Component Failure

24-370 - Spring 2011
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Announcements

• HW2 assigned, due next Wednesday
  – Homeworks guide projects
• Project 1 Questions?
• Where we are in the course...
  – Tools: sketching, SolidWorks
  – Analyses: stress, failure (today)
  – Next up: Design!
  – First loop: geometry, mass, and factor of safety
• But first... Failure!

Failure in Mechanical Engineering

• What is failure?
  – Compromisation or degradation of any kind
  – Commonly: breaking or permanently bending
  – Also: bending too much, wear, dynamic failure
• Today: custom component failure
  – Common & most-easily estimated modes
  – Later: catalog component failure

Failed Mechanical Parts

Avoiding Failure

• Reality can be a tricky place to operate...
  – Failure is very difficult to estimate accurately
  – Models: simple, generalizable, but miss details
  – Empirical data: accurate, but case specific
  – Prototyping: best data, but expensive
• What does a Mechanical Design Engineer do?
  – Use models or empirical data first
  – Use appropriate Factors of Safety
  – Prototype and test
Common Failure Modes

- **Static loading**
  - Primarily related to stress
  - Ultimate stress exceeded → breakage
  - Yield stress exceeded → plastic deformation
  - Impermissible deflection
    - Buckling → breakage
- **Cyclic loading: fatigue**
  - Similar stress analysis, very different behaviour
  - Fatigue life (# cycles) exceeded → breakage

Failure due to static loading

- Related to development of excessive stress
  - Exceeding yield stress is often unacceptable
  - Which maximum stress to use?
    - Many models, depending on material & risks
    - Typically, von Mises equivalent (distortion-energy)
      \[ \sigma^2 = \frac{1}{2} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) \]
      \[ S_y = 0.58 S_t \]
      - Pure normal stress equivalents

Models of Failure due to Static Loading

Failure due to static loading

- Related to development of excessive stress
  - Exceeding yield stress is often unacceptable
  - Which maximum stress to use?
    - Many models, depending on material & risks
    - Typically, von Mises equivalent (distortion-energy)
  - Stress concentrations?
    - In Ductile materials, usually not
      - Stress distributing, strain hardening
    - In Brittle materials, definitely yes
      - (later: also fatigue)
    - And... stress concentrators are everywhere!

Ductile vs Brittle Failure

- Ductile materials strain a lot, then fail
- Brittle materials just fail
  - Typically, \( \varepsilon_f \leq 0.05 \)
  - Often, only \( \sigma_f \) reported

Fracture Mechanics

- Failure usually occurs along cracks
- Cracks are everywhere!
- Brittle materials particularly susceptible
- Research on crack propagation ongoing
Static Failure Analysis Summary

- Use empirical data to determine yield strength
  - Or ultimate strength for brittle materials
- Determine maximum stress in design
  - Simple models (last week)
    • Stress concentrations for conservative or brittle cases
  - FEA analysis (last week)
- Compare to obtain factor of safety
- Design: \( \sigma_m \leq S_Y \text{ FOS}^{-1} \)
  - Obtaining this relationship... that’s the fun part!

Special Case: Contact Stress

- Strong analytical models for some types, e.g. cylindrical:
- I’ve typically used FEA
- SolidWorks example
  - Pin in hole

SolidWorks contact example

Deflection

- Possible failure mode: excessive displacement
- Compression or tension: \( \delta = F L A^{-1} E^{-1} \)
  - Load, length, area, and material stiffness
- Torsion: \( \Theta = T L J^{-1} G^{-1} \)
  - Rotational equivalents
- Bending: depends upon loading
  - Many approaches: see Shigley, others
  - Cantilever: \( \delta_w = \frac{3}{2} F L^3 E^{-1} I^{-1} \)
- Stiffness: rearrange terms

Buckling

- Special deflection
- Long, thin elements in compression
  - Unstable
  - Small deformations increase leverage
- Simple model
  - \( F_{cr} = \frac{C \pi^2 E I}{L^2} \)
- FEA: SolidWorks

SolidWorks Buckling Example
Failure due to Fatigue

- Everyone knows: push hard and things break
- But lots of little pushes breaking something?
  - Crazy, but true. This is fatigue
- Fatigue occurs below (apparent) yield stresses
  - Same stresses, different effects
- After a number of cycles, the part fails
- Predicting fatigue failure:
  - Difficult to model, but empirical data useful
  - Insight into mechanisms helpful too...

Stages of Fatigue Failure

- Stages:
  - A - Crack initiation
    - At concentrator
    - Or... at crack
  - B - Propogation
    - Often hidden
  - C - Final failure
    - Often brittle

Patterns tell a story

Fatigue Life and Strength

- Need to account for # cycles as well as load
  - Peak sustainable stress, S
  - Number of cycles, N
- Diagrams of S vs. N
  - Material-specific
  - Detailed fatigue data
  - Idealized conditions
- Scalar fatigue strength
  - Peak stress S that may be fully reversed N times

Fatigue Life: Steel S-N Diagram

Endurance Limit for Steels
Interpreting S-N Diagrams

- Steel has infinite fatigue life region
  - Fatigue strength corresponds to infinite life
- Aluminum (and most other materials) do not
  - Fatigue strength typically means $N = 10^8$
- S-N diagrams useful for sub-limit performance
- Often, we just want $\sigma(N=10^8)$
- Careful: this data is for specific conditions

Fatigue Life Analysis

- Stress-life method
  - Simplest, most data, $N \geq 10^3$, but least accurate
  - Other methods: strain life (low $N$), fracture (crack)
- Obtain material-specific fatigue strength
  - Generally, use empirical data
  - Steel endurance limit: $S_e' = 0.5 S_{ut}$
- Apply strength-reducing factors
  - Due to surface, size, load, temp, freq, etc.
- Compare to peak stress in component
  - Use fatigue stress concentration factors

Fatigue Analysis Example

- Material: Alum 7075-T6, $S_f' = 23$ ksi, $S_{ut} = 83$ ksi
- Fatigue strength modification factors:
  - Loading: bending $\Rightarrow k_l = 1$
  - Surface: ground
    - $k_s = a S_e^b$, $a = 1.34$, $b = -0.085 \Rightarrow k_s = 0.92$
  - Size: diameter $D = 1''$
    - $K_b = 0.879 D^{-0.107} \Rightarrow k_b = 0.88$
- Part stress
  - Peak bending stress $\sigma_{max} = 9$ ksi (no concentrators)
- Synthesis: $S_f = k_s k_b k_l S_f' = 18$ ksi $\Rightarrow FOS_f = 2$

Final Caveats

- Failure is stochastic
  - In many ways
    - e.g. actual yield stress
  - In design:
    - Modeling useful
    - Empirical data useful
    - Prototyping crucial
    - Iteration is reality

Suggested Reading

- Shigley Chapters 4, 5 & 6
  - Basics for the uninitiated
  - Details for the expert
  - Includes formulae from lecture
  - Accompaniment to HW3 (next week)