Simple and Detailed Stress Analysis

24-370 - Spring 2011
Professor Steve Collins

Announcements

- HW1:
  - Class average grade: 92%
  - Wednesday last chance…
  - Good practice for project
- Project 1:
  - Simple models in manner of today's lecture
  - Questions?

Suggested Reading

- Shigley Chapter 3: Load & Stress Analysis
  - Basics for the uninitiated
  - Details for the expert
  - Includes formulae from lecture
  - Accompaniment to homework

Simple vs. Detailed

- When should we use simple analyses?
- What are advantages of simple analysis?
  - Quicker answers
  - Separate combined stress effects
  - Formal connection between variables
- When should we use detailed analyses?
- What are advantages?
  - Accounts for greater complexity
  - Improved accuracy

Example: This Old I-Beam

Given geometry, load & material, what is F.O.S.?

Example: I-Beam Geometry

- I-beam cross-section
  - Area moment of inertia I
  - 10 in tall, 5 in wide, 0.5 in thick
  - 100 in long
- Big holes along center
  - 5 in ID
- Smaller holes along top
  - 0.5 in ID
- Let's sketch this…

Example: I-Beam Geometry

Example: I-Beam Load & Material

- Cantilever loading
  - Rigidly supported on one end
  - Load applied at other end
  - Expected load = 5,000 lbf (at 100 inches)
- Material: Alloy Steel
  - ASTM A36
  - Yield stress: 36 ksi tension
  - Ductile (20% elongation at failure)

Example: I-Beam Stress Analysis

- What is the simplest model of interest?
  - Beam in bending
  - Beam in shear?
- What other simple models could we use?
  - Stress concentration factor from holes
- Where will stress be greatest
  - Simplest: upper and lower faces
  - 1^{st} order: upper and lower hole edges
- Where will beam fail?
  - Near cantilever, near hole

Example: I-Beam Simple Analysis

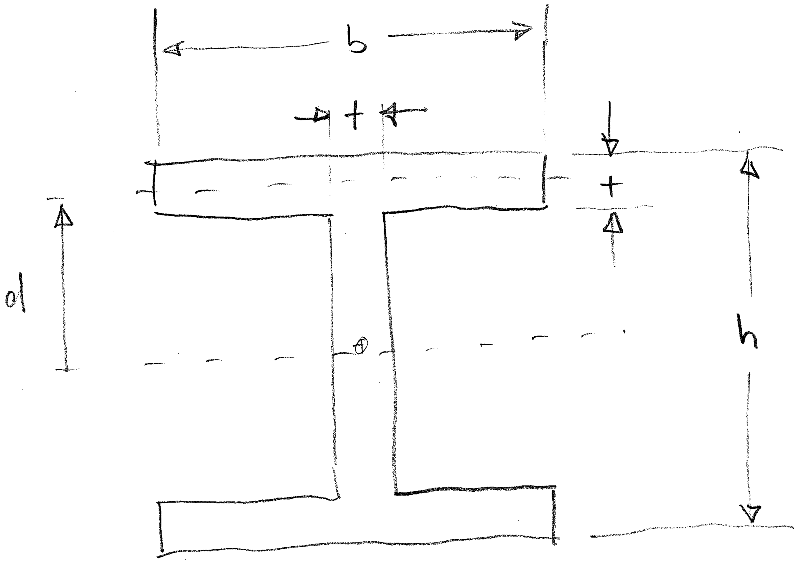
- Sketch simple models
  - Free Body Diagram
  - Cross-section and bending stress
- What is/are governing equation(s)?
- Substitute until only free parameters are left
- Can we now solve for the factor of safety?
- Other relationships of interest?
  - Mass as function of h, t, & b for pre-defined F.O.S.?

Example: I-Beam Detailed Analysis

- Model part in SolidWorks
- Perform stress analysis using Simulation
- What is peak stress?
- What is Factor of Safety?

Homework Assignment

- By email this evening
- Analyze stress in two parts, two ways:
  - Simple models with analytical stress analysis
  - Detailed model with FEA stress analysis
- If you don't recall governing equations…
  - Try Shigley, Chapter 3
  - Or go through old course notes



$I_{RECT} = \frac{1}{12} b h^3 + d^2 \cdot A$ ($A = b \cdot h$) (e.g. SHIGLEY)

$\therefore I_{TOTAL} = 2 \cdot \left(\frac{1}{12} b \cdot t^3 + \left(\frac{h}{2} + \frac{t}{2} \right)^2 \cdot b \cdot t \right) + \frac{1}{12} t (h-2t)^3$ (→0 terms)

$\approx \frac{1}{6} b t^3 + \frac{1}{2} b t h^2 - b t^2 h + \frac{1}{2} b t^3$

$I \approx \frac{1}{2} b t h^2$ (1)

$\sigma_m \approx \frac{My}{I}$ (e.g. SHIGLEY Ch. 3)

$M = F \cdot l \quad y = \frac{1}{2} h \quad I \approx \frac{1}{2} b t h^2$

$\therefore \sigma_m \approx \frac{F \cdot l \cdot \frac{1}{2} h}{\frac{1}{2} b t h^2} = \frac{Fl}{bth}$ (2)

$F.O.S. = \frac{\sigma_y}{\sigma_m}$

$\sigma_m \approx 20{,}000 \ \frac{lb}{in^2}$ (or 20 ksi)

$\sigma_y \approx 36$ KSI (36,000 PSI)

$\therefore F.O.S \approx 1.8$

SHEAR: $\tau_{AVE} = \frac{F}{A}$

$A = 2 \cdot b \cdot t + t(h-2t)$

$= 2bt + ht - 2t^2$ (→0)

$\therefore \tau_{AVE} \approx \frac{F}{t(2b+h)} = 500$ PSI

$\tau_{max_I} = \frac{F}{A_{WEB}} \approx \frac{F}{t \cdot h} = 1000$ PSI (NEGLIGIBLE)

STRESS CONCENTRATIONS:

HOLE IN PLATE (e.g. SHIGLEY 3-29)

$K_t \approx 2.5$

$\therefore \sigma_{max} \approx 20 \text{ KSI} \cdot 2.5 = 50$ KSI

$F.O.S.^* \approx \frac{36}{50} \approx 0.7 < 1$

BUT, DUCTILE MATERIAL...

MASS: $m = \rho \cdot l \cdot (2bt + t(h-2t))$

$\approx \rho \cdot l \cdot t \cdot (2b+h)$

(2) $\Rightarrow t \approx \frac{Fl}{\sigma_m bh}$

$\therefore m^* \approx \frac{\rho F l^2}{\sigma_m} \cdot \frac{(2b+h)}{bh}$

* FOR $t = f(b,h)$; F, l, σ_m GIVEN.