Assignment 4: Material Selection and High-Level Assembly Design

24-370 Engineering Design I
Due @ 12:30, Wednesday February 23nd 2011

Part 1: Kinematics and Dynamics

Derive equations relating Cartesian positions to joint angles for a simple assembly. Determine the dynamic loads that result from angular velocities or accelerations.

A robot arm (Figure 1) has an upper arm that rotates about point $\boldsymbol{O}$ and a lower arm that rotates about an elbow joint at point $\boldsymbol{a}$. The angle of the first link, θ_1, is defined with respect to the vertical, and the angle of the second link, θ_2, is defined with respect to the first link. The links have centers of mass located a distance c_1 and c_2 from their joints, respectively. For simplicity, approximate the mass of each link, m_1 and m_2, respectively, being concentrated at the center of mass. That is, approximate the mass moment of inertia I as zero for both links. The links have length l_1 and l_2, respectively, height h, cross-sectional area A, and cross-sectional area moment of inertia I_A.

1.a - Hand kinematics. Derive an equation relating the Cartesian coordinates (x- and y-position) of the hand as a function of the joint angles and model parameters.

1.b - Locked-joint mass properties. If the elbow joint is locked at zero at all times (i.e. $\theta_2 = 0$), the two links can be approximated as a single link with a single center of mass. Determine the total mass, m. Use a weighted average to determine the location of this center of mass, c. We will again approximate the mass moment of inertia I as zero.

1.c - Dynamic loading due to acceleration from rest. While the robot arm is still ($\dot{\theta}_1 = 0$) an instantaneous torque τ is applied at the joint $\boldsymbol{O}$ that results in an angular acceleration $\ddot{\theta}_1$. What is the magnitude of the acceleration of the center of mass? (Neglect rotational inertia about the center of mass.) Draw (a) free body diagram(s), and solve for the torque that was applied to the joint. Describe the qualitative loading condition that results from this dynamic load (in 5 words or less, e.g. column in compression). What is the peak stress in the upper link? You can neglect detailed effects such as from the features near the joint.

1.d - Dynamic loading due to constant angular velocity. At a later instant, the angular acceleration is zero, $\ddot{\theta}_1 = 0$, but the arm is rotating at a high velocity $\dot{\theta}_1$. What is the magnitude of the acceleration of the center of mass? Draw a free body diagram and determine the reaction forces and/or torques on the robot arm between the center of mass and the joint $\boldsymbol{O}$. What is the qualitative loading condition? What is the peak stress in the upper link that results from this dynamic loading? You can neglect detailed effects such as from the features near the joint.

1.e - Interpretation. Which of the two scenarios (c or d) will result in more worrisome loading? Which of the two analyses includes poorer assumptions?

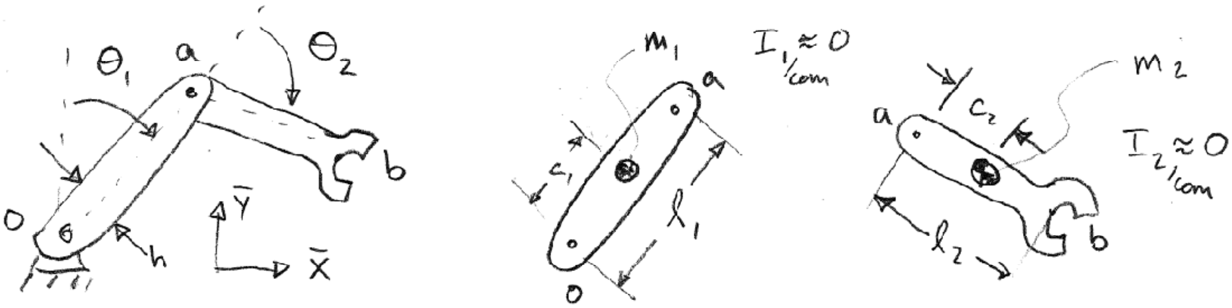

Figure 1. Robot arm diagram and variable definitions.

Part 2: Gear Ratios

Relate the position and torque of a driven component to the position and torque of a driving component using gear ratios in the drive train.

The transmission in Figure 2 has two gears, two pulleys and one belt. The first gear has radius r_1 and contacts the second gear at its outer edge without slipping. The second gear has radius r_2 and is rigidly connected to the first pulley. The first pulley has radius r_3 and contacts the belt at its outer edge without slipping. The belt contacts the second pulley at its outer surface without slipping. The second pulley has radius r_4. The angles of the first gear and second pulley are $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_4}$, respectively. Torques $\boldsymbol{\tau_1}$ and $\boldsymbol{\tau_4}$ are applied to the first gear and second pulley, respectively.

2.a - Kinematic ratios. What is $\boldsymbol{\theta_4}$ as a function of $\boldsymbol{\theta_1}$?

2.b - Torque ratios. If the system is in equilibrium, what is $\boldsymbol{\tau_4}$ as a function of $\boldsymbol{\tau_1}$?

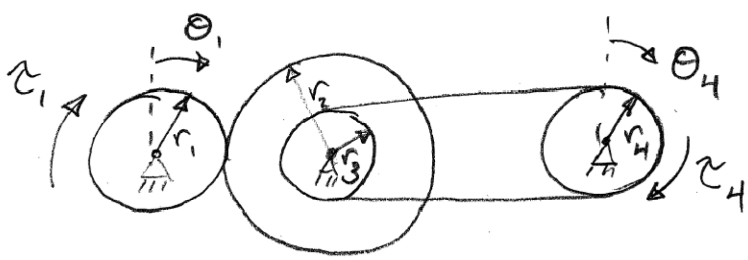

Figure 2. Transmission diagram and variable definitions.