Politically credible social insurance.[*]

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Abstract

This paper considers political credibility of allocations in settings with dynamic private information. It embeds a benchmark dynamic moral environment into political economy games which feature repeated voting over mechanisms. Optimal politically credible allocations are shown to solve virtual planning problems with social discount factors in excess of the private one.

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1 Introduction

A large literature has studied the efficient evolution of consumption and utility distributions in the presence of private information. It has considered different arrangements for implementing efficient incentive-compatible

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allocations and these have informed recent thinking on the design of wealth taxation, unemployment insurance and bankruptcy law. However, many of the models in this literature prescribe relentlessly increasing levels of inequality. Indeed, under standard restrictions on social and individual preferences and for various sorts of private information, an immiseration result obtains. The result asserts that at the social optimum, an agent's continuation lifetime utility almost surely converges to its lowest bound. The immiseration result makes the political viability of such optima doubtful. If a society has access to a political device for revising allocations, can it commit to an allocation that consigns agents almost surely to eventual misery? If political credibility precludes such commitment, what can be achieved?

This paper takes a first step in addressing such questions. It embeds a benchmark normative private information environment (namely that of Atkeson-Lucas (1992)) into a variety of political economy games.[1] All of these games incorporate repeated probabilistic voting over rival political parties and their proposals for allocating resources.[2] They allow societies to revise allocations via elections and, since neither political parties nor voters can commit in advance to propose or vote for a given allocation, politically credible allocations must be immune to such revision. In particular, such allocations must be induced by strategies for political parties and agent-voters that remain optimal after each history. Our focus is on Pareto optimal politically credible allocations. Our main result is that for each of the games we consider these allocations solve a "virtual planning problem" that shares the same constraint set as the benchmark normative problem, but has a perturbed social objective. This objective incorporates discount factors that (weakly) exceed those of agents and agent Pareto weights that tend to converge

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[1] See Acemoğlu et al (2007) for a related treatment of these questions.

[2] Probabilistic voting is a standard formulation in many applied political economy models, see Persson and Tabellini (2000) and, for applications to dynamic macroeconomic settings with Ramsey tax schemes, Azzimonti (2004) and Hassler et al (2005).

to a common value of 1. Elsewhere Farhi and Werning (2007) and Sleet and Yeltekin (2007a) have considered normative problems in which the planner is assumed to use a time invariant discount factor in excess of agents. For an appropriate initial Pareto weight distribution such normative problems emerge as virtual planning problems in our setting. However, we do not assume - or make a normative case for - a societal discount factor in excess of agents. Rather such effective societal discounting emerges endogenously as an equilibrium phenomenon in the political economy games we consider. Despite - indeed because - politicians cannot commit they behave as if they are more concerned with the long run than are agents.[3]

The connection we obtain between political economy models and planning problems is valuable for two reasons. First, as indicated above, it provides a micro-political rationale for planning problems with differential societal and agent discounting. Second, it implies that optimal politically credible allocations can be obtained as solutions to such planning problems. Existing work has established[4] that such solutions are non-immiserating - in fact, a characteristic of all politically credible allocations in the games we analyze, not just the Pareto optimal ones - and that for the right initial conditions, they exhibit stationary and sometimes ergodic utility and consumption distributions. For an appropriately chosen distribution of Pareto weights, optimal politically credible allocations display the same stationarity and ergodicity with important implications for inequality and social mobility.

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[3] Phelan (2006) considers a normative problem in which the planner's discount factor exceeds that of agents and equals one. We recover his problem in our political economy games when we alter our equilibrium concept so that, in the spirit of Pearce's (1987) renegotiation-proofness, it is robust to politically orchestrated joint deviations by parties and agents. Solutions to Phelan's planning problem are then equilibrium outcomes of our game and, conversely, any equilibrium outcome satisfying a stationarity requirement solves Phelan's planning problem. See Sleet and Yeltekin (2007b)

[4] E.g. Farhi-Werning (2007), Sleet-Yeltekin (2007a) and Phelan (2006).

The logic underlying our results is straightforward. The set of necessary and sufficient conditions for a politically credible allocation augments the constraint set from the benchmark Atkeson-Lucas problem with a sequence of political constraints. The latter ensure that the strategies which implement the allocation admit no profitable deviation for the political parties. Since the probabilistic voting games we consider tie electoral success to weighted aggregates of agent-voter continuation utilities, our political constraints require that these aggregates remain above some lower bound. If they fell below this bound, no continuation play could deter one or more political parties from defecting away from the allocation. A Pareto optimal politically credible allocation maximizes a standard Pareto weighted sum of agent utilities subject to the necessary and sufficient conditions for political credibility. When the political constraints bind they raise the shadow value of the weighted aggregate of agent continuation utilities. It is as if there is a planner who discounts future agent utility (possibly reweighted according to some modified Pareto scheme) less heavily.

Our basic results are obtained in the context of a number of examples that impose assumptions which, while standard in the political economy literature, are stylized. Section 4 of the paper provides some generalization. We show that as long as the political constraints implied by a given political economy game can be represented as bounds on smooth, increasing, concave functionals of agent-voter continuation payoff functions, then the optimal politically credible allocation solves a virtual planning problem with societal discount factors weakly in excess of private ones, and strictly in excess when the political constraints bind.

2 An environment with commitment

A continuum of infinitely-lived agents inhabit an economy. The population is initially partitioned into a measure space $(\mathbb{R}, \mathcal{B}, \Psi)$ of types w, where $\mathcal{B}$ is the Borel sigma algebra. In each period, agents receive a random taste shock $\theta_t \in \Theta := \{\widehat{\theta}_k\}_{k=1}^{K}$. These shocks are i.i.d. across agents and time with distribution π. Let $\theta^t := \{\theta_1, ..., \theta_t\} \in \Theta^t$ denote a t-period history of shocks; let π^t denote the corresponding probability distribution. We assume that for all t and sets $E \subset \Theta^t$, $\pi^t(E)$ gives the fraction of agents with shock history in E.[5]

After each realized history θ^t, an agent receives an allocation of consumption and, hence, utility. In the sequel, it will be convenient to describe allocations directly in terms of the stream of utility they provide rather than stream of resources they use. Define an *individual allocation* to be a sequence of functions $\{\psi_t\}_{t=1}^{\infty}$, $\psi_t : \Theta^t \to D$ with D a bounded subset of $\mathbb{R}$ and denote an agent's payoff from $\{\psi_t\}_{t=1}^{\infty}$ by

$$U(\{\psi_t\}_{t=1}^{\infty}) = (1-\beta)\sum_{t=1}^{\infty}\sum_{\Theta^t}\beta^{t-1}\theta_t\psi_t(\theta^t)\pi^t(\theta^t),$$

where $\beta \in (0,1)$ is the agent's discount factor. We define a *(population) allocation* $\{\varphi_t\}_{t=1}^{\infty}$ to be a sequence of measurable functions $\varphi_t : \mathbb{R} \times \Theta^t \to D$. A population allocation implies a collection of continuation individual allocations $\{\varphi_{t+r}|w, \theta^{t-1}\}_{r=0}^{\infty}$, $t \geq 1$, obtained after each individual history (w, θ^{t-1}). We denote the range of U by W and the set of population allocations by A.

Let $C : D \to \mathbb{R}$ denote the cost of delivering current utility to an agent. We assume that C is strictly increasing, strictly convex and differentiable on the interior of D. C implies a utility function over consumption: $u = C^{-1}$.

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[5] In making this assumption, we rely on the construction of Sun (2006).

2.1 Feasible Allocations

In period t, the economy possesses a quantity of resources $R_t \in [\underline{R}, \overline{R}]$, $\underline{R} > 0$. An allocation is *resource-feasible* if:

$$\forall t, \;\; R_t \geq \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw). \tag{1}$$

Allocations must provide agents with incentives to reveal their privately observed shocks. Let $\alpha = \{\alpha_t\}_{t=1}^{\infty}$ denote a reporting strategy for the agents with for each t, $\alpha_t : \Theta^t \to \Theta$. Let $\alpha^t(\theta^t)$ denote the history of reports induced by α given the shock history θ^t. An allocation is *incentive-compatible* if for Ψ-a.e. w and all α,

$$(1-\beta)\sum_{t=1}^{\infty}\sum_{\Theta^t}\beta^{t-1}\theta_t\varphi_t(w, \theta^t)\pi^t(\theta^t) \geq (1-\beta)\sum_{t=1}^{\infty}\sum_{\Theta^t}\beta^{t-1}\theta_t\varphi_t(w, \alpha^t(\theta^t))\pi^t(\theta^t). \tag{2}$$

Let $\Gamma(\{R_t\}, \Psi)$ denote the set of resource-feasible and incentive-compatible allocations.

2.2 A benchmark planning problem

We may define a family of Pareto planning problems by

$$\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w, \cdot)\}_{t=1}^{\infty})\Psi(dw), \tag{3}$$

for some measurable Pareto weighting function $\gamma : \mathbb{R} \to \mathbb{R}_+$ with $\int \gamma(w)\Psi(dw) = 1$. This is essentially a primal version of the problem considered by Atkeson and Lucas (1992). For a large class of cost functions C (or utility functions C^{-1}), any solution$\{\varphi_t^*\}_{t=1}^{\infty}$ to (3) satisfies the following *agent immiseration property*: for Ψ-a.e. w, π^{∞}-a.e. θ^{∞}, $\lim_{t\to\infty} U(\{\varphi_{t+s}^*(w, \theta^t, \cdot)\}_{s=1}^{\infty})$ exists and equals $E[\theta] \inf D$. Since D is bounded, this implies the *utilitarian immiseration property* $\lim_{t\to\infty} \int_{\mathbb{R}} \gamma(w) \sum_{\Theta^t} U(\{\varphi_{t+s}^*(w, \theta^t, \cdot)\}_{s=1}^{\infty}) \; \pi^t(\theta^t) \; \Psi(dw) = E[\theta] \inf D$.

3 A political economy game with office-motivated politicians

3.1 Basic environment

We embed the basic environment of Section 2 into a political economy game with probabilistic voting over political parties. As is typical in the probabilistic voting literature, we assume that agents have heterogeneous and time varying biases towards a particular party. Although stylized, these biases capture the ideas that election outcomes are uncertain and are not solely determined by economic policy platforms. We start with the simplest formulation in which 1) political bias distributions are uniform, 2) there are no incumbency advantages and 3) parties are operated by impatient, office-motivated politicians. Later we extend the model by relaxing some of these assumptions.

3.1.1 Players

Politicians There are two political parties $i \in \{A, B\}$. In each period t, politicians from the respective parties propose political mechanisms that describe how the current resource aggregate R_t will be allocated. Agents vote over the two proposed mechanisms and the election-winning mechanism is implemented. Politicians are *office-motivated,* i.e. concerned solely with winning elections. The objective of party i's politician is:

$$\sum_{t=1}^{\infty} \chi^{t-1} p_t^i,$$

where p_t^i is the probability that party i wins the election at date t and χ is the politician's discount factor. For now, we assume that $\chi = 0$; the assumption that politicians are completely myopic only makes our results starker.

Agents Agents have the same preferences over allocations as before; we augment them with biases for one or the other party. Suppose that the two political parties A and B are distinguished by fixed "ideologies" that are non-economic in nature and difficult to change. Agent preferences over ideologies are described by random variables $\{\xi\}$ and $\{\delta_t\}_{t=1}^{\infty}$. ξ represents a permanent and idiosyncratic shock to each agent's relative preference for party B's ideology, it is i.i.d. across agents according to an atomless distribution with c.d.f. F. δ_t represents a common, time varying shock to the population's preference for party B's ideology, it is i.i.d. across time according to an atomless distribution with c.d.f. G. These political preferences give rise to a lifetime "political" utility which augments the agent's payoff from an allocation to give a total payoff:

$$U(\{\varphi_t\}_{t=1}^{\infty}) + (1-\beta)E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left[\xi+\delta_t\right]1_t^B\right]. \tag{4}$$

In (4), 1_t^B is a random variable that equals 1 if party B is in power in period t and 0 otherwise. For now we make the following assumption on the distributions F and G. It is conventional in applied probabilistic voting models.

Assumption 1 *1) F is uniform with range $\Xi = \left[-\frac{1}{2\widehat{\xi}}, \frac{1}{2\widehat{\xi}}\right]$ and density $\widehat{\xi}$. 2) G is uniform with range $\Delta = \left[-\frac{1}{2\widehat{\delta}}, \frac{1}{2\widehat{\delta}}\right]$ and density $\widehat{\delta}$. 3) $\widehat{\delta}$ and $\widehat{\xi}$ satisfy $d + \frac{1}{2\widehat{\delta}} \in \left(0, \frac{1}{2\widehat{\xi}}\right)$ and $d \in \left(0, \frac{1}{2\widehat{\delta}}\right)$, where $d = E[\theta](\sup D - \inf D)$.*

The last part of the assumption rules out inconvenient boundary solutions by ensuring that within each history-contingent sub-population there are some agents who will vote for either party no matter what and at the aggregate level, there is a probability (possibly very small) that a party will win an election no matter what.

3.1.2 The stage game

Mechanisms At the beginning of period $t \geq 1$, each agent is publicly identified by its type w and a history of past messages m^{t-1}, where $m^{t-1} \in M^{t-1} := \prod_{r=1}^{t-1} M_r$ and M_r is a finite *message space* for prior period r. A period t mechanism proposal from party i is a pair $S_t^i = (M_t^i, \varphi_t^i)$. The first piece of the mechanism is a finite message space M_t^i that agents can use to communicate with the government in the current period, the second is a (measurable) *utility allocation function* $\varphi_t^i : \mathbb{R} \times M^{t-1} \times M_t^i \to D$ that describes how utility will be awarded to an agent contingent on its type and message history inclusive of current message.

Note that this specification allows for a much richer collection of mechanisms than is typically permitted in the probabilistic voting-political economy literature. In principle, these mechanisms can be used to implement history dependent allocations that provide future rewards and penalties for current behavior. In particular, they can be used to implement the optimal allocation of a committed utilitarian planner.

Timing The ensuing stage game consists of three sub-periods. In the first, politicians propose mechanisms. In the second the δ_t political preference shock is realized and agents vote over mechanisms. In the third sub-period, the election-winning mechanism is executed. Agents receive their taste shocks, they choose a probability distribution over the current message space and draw a message from it. They transmit this message to the government and receive the utility award implied by the election-winning utility allocation function.

3.1.3 Histories and strategies

We place some restrictions on strategies at the outset to rule out especially implausible equilibria. In particular, we assume that no player conditions her behavior on either past election-*loosing* mechanisms or the political identity of past election winners.[6] This assumption precludes strategies that would effectively punish the economy for failing to elect a particular party even if that party makes a very undesirable mechanism proposal. However, it still allows for considerable history dependence of equilibria and is much weaker than a Markov restriction.

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[6] This assumption resembles Duggan and Fey's (2006) assumption of outcome stationarity.

Policy strategies We say that a period t mechanism $S_t = (M_t, \varphi_t)$ is *compatible* with a sequence of past mechanisms $\{M_r, \varphi_r\}_{r=1}^{t-1}$ if $\varphi_t : \mathbb{R} \times \prod_{r=1}^{t} M_r \to D$. For $t \geq 2$, we define a t-period *aggregate history* H_t to be a sequence of $t-1$ compatible mechanisms, i.e. a sequence $\{M_r, \varphi_r\}_{r=1}^{t-1}$ such that each (M_r, φ_r) is compatible with $\{M_s, \varphi_s\}_{s=1}^{r-1}$. H_1 is set to the null history. The sets of t-period histories and mechanisms are denoted, respectively, $\mathcal{H}^t$ and $\mathcal{S}_t$; $\mathcal{S}_t(H_t)$ denotes the set of period t mechanisms compatible with H_t. A *policy strategy* for party i is a sequence of functions $\sigma^i = \{\sigma_t^i\}_{t=1}^{\infty}$, where $\sigma_t^i : \mathcal{H}^t \to \mathcal{S}_t$ gives party i's t-period mechanism as a function of the aggregate history and for all i, t, $\text{Graph}(\sigma_t^i) \subset \mathcal{H}^{t+1}$. Let $\sigma = \{\sigma^A, \sigma^B\}$ denote a profile of policy strategies.

Message strategies A period t *individual history* of an agent $h_t = \left(w, m^{t-1}\right)$ gives the agent's w-type and past message history. We say that an aggregate and individual history pair $(H_{t+s}, h_t) = (\{M_r, \varphi_r\}_{r=1}^{t+s}, (w, m^{t-1}))$, $s \geq 0$, are consistent if $m^{t-1} \in \prod_{r=1}^{t-1} M_r$. Let $\mathcal{J}^1 = \emptyset$ and for $t > 1$, let $\mathcal{J}^t$ denote the set of consistent period $t+1$ aggregate and period t individual histories. Let $\mathbb{P}$ denote the space of finite element probability distributions and $\mathbb{P}(X)$ the set of probability distributions on the finite set X. The first component of agent behavior is a message strategy, $\lambda = \{\lambda_t\}_{t=1}^{\infty}$, where $\lambda_t : \mathcal{J}^t \times \Theta \to \mathbb{P}$ and $\lambda_t(H_t, M_t, \varphi_t, h_t, \theta_t) \in \mathbb{P}(M_t)$. A message strategy maps a consistent aggregate-individual history pair and a current taste shock to a lottery over the current (election-winning) message space M_t. Let $Q_t(H_t, \lambda)$ denote the induced probability measure over h_t.

Voting strategies A period t *individual political history* of an agent $h_t^p = \left(\{H_t, S_t^A, h_t\},\ \{H_t, S_t^B, h_t\}, \xi, \delta_t\right) \in \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta$ includes two consistent aggregate-individual histories that coincide up to date $t-1$, but terminate with the current mechanism proposals of the parties. h_t^p also includes an agent's current political preference shocks. An agent's *voting strategy* is given by $\zeta = \{\zeta_t\}_{t=1}^{\infty}$, where $\zeta_t : \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta \to \mathbb{P}(\{A, B\})$ maps individual political histories to a probability distribution over $\{A, B\}$. After each h_t^p, the agent draws the name of a party from $\zeta_t\left(h_t^p\right)$ and then votes for that party. Note that an agent's voting strategy does not condition on her past votes since these are anonymous and do not affect her preferences over the parties.

Resource-feasibility To be resource feasible given λ, a mechanism must consume less than R_t in each period.

Definition 1 *Let λ denote a message strategy. A t-period mechanism $S = (M, \varphi) \in \mathcal{S}_t(H_t)$ is **resource-feasible** at H_t given λ if*

$$\int_{\mathbb{R}\times M^{t-1}\times M} C(\varphi(h_{t+1}))Q_{t+1}(H_t, S, \lambda)(dh_{t+1}) \leq R_t.$$

*Let $\mathcal{S}_t(\lambda, H_t)$ denote the set of compatible and resource-feasible mechanisms at H_t given λ. An aggregate history $H_t = \{S_r\}_{r=1}^{t-1}$ is **resource-feasible** given λ if each $S_r \in \mathcal{S}_r(\lambda, \{S_s\}_{s=1}^{r-1})$. Let $\mathcal{H}^t(\lambda)$ denote the set of t-period resource-feasible aggregate histories given λ. A policy strategy σ^i is **resource-feasible** given λ if after each $H_t \in \mathcal{H}^t(\lambda)$, $\sigma_t^i(H^t) \in \mathcal{S}_t(\lambda, H_t)$. Let $\Sigma(\lambda)$ denote the set of resource-feasible policy strategy profiles given λ.*

We do not allow political parties to make resource-infeasible policy proposals and, henceforth, restrict attention to resource-feasible policy strategies.

Strategy profile outcomes A mechanism is implemented at date t if it either wins more than 50% of the vote or, in the event of a tie, it wins a fair coin toss. A strategy profile (σ, ζ, λ) and the shock distributions F and G induce a family of conditional probability distributions over current and future mechanisms, incumbent governments and agent utilities. We call this the outcome of the profile and isolate two components. First, let $p_t^i(H_t, S_t^i, S_t^j|\zeta)$ denote the probability that mechanism S_t^i wins an election for party i when the aggregate history is

H_t, party j proposes S^j_t and the voting strategy is ζ. Second, let $U_t(H_{t+1}, h_t|\sigma, \zeta, \lambda)$ denote the agent continuation payoff implied by (σ, ζ, λ) after (H_{t+1}, h_t) (i.e. after the period t election, but before the period t taste shock realization).

3.2 Politically credible equilibria

We describe the elements of an equilibrium below and then collect these elements into a formal definition.

Policy strategies Since the politicians operating party i are concerned only with winning the current election, they select σ^i such that:

$$\forall t,\ H_t \in \mathcal{H}^t(\lambda),\ j \neq i, \qquad \sigma^i_t(H_t) \in \arg \sup_{S \in \mathcal{S}_t(\lambda, H_t)} p^i_t(H_t, S, \sigma^j_t(H_t)|\zeta). \tag{5}$$

In doing so they ignore the fact that their proposals may reduce the lifetime payoffs obtained by agents in previous periods and may, hence, contribute to electoral failure in those periods.

Message strategies Agents choose message strategies that are optimal after each history.

$$\forall t, (H_{t+1}, h_t) \in \mathcal{J}^t,\ \widehat{\lambda}, \qquad U_t(H_{t+1}, h_t|\sigma, \zeta, \lambda) \geq U_t(H_{t+1}, h_t|\sigma, \zeta, \widehat{\lambda}). \tag{6}$$

Voting strategies Let S^i_t denote the mechanism proposed by party i at time t. The total continuation payoff to an agent after individual political history $h^p_t = ((H_t, S^A_t, h_t), (H_t, S^B_t, h_t), \xi, \delta_t)$ and a period t election victory for party i is:

$$U_t(H_t, S^i_t, h_t|\sigma, \zeta, \lambda) + (1-\beta)(\xi + \delta_t)1_{\{i=B\}} + (1-\beta)E\left[\sum_{s=1}^{\infty} \beta^s (\xi + \delta_{t+s})1^B_{t+s}|H_t, S^i_t; \sigma, \zeta, \lambda\right].$$

where $1_{\{i=B\}}$ takes the value 1 if i equals B and is 0 otherwise. Let $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) := U_t(H_t, S_t^A, h_t|\sigma, \zeta, \lambda)$ $- \ U_t(H_t, S_t^B, h_t|\sigma, \zeta, \lambda)$ denote the difference in economic payoffs and

$$\frac{D(H_t, S_t^A, S_t^B \xi, \delta_t|\sigma, \zeta, \lambda)}{1-\beta} := \xi + \delta_t + E\left[\sum_{s=1}^{\infty} \beta^s (\xi + \delta_{t+s}^B)|H_t, S_t^B; \sigma, \zeta, \lambda\right] - E\left[\sum_{s=1}^{\infty} \beta^s (\xi + 1_{t+s}^B)|H_t, S_t^A; \sigma, \zeta, \lambda\right]$$

the difference in ideological payoffs across the two proposals to an agent with history h_t^p. We assume that agents vote as if they were pivotal in the current period. Thus, their voting strategy satisfies for all t, $h_t^p \in \mathcal{J}^t \times \mathcal{J}^t \times \Xi \times \Delta$,

$$\zeta_t(h_t^p; A) \in \begin{cases} \{1\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) > D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ [0,1] & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) = D(H_t, S_t^A, S_t^B, \xi, \delta_t) \\ \{0\} & \text{if} \quad \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) < D(H_t, S_t^A, S_t^B, \xi, \delta_t). \end{cases} \tag{7}$$

Politically credible equilibria The following definition collects the components of an equilibrium together.

Definition 2 (σ, ζ, λ) *is a* ***politically credible equilibrium*** *(PCE) if* $\sigma \in \Sigma(\lambda)$ *and 1) (party optimality)* $\forall$ $i \in \{A, B\}$, σ^i *satisfies (5); 2) (Agent optimality: messages)* λ *satisfies (6); 3) (Agent optimality: voting)* ζ *satisfies (7). A* ***symmetric politically credible equilibrium*** *(SPCE) is a PCE that satisfies* $\sigma^A = \sigma^B$.

This definition is in the spirit of Chari and Kehoe's sustainable plans equilibrium concept. They require optimality of player strategies after all feasible histories.

Characterization of equilibria Recall that continuation play of the game depends only on which mechanism wins the election, not which party. If the two parties propose the same mechanism then the continuation outcome path is independent of the electoral outcome and an agent's vote is determined solely by its current political shocks: $\zeta_t(h_t^p; A) = 1$ (resp. 0) if $0 > \xi + \delta_t$ (resp. $0 < \xi + \delta_t$). Party A (resp. B) then wins the election with probability

$p = G(-F^{-1}(1/2))$ (resp. $1-p$). Since party A is always able to mimic B and choose the same mechanism, p places a lower bound on its probability of winning. Conversely, party B can always mimic party A and secure a probability of winning of $1-p$. Hence, in any (continuation) equilibrium party A must win with probability p in each period and we have the following lemma.

Lemma 1 *In equilibrium, party A wins the election with probability $p = G(-F^{-1}(1/2))$ in each period. When F and G satisfy Assumption 1, $p = \frac{1}{2}$.*

Let $p_t^{*i}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda)$ denote the probability that party i wins the election at t following H_t, play of the current feasible mechanisms S_t^A and S_t^B and subsequent reversion to the equilibrium strategy profile (σ, ζ, λ). Proposition 1 below uses the optimality of the voting strategy at t to tie $p_t^{*i}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda)$ and, hence, the incentives of parties to agent payoffs. In particular, it identifies $p_t^{*i}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda)$ with a functional over the payoff difference functions $\Delta U_t(H_t, S_t^A, S_t^B, \cdot|\sigma, \zeta, \lambda)$. When Assumption 1 holds the result specializes to give party i's electoral success probabilities as affine functions of the continuation utilitarian payoff $\int U_t(H_t, S_t^i, h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda; dh_t)$.

Proposition 1 *The probability that party A wins the election at t given H_t, the current proposals S_t^A and S_t^B and reversion to the equilibrium strategy profile (σ, ζ, λ) is*

$$p_t^{*A}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda) = G\left(\delta^* : \int F\left(\frac{\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda)}{1-\beta} - \delta^*\right) Q_t(H_t, \lambda; dh_t) = 1/2\right).$$

For party B the probability is $p_t^{*B}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda) = 1 - p_t^{*A}(H_t, S_t^A, S_t^B|\sigma, \zeta, \lambda)$. When F and G satisfy Assumption 1, for $i \in \{A, B\}$, $j \neq i$, $p_t^{*i}(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) = K_0(H_t, S_t^j|\sigma, \zeta, \lambda) + K_1 \int U_t(H_t, S_t^i, h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda; dh_t)$.

Proof: By Lemma 1, in any (continuation) equilibrium party A must win with probability p in each period. Thus, *regardless of current play*, $D(H_t, S_t^A, S_t^B, \xi, \delta_t|\sigma, \zeta, \lambda) = (1-\beta)(\xi + \delta_t)$ and the current electoral outcome does not affect expected future *political* payoffs. Hence, given δ_t, all agents with pairs of histories and shocks (h_t, ξ) such that $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) > (1-\beta)(\xi + \delta_t)$ vote for party A. The fraction of the h_t sub-population that votes for A is then $F\left(\frac{\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta_t\right)$ and the fraction of the general population voting for A is $\int F\left(\frac{\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta_t\right) Q_t(H_t, \lambda; dh_t)$. The probability that this fraction is above $1/2$, resulting in an election victory for A is $G\left(\delta^* : \int F\left(\frac{\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma,\zeta,\lambda)}{1-\beta} - \delta^*\right) Q_t(H_t, \lambda; dh_t) = 1/2\right)$. In particular, when F and G satisfy Assumption 1, for each $i \in \{A, B\}$, $j \neq i$, $p_t^{*i}(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) = \frac{1}{2} + \frac{\widehat{\delta}}{1-\beta} \int \Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda)$ $Q_t(H_t, \lambda; dh_t) = K_0(H_t, S_t^j|\sigma, \zeta, \lambda) + K_1 \int U_t(H_t, S_t^i, h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda; dh_t)$, where $K_0(H_t, S_t^j|\sigma, \zeta, \lambda) = \frac{1}{2} - \frac{\widehat{\delta}}{1-\beta}$ $\int U_t(H_t, S_t^j, h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda; dh_t)$ and $K_1 = \frac{\widehat{\delta}}{1-\beta}$. ■

It follows from this proposition that under Assumption 1, the policy optimality condition (5) reduces to

$$\forall t,\ H_t \in \mathcal{H}^t(\lambda),\ j \neq i, \quad \sigma_t^i(H_t) \in \arg \sup_{S \in \mathcal{S}_t(\lambda, H_t)} \int U_t(H_t, S, h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda; dh_t). \tag{8}$$

Thus, the parties behave as if they are utilitarian planners who take the future message and policy strategies as given.[7] Sleet and Yeltekin (2006) assume a single, uncommitted policymaker who directly cares about utilitarian payoffs. They obtain an equilibrium condition similar to (8) except that the payoff function U_t depends on the message and policy strategies of an agent and a single policymaker and not on the strategy of a rival policymaker. The argument in the following subsections adapts that in Sleet and Yeltekin to the current setting.

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[7] In the context of static probabilistic voting models it is well known that if political bias distributions are uniform with wide enough support then politicians behave as if they are utilitarian. Our arguments generalize this result to a dynamic environment with an evolving distribution over publicy observable characteristics - in this case, message histories.

3.3 Necessary and sufficient conditions for politically credible outcome paths

We define an *outcome path* to be an event tree of compatible mechanisms, a probability distribution over the tree and a reporting strategy restricted to aggregate histories on the tree. We give necessary and sufficient conditions for such paths to be the outcomes of PCE. These conditions ensure that outcome paths are resource-feasible and incentive-compatible (i.e. respect optimal message-sending by agents). In addition, a sequence of "political constraints" require that continuation utilitarian payoffs along a path remain above a lower bound. Sufficiency of these conditions is obtained via the construction of trigger strategy equilibria which revert to a utilitarian payoff-minimizing no insurance PCE following the electoral success of a mechanism proposal that is not on the path. The political constraints then imply that continuation utilitarian payoffs and, hence, a party's reelection probabilities are reduced by such defections. Outcome paths are more complicated than allocations as defined in Section 2: additional randomness is introduced into utility awards by agent message strategies and by the uncertain nature of elections. However, we show that under Assumption 1 for any PCE there is a payoff-equivalent PCE that induces an allocation.

No insurance PCE In a no insurance PCE, politicians repeatedly propose mechanisms that provide no insurance against taste shocks and agents send messages that maximize their current payoff. Formally, a mechanism $S_t = (M_t, \varphi_t)$ offers no insurance against current taste shocks if it gives the same utility award regardless of the agent's current message, i.e. if for each h_t and m_t, m'_t in M_t, $\varphi_t(h_t, m_t) = \varphi_t(h_t, m'_t)$. A policy strategy provides no insurance if each $\sigma^i_t(H_t)$ is a no insurance mechanism. We focus on one particular no insurance policy σ^{NI} such that each $\sigma^{NI,i}_t(H_t) = (\Theta, \varphi^{NI}_t(H_t)) \in \mathcal{S}_t(H_t)$ and for all consistent history pairs H_t and h_t and all θ, $\varphi^{NI}_t(H_t, h_t, \theta)$

$= u(R_t)$. The mechanisms prescribed by σ^{NI} are direct - they use Θ as the current message space - and resource-feasible. If the future play of politicians conforms to σ^{NI}, then an agent's continuation payoff is unaffected by either the current election-winning mechanism or the agent's current message. It is then optimal for an agent to send a message and vote for a mechanism that maximizes her current payoff. This motivates our definition of the *no insurance message* λ^{NI} *and voting* ζ^{NI} *strategies.* For all t, $(H_t, M_t, \varphi_t, h_t) \in \mathcal{J}^t$ and θ, $\lambda_t^{NI}(H_t, M_t, \varphi_t, h_t, \theta)$ $= 1_\theta$ if $\theta \in \arg\max_{m \in M_t} \varphi_t(h_t, m)$ and $\lambda_t^{NI}(H_t, M_t, \varphi_t, h_t, \theta) = 1_{m'}$ some $m' \in \arg\max_{m \in M_t} \varphi_t(h_t, m)$ otherwise. Thus, λ^{NI} requires that an agent sends the message that maximizes her current payoff and if reporting her current shock achieves this maximum she is truthful. For all t and h_t^p, $\zeta_t^{NI}(h_t^p; A) = 1$ if $E[\theta_t] \max_{m \in M_t^A} \varphi_t^A(h_t, m)$ $\geq E[\theta_t] \max_{m \in M_t^B} \varphi_t^B(h_t, m) + \xi + \delta_t$ and $\zeta_t^{NI}(h_t^p; A) = 0$ otherwise, i.e. the agent votes for the proposal that maximizes her current payoff given λ^{NI}. Clearly, ζ^{NI} and λ^{NI} are optimal for an agent given σ^{NI}. Also, if agents send messages and vote for mechanisms that maximize their current payoff, then it is optimal for the parties to select σ^{NI}. Hence, $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ is a PCE. We have the following result where the proof of the second part, along with all subsequent proofs that are not stated in the main text, is deferred to the Appendix.

Lemma 2 *Let Assumption 1 hold. 1)* $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ *is a PCE with continuation utilitarian payoffs* $\{\underline{W}_t\}_{t=1}^{\infty}$, $\underline{W}_t := (1-\beta)E[\theta]\sum_{r=0}^{\infty} \beta^r u(R_{t+r})$. *2) Amongst PCE's* $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ *delivers the lowest continuation utilitarian payoff at each date.*

Outcome paths: Definition An outcome path is a tuple $\Upsilon = \{\mathcal{H}_t^\Upsilon, \mathcal{J}_t^\Upsilon, \sigma_t^\Upsilon, p_t^\Upsilon, \lambda_t^\Upsilon\}$ comprised of aggregate $\mathcal{H}_t^\Upsilon$ and consistent aggregate-individual $\mathcal{J}_t^\Upsilon$ history sets and functions that give mechanism proposals σ_t^Υ, probability distributions over these proposals p_t^Υ and lotteries over λ_t^Υ message spaces. The history sets are induced recursively by $\{\sigma_t^\Upsilon\}$, while the domains of the functions σ_t^Υ, p_t^Υ and λ_t^Υ are obtained from $\mathcal{H}_t^\Upsilon$ and $\mathcal{J}_t^\Upsilon$. Formally, the initial components of Υ are given by $\mathcal{H}_1^\Upsilon = \mathcal{H}_1$, for $i \in \{A, B\}$, $\sigma_1^{\Upsilon,i} \in \mathcal{S}_1$ and $\mathcal{J}_1^\Upsilon = \{(M_1, \varphi_1, m_1) : (M_1, \varphi_1) = \sigma_1^{\Upsilon,i}\ i \in \{A, B\}, m_1 \in M_1\}$, while the subsequent components evolve recursively according to $\mathcal{H}_{t+1}^\Upsilon = \{(H_t, S_t) : H_t \in \mathcal{H}_t^\Upsilon$ and $S_t = \sigma_t^{\Upsilon,i}(H_t)\ i \in \{A, B\}\}$, $\sigma_{t+1}^{\Upsilon,i} : \mathcal{H}_{t+1}^\Upsilon \to \mathcal{S}_{t+1}$ with for each $H_{t+1} \in \mathcal{H}_{t+1}^\Upsilon$, $\sigma_{t+1}^{\Upsilon,i}(H_{t+1}) \in \mathcal{S}_{t+1}(H_{t+1})$ and $\mathcal{J}_{t+1}^\Upsilon = \{(H_{t+1}, M_{t+1}, \varphi_{t+1}, h_t, m_{t+1}) : (H_{t+1}, h_t) \in \mathcal{J}_t^\Upsilon$, $(M_{t+1}, \varphi_{t+1}) = \sigma_{t+1}^{\Upsilon,i}(H_{t+1})$ some $i \in \{A, B\}$, $m_{t+1} \in M_{t+1}\}$. Additionally, $p_t^\Upsilon : \text{Graph}(\sigma_t^\Upsilon) \to \mathbb{P}(\{A, B\})$, where $\text{Graph}(\sigma_t^\Upsilon) = \{(H_t, \sigma_t^A(H_t), \sigma_t^B(H_t)) : H_t \in \mathcal{H}_t^\Upsilon\}$, and $\lambda_t^\Upsilon : \mathcal{J}_t^\Upsilon \times \Theta \to \mathbb{P}$ with $\lambda_t^\Upsilon(H_t, M_t, \varphi_t, h_t, \theta) \in \mathbb{P}(M_t)$. An outcome path is simpler than a strategy profile. First, it does not describe what happens after a history that is not generated by the path. Second, it replaces the voting strategy with the sequence of functions $\{p_t^\Upsilon\}$ that give the probability that each party wins the current election given an aggregate history on the path. Let $Q_t^\Upsilon(H_t)$ denote the cross sectional distribution of individual histories implied by the outcome path at aggregate history H_t.

Outcome paths: Necessary and sufficient conditions We now state and discuss the necessary and sufficient conditions for an equilibrium outcome path.

Proposition 2 *Let Assumption 1 hold. An outcome path* $\Upsilon = \{\{\mathcal{H}_t^\Upsilon\}, \{\mathcal{J}_t^\Upsilon\}, \{p_t^\Upsilon\}, \{\sigma_t^\Upsilon\}, \{\lambda_t^\Upsilon\}\}$ *is induced by a PCE if and only if:*

A. For all t, $H_{t+1} = \{M_r, \varphi_r\}_{r=1}^{t} \in \mathcal{H}_{t+1}^\Upsilon$, $\int_{\mathbb{R}\times M^t} C(\varphi_t(h_{t+1})) Q_{t+1}^\Upsilon(H_{t+1}, dh_{t+1}) \leq R_t$;

B. For all t, $(H_{t+1}, h_t) \in \mathcal{J}_t^\Upsilon$, $\forall \widehat{\lambda}^\Upsilon$, $U_t^\Upsilon(H_{t+1}, h_t | \sigma^\Upsilon, \lambda^\Upsilon) \geq U_t^\Upsilon(H_{t+1}, h_t | \sigma^\Upsilon, \widehat{\lambda}^\Upsilon)$;

C. For all t, $H_t = \{M_r, \varphi_r\}_{r=1}^{t-1} \in \mathcal{H}_t^\Upsilon$, $\int_{\mathbb{R}\times M^{t-1}} U_t^\Upsilon(H_t, \sigma_t^{\Upsilon,i}(H_t), h_t | \sigma^\Upsilon, \lambda^\Upsilon) Q_t^\Upsilon(H_t, dh_t) \geq \underline{W}_t$;

D. *For all* t, $H_t = \{M_r, \varphi_r\}_{r=1}^{t-1} \in \mathcal{H}_t^{\Upsilon}$, $\int_{\mathbb{R}\times M^{t-1}} U_t^{\Upsilon}(H_t, \sigma_t^{\Upsilon,A}(H_t), h_t|\sigma^{\Upsilon}, \lambda^{\Upsilon})\ Q_t^{\Upsilon}(H_t, dh_t) = \int_{\mathbb{R}\times M^{t-1}} U_t^{\Upsilon}(H_t, \sigma_t^{\Upsilon,B}(H_t), h_t\ |\sigma^{\Upsilon}, \lambda^{\Upsilon})\ Q_t^{\Upsilon}(H_t, dh_t)$.

E. *For all* t, $H_t \in \mathcal{H}_t^{\Upsilon}$, $i \in \{A, B\}$, $p_t^{\Upsilon}(H_t, \{i\}) = 1/2$.

Conditions A and B above ensure resource-feasibility and message optimality along the outcome path. They are clearly necessary for a political credibility. Condition C requires that each party's proposal generates a utilitarian payoff in excess of the no insurance one at all dates. Since, by Lemma 2, $\underline{W}_t$ is the lowest continuation utilitarian payoff amongst PCE's this is also necessary. Condition D implies that the mechanism proposal of party A generates the same continuation utilitarian payoff and, hence, the same probability of electoral success, as that of party B at each date. Thus, neither party has an incentive to defect to the proposal of the other. Finally, Condition E ensures that the probability that either party is elected is consistent with the necessary condition given in Lemma 1. Conversely, if Υ is an outcome path satisfying A-E, then it is possible to construct a PCE that induces it. In this PCE, following the electoral success of a mechanism not implied by Υ, agents assume that no insurance will be offered in future periods and revert to their no insurance message strategies. Consequently, the best utilitarian payoff attainable from a defection is $\underline{W}_t$ and, using Conditions C and D, the best election-winning probability is $p_t^i = \frac{1}{2} - \frac{\widehat{\delta}}{1-\beta} \int_{\mathbb{R}\times M^{t-1}} U_t^{\Upsilon}(H_t, \sigma_t^{\Upsilon,i}(H_t), h_t|\sigma^{\Upsilon}, \lambda^{\Upsilon})\ Q_t^{\Upsilon}(H_t, dh_t) + \frac{\widehat{\delta}}{1-\beta}\underline{W}_t \leq \frac{1}{2}$, ensuring political credibility.

3.4 Outcome paths to allocations: Two simplifications

A Revelation principle Our definition of PCE does not require that agents truthfully reveal their type. However, any politically credible distribution of payoffs can be implemented by an equilibrium in which 1) both political parties propose the same history-contingent mechanisms, 2) these mechanisms are direct and 3) agents are truthful along the equilibrium outcome path. Below, we give a (symmetric) Revelation principle for our environment. Some formal definitions precede the statement of the principle.

Definition 3 *A mechanism* $S_t = (M_t, \varphi_t)$ *is direct if* $M_t = \Theta$. *A policy strategy* σ *is direct if for all* i, t *and* $H_t \in \mathcal{H}^t$, $\sigma_t^i(H_t)$ *is direct.*

Definition 4 *A policy strategy* σ *is symmetric if for all* t *and* $H_t \in \mathcal{H}^t$, $\sigma_t(H_t) := \sigma_t^A(H_t) = \sigma_t^B(H_t)$.

Definition 5 *Let* $H_{t+1} = (H_t, S_t) \in \mathcal{H}^{t+1}$ *with* S_t *a direct mechanism. A message strategy* λ *is truthful at* H_{t+1} *if for all individual histories* h_t *such that* $(H_{t+1}, h_t) \in \mathcal{J}^t$ *and* θ, $\lambda_t(H_t, S_t, h_t, \theta) = 1_\theta$.

Definition 6 (σ, ζ, λ) *is a symmetric, truthful PCE if it is 1) a PCE, 2)* σ *is direct and symmetric, 3) for all* $H_{t+1} = (H_t, \sigma_t(H_t))$, λ *is truthful at* H_{t+1}.

Proposition 3 ***(Revelation Principle)*** *Let* (σ, ζ, λ) *be a PCE. Then there exists a symmetric, truthful PCE* $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ *that delivers the same lifetime payoff to* Ψ*-a.e.* w*-type of agent as* (σ, ζ, λ).

The proof is similar to one given in Sleet and Yeltekin (2006) and is not repeated. It utilizes Proposition 2 above; by this proposition any PCE outcome path can be supported by trigger strategies that revert to the no insurance equilibrium following the electoral victory of a defecting political party. A standard Revelation Principle argument can then be applied to replace the mechanisms along the outcome path with symmetric, truthful ones that do not lower an agent's payoff.

It follows from Proposition 3 that, from a welfare perspective, there is no loss of generality in restricting attention to symmetric, truthful PCE's. In the remainder of this section we do so. For these equilibria neither elections nor agent's message strategies introduce any additional uncertainty over an agent's utility award; these awards can be expressed as functions of the agent's w-type and her (truthfully reported) sequence of shocks. Thus, symmetric, truthful PCE's induce allocations (as we have previously defined them). We call an allocation *politically credible* if it is induced by a symmetric, truthful PCE.

3.5 Politically credible allocations

Proposition 2 and the definition of a politically credible allocation have the following immediate corollary.

Corollary 1 *Let Assumption 1 hold.* $\{\varphi_t\}_{t=1}^{\infty}$ *is a politically credible allocation if and only if it satisfies (1), (2) and the political constraints*

$$\forall t, \qquad Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})) := \int_{\mathbb{R}} \sum_{\Theta^{t-1}} U(\{\varphi_{t+r}|w, \theta^{t-1}\}_{r=0}^{\infty})\pi^{t-1}\left(\theta^{t-1}\right)\Psi(dw) - \underline{W}_t \geq 0. \tag{9}$$

Furthermore, no immiserating allocation is politically credible.

Anticipating later generalizations, (9) may be interpreted as a lower bound on a sequence of agent payoff aggregates $\{Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty}))\}_{t=1}^{\infty}$, where the aggregators Z_t stem from the assumptions that connect agents' economic payoffs $U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})$ to their voting behavior and, hence, the election-winning probabilities of the parties. In the current case, these aggregators are linear utilitarian ones, though more generally they may be non-linear. The latter part of the above corollary follows from the fact that a political party will eventually be able to increase the continuation utilitarian payoff of agents and its probability of electoral success by defecting to a no insurance mechanism and away from an immiserating allocation.

Corollary 1 implies that Pareto optimal politically credible allocations can be obtained by solving[8]:

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} (1-\beta)\int \gamma(w)\sum_{t=1}^{\infty}\beta^{t-1}\sum_{\Theta^t}\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) \text{ s.t. } (1),\ (2),\ \text{and } (9). \tag{10}$$

Proposition 4 below shows that any solution to (10) solves a *virtual planning problem* with the same constraint set as our benchmark planning problem (3) but a perturbed objective with societal discount factors (weakly) greater than the agents'. The proposition also provides a partial converse that gives conditions on the virtual planning problem sufficient for its solution to be an optimal politically credible allocation. The proof relies on the manipulation of a Lagrangian suggested by (10) that incorporates the political constraints. For all $\{\varphi_t\}_{t=1}^{\infty} \in A$ and $\{\mu_t\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\}) = \{\{x_t\} \in \mathbb{R}_+^{\infty} | \sum_{t=1}^{\infty}\beta^{t-1}x_t < \infty\}$, this Lagrangian is given by:

$$\mathcal{L}^*(\{\varphi_t\}_{t=1}^{\infty},\{\mu_t\}_{t=1}^{\infty}) = \int_{\mathbb{R}}\gamma(w)\sum_{t=1}^{\infty}\beta^{t-1}\sum_{\Theta^t}\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) + \sum_{t=1}^{\infty}\beta^{t-1}\mu_t Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})). \tag{11}$$

Proposition 4 *Suppose that $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (10), then there is a positive-valued sequence $\{B_1^{*t}\}_{t=0}^{\infty}$ such that $\{\varphi_t^*\}_{t=1}^{\infty}$ solves:*

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}\in\Gamma(\{R_t\},\Psi)}\int_{\mathbb{R}}\sum_{t=1}^{\infty}B_1^{*t}\gamma_t^*(w)\sum_{\Theta^t}\theta_t\varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw), \tag{12}$$

where $\forall t \geq 0$, $\gamma_{t+1}^*(w) = (1-\omega_{t+1}) + \omega_{t+1}\gamma_t^*(w)$, $\gamma_0^*(w) = \gamma(w)$ with $\omega_{t+1} = \beta[B_1^{*t}/B_1^{*t+1}]$. The sequence $\{B_1^{*t}\}_{t=0}^{\infty}$ satisfies 1) $\sum_{t=0}^{\infty}B_1^{*t} < \infty$, 2) $B_1^{*0} = 1/\beta$ and $\forall t \geq 0$, $B_1^{*t+1}/B_1^{*t} \geq \beta$ and 3) $B_1^{*t+1}/B_1^{*t} > \beta$ and $\omega_{t+1} < 1$ if the period $t+1$ political constraint binds in (10).

Conversely, if $\{\varphi_t^*\}_{t=1}^{\infty}$ solves (12) at $\{B_1^{*t}\}_{t=0}^{\infty}$ (with $\{\gamma_t^*\}_{t=0}^{\infty}$ given as above) and if $\{B_1^{*t}\}_{t=0}^{\infty}$ satisfies 4) $B_0^{*1} = 1/\beta$ and $\{\frac{1}{\beta^{t-1}}(B_1^{*t} - \beta B_1^{*t-1})\} \in \arg\inf_{\{\mu_t\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\})} \mathcal{L}^*(\{\varphi_t^*\}_{t=1}^{\infty},\{\mu_t\}_{t=1}^{\infty})$ then $\{\varphi_t^*\}_{t=1}^{\infty}$ solves (10).

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[8] After any aggregate history, the weighted average ideological payoff to agents is constant across equilibria. This component of agent payoffs can be dropped from the ranking criterion.

The *virtual planning problem* (12) retains the resource and incentive-compatibility constraints, but absorbs the political constraints into the planner's objective. This objective features a "perturbed" societal discounting scheme $\{B_1^{*t}\}_{t=0}^{\infty}$. The period t societal discount factor is given by $B_1^{*t}/B_1^{*t-1} = \beta(1+\mu_t^*) \geq \beta$, where μ_t^* is the optimal (normalized) Lagrange multiplier on the current political constraint (9). Intuitively, when the current political constraint binds, the shadow value of the continuation utilitarian payoff is increased and it is as if allocations are selected by a politically unconstrained (virtual) planner who weights the future more heavily than agents. Of course, if the political constraints never bind, then the virtual planner's preferences coincide with those in the benchmark planning problem (3). However, as noted previously, for many choices of C the solution to (3) is immiserating and eventually violates the political constraints. This implies that for at least some dates t, the political constraints will bind and $B_1^{*t}/B_1^{*t-1} > \beta$.[9]

The other feature of the virtual planner's problem is the modified Pareto weighting scheme $\{\gamma_t^*(w)\}_{t=0}^{\infty}$. This scheme implies that if the political constraints repeatedly bind (so that ω_t is repeatedly in $(0,1)$), then each agent's Pareto weight converges towards one and any initial differential weighting of agents implied by γ and Ψ washes out. Such convergence reflects the equal weighting of agents in the political constraints.

Farhi and Werning (2007) consider similar planning problems to (12). They emphasize a normative rationale for societal discount factors in excess of agents'. We do not make such a normative case. Rather such social criteria emerge as equilibrium phenomena in our setting. Simply put, *in an optimal PCE politicians concerned only with winning current elections implement the same allocation as a planner who is more patient than agents.*

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[9] In fact, as we discuss below, in these cases $B_1^{*t}/B_1^{*t-1} > \beta$ for infinitely many dates t. In addition, in some numerical examples, $B_1^{*t}/B_1^{*t-1} > \beta$ eventually and in others the inequality holds at all dates.

3.6 Properties of politically credible allocations

3.6.1 Political constraints and inequality

Let $\mathcal{L}^{VP}$ be a Lagrangian for the virtual planning problem that incorporates the resource constraints, i.e. for $\{\varphi_t\}_{t=1}^{\infty} \in A$ and $\{q_t\}_{t=1}^{\infty} \in L(\{B_1^{t*}\}) = \{\{x_t\} \in \mathbb{R}_+^{\infty} | \sum_{t=1}^{\infty} B_1^t x_t < \infty\}$,

$$\mathcal{L}^{VP}(\{\varphi_t\}_{t=1}^{\infty}, \{q_t\}_{t=1}^{\infty}) = \int_{\mathbb{R}} \sum_{t=1}^{\infty} B_1^{t*} \sum_{\Theta^t} [\gamma_t^*(w)\theta_t\varphi_t(w,\theta^t) - q_t C(\varphi_t(w,\theta^t))]\pi^t(\theta^t)\Psi(dw). \tag{13}$$

If for some $\{q_t^*\}_{t=1}^{\infty} \in L(\{B_1^{t*}\})$, $\{\varphi_t^*\}_{t=1}^{\infty}$ maximizes (13) subject to $\{\varphi_t\}_{t=1}^{\infty} \in A$ and the incentive constraint (2), and if $\{\varphi_t^*\}_{t=1}^{\infty}$ satisfies the resource constraint with equality then $\{\varphi_t^*\}_{t=1}^{\infty}$ solves (12). The maximization (13) features no cross-agent "aggregate" constraints. Consequently, it can be decomposed into a collection of component planning problems in each of which a virtual planner selects an individual allocation $\{\psi_t\}_{t=1}^{\infty}$, $\psi_t : \Theta^t \to D$, to maximize the net-of-cost utility of a single agent subject to incentive constraints. The component virtual planners may be interpreted as trading resources amongst themselves at equilibrium prices $\{q_t^*\}_{t=1}^{\infty}$.

The component virtual planner problems admit a recursive formulation that utilizes "effective" Pareto weights as state variables.[10] Since D is bounded, the incentive constraint (2) can be replaced with an equivalent collection of simpler *temporary incentive constraints* that facilitate this formulation. For $T = 1, 2, \ldots$ let Ω_T be the set of individual allocations satisfying the temporary incentive constraints $t \geq T$, $\theta^{t-1} \in \Theta^{t-1}$, $k, k+j \in \{1, \ldots, K\}$, $j \in \{-1, 1\}$,

$$\widehat{\theta}_k\psi_t(\theta^{t-1}, \widehat{\theta}_k) + \sum_{r=1}^{\infty} \beta^r \sum_{\Theta^r} \theta_t\psi_{t+r}(\theta^{t-1}, \widehat{\theta}_k, \theta^r)\pi^r(\theta^r) \geq \widehat{\theta}_k\psi_t(\theta^{t-1}, \widehat{\theta}_{k+j}) + \sum_{r=1}^{\infty} \beta^r \sum_{\Theta^r} \theta_t\psi_{t+r}(\theta^{t-1}, \widehat{\theta}_{k+j}, \theta^r)\pi^r(\theta^r). \tag{14}$$

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[10] The formulation builds on the approach of Marcet and Marimon (1998); full details are given in Sleet and Yeltekin (2007a).

The component virtual planner problem is then:

$$V_1(\gamma_1(w)) = \sup_{\{\psi_t\}_{t=1}^{\infty} \in \Omega_1} \sum_{t=1}^{\infty} \frac{B_1^{t*}}{B_1^{1*}} \sum_{\Theta^t} [\theta_t \gamma_t^*(w) \psi_t(\theta^t) - q_t^* C(\psi_t(\theta^t))] \pi^t(\theta^t) \tag{15}$$

Manipulating the Lagrangian that incorporates the period 1 temporary incentive constraints gives:

$$V_1(\gamma_1(w)) = \inf_{\eta \in \mathbb{R}_+^{2K}} \sup_{\{\psi_t\}_{t=1}^{\infty} \in \Omega_2} \sum_{\Theta} [\theta \rho(\gamma_1^*(w), \theta, \eta) \psi_1(\theta) - q_1^* C(\psi_1(\theta)) + \frac{B_1^{2*}}{B_1^{1*}} V_2(\gamma_2'(\gamma_1^*(w), \theta, \eta))] \pi(\theta) \tag{16}$$

where V_2 is the value function from a collection of problems analogous to (15) evaluated at the continuation discount and price sequences $\{B_1^{t*}/B_1^{2*}\}_{t \geq 2}$ and $\{q_t^*\}_{t \geq 2}$. The current utility weights ρ are defined as:

$$\rho(\gamma_1^*(w), \widehat{\theta}_k, \eta) = \gamma_1^*(w) + \sum_{j \in \{-1,1\}} \eta_{k,k+j} - \sum_{j \in \{-1,1\}} \eta_{k+j,k} \frac{\widehat{\theta}_{k+j}}{\widehat{\theta}_k} \frac{\pi(\widehat{\theta}_{k+j})}{\pi(\widehat{\theta}_k)}, \tag{17}$$

and the "effective" Pareto weights in period 2 are:

$$\gamma_2'(\gamma_1^*(w), \widehat{\theta}_k, \eta) = (1 - \omega_2) + \omega_2 \gamma_1^*(w) + \omega_2 \left[\sum_{j \in \{-1,1\}} \eta_{k,k+j} - \sum_{j \in \{-1,1\}} \eta_{k+j,k} \frac{\pi(\widehat{\theta}_{k+j})}{\pi(\widehat{\theta}_k)} \right]. \tag{18}$$

Here $\eta_{k,k+j}$ denotes the Lagrange multiplier from the $(k, k+j)$-th period 1 temporary incentive constraint.[11] By a similar logic the continuation value functions at all periods t, $t+1$ satisfy Bellman equations of the form (16) updated to include q_t^*, B_1^{t+1*}/B_1^{t*} and an augmented law of motion for effective Pareto weights γ_{t+1}' of the form (18) with parameter ω_{t+1}. These Bellman equations give rise to optimal policy functions $\{\widetilde{\eta}_t, \widetilde{\psi}_t, \widetilde{\gamma}_{t+1}, \}_{t=1}^{\infty}$ that give multiplier, utility and effective Pareto weight choices as functions of the current effective Pareto weight and shock and that, coupled with an initial $\gamma_0^*(w)$ induce an optimal allocation for the associated virtual component

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[11] $\eta_{0,1}$ and $\eta_{K,K+1}$ are normalized to 0.

planner. Notice that the optimal laws of motion for effective Pareto weights can be expressed as:

$$\begin{aligned}\widetilde{\gamma}_{t+1}(\gamma, \widehat{\theta}_k) &= \gamma'_{t+1}(\gamma, \widehat{\theta}_k, \widetilde{\eta}_t(\gamma, \widehat{\theta}_k)) \\ &= (1 - \omega_{t+1}) + \omega_{t+1}\gamma + \varepsilon^{\eta}_t(\gamma, \widehat{\theta}_k), \end{aligned} \tag{19}$$

where γ and $\widehat{\theta}_k$ are interpreted as the t-th period effective Pareto weight and taste shock and the "incentive shock" ε^{η}_t is obtained from the optimal multiplier policy function $\widetilde{\eta}_t$. The laws of motion (19) augment the earlier deterministic sequence of Pareto weights $\{\gamma^*_t(w)\}_{t=1}^{\infty}$ with incentive shocks $\{\varepsilon^{\eta}_t\}_{t=1}^{\infty}$ that incorporate the future rewards and penalties for a current report. For example, in a two shock problem with $\Theta = \{\widehat{\theta}_k\}_{k=1}^{2}$, the optimal allocation provides more current resources to an agent who reports the high taste shock $\widehat{\theta}_2$. To induce truthful reporting, agents who report a high taste shock receive a relative reduction in their future effective Pareto weight: $\varepsilon^{\eta}_t(\gamma, \widehat{\theta}_2) < 0$, while those who report the low shock receive a relative increase: $\varepsilon^{\eta}_t(\gamma, \widehat{\theta}_1) > 0$. In this way the optimal provision of incentives contributes to a dispersion in effective Pareto weights. In addition, $E[\varepsilon^{\eta}_t(\gamma, \widehat{\theta}_k)] = 0$ and so $\{\widetilde{\gamma}_{t+1}\}$ and π define an AR(1) process with time varying coefficient $\omega_{t+1} \in (0, 1]$. Recall that $\omega_{t+1} = \frac{B^{t*}_1}{B^{t+1*}_1} < 1$ if the political constraint binds at $t + 1$. Thus, the political constraints introduce a force for mean reversion into the evolution of effective Pareto weights. Although, increased future dispersion in these weights and, hence, agent utilities may be useful in providing current incentives, it lowers future continuation utilitarian payoffs. When the political constraints bind such dispersion is damped: political constraints are a force for equality.

3.6.2 The pattern of binding political constraints

Suppose there exist a sequence of optimizing resource constraint multipliers $\{q^*_t\}_{t=1}^{\infty}$ and policy functions $\{\widetilde{\eta}_t, \widetilde{\psi}_t, \widetilde{\gamma}_{t+1}\}_{t=1}^{\infty}$ for the virtual planning problem and let $\{\varphi^*_t\}_{t=1}^{\infty}$ denote the optimal allocation implied by the policy functions.

It is easy to check that Ψ, γ, π and the policy functions $\{\widetilde{\gamma}_{t+1}\}_{t=1}^{\infty}$ induce a sequence of effective Pareto weight distributions at each successive date $\{\Phi_t\}_{t=1}^{\infty}$ and that the optimal continuation allocation $\{\varphi^*_{T+t}\}_{t=1}^{\infty}$ solves a virtual planning problem with initial Pareto weight distribution Φ_{T+1} and discount sequence $\{B_1^{T+r*}/B_1^{T*}\}_{r=1}^{\infty}$.[12] It follows that if the political constraints do not bind after date T, then $\{\varphi^*_{T+t}\}_{t=1}^{\infty}$ solves a politically unconstrained problem of the form (3) with initial Pareto weight distribution Φ_{T+1}. As noted above, for a large class of cost functions C, all solutions to (3) are immiserating. Thus, for such C, if the political constraints do not bind after some date, then $\{\varphi^*_t\}_{t=1}^{\infty}$ must converge to an immiserating allocation. But this implies that $\{\varphi^*_t\}_{t=1}^{\infty}$ must eventually violate the political constraints, a contradiction. We have the following result.

Proposition 5 *Suppose that all solutions to (3) are immiserating. Let $\{B_1^{t*}\}_{t=0}^{\infty}$ be a discount factor sequence implied by (10) and assume the existence of a sequence of optimizing resource constraint multipliers $\{q^*_t\}_{t=1}^{\infty}$ and policy functions $\{\widetilde{\eta}_t, \widetilde{\psi}_t, \widetilde{\gamma}_{t+1}, \}_{t=1}^{\infty}$ for the corresponding virtual planner's problem. Then the political constraints in (10) bind infinitely often, i.e. for all T, there is a $t > T$ such that these constraints bind and the corresponding effective societal discount factor B_1^{t*}/B_1^{t-1*} is greater than β.*

It would be desirable to strengthen this proposition to show that political constraints bind eventually, i.e. there is some T such that for all $t > T$, $B_1^{t*}/B_1^{t-1*} > \beta$. Sleet and Yeltekin (2007b) give numerical examples in which this is the case. In one example, the allocation is a stationary one with a constant societal discount factor in excess of the agents' and a stationary utility/effective Pareto weight distribution. It corresponds to the stationary allocations considered by Farhi and Werning (2007) and Sleet and Yeltekin (2007a) in settings with a constant

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[12] More precisely, there is an allocation $\{\widetilde{\varphi}_r\}_{r=1}^{\infty}$ with for all r, w, θ^T, θ^r, $\widetilde{\varphi}_r(\widetilde{\gamma}_{T+1}(w, \theta^T), \theta^r) = \varphi^*_{T+r}(w, \theta^T, \theta^r)$ and $\widetilde{\gamma}_{T+1}(w, \theta^T)$ the induced period $T+1$ effective Pareto weight for an agent with individual history (w, θ^T) which solves this problem.

exogenously given societal discount factor in excess of the agents'. It inherits the ergodicity proved analytically in these papers and has the additional property that the constant utilitarian payoff equals $E[\theta]u(R)$, where for all t, $R_t = R$. In another example, the initial Pareto weight function (and distribution) is degenerate at 1. The effective Pareto weight distributions then fan out over time and consumption and utility inequality increase until the political constraints bind and this increase in inequality is arrested. Thereafter, the allocation converges to a stationary one.

3.6.3 Intertemporal first order conditions

Dynamic private information economies give rise to "inverted Euler equations", i.e. intertemporal first order conditions involving the reciprocal of the marginal utility of consumption at successive dates. These equations are usually derived in the context of models with privately observed taste shocks to leisure or, equivalently, privately observed productivity shocks. In such settings they imply a positive wedge between an agents' expected intertemporal marginal rate of substitution and the shadow social marginal rate of transformation. In the present setting, with privately observed taste shocks to the marginal utility of consumption, these relationships are more complicated. Absent political constraints, we obtain for almost all w, for all t and all $(\theta^{t-1}, \widehat{\theta}_k, \theta')$,

$$\frac{1}{E[\theta]} E_{\theta'} \left[\frac{1}{\beta L_{t+1} u'(c^*_{t+1}(w, \theta^{t-1}, \widehat{\theta}_k, \theta'))} \right] = M_t + \frac{1}{\widehat{\theta}_k u'(c^*_t(w, \theta^{t-1}, \widehat{\theta}_k))}, \tag{20}$$

where $c^*_t(w, \theta^t) = C(\varphi^*_t(w, \theta^t))$ is an optimal consumption award to an agent with history (w, θ^t), $u' = (\partial C/\partial u)^{-1}$, $M_t = \frac{1}{q^*_t} \sum_{j \in \{-1,1\}} \eta^*_{t,k+j,k}(w, \theta^{t-1}) \left(\frac{\widehat{\theta}_{k+j} - \widehat{\theta}_k}{\widehat{\theta}_k} \right) p_{k+j,k}$ and L_{t+1} is the shadow return $\left(\beta \frac{q^*_{t+1}}{q^*_t} \right)^{-1}$. In general, (20) implies a wedge between the intertemporal marginal rate of substitution $\frac{\beta E[\theta' u'(c^*_{t+1}(w, \theta^{t-1}, \widehat{\theta}_k, \theta'))]}{\widehat{\theta}_k u'(c^*_t(w, \theta^{t-1}, \widehat{\theta}_k))}$ and the shadow price L^{-1}_{t+1}. The sign of this wedge is driven by several factors. First, as is typical with inverse Euler equations, Jensen's inequality implies that $E[\theta]E_{\theta'}[u'(c^*_{t+1}(w,\theta^{t-1},\widehat{\theta}_k,\theta'))] \geq E[\theta]\left\{E_{\theta'}\left[\frac{1}{u'(c^*_{t+1}(w,\theta^{t-1},\widehat{\theta}_k,\theta'))}\right]\right\}^{-1}$, a force for a positive intermporal wedge. On the other hand, the optimal allocation generally provides some insurance against shocks and so $E[\theta]E_{\theta'}[u'(c^*_{t+1}(w,\theta^{t-1},\widehat{\theta}_k,\theta'))] \geq E_{\theta'}[\theta' u'(c^*_{t+1}(w,\theta^{t-1},\widehat{\theta}_k,\theta'))]$, a force in the reverse direction. Finally, M_t captures the impact of the contemporaneous incentive constraints at t. Specifically, if the $(k+1,k)$-th constraint is binding, the wedge is reduced: intuitively, to deter a $\widehat{\theta}_k$ lie given a $\widehat{\theta}_{k+1}$ shock realization, fewer resources are paid out in the present relative to the future contingent on a $\widehat{\theta}_k$ report. The reverse logic holds if the $(k-1,k)$-th constraint binds.

For virtual planning problems with a discount sequence $\{B_1^{t*}\}_{t=0}^{\infty}$ (20) becomes:

$$\frac{1}{E[\theta]}E_{\theta'}\left[\frac{1}{\beta L_{t+1}u'(c^*_{t+1}(w,\theta^{t-1},\widehat{\theta}_k,\theta'))}\right] = \frac{1}{q^*_t}\frac{(B_1^{t+1*}/B_1^{t*})-\beta}{\beta} + M_t + \frac{1}{\widehat{\theta}_k u'(c^*_t(w,\theta^{t-1},\widehat{\theta}_k))}. \tag{21}$$

where now $L_{t+1} = \left(\frac{B_1^{t+1*}}{B_1^{t*}}\frac{q^*_{t+1}}{q^*_t}\right)^{-1}$. In this case, the right hand side is augmented with the additional term $\frac{1}{q^*_t}\frac{(B_1^{t+1*}/B_1^{t*})-\beta}{\beta} > 0$ which depresses the intertemporal wedge.

4 Variations and extensions

We now describe several variations of the previous model. The variations alter agents' political preferences to allow for incumbency advantages and the parties' preferences to allow for parties motivated by the rents accruing from office rather than office itself. In each case we show that the variation gives rise to political constraints related, and in some cases identical, to those from our initial model. In all cases, Pareto optimal politically credible allocations solve virtual planning problems with discount factors that weakly exceed those of agents.

4.1 Incumbency advantages

Incumbency advantage can be introduced in a simple way by respecifying an agent's political preferences as:

$$(1-\beta)E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left[\xi+\delta_t+\phi_t\right]1_t^B\right]$$

where $\phi_t = \phi$ if party B is in power at $t-1$, $\phi_t = -\phi$ otherwise and augmenting Assumption 1 with $\phi \in (0, \min(-1/(2\widehat{\delta}) + 1/(2\widehat{\xi}) - d, 1/(2\widehat{\delta}) - d))$. All else equal, the introduction of the ϕ_t variable raises the probability of an incumbent party being re-elected. Applying the argument that underlies Lemma 1, we obtain that an incumbent party wins re-election with probability $G(\phi) > \frac{1}{2}$ independently of the specific history of election-winning mechanisms generated by past play. A very similar argument to that given in the previous section then implies that parties seeking to maximize their probability of re-election propose mechanisms that maximize the utilitarian payoff to agents given future play. After specializing to symmetric PCE with truth-telling, we recover the political constraints (9) and Proposition 4. The only change in this setting is to the probability of any given party winning an election conditional on which party was in office in the prior period.

4.2 Patient political parties

We revert to our earlier model (without incumbency advantages), but now suppose that political parties (or politicians) have a discount factor $\chi \in (0,1)$. Let $V_t^i(\sigma,\zeta,\lambda|H_t)$ denote the continuation payoff to party i from (σ,ζ,λ) after the history H_t; V_t^i satisfies the recursion:

$$V_t^i(\sigma,\zeta,\lambda|H_t) = \sum_{j\in\{A,B\}} p_t^j(H_t,\sigma_t(H_t)|\zeta)[(1-\chi)I(j) + \chi V_{t+1}^j(\sigma,\zeta,\lambda|H_t,\sigma_t^j(H_t))],$$

where $I(j)=1$ if $j=i$ and 0 otherwise. The natural extension of our earlier equilibrium definition is:

Definition 7 *(σ, ζ, λ) is a PCE if $\sigma \in \Sigma(\lambda)$, if for all t, H_t, $i, j \in \{A, B\}$ and all $\widehat{\sigma}^i$ that are resource-feasible given λ, $V_t^i(\sigma^i, \sigma^j, \zeta, \lambda | H_t) \geq V_t^i(\widehat{\sigma}^i, \sigma^j, \zeta, \lambda | H_t)$ and if the agent optimality conditions from Definition 2 are satisfied.*

All that has changed here is the generalization of our earlier political optimality requirement to allow for political discount factors in excess of 0. This generalization initially appears more complicated than before. A policy choice effects both the probability of winning in the current period and, through its effect on the game's history, its probability of winning in subsequent periods. Potentially, a party might trade these probabilities off against each other, reducing its chance of winning today, in order to improve its future electoral prospects. In fact, this does not happen. Even with these more complicated preferences, each party chooses its current mechanism to maximize the current utilitarian payoff, given its future play and the play of its rival.

Proposition 6 *(σ, ζ, λ) is a PCE of a game with patient parties who have discount factor $\chi > 0$ if and only if it is a PCE of a game with impatient parties who have discount factor $\chi = 0$.*

Thus, the set of PCE's is independent of the politicians' discount factor. This independence extends to symmetric, truth-telling PCE's and, hence, the conditions for a politically credible allocation are unaltered. In particular, the political constraints (9) continue to hold as does Proposition 4. We conclude that in an optimal PCE, politicians implement the same allocation as a planner who is more patient than agents regardless of their discount factor. Such politicians cannot commit to severe limiting allocations even if they raise their electoral prospects. Note that one cannot appeal to a folk theorem-type result to the effect that political constraints are relaxed as politicians become more patient. Although the punishment for a defection is immediate, the force for immiseration ensures that political constraints (eventually) bind.

4.3 Political rents

Suppose that any resources not allocated to agents by an election-winning mechanism are appropriated by the winning politician as rent. Let r_t^i and p_t^i denote, respectively, party i's political rents and probability of electoral success at date t and assume that this party's date t objective is $r_t^i p_t^i$. Such an objective implies that parties are impatient caring only about their contemporaneous rent and that they are willing to trade the probability of winning office off against the amount of rent they extract if they win. The following definition redefines a PCE to accommodate this new political objective.

Definition 8 *(σ, ζ, λ) is a PCE in the political rents model if $\sigma \in \Sigma(\lambda)$, if $\forall t$, H_t, $i \in \{A, B\}$, σ^i satisfies*

$$\sigma_t^i(H_t) \in \arg \sup_{S_t^i \in \mathcal{S}_t(\lambda, H_t)} r(S_t^i, \lambda | H_t) p_t^i(H_t, S_t^i, \sigma_t^j(H_t) | \zeta). \tag{22}$$

where for $S_t^i = (M_t^i, \varphi_t^i)$, $r(S_t^i, \lambda | H_t) := R_t - \int_{\mathbb{R} \times M^{t-1} \times M_t^i} C(\varphi_t^i(h_{t+1})) Q_{t+1}(H_t, S_t^i, \lambda)(dh_{t+1})$ and if the agent optimality conditions from Definition 2 are satisfied.

Our previous analysis of PCE's used the fact that in equilibrium the probability with which a party wins an election is independent of the past aggregate history. In general, this is no longer true in the model with political rents. To simplify the analysis, we focus from the outset on symmetric PCE. Invoking Assumption 1 and using the optimality of the voting strategy, the party optimality condition (22) can be rewritten as:

$$\sigma_t^i(H_t) \in \arg \sup_{S_t^i \in \mathcal{S}_t(\lambda, H_t)} r(S_t^i, \lambda | H_t) \left[\frac{1}{2} + \frac{\widehat{\delta}}{1-\beta} \int [U_t(H_t, S_t^i, h_t | \sigma, \zeta, \lambda) - U_t(H_t, \sigma_t^j(H_t), h_t | \sigma, \zeta, \lambda)] Q_t(H_t, \lambda | dh_t) \right],$$

where the bracketed term gives the probability that party i wins. Let $\underline{W}_t$ denote the infimum over utilitarian payoffs attained by symmetric PCE's at t in the political rents model and assume that $\underline{W}_t$ is itself attained. One can then show that for any aggregate history H_t there is a continuation symmetric PCE that attains $\underline{W}_t$ and that makes no use of information revealed by agents in prior periods. Moreover, any given symmetric PCE payoff distribution can be implemented with a truthful symmetric PCE that reverts to this continuation equilibrium after a political defection. Formally, we have the following result.

Proposition 7 ***(Revelation Principle)*** *Let (σ, ζ, λ) be a symmetric PCE in the political rents model. Then there is a symmetric, truthful PCE $(\hat{\sigma}, \hat{\zeta}, \hat{\lambda})$ that gives the same lifetime payoff to Ψ-a.e. w-type agent as (σ, ζ, λ).*

As in Section 3, this Revelation Principle allows us, without loss of generality, to restrict attention to equilibria that rely on direct mechanisms and that induce truth-telling along their equilibrium paths. The next proposition provides necessary and sufficient conditions for the politically credible allocations induced by such equilibria. Once again these conditions feature political constraints on aggregates of agent payoffs. In particular, a party's best defection payoff is given by:

$$Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})) = -\sup_{r''} r'' \left\{ \frac{1}{2} + \frac{\widehat{\delta}}{1-\beta} \left[E[\theta] u(R_t - r'') + \beta \underline{W}_{t+1} - \int \sum_{\Theta^{t-1}} U(\{\varphi_{t+r}|w, \theta^{t-1}\}_{r=0}^{\infty}) \pi^{t-1}\left(\theta^{t-1}\right) \Psi(dw) \right] \right\},$$

where the bracketed term gives the party's re-election probability. Political credibility requires that this payoff exceeds the expected rents attainable by adhering to the allocation: $\frac{1}{2}\left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t)) \pi^t(\theta^t) \Psi(dw)\right)$. In contrast to before, the aggregators $\{Z_t\}$ are non-linear, but they remain concave.

Proposition 8 *$\{\varphi_t\}_{t=1}^{\infty}$ is a politically credible allocation in the model with political rents if and only if it satisfies (2) and the political constraints*

$$\forall t, \qquad Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^{\infty})) + \frac{1}{2}\left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t)) \pi^t(\theta^t) \Psi(dw)\right) \geq 0, \tag{23}$$

where Z_t is defined as above.

Proof: (Necessity) Let $\{\varphi_t\}_{t=1}^{\infty}$ denote a politically credible allocation. Agent message optimality implies (2). A party can always feasibly defect at t to a mechanism $(\Theta, \widetilde{\varphi}_t)$, where for all (w, θ^t), $\widetilde{\varphi}_t(w, \theta^t) = u(R_t - r)$, $r \in [0, R_t]$. Let $W_{t+1} \geq \underline{W}_{t+1}$ denote the continuation utilitarian payoff following the defection. Party optimality implies that:

$$\begin{aligned}\frac{R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw)}{2} \quad &\geq \quad r\left\{\frac{1}{2} + \frac{\widehat{\delta}}{1-\beta}\,[E[\theta]u(R_t - r) + \beta W_{t+1} - W_t(\{\varphi_t\}_{t=1}^{\infty})]\right\} \\ &\geq \quad r\left\{\frac{1}{2} + \frac{\widehat{\delta}}{1-\beta}\,\left[E[\theta]u(R_t - r) + \beta \underline{W}_{t+1} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\right\}.\end{aligned}$$

Since r was an arbitrary element of $[0, R_t]$, we have (23).

(Sufficiency) Suppose $\{\varphi_t\}_{t=1}^{\infty}$ satisfies (2) and (23). We construct a PCE that supports it as follows. For all i, t, $H_t = \{\Theta, \varphi_s\}_{s=1}^{t-1}$, $h_t = (w, \theta^{t-1})$ and $h_t^p = ((\{H_t, S_t^A, h_t\},\ \{H_t, S_t^B, h_t\}, \xi, \delta_t\big)$, set $\sigma_t^i(H_t) = (\Theta, \varphi_t)$, $\zeta_t(h_t^p; A) = 1$ if $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) \geq \delta_t + \xi$ and $\zeta_t(h_t^p; A) = 0$ otherwise and $\lambda_t(H_t, h_t, \theta_t) = 1_{\theta_t}$. For all $H_{t+1} = (\{\Theta, \varphi_s\}_{s=1}^{t-1}, M_t', \varphi_t')$, $(M_t', \varphi_t') \neq (\Theta, \varphi_t)$, set $\lambda_t(H_{t+1}, w, \theta^t) = 1_{m^*}$, where $m^* \in \arg\max_{m \in M_t'} \varphi_t'(h_t, m)$. Following a defection in t, set the continuation strategies from $t+1$ onwards to conform to those of a continuation PCE with payoff $\underline{W}_{t+1}$. The strategy profile (σ, ζ, λ) constructed in this way induces the desired allocation.

Since

$$\frac{R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw)}{2} \geq \sup_{r'' \in [0, R_t]} r''\left\{\frac{1}{2} + \frac{\widehat{\delta}}{1-\beta}\left[E[\theta]u(R_t - r'') + \beta \underline{W}_{t+1} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\right\} \geq 0,$$

the profile is resource-feasible along its outcome path. (2) ensures that λ is optimal for agents provided there has been no election-winning political defection. If a defecting party wins the period t election, subsequent play reverts to the equilibrium with utilitarian payoff $\underline{W}_{t+1}$; this continuation equilibrium makes no use of previously revealed information and so it is optimal for agents to send the message that maximizes their current payoff in period t. Consequently, $(\Theta, \widetilde{\varphi}_t)$ with for Ψ-a.e. w and all θ^t, $\widetilde{\varphi}_t(w, \theta^t) = u(R_t - r_t^*)$ and $r_t^* \in \arg\sup_{r \in [0, R_t]} r \ \{\frac{1}{2} + \frac{\widehat{\delta}}{1-\beta}\left[E[\theta]u(R_t - r) + \beta \underline{W}_{t+1} - W_t(\{\varphi_t\}_{t=1}^{\infty})\right]\}$ is an optimal defection for a party. (23) then ensures that no party will undertake a defection (or any sequence of defections). The agents' voting strategy is optimal in the periods up to and including an election-winning defection. Finally, the use of a continuation equilibrium with payoff $\underline{W}_{t+1}$ to construct continuation strategies following a defection ensures the post-defection optimality of player decisions. Hence, (σ, ζ, λ) is a (symmetric) PCE and $\{\varphi_t\}_{t=1}^{\infty}$ is politically credible. ■

It follows from Proposition 8 that a Pareto-optimal, politically credible allocation in the model with political rents solves

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} \int_{\mathbb{R}} \gamma(w) \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1} \theta_t \varphi_t(w, \theta^t) \pi(\theta^t) \Psi(dw) \tag{24}$$

subject to (2) and (23). As in the model with office-motivated politicians, Pareto optimal politically credible allocations solve politically unconstrained planning problems with planner discount factors in excess of agents'. In this case, since the political constraints bind in all periods, the virtual planner's discount factor strictly exceeds the agents' discount factor in all periods. We obtain the following result, where the Lagrangian used in the converse is of the form (11) updated to include the (concave) $\{Z_t\}$ functions defined in Proposition 8.

Proposition 9 *Suppose that $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (24), then there are positive-valued sequences $\{B_1^{*t}\}_{t=0}^{\infty}$ and $\{R_t^*\}_{t=1}^{\infty}$ such that $\{\varphi_t^*\}_{t=1}^{\infty}$ solves*

$$V(\{R_t^*\}_{t=1}^{\infty}, \Psi) = \sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t^*\}, \Psi)} \int_{\mathbb{R}} \sum_{t=1}^{\infty} B_1^{*t} \gamma_t^*(w) \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \tag{25}$$

where $\forall t \geq 0$, $\gamma^*_{t+1}(w) = (1-\omega_{t+1}) + \omega_{t+1}\gamma^*_t(w)$, $\gamma^*_0(w) = \gamma(w)$ *with* $\omega_{t+1} = \beta[B_1^{*t}/B_1^{*t+1}]$. *The sequences* $\{B_1^{*t}\}_{t=0}^\infty$ *and* $\{R^*_t\}_{t=1}^\infty$ *satisfy 1)* $\sum_{t=1}^\infty B_0^{*t} < \infty$, *2)* $B_0^{*1} = 1/\beta$ *and* $\forall t$, $B_1^{*t+1}/B_1^{*t} > \beta$, *and 3)* $\forall t$, $R^*_t \in (0, R_t]$.

Conversely, if $\{\varphi^*_t\}_{t=1}^\infty$ *solves (25) at* $\{B_1^{*t}\}_{t=0}^\infty$ *and* $\{R^*_t\}_{t=1}^\infty$ *(with* $\{\gamma^*_t\}_{t=0}^\infty$ *given as above) and if A)* $\{B_1^{*t}\}_{t=0}^\infty$ *satisfies* $B_0^{*1} = 1/\beta$ *and* $\{[\widehat{\delta} r^*_t \beta^{t-1}]^{-1}(B_1^{*t} - \beta B_1^{*t-1})\} \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})} \mathcal{L}(\{\varphi^*_t\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty)$, *with* $\forall t$, r^*_t $= \arg\max_{\widehat{r}\in[0,R_t]} \widehat{r} \left[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R_t - \widehat{r}) + \beta \underline{W}_{t+1} - \int_{\mathbb{R}} \sum_{\Theta^{t-1}} U(\{\varphi^*_{t+r}|w, \theta^{t-1}\}_{r=0}^\infty)\pi^{t-1}\left(\theta^{t-1}\right)\Psi(dw)\} \right]$ *and if B)* $\{R^*_t\}_{t=1}^\infty$ *solves* $\max \frac{1}{2}\sum_{t=1}^\infty [\widehat{\delta} r^*_t]^{-1}(B_1^{*t} - \beta B_1^{*t-1})(R - R_t) + V(\{R_t\}, \Psi)$, *then* $\{\varphi^*_t\}_{t=1}^\infty$ *solves (24).*

5 Political constraints and patient virtual planners

In each of the preceding examples Pareto optimal politically credible allocations solved virtual planning problems that shared the same constraint set as the benchmark problem (3), but whose objectives featured higher societal discount factors and mean reversion of Pareto weights. In this section, we seek generalizations of these results. Specifically, we look for conditions on abstract political constraints that ensure that the benchmark problem (3) augmented with these constraints is equivalent to a virtual planning problem with perturbed discounting and Pareto weighting schemes. Our conditions generalize and nest those from each of the models described above. It follows that if political constraints satisfying our conditions emerge as equilibrium restrictions in a political economy game, then the implied Pareto optimal equilibrium allocations of this game solve virtual planning problems with planners who are (weakly) more patient than the agents.

5.1 A generalized politically-constrained Pareto problem

Abstract political constraints We now consider abstract political constraints of the form:

$$Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty})) + X\left(R_t - \int_{\mathbb{R}} \sum\nolimits_{\Theta^t} C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)\right) \geq 0, \tag{26}$$

where $X : [0, \overline{R}] \to \mathbb{R}$ and $Z_t : \mathcal{F}_t \to \mathbb{R}$, $\mathcal{F}_t = \{u_t| \int_{\mathbb{R}} \sum_{\Theta^t} |u_t(w,\theta^{t-1})| \; \pi^{t-1}(\theta^{t-1})\Psi(dw) < \infty\}$ We interpret these constraints as capturing the restrictions necessary to ensure that an allocation is not revised in the future by voters in an election. Notice that (26) nests the political constraints in the preceding sections where Z_t was either a linear utilitarian aggregator that gave the net effect on a party's probability of winning an election from its best defection or a non-linear aggregator that gave the (negative) of the expected political rents attainable from such a defection. X was either 0 or gave current expected equilibrium rents. We assume the following.

Assumption Z1 *For all t, Z_t is Fréchet differentiable. For $u \in \mathcal{F}_t$ its Fréchet derivative ∂Z_t is a linear operator of the form: $\partial Z_t(u;\cdot) = \langle z_t(u), \cdot \rangle$, where $\langle z_t(u), f\rangle = \int z_t(u; w, \theta^{t-1}) f(w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw)$, $f \in \mathcal{F}_t$ and the functions $\{|z_t(\cdot;\cdot)|\}_{t=1}^{\infty}$ are uniformly bounded by some $\overline{z} < \infty$.*

Assumption Z2 *For all t, Z_t is concave and $Z_t : \mathcal{U}_t \to Z$, where $\mathcal{U}_t = \{u_t | u_t : \mathbb{R} \times \Theta^t \to \mathbb{R}$, u_t measurable and Ψ-a.e. w, all θ^t, $u_t(w,\theta^t) \in W\} \subset \mathcal{F}_t$ and Z is a bounded subset of $\mathbb{R}$.*

Assumption Z3 *For all t, Z_t is increasing and for all $u \in \mathcal{U}_t$ and almost all w, θ^{t-1}, $z_t(u; w, \theta^{t-1}) > 0$.*

Assumption X *X is non-decreasing, continuous, smooth and concave.*

Assumption ZX *There exists an allocation $\{\varphi_t\}_{t=1}^{\infty}$ satisfying (1), (2) and $\inf_t Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty}) + X(R_t - \int C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw))) > 0$.*

Assumptions Z1, X and ZX are essentially technical. Assumption Z3 captures the intuitive idea that if an allocation is politically credible, then one that offers more to everyone is also politically credible.

Generalized politically-constrained Pareto problems We call an allocation politically credible if it satisfies (1), (2) and the political constraints (26). An optimal politically credible allocation then solves:

$$\sup_{\{\varphi_t\}_{t=1}^{\infty}} \int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^{\infty}) \Psi(dw) \quad \text{s.t. (1), (2) and (26).} \tag{27}$$

The virtual planner We now prove that (27) can be transformed into the problem of a virtual planner who uses a perturbed discounting $\{B_1^{*t}\}_{t=0}^{\infty}$ and Pareto weighting scheme $\{\gamma_t^*\}_{t=0}^{\infty}$ and who faces no political constraints. As before, the current effective societal discount factor B_1^{*t}/B_1^{*t-1}, $t = 1, \dots$ exceeds that of the agents whenever the current political constraint binds. The more general virtual planning problem derived below features an individualized, history-contingent Pareto weighting scheme emerges that can be interpreted as capturing the time and history dependent political influence of agents.

Proposition 10 *Let Assumptions Z1-ZX hold. Suppose that $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (27), then there are positive-valued sequences $\{B_1^{*t}\}_{t=0}^{\infty}$, $\{\gamma_t^*\}_{t=0}^{\infty}$ and $\{R_t^*\}_{t=1}^{\infty}$ such that $\{\varphi_t^*\}_{t=1}^{\infty}$ solves*

$$V(\{R_t^*\}_{t=1}^{\infty}, \Psi) = \sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t^*\}, \Psi)} \int_{\mathbb{R}} \sum_{t=1}^{\infty} B_1^{*t} \sum_{\Theta^t} \gamma_t^*(w, \theta^{t-1}) \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw). \tag{28}$$

The sequences $\{B_1^{*t}\}_{t=0}^{\infty}$, $\{\gamma_t^*\}_{t=0}^{\infty}$ and $\{R_t^*\}_{t=1}^{\infty}$ satisfy: 1) $\sum_{t=0}^{\infty} B_1^{*t} < \infty$, 2) $B_0^{*1} = \beta^{-1}$ and $\forall t$, $B_1^{*t+1}/B_1^{*t} \geq \beta$, 3) $\forall t$, $\gamma_{t+1}^*(w, \theta^t) = (1 - \omega_{t+1}) + \omega_{t+1} \gamma_t^*(w, \theta^{t-1}) + \varepsilon_{t+1}^z(w, \theta^t)$, $\gamma_0^*(w, \theta^{-1}) = \gamma(w)$ with $\omega_{t+1} = \beta[B_1^{*t}/B_1^{*t+1}]$ and $\int \sum \varepsilon_{t+1}^z(w, \theta^t)\ \pi^t(\theta^t)\ \Psi(dw) = 0$, 4) $B_1^{*t+1}/B_1^{*t} > \beta$ and $\omega_{t+1} < 1$ if the period $t+1$ political constraint binds in (27) and 5) $R_t^* \in [0, R_t]$.

Conversely, if $\{\varphi_t^*\}_{t=1}^\infty$ *solves (28) for a triple of sequences* $\{B_1^{*t}\}_{t=0}^\infty$, $\{\gamma_t^*\}_{t=0}^\infty$ *and* $\{R_t^*\}_{t=1}^\infty$ *satisfying 1)-3) and 5) above and the additional conditions: A)* $\{[\overline{z}_t\beta^{t-1}]^{-1}(B_1^{*t} - \beta B_1^{*t-1})\}_{t=1}^\infty \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})} \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty)$, *where* $\overline{z}_t = \int \sum_{\Theta^{t-1}} z_t^*(w,\theta^{t-1})\ \pi^{t-1}(\theta^{t-1})\ \Psi(dw)$ *and* $\langle z_t^*, \cdot\rangle = \partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot)$, *B)* $\forall t$, $\varepsilon_{t+1}^z(w,\theta^t) = (1-\omega_{t+1})\frac{z_{t+1}^*(w,\theta^t)-\overline{z}_{t+1}}{\overline{z}_{t+1}}$ *and C)* $\{R_t^*\}_{t=1}^\infty$ *solves* $\max_{\{\widehat{R}_t\}_{t=1}^\infty \in \Pi_{t=1}^\infty [0,R_t]} \sum_{t=1}^\infty [\overline{z}_t]^{-1}\ (B_1^{*t} - \beta B_1^{*t-1})X(R - \widehat{R}_t) + V(\{\widehat{R}_t\}_{t=1}^\infty, \Psi)$, *then* $\{\varphi_t^*\}_{t=1}^\infty$ *solves (27).*

6 Conclusion

Many dynamic normative models of incentive provision imply that it is ex ante optimal to almost surely immiserate agents. Implementation of the resulting optimal allocations thus requires a high degree of social commitment. This paper embeds a benchmark dynamic private information environment into a variety of political economy games that allow societies to vote over and, hence, revise allocations ex post. The probabilistic voting games considered directly connect the probabilities of electoral success and the payoffs of politicians to aggregates of agent utilities. The set of equilibrium restrictions on allocations augments the resource and incentive-feasibility conditions found in earlier normative contributions with a sequence of political constraints that impose lower bounds on these aggregates. Such bounds ensure that political parties adhere to the strategies underlying an allocation; they preclude immiserating outcomes. The paper shows that Pareto optimal politically credible allocations solve (politically unconstrained) virtual planning problems in which the effective societal discount factor weakly exceeds the private one in all periods and strictly exceeds it when the political constraints bind. More generally, whenever a political economy model implies a sequence of bounds on smooth, monotone, concave aggregates of agent utilities, then the corresponding effective societal discount factors weakly exceed those of the agents. These results provide alternative micro-political foundations for the assumption of a societal discount factor in excess of the private one made in several recent normative contributions. We conjecture that similar results will hold whenever political competition is strong enough to align sufficiently the preferences of politicians with those of the majority of agents.

7 Appendix: Proofs

7.1 Necessary and sufficient conditions for PCE

Lemma 2 By the argument preceding Lemma 2, $(\sigma^{NI}, \zeta^{NI}, \lambda^{NI})$ forms a PCE with continuation utilitarian payoffs $\{\underline{W}_t\}_{t=1}^{\infty}$. Consider an arbitrary PCE (σ, ζ, λ); denote its utilitarian payoff after H_t by $W_t(H_t|\sigma, \zeta, \lambda)$. Define the period t no insurance mechanism $S_t^{NI} = (\Theta, \varphi_t^{NI})$, $\forall \theta^t$, $\varphi_t^{NI}(\theta^t) = u(R_t)$. A party that proposes S_1^{NI} at date 1 delivers the utilitarian payoff: $(1-\beta)E[\theta]u(R_1) + \beta W_2(S_1^{NI}|\sigma, \zeta, \lambda)$. Since S_1^{NI} need not be the mechanism prescribed by the equilibrium σ, by Assumption 1 and Proposition 1: $W_1(\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R_1) + \beta W_2(S_1^{NI}|\sigma, \zeta, \lambda)$. Similarly, a party that proposes S_t^{NI} at date t after $\{S_s^{NI}\}_{s=1}^{t-1}$ attains the continuation utilitarian payoff $(1-\beta)E[\theta]u(R_t) + \beta W_{t+1}(\{S_s^{NI}\}_{s=1}^{t}|\sigma, \zeta, \lambda)$. Once more this proposal need not be that prescribed by σ and so, $W_t(\{S_s^{NI}\}_{s=1}^{t-1}|\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R_t) + \beta W_{t+1}(\{S_s^{NI}\}_{s=1}^{t}|\sigma, \zeta, \lambda)$. Combining these inequalities gives $W_1(\sigma, \zeta, \lambda) \geq (1-\beta)\sum_{t=1}^{T} \beta^{t-1} E[\theta]u(R_t) + \beta^{T+1}\, W_{T+1}(\{S_t^{NI}\}_{t=1}^{T}|\sigma, \zeta, \lambda)$ and so, taking the limit in T and using the boundedness of payoffs, $W_1(\sigma, \zeta, \lambda) \geq \underline{W}_1$. A similar logic implies that for all H_t, $W_t(H_t|\sigma, \zeta, \lambda) \geq \underline{W}_t$. ■

Proposition 2 (Necessity) Let (σ, ζ, λ) be a PCE and $\Upsilon(\sigma, \zeta, \lambda)$ it's outcome path. Since $\sigma \in \Sigma(\lambda)$, $\Upsilon(\sigma, \zeta, \lambda)$ satisfies A; since λ satisfies (6), $\Upsilon(\sigma, \zeta, \lambda)$ satisfies B. Defining S_t^{NI} as in the proof of Lemma 2, $\int U_t(H_t, \sigma_t^i(H_t), h_t|\sigma, \zeta, \lambda)\, Q_t(dh_t) \geq \int U_t(H_t, S_t^{NI}, h_t|\sigma, \zeta, \lambda)\, Q_t(dh_t) = (1-\beta)E[\theta]u(R_t) + \beta W_{t+1}(H_t, S_t^{NI}|\sigma, \zeta, \lambda) \geq (1-\beta)E[\theta]u(R_t)$

$+ \beta W_{t+1}(H_t, S_t^{NI}|\sigma^{NI}, \zeta^{NI}, \lambda^{NI}) = \underline{W}_t$, where the first inequality stems from $\sigma_t^i(H_t) \in \arg\max_{\mathcal{S}_t(H_t,\lambda)} \int U_t(H_t, S_t, h_t|\sigma, \zeta, \lambda)\; Q_t(dh_t)$ and $S_t^{NI} \in \mathcal{S}_t(H_t, \lambda)$, the equality follows from the definition of S_t^{NI} and the second inequality follows from Lemma 2. Hence, $\Upsilon(\sigma, \zeta, \lambda)$ satisfies C. Since, party i can always defect and select $\sigma_t^j(H_t)$ at H_t, so $\int_{\mathbb{R}\times M^{t-1}} U_t(H_t, \sigma_t^i(H_t), h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda, dh_t) \geq \int_{\mathbb{R}\times M^{t-1}} U_t(H_t, \sigma_t^j(H_t), h_t|\sigma, \zeta, \lambda) Q_t(H_t, \lambda, dh_t)$ and D follows. Lemma 1 implies E.

(Sufficiency) For the converse, we construct a strategy profile that induces $\Upsilon = \{\{\mathcal{H}_t^\Upsilon\}, \{\mathcal{J}_t^\Upsilon\}, \{\sigma_t^\Upsilon\}, \{\lambda_t^\Upsilon\}\}$ and verify that if Υ satisfies A-E, then the profile is an equilibrium one. For $H_t \in \mathcal{H}_t^\Upsilon$, set $\sigma_t(H_t) = \sigma_t^\Upsilon(H_t)$ and for $(H_t, w, m^{t-1}) \in \mathcal{J}_t^\Upsilon$ set $\lambda_t(H_t, w, m^{t-1}, \theta) = \lambda_t^\Upsilon(H_t, w, m^{t-1}, \theta)$. For $H_t \notin \mathcal{H}_t^\Upsilon$, set $\sigma_t(H_t) = \sigma_t^{NI}(H_t)$ and for $(H_t, w, m^{t-1}) \notin \mathcal{J}_t^\Upsilon$ set $\lambda_t(H_t, w, m^{t-1}, \theta) = \lambda_t^{NI}(H_t, w, m^{t-1}, \theta)$. Finally, set ζ so that for all t, h_t^p $\zeta_t(h_t^p; A) = 1$ if $\Delta U_t(H_t, S_t^A, S_t^B, h_t|\sigma, \zeta, \lambda) \geq D(H_t, S_t^A, S_t^B, \xi, \delta_t)$ and $\zeta_t(h_t^p; A) = 0$ otherwise. It is straightforward to verify that these strategies are resource-feasible and optimal for agents. To verify the optimality of σ note that by C and D, for $H_t \in \mathcal{H}_t^\Upsilon$, $i, j \in \{A, B\}$and all $S_t \in \mathcal{S}_t(H_t, \lambda^{NI}) \backslash \{\sigma_t^i(H_t)\}_{i\in\{A,B\}}$, $\int U_t(H_t, \sigma_t^i(H_t), h_t|\sigma, \zeta, \lambda) Q_t(H_t, dh_t) = \int U_t^\Upsilon(H_t, \sigma_t^{\Upsilon,i}(H_t), h_t|\sigma^\Upsilon, \lambda^\Upsilon) Q_t^\Upsilon(H_t, dh_t) = \int U_t^\Upsilon(H_t, \sigma_t^{\Upsilon,j}(H_t), h_t|\sigma^\Upsilon, \lambda^\Upsilon) Q_t^\Upsilon(H_t, dh_t) = \int U_t(H_t, \sigma_t^j(H_t), h_t|\sigma, \zeta, \lambda)$ $Q_t(H_t, dh_t) \geq \underline{W}_t = (1-\beta)E[\theta]u(R_t) + \beta W_{t+1}(H_t, S_t^{NI}|\sigma^{NI}, \zeta^{NI}, \lambda^{NI}) \geq \int U_t(H_t, S_t, h_t|\sigma, \zeta, \lambda)\; Q_t(H_t, dh_t)$, where the maximality of S_t^{NI} in $\mathcal{S}_t(H_t, \lambda^{NI})$ delivers the final inequality. ■

Proof of Proposition 6 Suppose that (σ, ζ, λ) is a PCE of a game with patient parties, then it is resource-feasible and satisfies agent optimality. It remains to check that $\sigma_t^i(H_t) \in \arg\sup_{S_t^i \in \mathcal{S}_t(\lambda, H_t)} p_t^i(H_t, S_t^i, \sigma_t^j(H_t)|\zeta)$. As before, party A (resp. B) can always win elections with probability p (resp. $1-p$) simply by playing σ^B (resp. σ^A). Thus, in the game with patient parties, p (resp. $1-p$) places a lower bound on party A's (resp. B's) payoff. Hence, $p \leq (1-\chi)\sum_{t=1}^{\infty}\chi^{t-1}p_t^A = 1-(1-\chi)\sum_{t=1}^{\infty}\chi^{t-1}p_t^B \leq p$ and so party A always earns a payoff of p in equilibrium. By the same argument, party A's continuation equilibrium payoff after any history is also p. If after some H_t, A defects to S_t^A and then reverts to equilibrium play its payoff is: $(1-\chi)p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta) + \chi p$. Whether party A wins or looses in the present does not affect its equilibrium continuation payoff and, so, it chooses policy simply to maximize its current payoff. It follows that equilibrium strategies must maximize the per period probability of winning. The same logic applies to party B.

Conversely, if (σ, ζ, λ) is a PCE of a game with impatient parties ($\chi = 0$), then it is resource-feasible and satisfies the agent optimality conditions. It remains to check that σ satisfies for all t and H_t, $V_t^i(\sigma, \zeta, \lambda|H_t) \geq V_t^i(\widehat{\sigma}^i, \sigma^j, \zeta, \lambda|H_t)$, where the V_t^i are defined for some $\chi > 0$. In the game with impatient parties, party A wins an election with probability p in each period and so for all t, H_t, $V_t^A(\sigma, \zeta, \lambda|H_t) = p$. It follows that for all t, H_t, since $\sigma_t^A(H_t)$ maximizes $p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)$, $\sigma_t^A(H_t) \in \arg\sup_{S_t^A \in \mathcal{S}_t(\lambda, H_t)} (1-\chi)\; p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)$ $+\; \chi(1 - p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta))\; V_{t+1}^A(\sigma^A, \sigma^B, \zeta, \lambda|H_t, \sigma_t^B(H_t)) + \chi\; p_t^A(H_t, S_t^A, \sigma_t^B(H_t)|\zeta)\; V_{t+1}^A(\sigma^A, \sigma^B, \zeta, \lambda|H_t, S_t^A)$. Thus, even with $\chi > 0$, party A has no incentive to defect from σ^A for one period nor, given the boundedness of payoffs, to undertake any series of deviations from σ^A. Hence, for all t, H_t, $V_t^A(\sigma, \zeta, \lambda|H_t) \geq V_t^i(\widehat{\sigma}^A, \sigma^B, \zeta, \lambda|H_t)$. The same argument implies that for all t, H_t, $V_t^B(\sigma, \zeta, \lambda|H_t) \geq V_t^i(\sigma^A, \widehat{\sigma}^B, \zeta, \lambda|H_t)$. ■

7.2 Virtual planning formulations

Propositions 4 and 9 are proven as special cases of the more general result Proposition 10. The proof of this result is obtained in three steps. In the first, an allocation is shown to be an optimal politically credible one if and only if it attains a saddle point of a Lagrangian. In the second step this Lagrangian is "linearized", in the third, the linearized Lagrangian is reconfigured to give the virtual planner's objective. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L}(\{\varphi_t\}_{t=1}^{\infty}, \{\mu_t\}_{t=1}^{\infty}) = &\int_{\mathbb{R}} \gamma(w) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \\ &+ \sum_{t=1}^{\infty} \beta^{t-1} \mu_t \left[Z_t \left(\sum_{r=0}^{\infty} \beta^r \sum_{\Theta^{t+r}} \theta_{t+r} \varphi_{t+r}(\cdot, \cdot, \theta^r) \pi^r(\theta^r) \right) + X \left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t)) \pi^t(\theta^t) \Psi(dw) \right) \right],\end{aligned} \tag{29}$$

where $\{\mu_t\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\})$. It is convenient to re-express the sequence of constraints (26) as a single constraint $G^{POL}(\{\varphi_t\}_{t=1}^{\infty}) \geq 0$, where $G^{POL} : A \to \ell_{\infty}$ and $G^{POL}(\{\varphi_t\}_{t=1}^{\infty}) = \{Z_t(U(\{\varphi_{t+r-1}|\cdot\}_{r=1}^{\infty})) + X(R_t - \int \sum C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Psi(dw))\}_{t=1}^{\infty}$. We have the following.

Proposition A1 *Let Assumptions Z2, X and ZX hold and let U^* denote the optimal payoff from* (27)*. Then, there is a $\{\mu_t^*\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\})$ such that $U^* = \sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)} (1 - \beta)\ \mathcal{L}(\{\varphi_t\}_{t=1}^{\infty}, \{\mu_t^*\}_{t=1}^{\infty})$. Furthermore, if $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in* (27)*, then $\forall\ \{\mu_t\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\})$, $\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)$,*

$$\mathcal{L}(\{\varphi_t\}_{t=1}^{\infty}, \{\mu_t^*\}_{t=1}^{\infty}) \leq \mathcal{L}(\{\varphi_t^*\}_{t=1}^{\infty}, \{\mu_t^*\}_{t=1}^{\infty}) \leq \mathcal{L}(\{\varphi_t^*\}_{t=1}^{\infty}, \{\mu_t\}_{t=1}^{\infty}). \tag{30}$$

Conversely, if $\{\mu_t^*\}_{t=1}^{\infty} \in L(\{\beta^{t-1}\})$ and $\{\varphi_t^*\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)$ satisfy (30), then $\{\varphi_t^*\}_{t=1}^{\infty}$ solves (27).

Proof: By Assumption ZX, there is some $\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)$ such that $G^{POL}(\{\varphi_t\}_{t=1}^{\infty}) > 0$. By Assumption Z2 and X, $\{\{\varphi_t\}_{t=1}^{\infty} | G^{POL}(\{\varphi_t\}_{t=1}^{\infty}) \geq 0\}$ is convex. Since $\Gamma(\{R_t\}, \Psi)$ is convex and $\int_{\mathbb{R}} \gamma(w) U(\{\varphi_t(w, \cdot)\}_{t=1}^{\infty}) \Psi(dw)$ is concave, it follows from Luenberger (1969), Theorem 1, p.217 that there is an element $\mu^* \in \ell'_{\infty}$ such that

$$\begin{aligned}\frac{U^*}{1 - \beta} = &\sup_{\{\varphi_t\}_{t=1}^{\infty} \in \Gamma(\{R_t\}, \Psi)} \int_{\mathbb{R}} \gamma(w) \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\Theta^t} \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \\ &+ \left\langle \mu^*, \left\{ Z_t \left(\sum_{\Theta^{t+r}} \theta_{t+r} \varphi_{t+r}(\cdot, \cdot, \theta^r) \pi^r(\theta^r) \right) + X \left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \right) \right\}_{t=1}^{\infty} \right\rangle\end{aligned} \tag{31}$$

By Luenberger (1969), Corollary 1, p. 219, if $\{\varphi_t^*\}_{t=1}^{\infty}$ attains the supremum in (27), then the Lagrangian on the right hand side of (31) has a saddle point at $(\mu^*, \{\varphi_t^*\}_{t=1}^{\infty})$. That μ^* can be represented by an element in $L(\{\beta^{t-1}\})$

follows from Rustichini (1998), Corollary 5.6. Thus, the saddle point condition (30) holds. The converse follows from Luenberger Theorem 2, p. 221 and Rustichini (1998), Corollary, 5.6. ■

Proposition A2 *Let Assumptions Z1, Z2, X and ZX hold. If* $\{\varphi_t^*\}_{t=1}^\infty$ *attains the supremum in (27), then there is a pair of sequences* $\{\mu_t^*\}_{t=1}^\infty \in L(\{\beta^{t-1}\})$ *and* $\{z_t^*\}_{t=1}^\infty$, $z_t^* : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_+$ *such that* $\{\varphi_t^*\}_{t=1}^\infty$ *solves*

$$\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty), \tag{32}$$

$\mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty) := \int \gamma(w) U(\{\varphi_t(w,\cdot)\}_{t=1}^\infty)\Psi(dw) + \sum_{t=1}^\infty \beta^{t-1}\, \mu_t^* \Big[\int \sum z_t^*(w,\theta^{t-1})\, U(\{\varphi_{t+r-1}|w,\theta^{t-1}\})$ $\pi^{t-1}(\theta^{t-1})\, \Psi(dw) + X\left(R_t - \int \sum C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)\right) \Big]$. *Conversely, if* $\{\varphi_t^*\}_{t=1}^\infty$ *solves* (32) *for some* $\{\mu_t^*\}_{t=1}^\infty$ $\in L(\{\beta^{t-1}\})$ *and* $\{z_t^*\}_{t=1}^\infty$, $z_t^* : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_+$ *such that* $\partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot) = \langle z_t^*, \cdot \rangle$ *and* $\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})}$ $\mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty)$*, then* $\{\varphi_t^*\}_{t=1}^\infty$ *solves (27).*

Proof: If $\{\varphi_t^*\}_{t=1}^\infty$ attains the supremum in (27), then by Proposition A1 there is a sequence $\{\mu_t^*\}_{t=1}^\infty \in L(\{\beta^{t-1}\})$ such that $\{\varphi_t^*\}_{t=1}^\infty \in \arg\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}(\{\varphi_t\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$. Fix $\{\varphi_t'\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)$ and for $\alpha \in [0,1]$, let $\{\varphi_t^\alpha\}_{t=1}^\infty = \{(1-\alpha)\varphi_t^* + \alpha\varphi_t'\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)$ and $J(\alpha) = \mathcal{L}(\{\varphi_t^\alpha\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$. Then, $J : [0,1] \to \mathbb{R}$ is smooth and concave with a maximal value at $\alpha = 0$ and $\lim_{\alpha\downarrow 0} J'(\alpha) \le 0$. The latter inequality, the definitions of J and $\mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty)$, Assumption Z1 and the concavity of X and $\int_{\mathbb{R}} \gamma(w)U(\cdot)\Psi(dw)$, imply that $\mathcal{L}^*(\{\varphi_t^*\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty) \ge \mathcal{L}^*(\{\varphi_t'\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty)$, where $\langle z_t^*, \cdot\rangle = \partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot)$. For the converse, suppose that $\mathcal{L}^*(\{\varphi_t^*\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty) \ge \mathcal{L}^*(\{\varphi_t'\}_{t=1}^\infty; \{\mu_t^*\}_{t=1}^\infty, \{z_t^*\}_{t=1}^\infty)$, where $\langle z_t^*, \cdot\rangle = \partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot)$ and $\mathcal{L}(\{\varphi_t'\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty) > \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t^*\}_{t=1}^\infty)$. Combining these inequalities, $\sum_{t=1}^\infty \beta^{t-1}\, \mu_t^* \int_{\mathbb{R}} \sum_{\theta^{t-1}} \widehat{z}_t^*(w,\theta^{t-1})$ $[U(\{\varphi_{t+r-1}^*|w,\theta^{t-1}) - U(\{\varphi_{t+r-1}'|w,\theta^{t-1})]\, \pi^t(\theta^t)\Psi(dw) > \sum_{t=1}^\infty \beta^{t-1}\, \mu_t^* \left[Z_t\left(U(\{\varphi_{t+r-1}^*|\cdot,\cdot)\right) \; - \; Z_t\left(U(\{\varphi_{t+r-1}'|\cdot,\cdot)\right)\right]$. But this contradicts the concavity of the Z_t and so $\{\varphi_t^*\}_{t=1}^\infty \in \arg\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \mathcal{L}(\{\varphi_t\}_{t=1}^\infty,\, \{\mu_t^*\})$. Since by assumption, $\{\mu_t^*\} \in \arg\inf_{\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})} \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty)$, it follows that $\{\varphi_t^*\}_{t=1}^\infty$ and $\{\mu_t^*\}$ satisfy the saddle point condition (30) and so by Proposition A1, $\{\varphi_t^*\}_{t=1}^\infty$ is optimal in (27). ■

The "linearized" Lagrangian $\mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t^*\}, \{z_t^*\})$ incorporates a history-specific multiplier scheme $\{\mu_t^* z_t^*\}$. We use this to construct a sequence of societal discount factors and history contingent Pareto weights. Given $\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})$ and a sequence of integrable functions $\{z_t\}_{t=1}^\infty$, $z_t : \mathbb{R} \times \Theta^{t-1} \to \mathbb{R}_{++}$, let $\overline{z}_t := \int_{\mathbb{R}} \sum_{\Theta^{t-1}} z_t(w, \theta^{t-1})\ \pi^{t-1}(\theta^{t-1})\ \Psi(dw)$ and define the discount factor sequence: $B_1^0 = \beta^{-1}$ and $B_1^t = \beta^{t-1}\ [1 + \sum_{s=1}^t \mu_s \overline{z}_s] = \beta B_1^{t-1} + \beta^{t-1}\mu_t \overline{z}_t$. Define the Pareto weight sequence: $\gamma_0(w, \theta^{-1}) = \gamma(w)$ and for all t, θ^{t-1}, $\omega_t = \beta[B_1^{*t-1}/B_1^{*t}]$, $\varepsilon_t^z(w, \theta^{t-1}) = (1 - \omega_t) \left(\frac{z_t(w,\theta^{t-1}) - \overline{z}_t}{\overline{z}_t}\right)$ and $\gamma_t(w, \theta^{t-1}) = (1 - \omega_t) + \omega_t \gamma_{t-1}(w, \theta^{t-2}) + \varepsilon_t^z(w, \theta^{t-1})$. By construction the average value of the weights $\gamma_t(w, \theta^{t-1})$ in the population is always 1 and $E[\varepsilon_t^z] = 0$. The following lemma establishes that the Lagrangian in (32) evaluated at a (bounded) allocation and multiplier sequence can be rearranged to obtain an alternative objective. It's proof, essentially an application of Abel's lemma, is omitted.

Lemma A1 *Given $\{\mu_t\}_{t=1}^\infty \in L(\{\beta^{t-1}\})$ and $\{z_t\}_{t=1}^\infty$, $z_t : \mathbb{R} \times \Theta^{t-1} \to (0, \overline{z}]$ with each z_t integrable, define the sequences $\{\gamma_t\}_{t=0}^\infty$ and $\{B_1^t\}_{t=0}^\infty$ as above. Then, for each $\{\varphi_t\}_{t=1}^\infty \in A$, we have:*

$$\begin{aligned}\mathcal{L}^*(\{\varphi_t\}_{t=1}^\infty; \{\mu_t\}_{t=1}^\infty, \{z_t\}_{t=1}^\infty) \quad = \quad & \int_{\mathbb{R}} \sum_{t=1}^\infty B_1^t \sum_{\Theta^t} \gamma_t(w, \theta^{t-1}) \theta_t \varphi_t(w, \theta^t) \pi^t(\theta^t) \Psi(dw) \\ & + \sum_{t=1}^\infty \beta^{t-1} \mu_t X \left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t)) \pi^t(\theta^t) \Psi(dw) \right).\end{aligned}$$

We now prove Proposition 10.

Proposition 10 Given Assumptions Z1-ZX, Propositions A1 and A2, Lemma A1 and the renormalizations preceding Lemma A1 if $\{\varphi_t^*\}_{t=1}^\infty$ attains the supremum in (27), then there are positive-valued sequences $\{B_1^{*t}\}_{t=0}^\infty$

and $\{\gamma_t^*\}_{t=0}^\infty$ such that $\{\varphi_t^*\}_{t=1}^\infty$ solves

$$\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t\},\Psi)} \int_{\mathbb{R}} \sum_{t=1}^\infty B_1^{*t} \sum_{\Theta^t} \gamma_t^*(w,\theta^{t-1})\theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) + \sum_{t=1}^\infty \beta^{t-1}\mu_t^* X\left(R_t - \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w,\theta^t))\pi^t(\theta^t)\Psi(dw)\right). \tag{33}$$

where for $t \geq 1$, $\mu_t^* = \frac{1}{\overline{z}_t \beta^{t-1}}(B_1^t - \beta B_1^{t-1}) \geq 0$, $\overline{z}_t = \int_{\mathbb{R}} \sum_{\Theta^{t-1}} z_t^*(w,\theta^{t-1})\ \pi^{t-1}(\theta^{t-1})\ \Psi(dw) > 0$, $\langle z_t^*, \cdot\rangle = \partial Z_t(\{\varphi_t^*\}_{t=1}^\infty; \cdot)$ and the sequences $\{B_1^{*t}\}_{t=0}^\infty$ and $\{\gamma_t^*\}_{t=0}^\infty$ satisfy conditions 1)-4) in the proposition. Also, $\{\mu_t^*\}_{t=1}^\infty$ is the optimizing multiplier sequence $\min_{\{\mu_t\}_{t=1}^\infty \in L(\{\beta^t\})} \mathcal{L}(\{\varphi_t^*\}_{t=1}^\infty, \{\mu_t\}_{t=1}^\infty)$. Define for all t, $R_t^* = \int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t^*(w,\theta^t))\ \pi^t(\theta^t)\ \Psi(dw) \in [0, R_t]$. Clearly then $\{\varphi_t^*\}_{t=1}^\infty$ solves $\sup_{\{\varphi_t\}_{t=1}^\infty \in \Gamma(\{R_t^*\},\Psi)} \int_{\mathbb{R}} \sum_{t=1}^\infty\ B_1^{*t} \sum_{\Theta^t} \gamma_t^*(w,\theta^{t-1})\ \theta_t \varphi_t(w,\theta^t)\ \pi^t(\theta^t)\Psi(dw)$, where $\{R_t^*\}$ satisfies (5) in the proposition. The converse of Proposition A2 coupled with lemma A1 implies that if $\{\varphi_t^*\}_{t=1}^\infty$ solves (33) for a pair of sequences $\{B_1^{*t}\}_{t=1}^\infty$ and $\{\gamma_t^*\}_{t=1}^\infty$ satisfying 1)-3), 5), A) and B) then it attains the supremum in (27). We may restate the optimization in (33) as

$$\sup_{\{\widehat{R}_t\} \in \Pi_{t=1}^\infty [0,R_t], \{\varphi_t\}_{t=1}^\infty \in \Gamma(\{\widehat{R}_t\},\Psi)} \int_{\mathbb{R}} \sum_{t=1}^\infty B_1^{*t} \sum_{\Theta^t} \gamma_t^*(w,\theta^{t-1})\theta_t \varphi_t(w,\theta^t)\pi^t(\theta^t)\Psi(dw) + \sum_{t=1}^\infty \beta^{t-1}\mu_t^* X\left(R_t - \widehat{R}_t\right).$$

The converse and condition C) in the proposition follow from this. ■

Proposition 4 is a direct consequence of this with $Z_t(u) = \int \sum_{\Theta^{t-1}} u(w,\theta^{t-1})\pi^{t-1}(\theta^{t-1})\Psi(dw)$ and $X = 0$. These clearly satisfy Assumptions Z1-ZX. Proposition 9 follows with $Z_t(U(\{\varphi_{t+r}|\cdot\}_{r=0}^\infty)) = \widehat{Z}_t(\int_w \sum_{\Theta^t} \sum_{r=0}^\infty U(\{\varphi_{t+r}|w,\theta^{t-1}\}_{r=0}^\infty)\ \pi^{t-1}(\theta^{t-1})\ \Psi(dw))$, where $\widehat{Z}_t(B) := -\max_{\widehat{r}\in[0,R_t]} \widehat{r}[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R_t - \widehat{r}) + \beta \underline{W}_{t+1} - B\}]$ and $X(r) = r$. These functions also satisfy Assumptions Z1-ZX. In particular, assuming that $r_t^*(B) = \arg\max_{\widehat{r}\in[0,R_t]} \widehat{r}\,[\frac{1}{2} + \widehat{\delta}\{(1-\beta)E[\theta]u(R_t - \widehat{r}) + \beta \underline{W}_t - B\}]$ is interior, we see that $\frac{\partial^2 \widehat{Z}_t}{\partial B^2}(B) = \frac{1}{-2(1-\beta)E[\theta]u'(R_t - r_t^*(B)) + r_t^*(B)(1-\beta)}$ < 0 and $\widehat{Z}_t$ is concave. ■

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