Design with Electric Motors: Analysis and Selection

24-370 - Spring 2011
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Reminders and Announcements

• Project 2 testing Monday
  – Silly hat competition... continues :D
  – Robotic arm and wrist available in ME Shop
  – Saturday ME shop hours: 9:00 - 3:00
  – Reimbursement forms now online (Item 6)
  – Quick Project 2 questions?
What are Electric Motors?

• Electromechanical devices that transform between electrical and mechanical domains
  – For instance, from current to torque
  – Based on electromagnetism, i.e. magnetic fields
Types of Electric Motors

- DC motors
  - Commonly used in robotics applications
  - Variable speed and torque
  - Permanent magnets (usually)
  - Variants: brushless (electronic commutation), coreless (lighter rotor),

- AC motors
  - Commonly used for high power applications
  - Constant (high) speed, poor low-speed pos control
  - Magnetic field typically induced in rotor (induction)

Other mech power sources vs. motors

- Gasoline engines
  - Higher fuel energy density ($5 \times 10^7$ vs. $5 \times 10^5$ J·kg$^{-1}$)
  - Lower efficiency (~25% vs. ~75%)
  - More noise, local pollution, minimum (stall) speed

- Gas turbines and jet engines
  - Similar to gasoline engines, but efficiency ~40%

- Nuclear reactors
  - Very high fuel energy density ($3 \times 10^{12}$ J·kg$^{-1}$)
  - Lower efficiency: ~40%
  - Generally steam-based, radiation issues
Other mech power sources vs. motors

- Mammalian muscle tissue
  - Higher fuel energy density ($4 \times 10^7$ vs. $5 \times 10^5$ J·kg$^{-1}$)
  - Lower efficiency (~25% vs. ~75%)
  - Equally quiet, low pollution, low speeds
  - Self-healing vs. gradually deteriorating

Design with Electric Motors

- Selection, rather than continuum design
  - Discrete set of options available by catalog
- Highly dependent on mechanical usage
  - Torque, speed, power
  - Efficiency
- Simultaneously choose gear box
- Many types available
  - We will primarily consider DC motors (brushed)
**Mechanical uses of electric motors**

- Provide mechanical input to system
- Open-loop drives:
  - Self-regulated by electrical or mechanical reactions
  - e.g. Project 2 drive shaft
- Closed-loop control of current:
  - Regulation of voltage to achieve desired current
  - e.g. electric car
- Closed-loop control of mechanical output:
  - Automatic control of current
  - Feedback control of output torque, velocity, or position
  - e.g. robotics applications

**Simple models of motor function**

- Isolated torque production
  - Neglect speed and inertial effects
- Maximum drive speed
  - Neglect external loads
- Maximum power production
  - Best combination of speed and torque
- Maximum control bandwidth
  - Account for speed, torque, and inertial effects
## Key properties of electric gear motors

- **Peak torque**
  - Peak current, $i_{\text{max}}$, motor torque constant, $K_t$
- **Maximum speed**
  - Rated voltage, $V$, motor voltage constant, $K_v$
- **Maximum power**
  - Optimal combination of torque and speed (rated)
- **Dynamic response (time constant, bandwidth)**
  - Torque, speed, inertia, and load
- **Efficiency**
  - Gearbox, backdrivability
- **Mass**

## General mathematical motor model

- **Derive from basic familiar equations:**
  - Newton’s second law: $\Sigma \tau = J \cdot \alpha$
  - Ohm & Faraday: $V = i \cdot R + L \cdot \frac{di}{dt}$
- **And two new ones:**
  - Motor torque equation: $\tau_m = K_t \cdot i$
  - Back EMF equation: $V_{\text{BEMF}} = K_v \cdot \omega$
- **Coupled equations for motor dynamics:**
  - $V = i \cdot R + L \cdot \frac{di}{dt} + K_v \cdot \frac{\omega}{J}$
  - $K_t \cdot i - \tau_a - b \cdot \omega = J \cdot \alpha$
### Simplified torque analysis

- **Simplifying assumptions:**
  - Velocity is zero $\Rightarrow$ no back EMF, no damping
  - No inductance, or steady state current
- **Implications:**
  - $i = V/R = \text{maximum (stall) current}$
  - $\tau_m = K_c \cdot V/R = \tau_{\text{stall}} \text{ maximum (stall) torque}$
- **How might we apply to the design process?**
  - Max applied torque, $\tau_a$, implies $\min \tau_{\text{stall}}$
- **Gearbox torque losses**
  - Gearbox inefficiency presents as friction torque loss

### Simplified velocity analysis

- **Simplifying assumptions**
  - Applied torque is zero $\Rightarrow$ no external resistance
  - Steady-state current $\Rightarrow$ no inductance effects
  - Neglect damping
- **Implications**
  - $\omega$ increases until reaching $V/K_v$, driving $i$ to 0
  - No current, no torque ($K_v \cdot i$), so acceleration is 0
  - $\omega \approx V/K_v \approx \text{maximum (no-load) motor speed}$
- **Application to design process?**
  - Max output speed must be less than no-load speed
## Simplified power analysis

- Relate max power, torque, and speed?
  - Not the product of max torque and max speed
- Simplifying assumptions
  - Constant current or no inductance
  - Constant optimal (nominal) speed and torque
  - Power = torque times velocity, or \( P_{\text{max}} \approx \tau_{\text{m,nom}} \cdot \omega_{\text{nom}} \)
- Implications for design
  - Constraint on minimum motor power
  - Optimal gear ratio for motor and application
  - Include losses in gearbox

## Efficiency analysis

- Efficiency, \( \eta = \frac{\text{energy out}}{\text{energy in}} \), \( 0 \leq \eta \leq 1 \)
- Typically applied to a single conversion
  - Electromechanical: work / electrical energy
  - Mechanical: work out / work in
- For typical DC motors, \( \eta \approx 0.9 \)
  - Additional gearbox term of 0.4-0.8 (from torque)
  - Where does energy go?
- Gross efficiency might also include:
  - Battery efficiency: operational energy / charge
  - Power plant efficiency: electrical / chemical
Dynamic analysis

- Dynamic response $\rightarrow$ full equations of motion
- Implications:
  - Basic properties ($R$, $L$, $K_v$, $K_p$) all contribute
  - Partially captured by mechanical time constant
  - External loads, i.e. $V_a$, $\tau_a$, contribute
- Implications for mechanical design
  - Dynamic motor evaluation needed
- Matlab example
  - See next lecture notes (ran out of time)