# The grammar of paradise: on the generation of Mughul gardens 

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#### Abstract

The conventions used to design Mughul gardens are characterized. These conventions are represented by a parametric shape grammar.


## The concept of paradise

The origin of the English word 'paradise' can be traced back via the Latin and the Greek to the Old Persian paira (around) and daeza (wall), and so has its roots in the simple concept of a walled garden (Moynihan,1979). Paradise recalls the culture of a desert people, who conceived of a garden as a protected place cut off from hostile surroundings, by contrast with the Whiggish self-confidence of eighteenth century English gardeners like William Kent, who famously "leaped the fence, and saw that all nature was a garden".

Now a garden in the desert needs a source of water. In the Persian desert, water has traditionally been provided by a well (quanat) supplied by an underground tunnel running down from the mountains (Crowe et al, 1972). So at the center of a walled Persian garden, one typically finds a source marked by a formal pool or reservoir.

Since this arrangement did not yield a sufficient head of water pressure to enable satisfactory piped distribution of water to the plants, some other mode of distribution was necessary. The traditional solution was to provide canals radiating out from the central reservoir, and to set in stone causeways raised slightly above the level of the planted areas. This divided the garden into four quadrants, as shown in figure 1, and gave rise to the traditional Persian name char-bagh (literally 'four gardens') for this motif.


Figure 1. A simple char-bagh or fourfold garden.

The quadrants were watered simply by flooding from the canals, whereas the raised stone causeways provided a permanently dry circulation system.

Not only is the char-bagh an eminently practical garden form for its desert setting, it also functions as a religious symbol. As Eliade (1961) has shown, a center intersected by cross-axes is employed as a religious, cosmological diagram in many cultures. But more explicitly, in this case, the reference surely is to the four rivers that flow out of Paradise, as described in the Book of Genesis.

Over the centuries the char-bagh was propagated far afield from its place of origin. Babur and his successors established it in Mughul India, and the Moors carried it to Spain. The form underwent various transformations to adopt it to particular settings and circumstances, but the fundamental functional organization and symmetrical patterning remained remarkably constant. The char-bagh, wherever one finds it exemplified, is a clearly defined and immediately recognizable architectural type.

## The design of paradise

In this paper, we examine the conventions used to design a char-bagh. We show that these conventions can be represented by a parametric shape grammar. This grammar


Figure 2. The ground plan for the garden of the Taj Mahal in Agra.


Figure 3. The ground plan for the Shalamar Bagh in Lahore.


Figure 4. The ground plan for the Tomb of Jahangir at Shahdara near Lahore.
defines a language of char-bagh ground plans by generating each one of them in complete detail. The development of the grammar was motivated by our belief that to understand an architectural style or type requires more than simply pointing to a few important examples of it and describing their essential properties. One can claim full understanding only when one can give rules to construct new instances of the style or type. These new instances must not only look plausible; they must truly constitute functionally and symbolically adequate designs.

For the purpose of this exercise, we have taken as our starting point a small corpus of Mughul char-baghs, from India and Pakistan. These examples were chosen because they are elaborately and very beautifully developed, well-preserved, and well-documented (see Gothein, 1926; Crowe et al, 1972; and Moynihan, 1979). Choosing a different corpus might yield a somewhat different grammar, but such differences would be relatively minor and of little conceptual impact.

The corpus consists of three char-bagh ground plans as drawn by the Baroness Gothein (1926): the Taj Mahal in Agra (figure 2), the Shalamar Bagh in Lahore (figure 3), and the Tomb of Jahangir at Shahdara near Lahore (figure 4). These are all seventeenth century royal gardens built by the great Mughul emperors.

## The geometry of paradise

The geometry of the char-bagh may be described in terms of an underlying site parti and its relationship to the canals and pathways separating landscaped areas in the garden and to the borders enclosing these areas.

## The site parti

In its simplest form, a char-bagh is based on a site parti consisting of a square perimeter in which four congruent squares are arranged as shown in figure 5. The four squares are separated by a pair of orthogonal arteries of constant width.

More complicated site partis may be derived from the site parti of figure 5. A site parti containing $4^{n+1}$ squares is obtained from one containing $4^{n}$ squares by subdividing each of these areas as shown in figure 6. The four squares produced in any such subdivision of a square are congruent. The width of the orthogonal arteries separating


Figure 5. The basic site parti for a char-bagh.


Figure 6. Subdivision of the internal squares in a site parti.
these squares is constant; it is typically a simple function of the length $l$ of the edges of the subdivided square, for example, a geometric function of the form $\mathrm{f}(l)=c l$, where $c$ is a constant such that $0<c<1$, or a constant function of the form $\mathrm{f}(l)=c$, where $c$ is a constant such that $0<c$.

To facilitate the detailed description of char-baghs, the following recursive definition is used. We will say that the pair of orthogonal arteries intersecting at the center of a site parti has depth 1. A pair of orthogonal arteries has depth $n+1$ whenever it subdivides a square that forms a quadrant defined by a pair of orthogonal arteries of depth $n$. A typical site parti for a char-bagh is given in figure 7. The depth of each pair of orthogonal arteries is indicated by the number at its intersection.

In the site parti for a char-bagh, all arteries of depth $n$ have the same width. The width of any artery is less than the length of the edges of any square in the site parti.

This underlying recursive organization of the char-bagh is not fortuitous; it follows from a simple principle of allometric growth, and thus is closely analogous to many recursive patterns found in nature [see Thompson's (1961) On Growth and Form for numerous striking examples]. The principle is this: When a simple char-bagh is enlarged beyond a certain size, it becomes impossible to irrigate the garden areas satisfactorily from four central canals. So each square is subdivided by arteries which may contain


Figure 7. A possible site parti for a char-bagh. The depth of each pair of orthogonal arteries in this site parti is indicated by the number at their intersection.


Figure 8. Examples of increasing articulation of the site parti corresponding to increasing area of the char-bagh.
secondary canals to form itself a miniature char-bagh. With further enlargement, this procedure is simply performed recursively, to whatever depth is considered necessary. Thus increasing scale implies increasing articulation of the site parti (figure 8).

## Canals and pathways

The arteries in the site parti for a char-bagh provide for the circulation of water and people in the garden.

Canal systems have the two characteristic forms drawn in figure 9. The motif of figure 9 (a) consists of four canals connected by a central reservoir running around the perimeter of a square. The motif of figure $9(b)$ also consists of four canals, but in this case they are connected by a central reservoir running around the perimeter of an octagon. In both motifs, the width of the canals and their reservoirs is the same.

Either of the motifs of figure 9 may be used in a char-bagh. The occurrence of canal systems is determined by the site parti in accordance with these rules:
(1) A canal system must occur in the pair of orthogonal arteries of depth 1.
(2) A canal system may occur in a pair of orthogonal arteries of depth $n+1$ whenever these arteries are bounded by a pair of orthogonal arteries of depth $n$ that already has a canal system in it.
(3) Canal systems are added so that the bilateral symmetry of the site parti with respect to each of the axes bisecting the edges of its perimeter is preserved.
(4) The dimensions of canal systems occurring in pairs of orthogonal arteries of the same depth are the same.

A canal system is inserted in a pair of orthogonal arteries as shown in figure 10.


Figure 9. Square and octagonal motifs for canal systems.


Figure 10. Inserting canal systems in a site parti.

The corners of the four squares with vertices at the intersection of the arteries are inflected so that these squares do not touch the reservoir. The canals run down the center of the arteries and do not touch their edges.

The termini of canals, where they meet bounding walls or other similar canals, may be ornamented in various ways. When the end of a canal is adjacent to the outer perimeter, it may be truncated to correspond with inflections of the corners of the two squares on its opposite edges as shown in figure 11(a). Usually the dimensions of these inflections are fixed by the dimensions of the inflections of the squares around the reservoir to which the canal leads. When the ends of two canals in arteries of the same depth are adjacent, each end may be divided into two branches as shown in figure 11(b). These branches line up with the reservoirs to which the canals lead. The corners of the squares around the branches are inflected to correspond to the inflections of the corners of the squares around the reservoirs. The ends of canals are ornamented so that the symmetry of the site parti is preserved.

A possible arrangement of canals in the site parti of figure 7 is given in figure 12. Those parts of arteries in which no canals occur are pathways. Sometimes the space


Figure 11. Motifs for ornamenting the ends of canals.


Figure 12. A possible arrangement of canal systems in a site parti.
enclosed by the reservoir at the center of the canal system occurring in the pair of orthogonal arteries of depth 1 is filled by a monument or tomb.

## Borders

The interior squares of a char-bagh are planted with trees and flowers. Often a square contains part of a formal planted border. The occurrence of borders depends on the underlying site parti for the garden.

We will say that a border frames a pair of orthogonal arteries in a char-bagh whenever it follows the peripheral edges of the squares in the four quadrants defined by the intersection of these arteries and has constant width that is always less than the length of the edges of these squares. The border indicated in each of the two examples of figure 13 frames the central pair of orthogonal arteries.

The occurrence of borders in a site parti is governed by these rules:
(1) A border may frame a pair of orthogonal arteries of depth 2.
(2) A border may frame a pair of orthogonal arteries of depth $n+1$ whenever these arteries are bounded by a pair of orthogonal arteries of depth $n$ that has already been framed by a border.
(3) Borders are added so that the symmetry of the site parti is preserved.
(4) The width of borders framing pairs of orthogonal arteries of the same depth is the same.

The width of a border depends on the depth of the pair of orthogonal arteries it frames. In any char-bagh, one of these two cases is allowed: the width of a border íraming a pair of orthogonal arteries of depth $n+1$ is either (1) always less than or (2) always greater than the width of a border framing a pair of orthogonal arteries of depth $n$. The interaction of multiple borders in these two cases is illustrated in figure 14.

A possible arrangement of borders in the site parti of figure 7 is given in figure 15.


Figure 13. Examples of borders that frame central pairs of orthogonal arteries.


Figure 14. The interaction of multiple borders of different widths.


Figure 15. A possible arrangement of borders in a site parti.

## The generation of paradise

The conventions for designing a char-bagh ground plan as outlined above can be represented concisely and rigorously in the shape grammar formalism (Stiny, 1977; Stiny, forthcoming).

A shape grammar is an algorithm defined in terms of labelled shapes made up of lines and symbols. In its standard form, it consists of some shape rules and an initial shape. A shape rule $\alpha \rightarrow \beta$ consists of two labelled shapes $\alpha$ and $\beta$. The rule can be used to change a given labelled shape $\gamma$ into a new one whenever there is an isometry or similarity transformation that makes $\alpha$ identical to some part of $\gamma$. In this case, that part of $\gamma$ can be replaced by the same transformation of $\beta$. The shape rules in the grammar are applied to the initial shape and to labelled shapes produced from it. In this way, a language of shapes is defined by generating its individual members.

Parametric shape grammars are an extension of shape grammars in which shape rules are defined by filling in the open terms in a general schema. A shape rule schema $\alpha \rightarrow \beta$ consists of parameterized labelled shapes $\alpha$ and $\beta$. Whenever specific values are given to all of the variables in $\alpha$ and $\beta$ by an assignment $g$ to determine specific labelled shapes, a new shape rule $g(\alpha) \rightarrow g(\beta)$ is defined. This shape rule can then be used to change a given labelled shape into a new one in the usual way. More precisely, if there is a transformation $\tau$ that makes $g(\alpha)$ a subshape of the given labelled shape, then this occurrence of the labelled shape $\tau[g(\alpha)]$ can be replaced with the labelled shape $\tau[g(\beta)]$. It will be said that a shape rule schema applies to a labelled shape whenever it defines a shape rule that does.

In this section, we specify a parametric shape grammar that defines a language of char-bagh ground plans. Each plan in this language is generated in four stages: in stage 1, a site parti is produced, and the intended occurrences of canals and borders are fixed; in stage 2 , canal systems are inserted where allowed; in stage 3, borders are determined; and in stage 4, the generation process is terminated.

## In the beginning

The initial shape of the grammar is drawn in figure 16. This labelled shape consists of a pair of orthogonal axes with end points labelled by the symbols N, S, E, and W. All char-bagh ground plans are generated from this initial shape by applying the shape
rule schemata specified below. All of these schemata apply under transformations that are isometries.


Figure 16. The initial shape of the grammar.

## Stage 1

The shape rule schemata applied in stage 1 of the generation process are given in figure 17. In this stage, the schemata are used to produce a site parti and to fix the intended occurrences of canals and borders in it.

The variables associated with the schemata of figure 17 and with subsequently specified ones are not given explicitly; they are implied by the previous discussion of the conventions for char-bagh design and the description of the schemata in the text. The detailed parameterization of schemata is straightforward and is left to the interested reader. Schema 1 allows for a basic site parti to be superimposed on the initial shape. The pair of orthogonal arteries in this site parti is distinguished by a corresponding pair of axes with arrowheads at their end points. This pair of axes indicates the intended occurrence of a canal system by fixing the length of its canals; it is used in stage 2 . The center of each square inside the perimeter of the site parti is labelled by the symbol A. This symbol and the symbols $\mathrm{B}, \mathrm{C}$, and D allow for the application of schemata $2-10$. The vertices of the square perimeter are labelled by the symbol $P$. This symbol is used again in stage 2 .

Schemata $2-10$ provide for the subdivision of squares inside the perimeter of a site parti. Each subdivision of a square into four squares is produced by a pair of orthogonal arteries. The width of these arteries is a function of the length of the edges of the subdivided square; it is always smaller than some constant $c$. As mentioned earlier, this function is often a geometric one or a constant one. The constant $c$ is used again in schemata $11-14$; it fixes the length of the edges of all squares inside the perimeter of a completed site parti.

A subdivision of a square may have associated with it the occurrence of a canal system or the occurrence of a border. Such occurrences are controlled by the symbols A, B, C, and D which label the centers of squares in schemata 2-10. The use of these symbols is determined by the rules for adding canal systems and borders to a site parti given above.

The pair of orthogonal arteries used to subdivide a square may have a canal system or a border occur with it in four possible ways: (1) it may contain a canal system and be framed by a border; (2) it may just contain a canal system; (3) it may just be framed by a border; or (4) it may neither contain a canal system nor be framed by a border. Cases 1-4 are all allowed whenever the symbol A labels the center of a square (schemata 2-5), cases 2 and 4 whenever the symbol B does so (schemata 6 and 7), cases 3 and 4 whenever the symbol C does so (schemata 8 and 9), and only case 4 whenever the symbol D does so (schema 10).

In the subdivisions of a square produced by schemata $2-10$, the occurrence of a canal system is indicated by a pair of axes with arrowheads at their end points. This pair


2

4

5



Figure 17. Shape rule schemata applied in stage 1 of the char-bagh generation process.
6



$\rightarrow$


Figure 17 (continued)
11

13


14


Figure 17 (continued)
of axes corresponds to the pair of orthogonal arteries used to subdivide the square; it determines the length of the canals in the system. The occurrence of a border is indicated by a square with the symbol $*$ at the end of one of its edges. The square follows the perimeter of the subdivided square. The distance between the edges of the square and the subdivided one (the width of the border) is a function of the length of the edges of the subdivided square, and is always less than the constant $c$. This function assigns either strictly decreasing or strictly increasing values to decreasing lengths of edges, thus ensuring that the width of a border framing a pair of orthogonal arteries of depth $n+1$ is appropriately related to the width of a border framing a pair of orthogonal arteries of depth $n$. The symbol $*$ allows those parts of the square determining the border but not actually part of it to be erased in stage 3 .

As specified in figure 17, schemata $2-10$ do not preserve the requisite symmetry of a site parti when they are applied. For these schemata to do so, they must be redefined by reflecting their left and right sides with respect to two orthogonal axes as illustrated in figure 18. We assume that schemata 2-10 always apply in this extended form, rather than redefining them in this more cumbersome way.

Schemata 11-14 of figure 17 provide for the termination of the stage 1 generation process. These schemata can be applied to replace the symbol A, B, C, or D at the


Figure 18. Redefining shape rule schemata to ensure bilateral symmetry with respect to two orthogonal axes.
center of a square with the symbol $\bullet$ and the square's diagonals whenever the length of the edges of the square is greater than the constant $c$ but smaller than $2 c$. This condition on the application of schemata 11-14 ensures that all of the squares in a site parti in which no square has the symbol $\mathrm{A}, \mathrm{B}, \mathrm{C}$, or D at its center are congruent. Further, the width of any artery or border in this site parti is less than the length of the edges of these squares. The symbol $\bullet$ and the diagonals added to a square by applying schemata $11-14$ are used in stage 3 .

A completed site parti with intended occurrences of canal systems and borders produced by applying schemata $1-14$ is drawn in figure 19.


Figure 19. A completed site parti with intended occurrences of canal systems and borders produced by applying schemata 1-14. Construction lines are drawn finer than others.

## Stage 2

The shape rule schemata applied in stage 2 to add canal systems and ornament the ends of canals are given in figure 20. Schemata 15 and 16 replace a pair of axes with arrowheads at their end points with the canal sy'stem motifs given in figures 9(a) and 9(b), respectively. The length of the canals in such systems is fixed by the length of the axes. To add a canal system, the corners of the squares at the intersection of the axes are inflected. The width of the canals and the reservoir in the added system and the dimensions of the inflected corners are a function of the width of the arteries separating the squares. Schema 17 allows the ends of the canals to be ornamented whenever they are adjacent to the perimeter of a site parti. Notice that the perimeter is distinguished by occurrences of the symbol P. Schema 18 provides for ornamentation when the ends of two canals are adjacent. The dimensions of ornamentations are a function of the width of the arteries through which the canals run. Thus the dimensions of ornamentations can be appropriately related to the dimensions of canal systems.

Application of schemata $15-18$ produces areas thăt are not squares.. Consequently for these schemata to be used generally, they must be modified to apply to squares that have one or more of their corners inflected. This modification may be effected by parameterizing the corners of squares as shown in figure 21. Consequent alterations to borders passing through:squares with inflected corners would be specified by schemata like those given in figure 22; these are also parameterized as in figure 21.

16

17


18



Figure 20. Shape rule schemata applied to insert canal systems in stage 2 of the char-bagh generation process.


Figure 21. Parameterizing the corners of squares to allow for inflections.


21


Figure 22. Examples of shape rule schemata used to alter borders passing through squares with inflected corners.

As specified in figure 20, schemata 15-18 do not preserve the symmetry of a site parti. We assume that these schemata are always applied in the extended form given in figure 18.

One possible result of applying schemata $15-18$ and $19-21$ to the stage 1 ground plan of figure 19 is drawn in figure 23.


Figure 23. One possible result of applying schemata 15-18 and 19-21 to insert canal systems in the stage 1 ground plan of figure 19.

## Stage 3

The shape rule schemata applied in stage 3 are given in figure 24. These schemata are used to erase those parts of squares defining borders that are not actually part of them. Each part of such a square that is erased overlaps either an artery in a site parti or borders that have already been determined.

Essentially, the symbol $*$ is used to trace around a square defining a border, going from square to square in a site parti. Such squares have their centers labelled by the symbol •. For all occurrences of the symbol $*$ to be erased, this tracing process must be performed on a given square defining a border only after all squares defining borders outside of it have already been traced.

Schema 22 is used to begin the trace of a given square defining a border by repositioning the symbol $*$. If this square is the outermost one, then schemata 23,24 , 31 , and 32 are applied to complete the trace. Otherwise, schemata $25-32$ are applied.

Schemata 23-30 apply to a square in a site parti with its center labelled by the symbol - that also has part of a border labelled by the symbol * passing through it. These schemata erase that part of the border that does not overlap the rectangular area distinguished by its diagonals inside the square. This area does not yet contain a part of a border that has been traced. During the application of schemata 23-30 that area of the square not yet occupied by a part of a border that has been traced is redefined in terms of the border currently being traced, and the symbol $*$ is advanced to the outside of the square.

Schema 31 is applied to erase the symbol $*$ after a square defining a border has been completely traced. During this trace, schema 32 is used to advance the symbol *
across an artery and to erase that part of the square defining the border that overlaps the artery. This schema applies to squares in the site parti that are less than the constant $c$ apart. Schema 33 is applied to erase diagonals in rectangular areas after all squares defining borders have been traced around.

Of course, schemata 22-33 are parameterized as indicated in figure 21 so that they may be applied to squares in a site parti that have one or more of their corners inflected. Notice that the application of schemata $22-33$ has no effect on the symmetry of a site parti, as these schemata add nothing new to it.

The result of applying schemata $22-33$ to the stage 2 ground plan of figure 23 is drawn in figure 25 .


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31
 $\rightarrow$


32


33


Figure 24. Shape rule schemata applied to determine borders in stage 3 of the char-bagh generation process.


Figure 25. The result of applying schemata $22-33$ to determine borders in the stage 2 ground plan of figure 22.

## Stage 4

The generation of the ground plan for a char-bagh is terminated in stage 4. Figure 26 shows the results of applying the shape rule schemata of figure 27 to the stage 3 ground plan of figure 25 . The schemata are used to erase the symbols $\bullet, \mathrm{N}, \mathrm{S}, \mathrm{E}, \mathrm{W}$, and $P$. In this way, the complete ground plan for a char-bagh is produced.

Only ground plans that can be generated to have no symbols associated with them are members of the language defined by the grammar developed in this section.


Figure 26. The completed ground plan for a char-bagh.
$34\left\langle s_{\phi},\{(0,0): \bullet\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$
$35\left\langle s_{\phi},\{(0,0): \mathrm{N}\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$
$36\left\langle s_{\phi},\{(0,0): \mathrm{S}\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$
$37\left\langle s_{\phi},\{(0,0): \mathrm{E}\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$
$38\left\langle s_{\phi},\{(0,0): W\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$
$39\left\langle s_{\phi},\{(0,0): \mathrm{P}\}\right\rangle \rightarrow\left\langle s_{\phi}, \varnothing\right\rangle$

Figure 27. Shape rule schemata applied in stage 4 to terminate the char-bagh generation process.

All of these ground plans are constructed in accordance with the conventions for char-bagh design given in the previous section. No other ground plans based on these conventions are possible.

## The dreams of Shah Jahan

Towards the end of his life, Shah Jahan, builder of the Taj Mahal, was deposed and imprisoned in the Red Fort at Agra by his son Aurangzib. This forcibly brought the great era of Mughal char-bagh building to a close. But from his pavilion in the fort, Shah Jahan could sit and look out across the Jumna to the Taj, and no doubt dream of char-baghs that might have been. The grammar that we have specified here enables us to dream those same dreams once again (figure 28).


Figure 28. 'II, Cyrus, am full of wonder at the beauty of everything (about your garden), but much more do I admire the one who has measured out and ordered each kind of thing for you."-Lysander as quoted by Socrates in Xenophon's Oeconomicus.

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