when the fabric type changes?

Develop a general method for predicting whether the stand will slip or tip as the horizontally applied force is increased.

**4.6 Distributed force between two bodies in contact and position of single force that represents distribution**

*Experiment 1: A student shifts his/her center of gravity*

Scale 1

Scale 2

shift weight towards the front and then towards the back scale

As the student shifts his/her weight, what is changing?

How is the force between his/her feet and the tracks changing (consider its magnitude and its position)?

In any single position of the body, given the two scale readings, how could you determine the magnitude of the force between the feet and the tracks and the position of this force?

*Experiment 2: A tipping stand subjected to a horizontally applied force affects the normal force*

Place weights on the tipping stand so the scales have equal readings. Then, push from the side.

Scale 1

Scale 2

Two-by-four tracks (tipping stand base)

Tipping stand

Fpush

FBD’s. For each case draw FBD of tipping stand and of tipping stand base. Be careful how and where you draw the forces between the tipping stand and the tipping stand base.

 Case A Case B

 (without Fpush) (with Fpush)

Sum of Forces (for tipping stand):

Sum of Moments (for tipping stand):

Using your sums of forces and moments, justify how you drew the forces between the tipping stand and tipping stand base in the two cases.

Explain why and by how the scale readings change when the horizontal force is applied.

What would eventually happen as the horizontally applied force is increased?

**4.7 Potential for slip or tip with distributed forces**

*Experiment 1: All of the forces on a box subjected to a horizontally applied force*

Place a box in an upright position, so that it would tip if the applied force was large enough. Imagine a horizontal force applied to the box that is not large enough to cause it to tip or slip.

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Draw an FBD of the box, taking care to label all distances and forces: Fpush, Weight (W), Normal (FN) and Friction (Ff). In particular, use an “x” to denote the horizontal distance of the normal force from the center of the box.

Treating the weight and pushing force symbolically, use the sums of forces and moments to determine mathematical expressions for the normal force, friction force, and the location of the normal force.

For the case in which the box does not slip, determine an inequality that relates Fpush, weight W, and the friction coefficient. (Dimensions may or may not matter.)

For the case in which the box does not tip determine an inequality that relates Fpush, and the weight W. (Dimensions may or may not matter.)

*Experiment 2:* *Whether a horizontally applied force causes a body to slip or tip*

Place a tall box on a flat surface. Obtain three fabrics which differ in terms of the amount of friction each can provide to a box sliding across the fabric’s surface. Place one of the fabric samples between the box and the horizontal surface (*i.e.* under the box). Apply an increasing horizontal force until the box either slips or tips. Investigate how each of the following conditions influences whether the box first slips or tips.

* Friction at surface, for example, with different fabrics
* Orientation of the box (possibly three different orientations)
* Position of force application relative the bottom

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Use the results from the force and moment summations in Experiment 1 to explain the influence of friction, box orientation, and position of force on the tendency to slip or tip first.