48-747 Shape Grammars

SHAPE GRAMMARS

READINGS FOR WEEK 1

george stiny

two exercises in formal composition

The possibility of a full-fledged science of form presupposes a set of basic tools for the **systematic** and **uniform** treatment of a host of varied problems in formal composition.

Formal composition consists of arranging or combining or putting together certain spatial elements in accordance with some system of rules.

on formal composition …

Spatial elements - the functional volumes of buildings—rooms, vestibules, exits, and staircases—and the **system of rules** determines the symmetrical disposition of these volumes about one or more axes.

The nature of formal composition depends on the spatial elements and the systems of rules involved.

Spatial elements are considered generally as shapes that may be 2-dimensional or 3-dimensional, rectilinear or curvilinear, or structural or functional components of rooms, buildings, or towns.

Systems of rules are considered generally as classes of spatial relations that may fix the disposition of shapes in any spatially conceivable way.

Beaux Arts composition …

A **shape** is a finite arrangement of lines of non-zero length which are specified by drawing them in a fixed, two-dimensional Cartesian coordinate system.

for our purpose, what is a shape?

A shape is a ***subshape*** of another
if all the lines in the first shape are lines in the second

A subshape identifies a ***part*** of a shape

A shape has ***indefinitely many*** subshapes

subshapes

A central notion in this definition of shapes is ***pictorial equivalence***

Two shapes are ***pictorially equivalent*** (identical) if and only if the first shape is a subshape of the second, and the second shape is a subshape of the first.

That is, the two shapes coincide point for point in the coordinate system in which they are drawn.

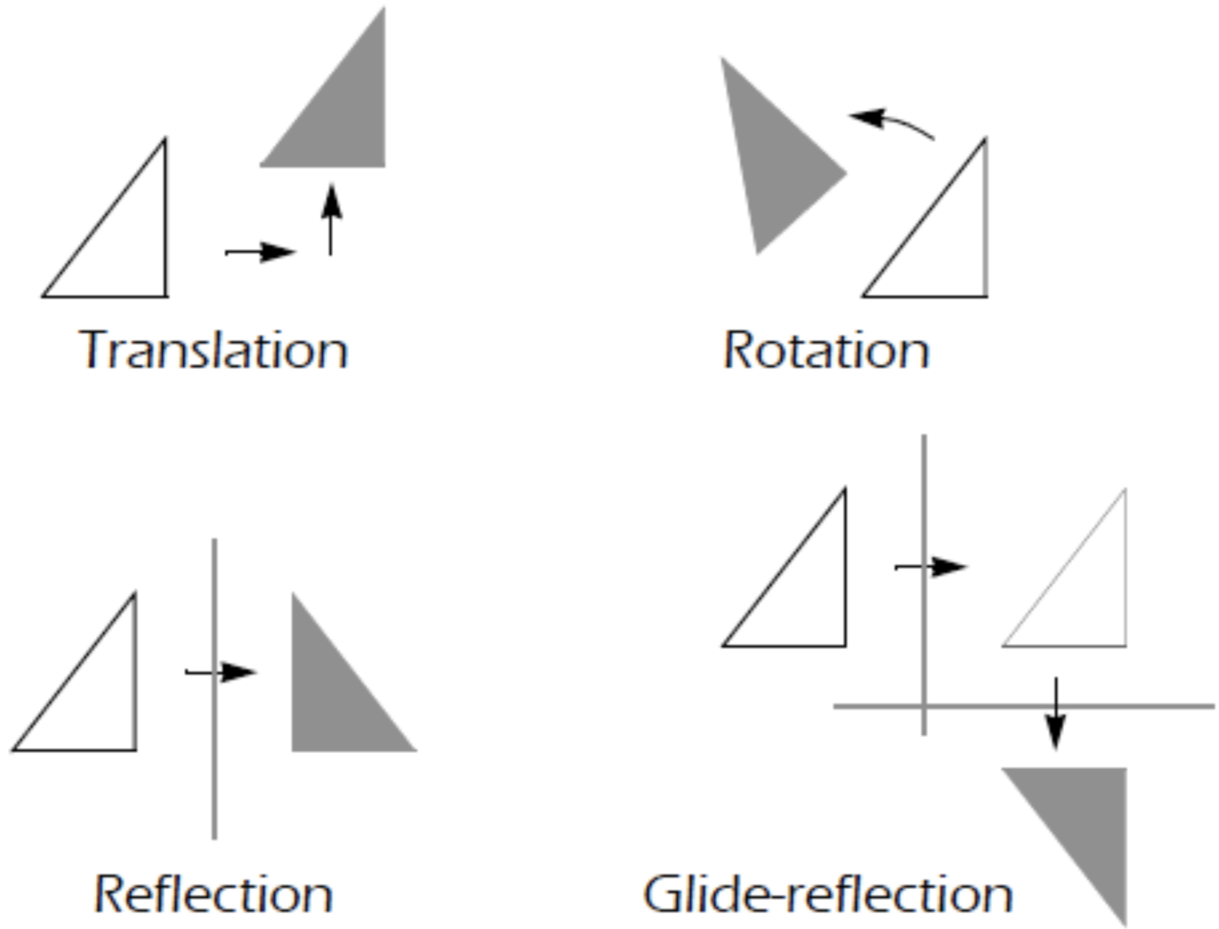
pictorial equivalence

Shapes can be formed by **union –** that is, by ***adding*** shapes – and consists of lines in both shapes.

Shapes can be formed by **difference** – that is, by ***substracting*** shapes – and consists of the lines in the first shape that are not in the second.

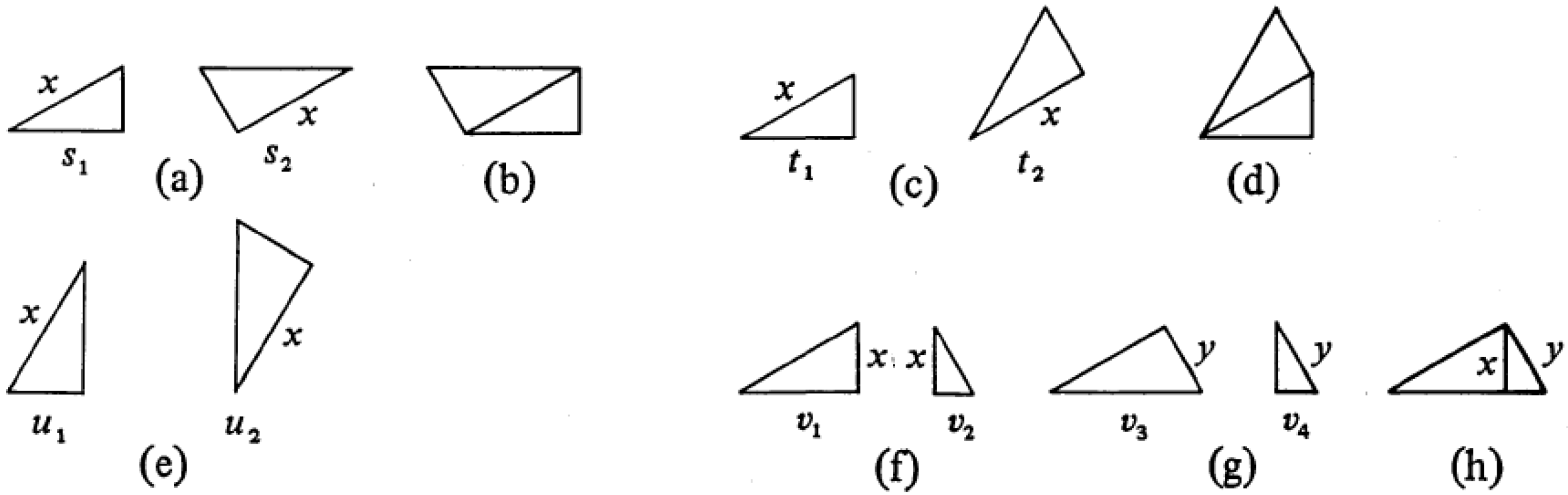
Shapes can be formed by combinations of the two under an ***euclidean transformation***

for our purposes, **shapes** can be formed by…



euclidean transformations + scale

spatial relations



same
spatial relation

different
spatial relations

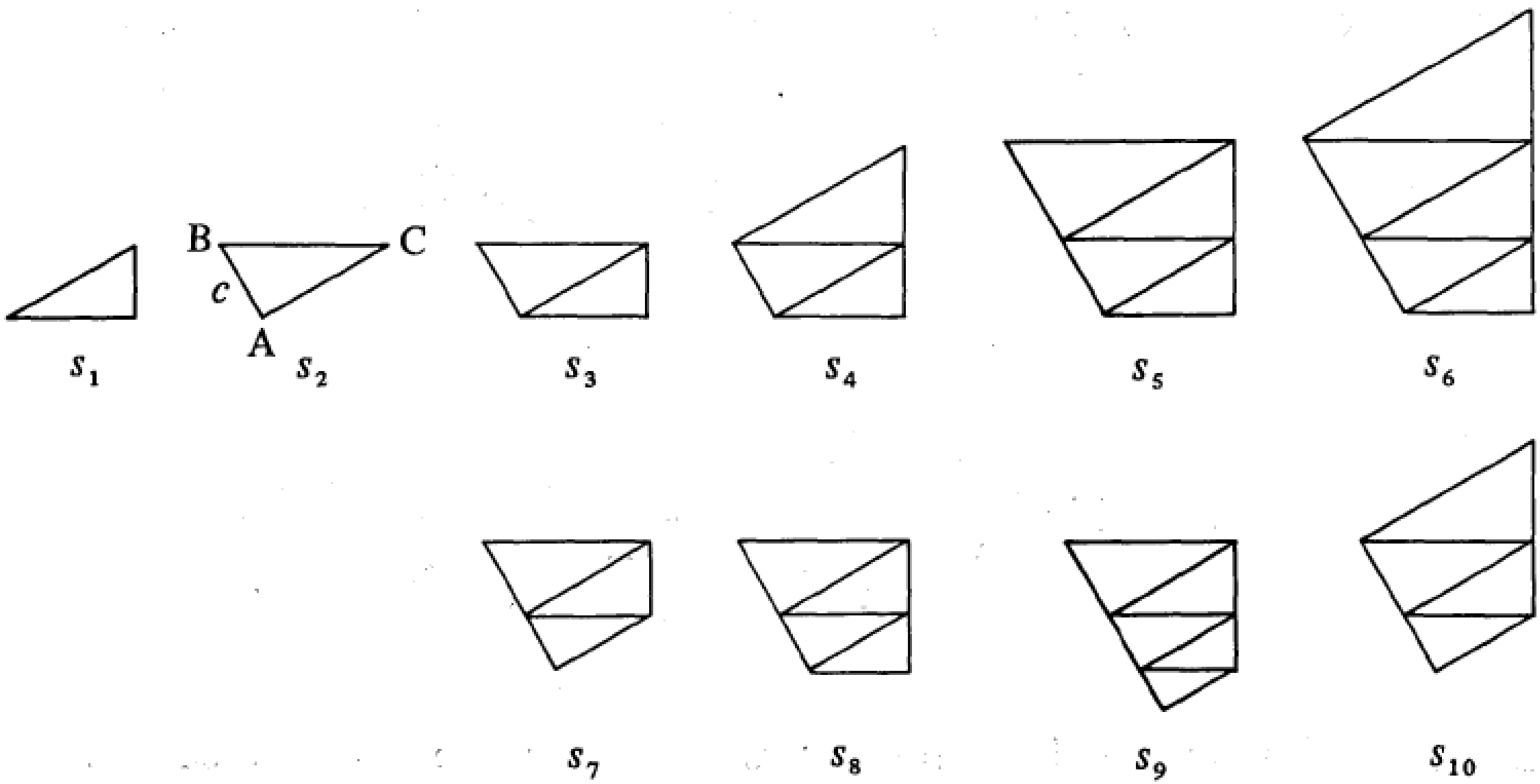
same
shape union

spatial relations

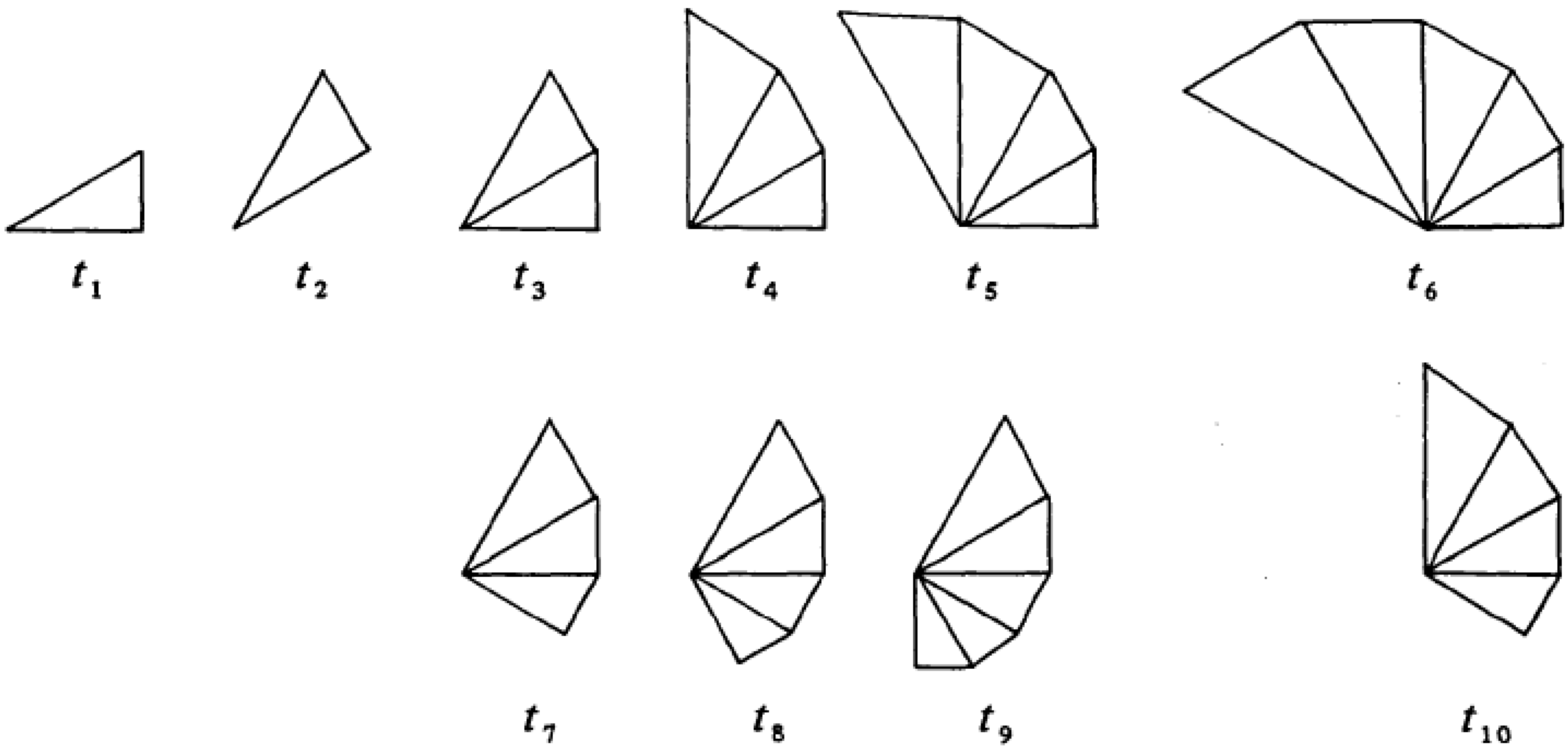
If we are given a set of shapes *S*, then we can create a class *C* of shapes in the following manner:

- A shape in *S* is also in *C*
- If *t* is in C and *s* is in S such that there is an euclidean transformation *F* such that F(s) is a subshape of t then replacing that subshape of t by F(union of shapes in *S*) is a shape in C
- No other shapes are in the class

class of shapes formed by shapes in a spatial relation



shapes in the class defined recursively using the spatial relation



shapes in the class defined recursively using the spatial relation shape grammar

shape grammar $G = (V_T, V_M, I, R)$

shape rules

initial or seed shape

finite set of marker shapes

finite set of terminal shapes

initial (seed) shape

contains a subshape of V_M

R contain shape rules of the form **u** → **v** where **u** and **v** are made up of shapes from $\mathbf{V_T}$ or $\mathbf{V_M}$. **u** must have a shape in $\mathbf{V_M}$.

formally, a shape grammar is …

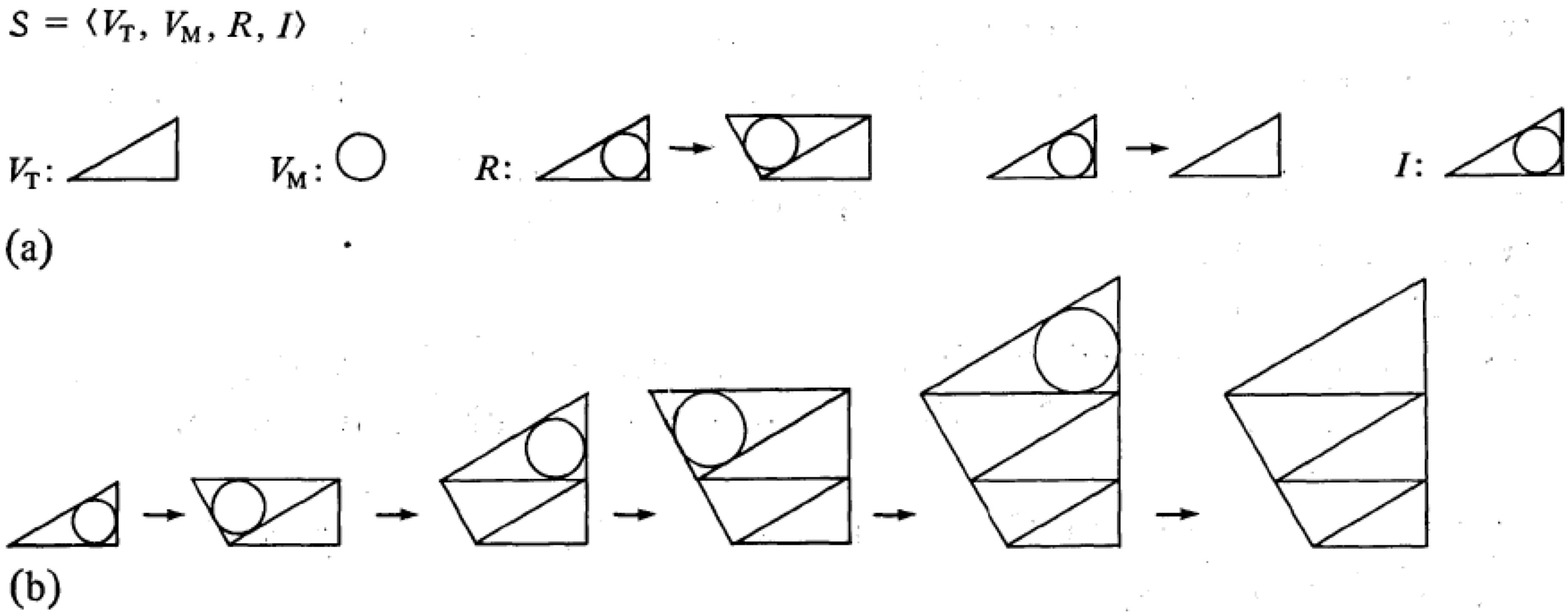
A shape is generated by a shape grammar $\mathbf{S = (V_T, V_M, R, I)}$ by beginning with the initial shape **I** and applying the shape rules in the set **R** until no shape rule can be applied.

A shape rule $\mathbf{u} \rightarrow \mathbf{v}$ applies to a shape **s** if and only if there is a Euclidean transformation *F* such that *F*(**u**) is a subshape of **s**.

The result of applying the shape rule $\mathbf{u} \rightarrow \mathbf{v}$ to the shape **s** under the Euclidean transformation *F* is the shape produced by replacing the occurrence of *F*(**u**) in **s** with *F*(**v**)

The shape generation process terminates when no shape rule in the set R can be applied.

generating shapes



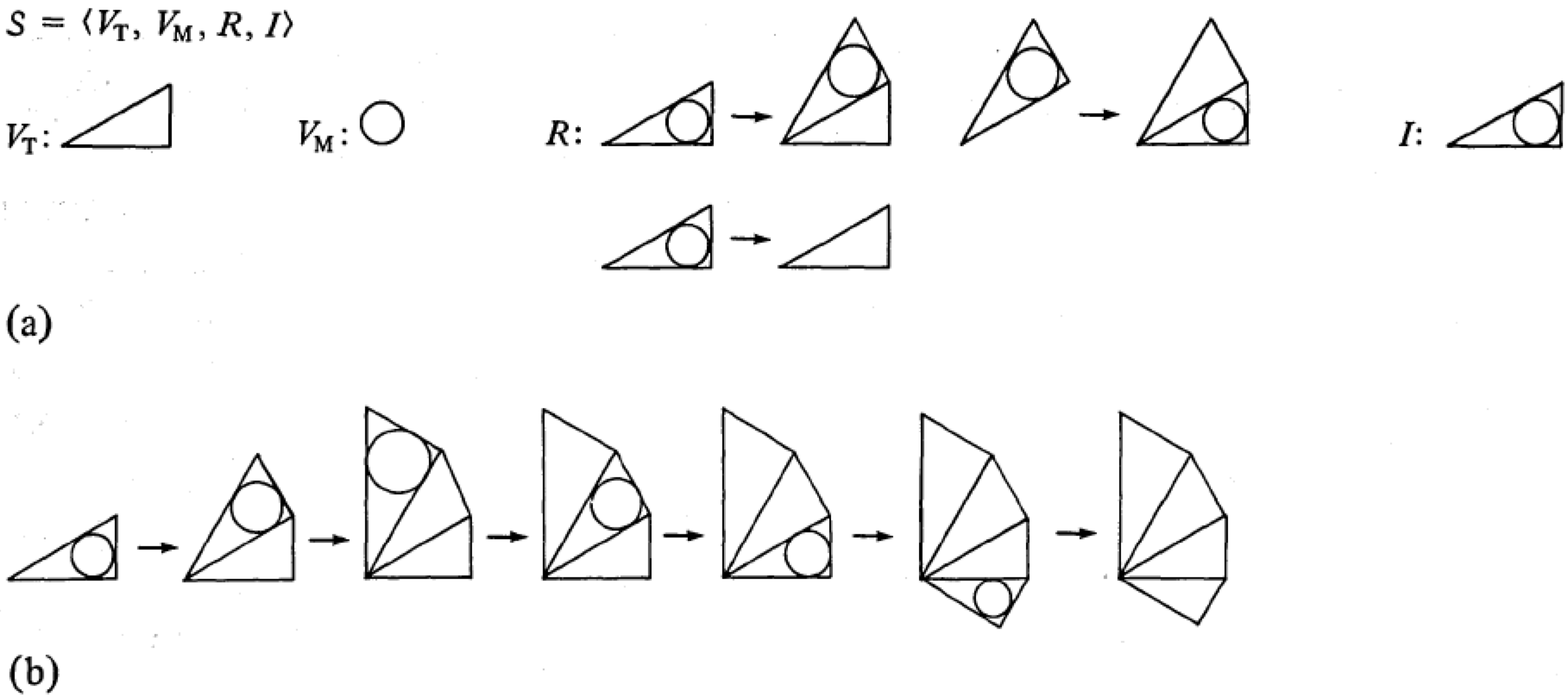
a shape grammar and the generation of shapes

(a) (b)

Adding this rule generates all shapes except $\mathbf{s}_2$ – see slide 13

Replacing this as the initial shape generates all shapes except $\mathbf{s}_1$ – see slide 13

by adding new rules a new shape grammar may be defined



another shape grammar for generating shapes in slide 14 two exercises

"Leonardo wanted to know in a general way what forms he could give to the central-plan church, and set about systematically to find the answer. He realized that if he began with the simplest spatial forms (square, octagon, circle, or dodecagon), he would arrive at every conceivable central-plan church ... by the mechanical addition of circular, semicircular, square, rectangular, or octagonal ancillary spaces to the principal and cross axes of his basic figures. A complete series of related central-plan churches could be developed from a basic schema. For example, he could begin with a Greek cross (four square arms added to the sides of a central square), and then either replace the square space by an octagon, a circle, or a dodecagon, or replace the square arms with rectangles, octagons, circles, semicircles, or dodecagons" (Frankl, 1914, pages 5-6).

exercise 1

A **vocabulary** of primitive shapes

which fixes the spatial elements that are to be used to make other shapes

the distinct **spatial relations** that govern the joint occurrence of these shapes in a legally constructed shape are enumerated – these spatial relations are specified using the shapes in the vocabulary

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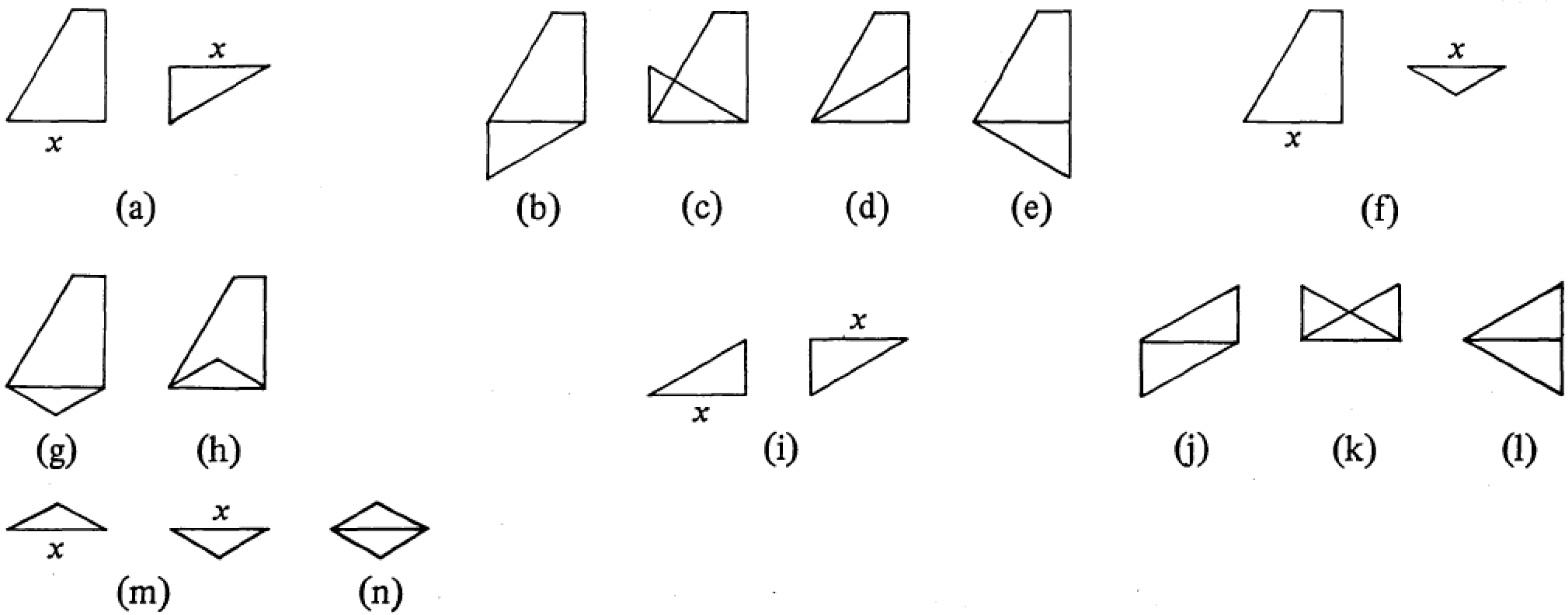
the **vocabulary** and **spatial relations** are used to define **shape grammars,** which generate shapes made up of shapes in the vocabulary in accordance with the spatial relations

developing a shape grammar

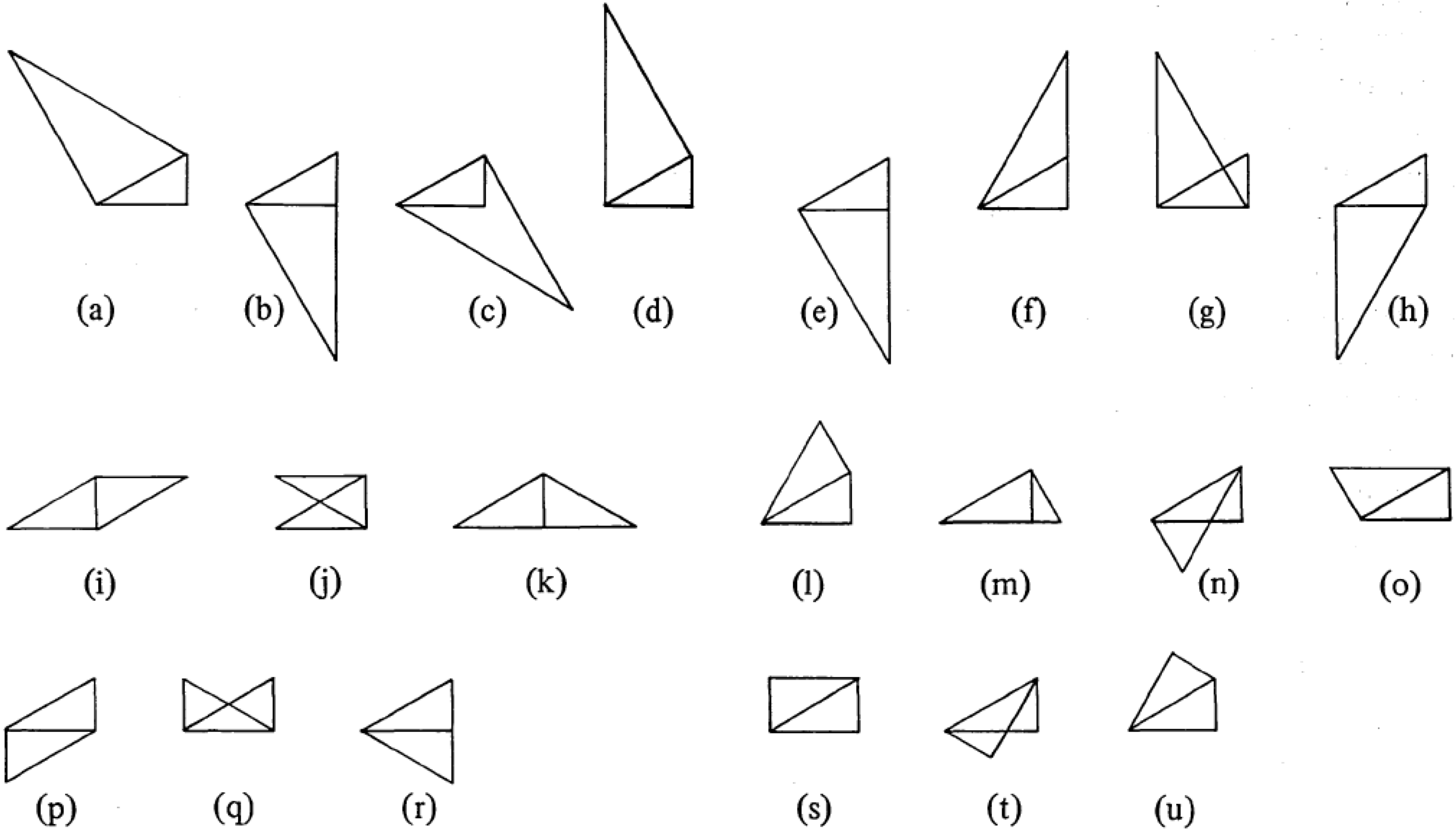
The construction of a shape begins with a polygon in the vocabulary.

A polygon can be added to a shape under construction when the polygon is similar to a polygon in the vocabulary and shares at least one of its edges with a polygon already added to the shape.

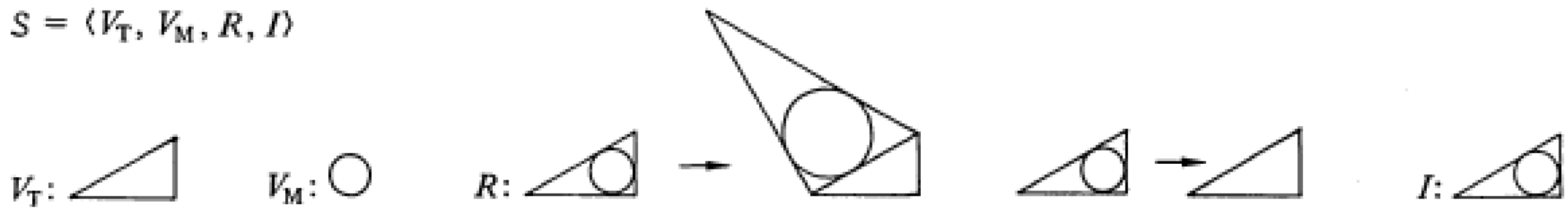
start …



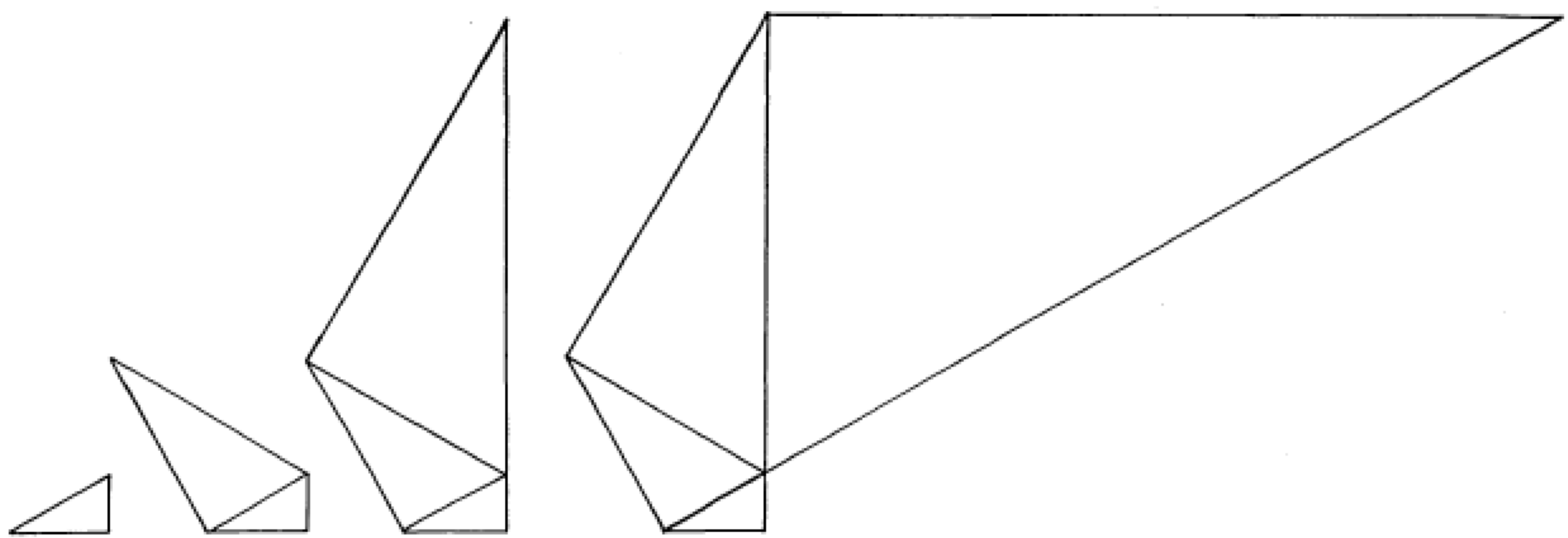
spatial relations specified using polygons that share a fixed edge



The twenty-one distinct spatial relations specified using the similar triangles drawn in figure (i)

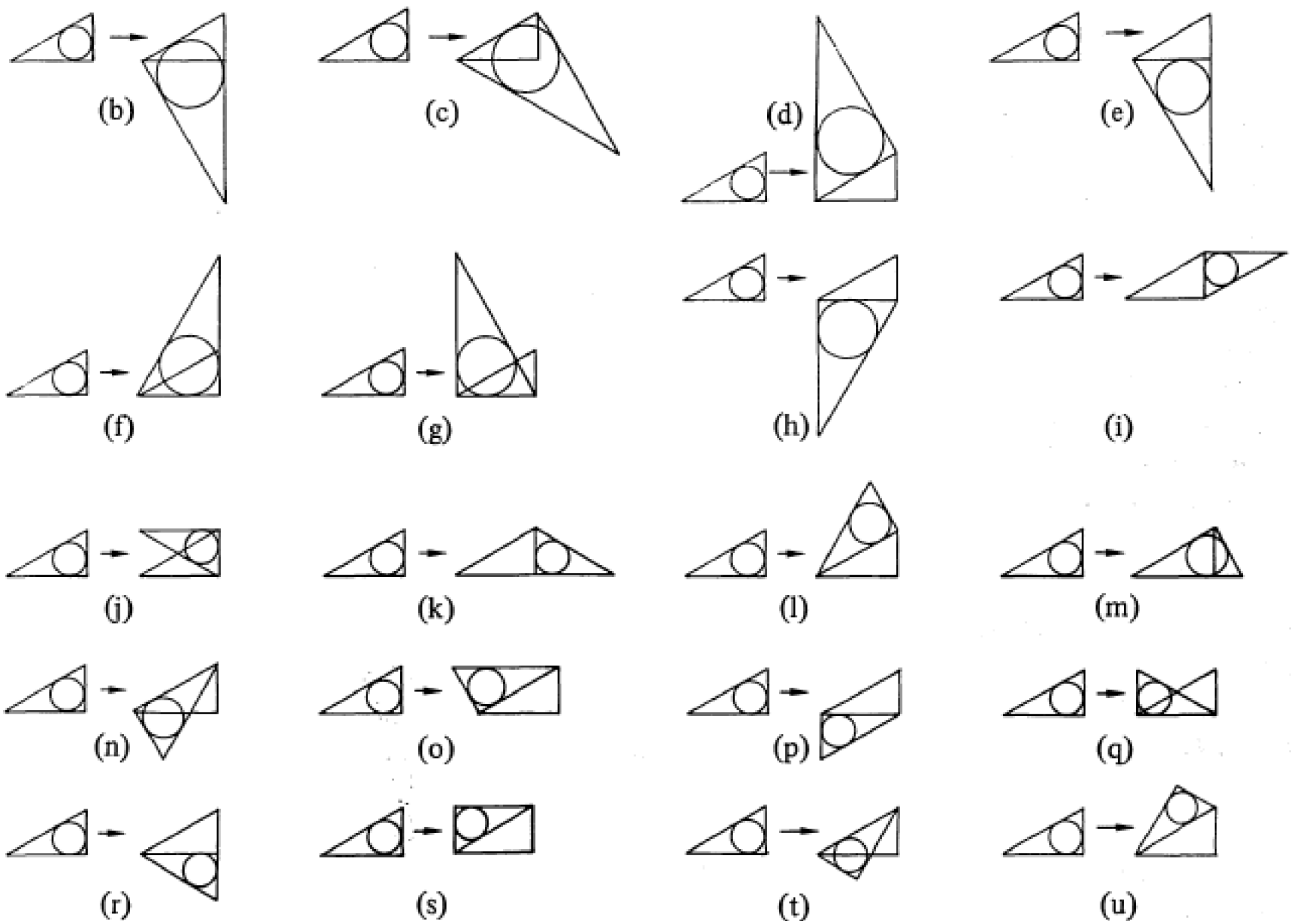


(a)

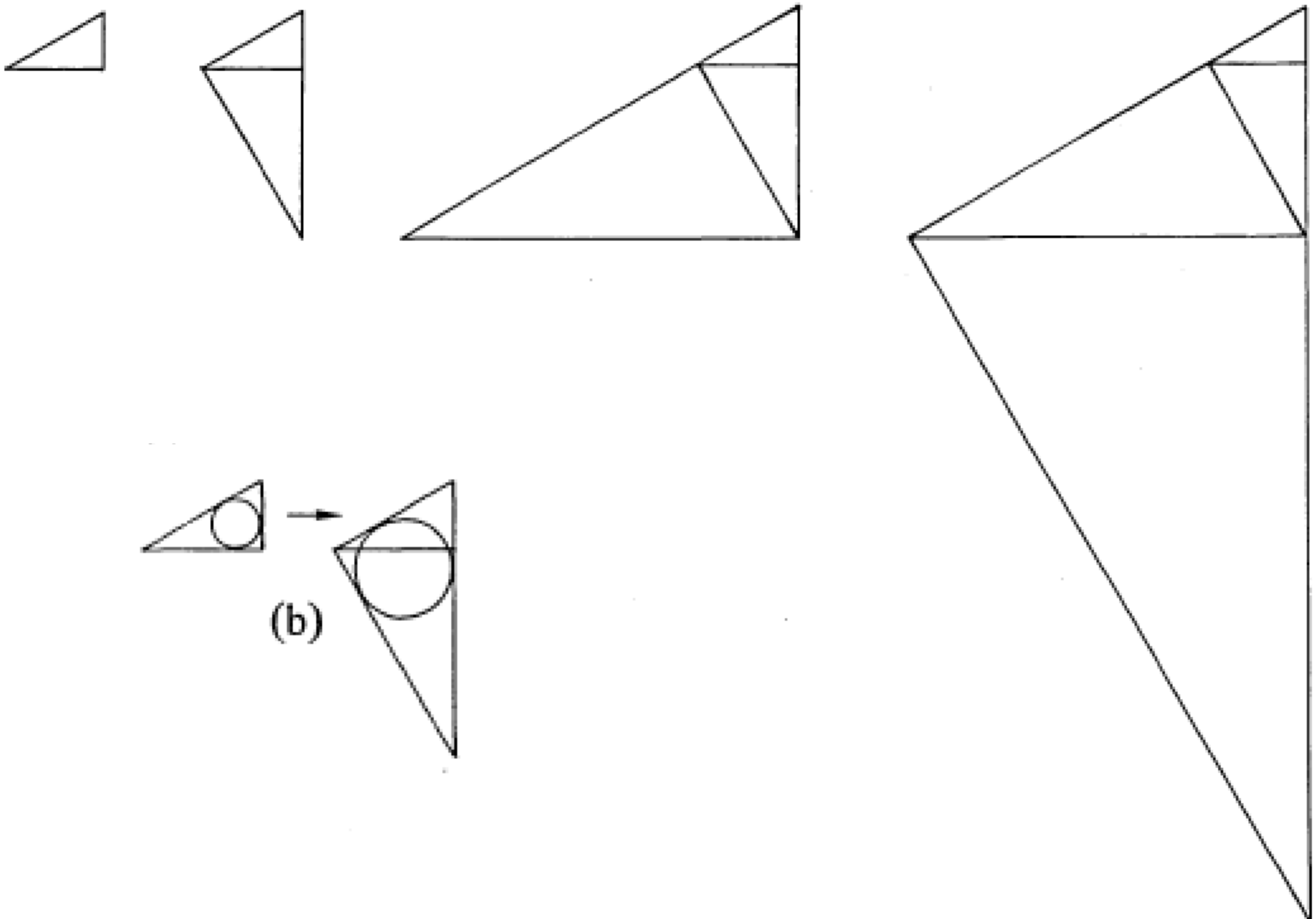


(a)

a shape grammar based on relation (a) and its language

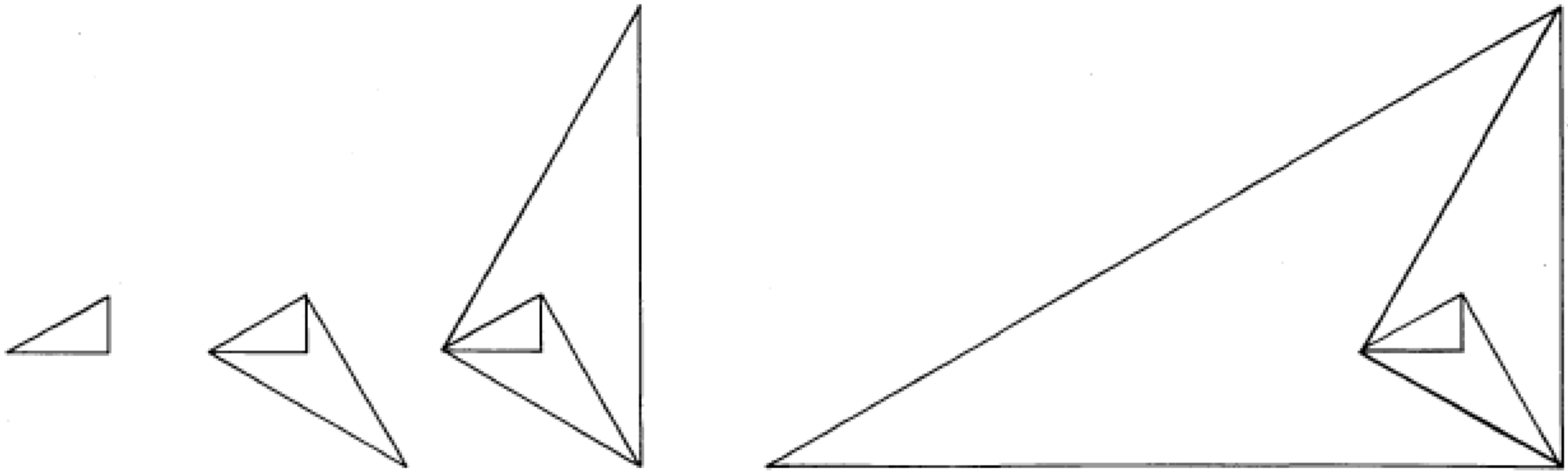


possible replacements for (a)

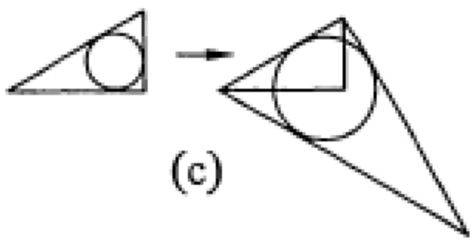


(b)

using spatial relation (b)



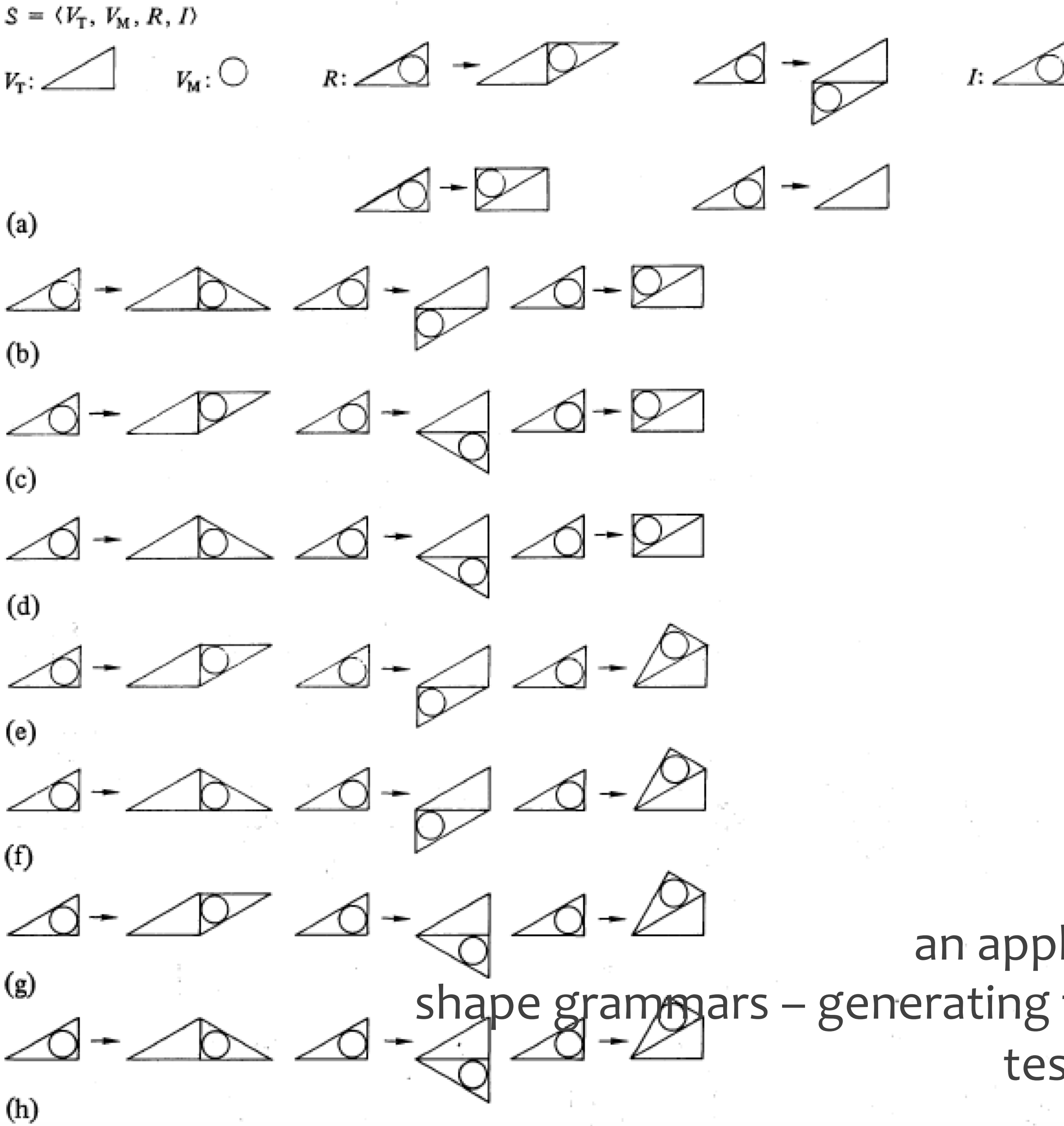
(c)



(c)

using spatial relation (c)

there are more in the paper see pp 200&201



an application of
shape grammars – generating triangular
tessellations

A rule is *applicable* to the *current shape*
which is either the initial shape
or a shape produced from the initial shape
whenever the left hand side of the rule '*occurs*' in the object
in which case
it is *replaced* by the right hand side of the rule under rule application

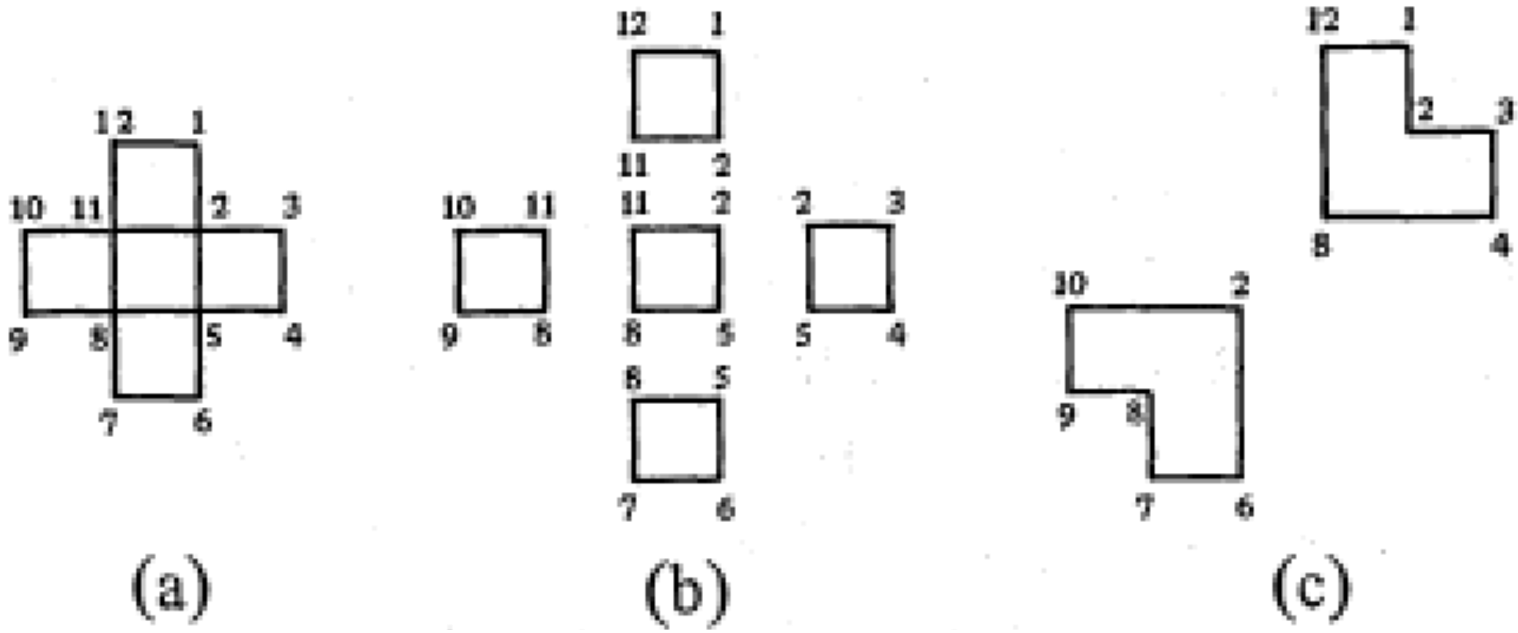
shape rule application

Another exercise consists of beginning with a given arrangement or arrangements of certain spatial elements and constructing or identifying additional arrangements of these elements that are in the same **style**.

First decompose a given form into its **principal** and **ancillary elements** and then recombine the ancillary elements with the principal one in as many new ways as possible.

Completion of the series would depend on what we inferred the principal and ancillary elements of the given form to be and what we inferred legal combinations of these elements to be.

Exercise 2 - The first inkling of 'style'



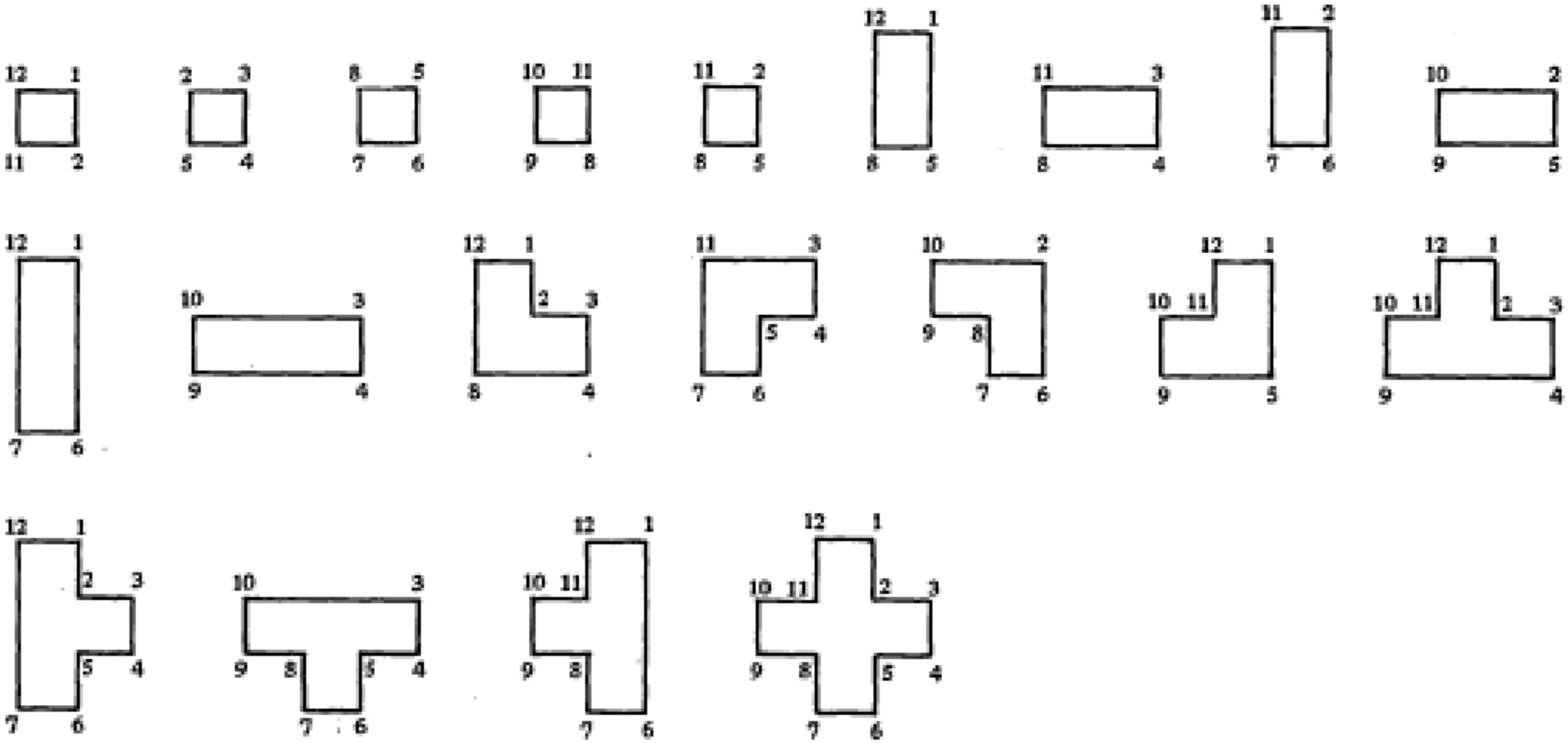
a shape and two of its possible decomposition

(1) Each polygon in a decomposition should be **similar** to as many other polygons in the decomposition as possible.
At the very least the decomposition should contain two polygons that are similar. In addition to this basic heuristic for selecting the most appropriate decompositions of a shape made up of polygons, these further heuristics are sometimes useful:

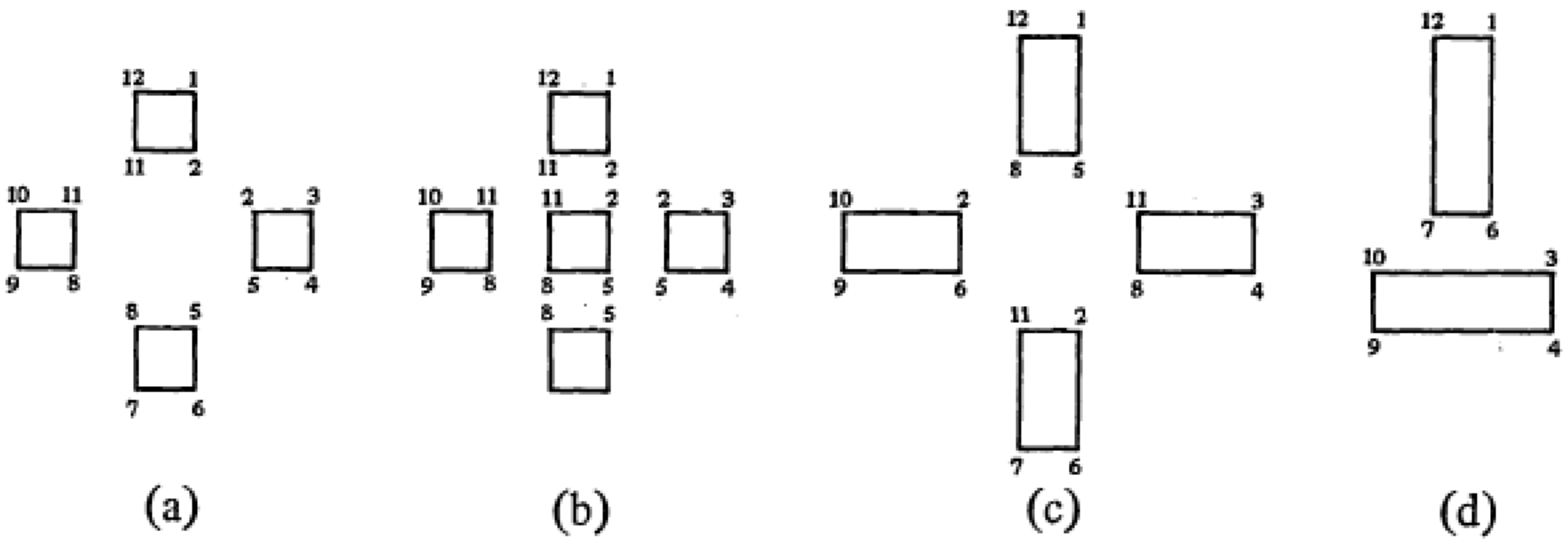
(2) No two polygons in the decomposition should overlap.

(3) The decomposition should contain the fewest number of non-similar polygons with the fewest number of edges.

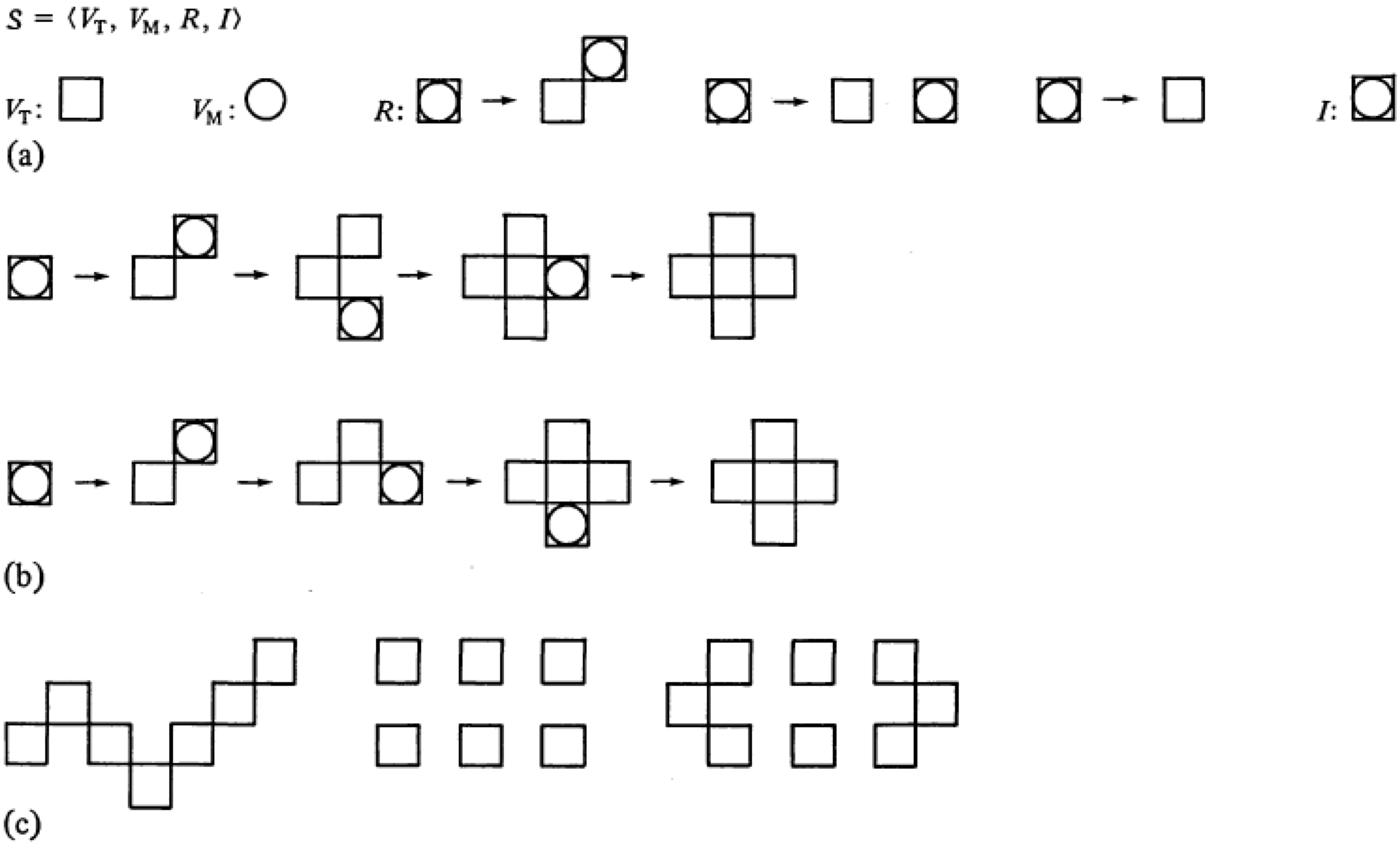
not all decompositions are interesting – use heuristics to choose



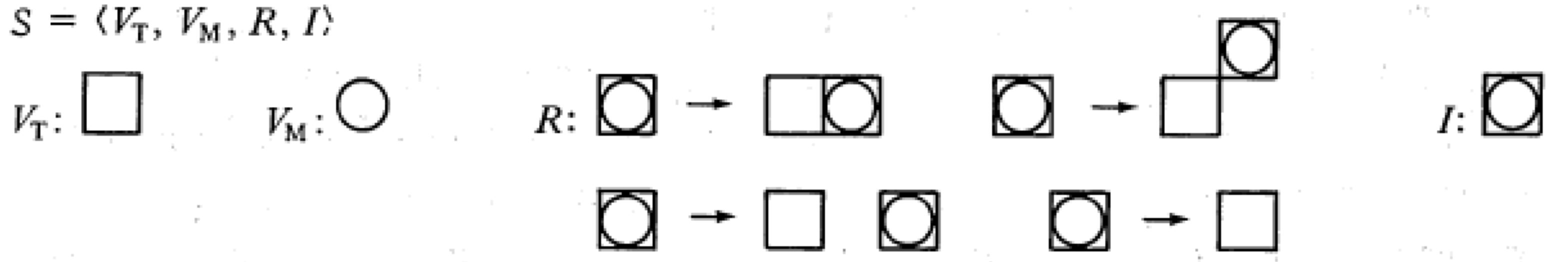
The twenty polygons in the shape drawn in figure (a) see side 35



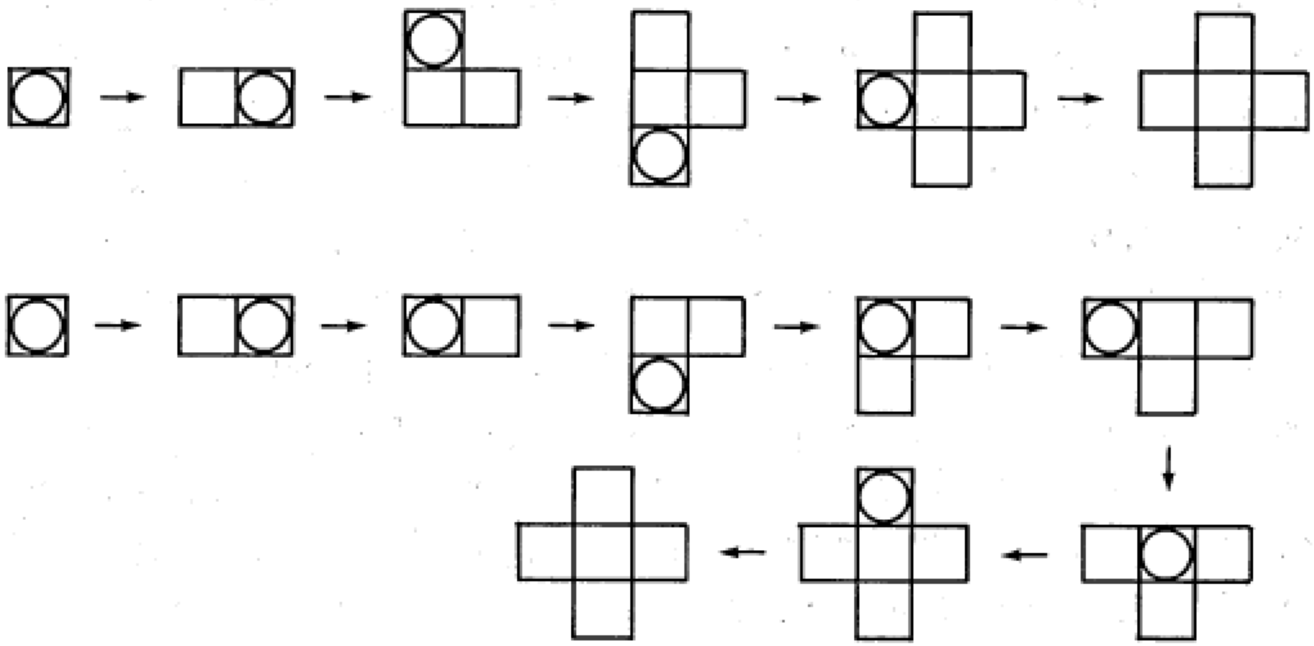
Decomposition of the shape drawn in figure (a) see side 35



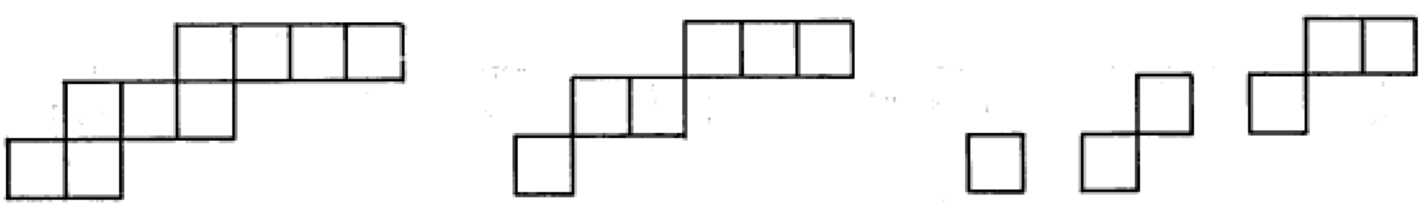
a shape grammar defined in terms of the spatial relations given in figures (a) and (b);



(a)



(b)



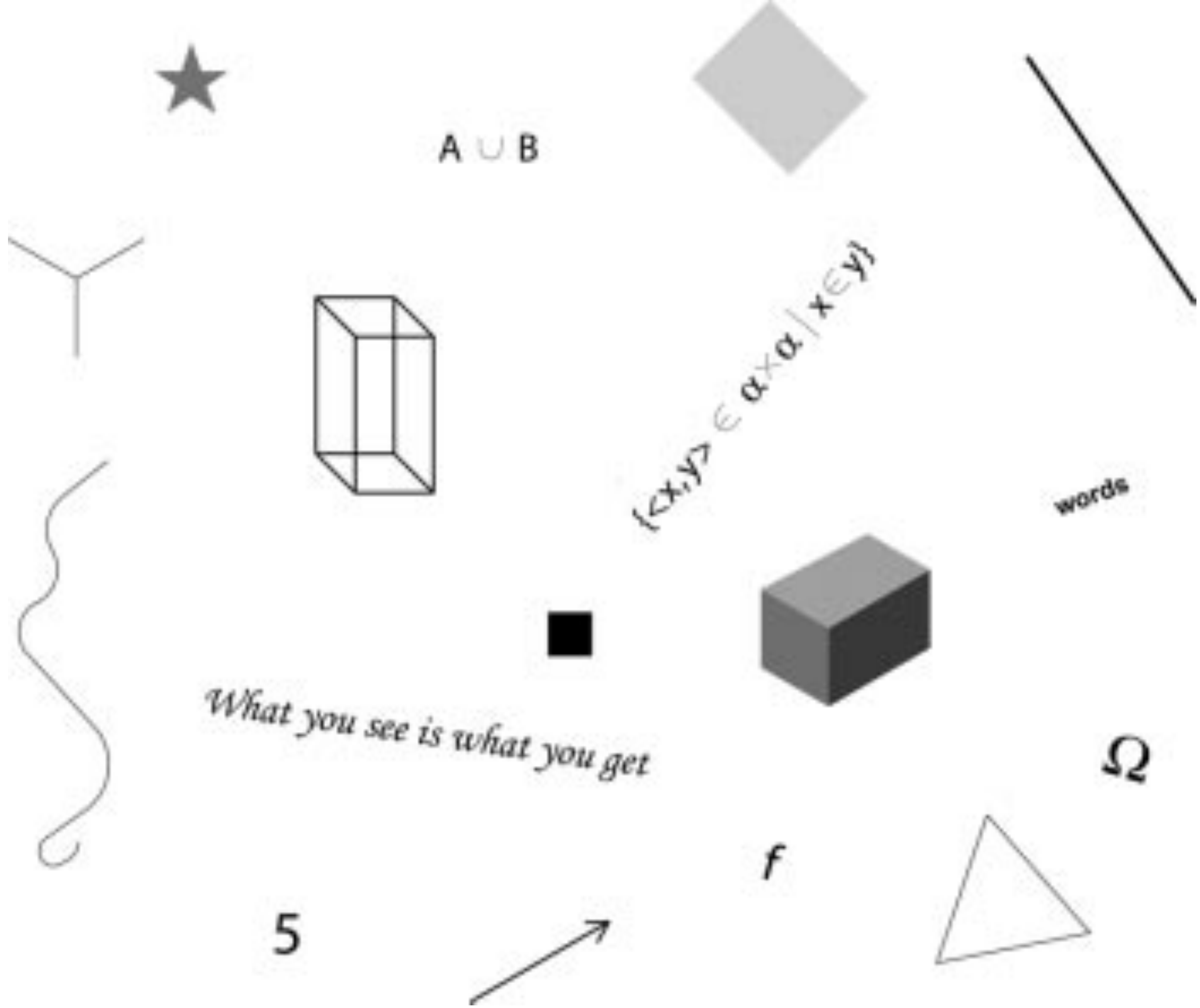
(c)

Shape grammars seem ideally suited as a basic research tool for the development of a science of form.

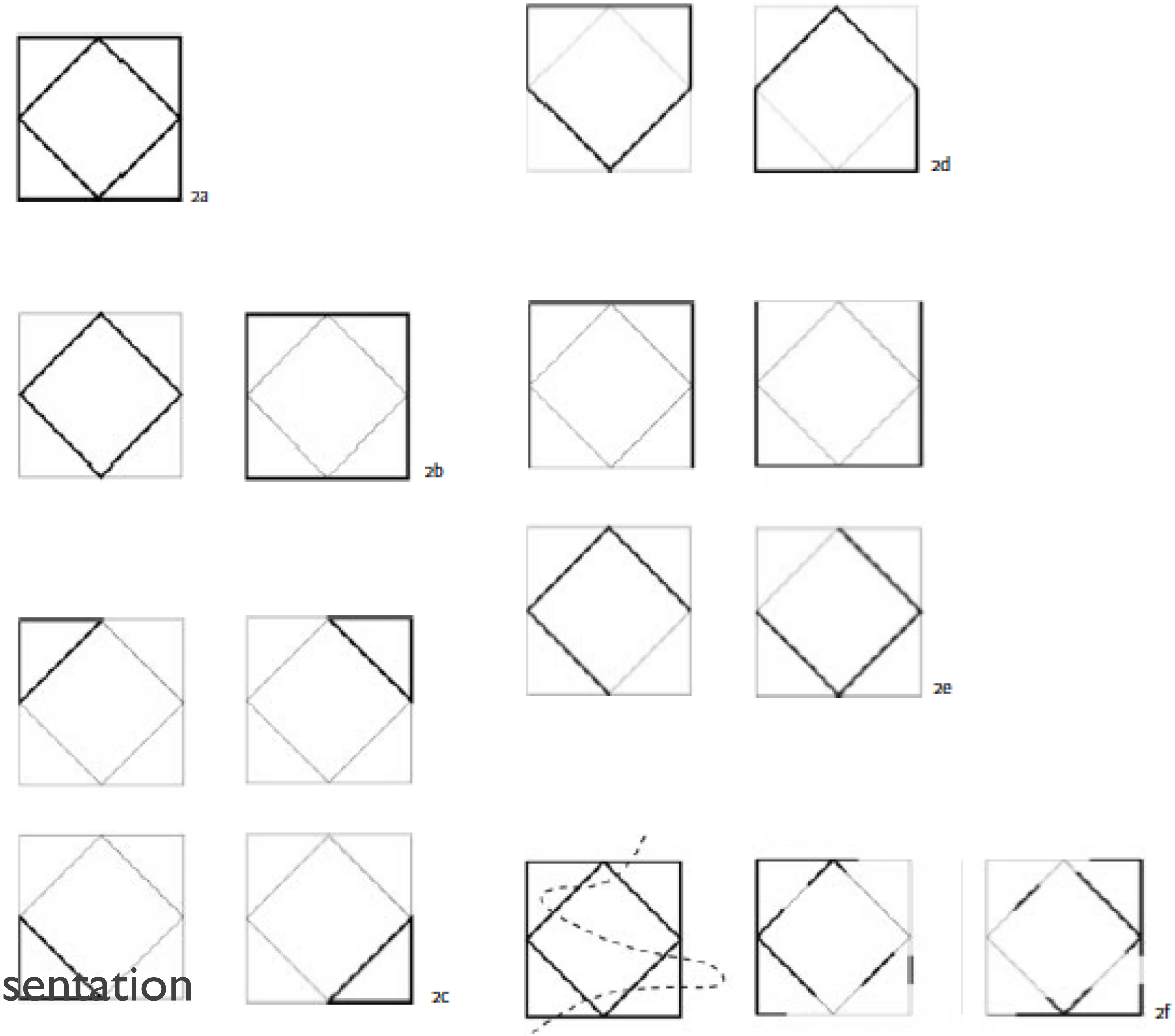
As shape grammars are defined in terms of shapes and not in terms of a fixed collection of primitives out of which all shapes must be tediously specified, they provide for the straightforward treatment of problems in formal composition in terms of the spatial elements that seem most natural to them.

george’s conclusion then were …

terry knight and george stiny
classical and non-classical computation



Assorted classical and non-classical **representations.**
Verbal and numerical ones are familiar – and classical.
Visual ones are non-classical

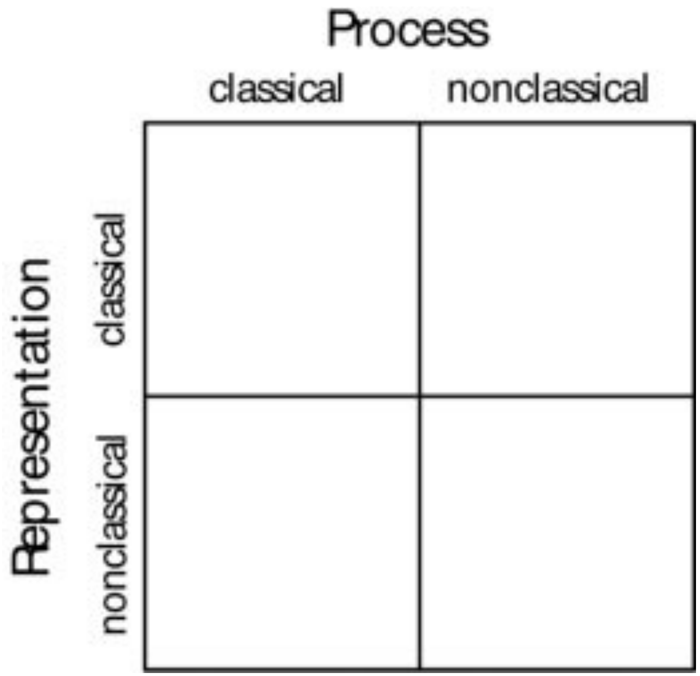


representation

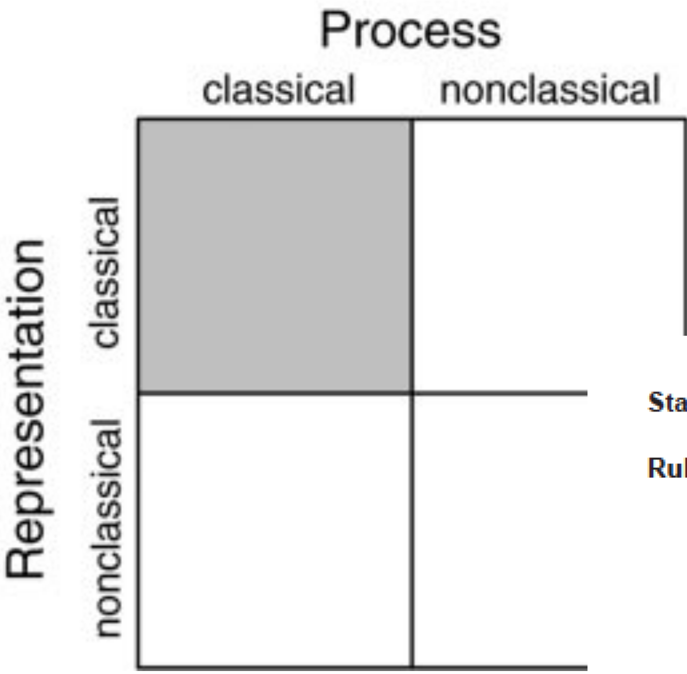
We think about a computational process in terms of **explanation** and **results**. If a computational process explains what is happening, if it provides the rules of the game and these are meant to be understandable, then the process is **classical**.

If what is of most interest is the **result** of a computation, and if there is little concern with understanding how a result is achieved, then the process is a **non-classical** one.

Computation is indifferent to this distinction.



computation – a mix of classical and non-classical representation and process



Start symbol: [SENTENCE]

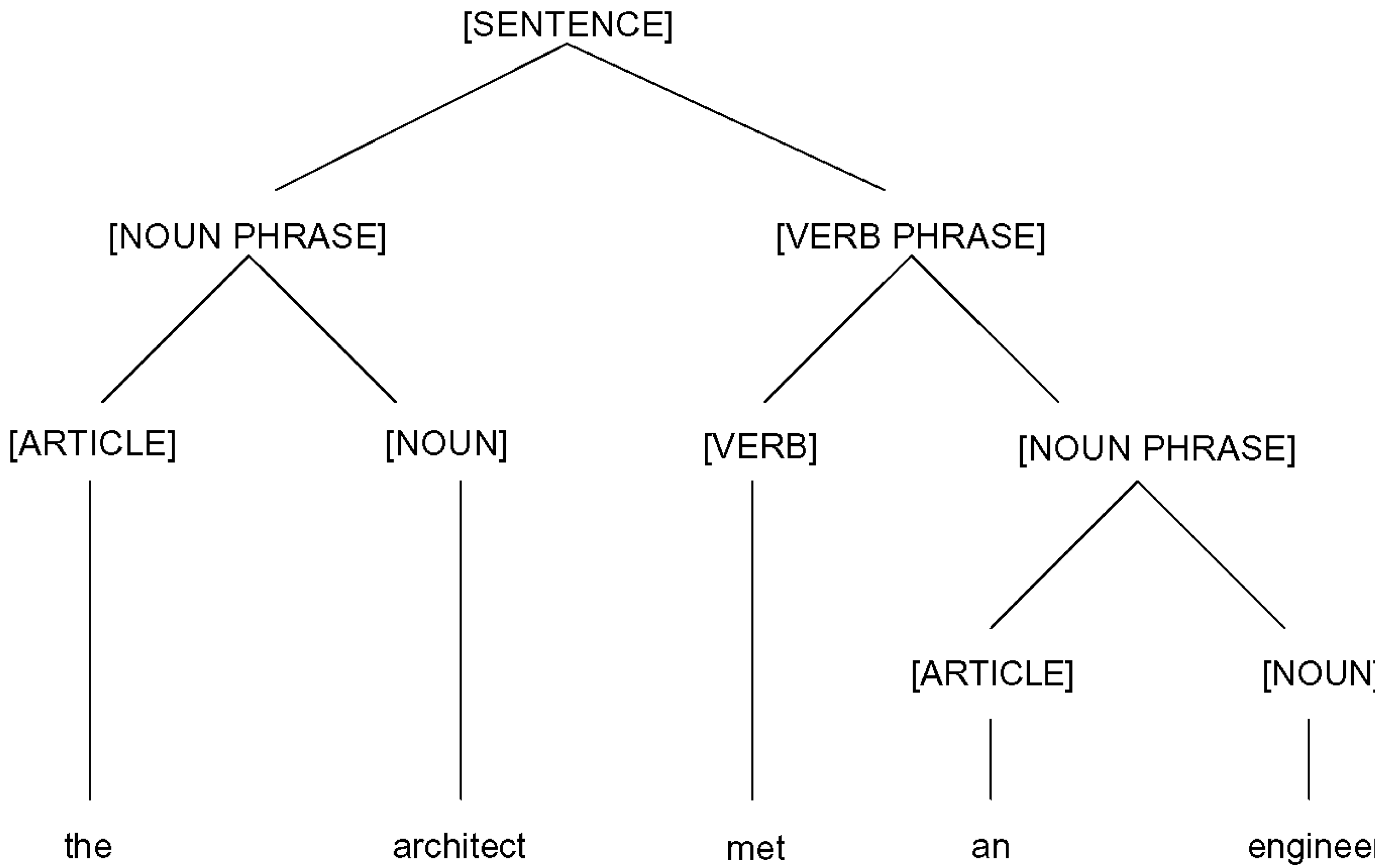
Rules:

[SENTENCE] → [NOUN PHRASE] [VERB PHRASE]
[NOUN PHRASE] → [ARTICLE] [NOUN]
[VERB PHRASE] → [VERB] [NOUN PHRASE]
[ARTICLE] → an
[ARTICLE] → the
[NOUN] → architect
[NOUN] → engineer
[VERB] → met
[VERB] → sued

Computation:

[SENTENCE] ⇒ [NOUN PHRASE] [VERB PHRASE] ⇒ [ARTICLE] [NOUN] [VERB PHRASE] ⇒
[ARTICLE] [NOUN] [VERB] [NOUN PHRASE] ⇒ [ARTICLE] [NOUN] [VERB] [ARTICLE] [NOUN] ⇒
[ARTICLE] architect [VERB] [ARTICLE] [NOUN] ⇒ . . . ⇒ the architect met an engineer

an example – chomsky’s approach to linguistics


[SENTENCE]
[NOUN PHRASE]
[VERB PHRASE]
[ARTICLE]
[NOUN]
[VERB]
[NOUN PHRASE]
[ARTICLE]
[NOUN]
the
architect
met
an
engineer

a parse tree

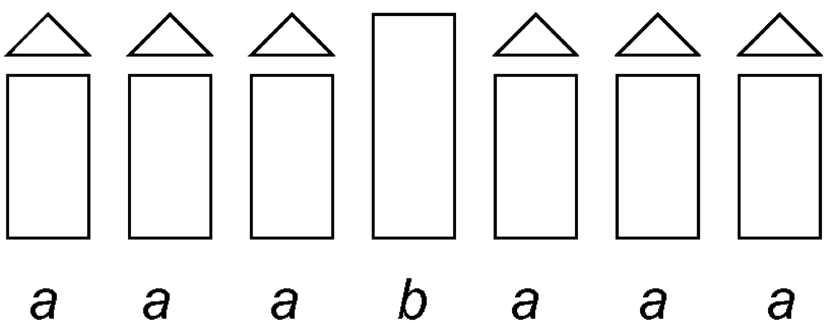
Start symbol: S

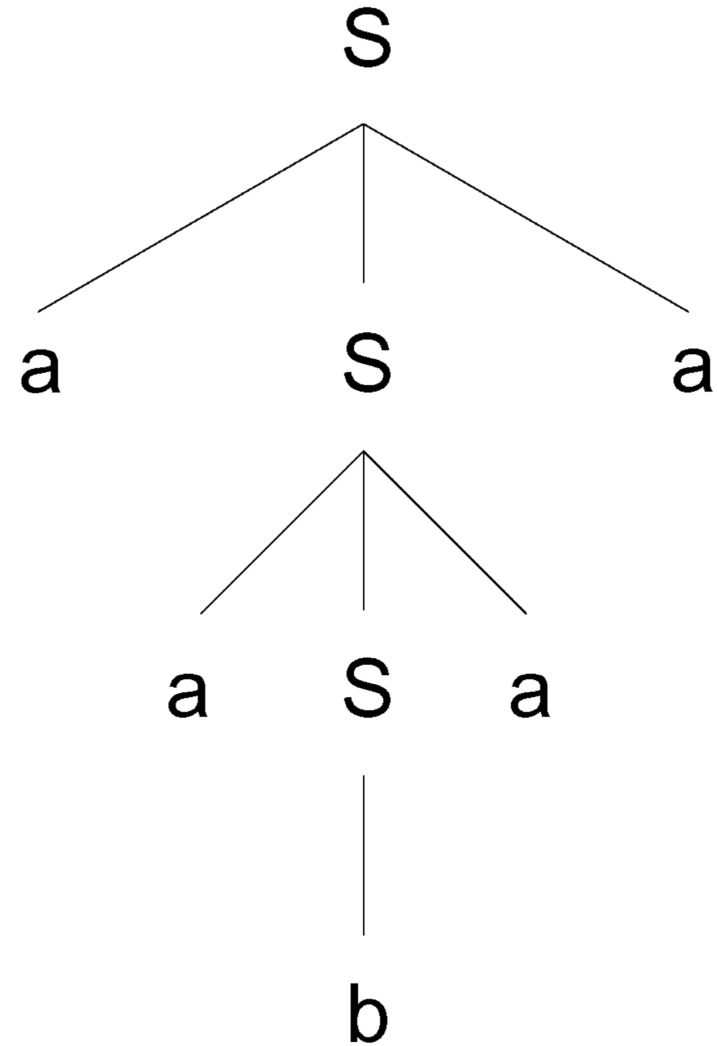
Rules: $S \rightarrow aSa$

$S \rightarrow b$

Language of bilaterally symmetric strings

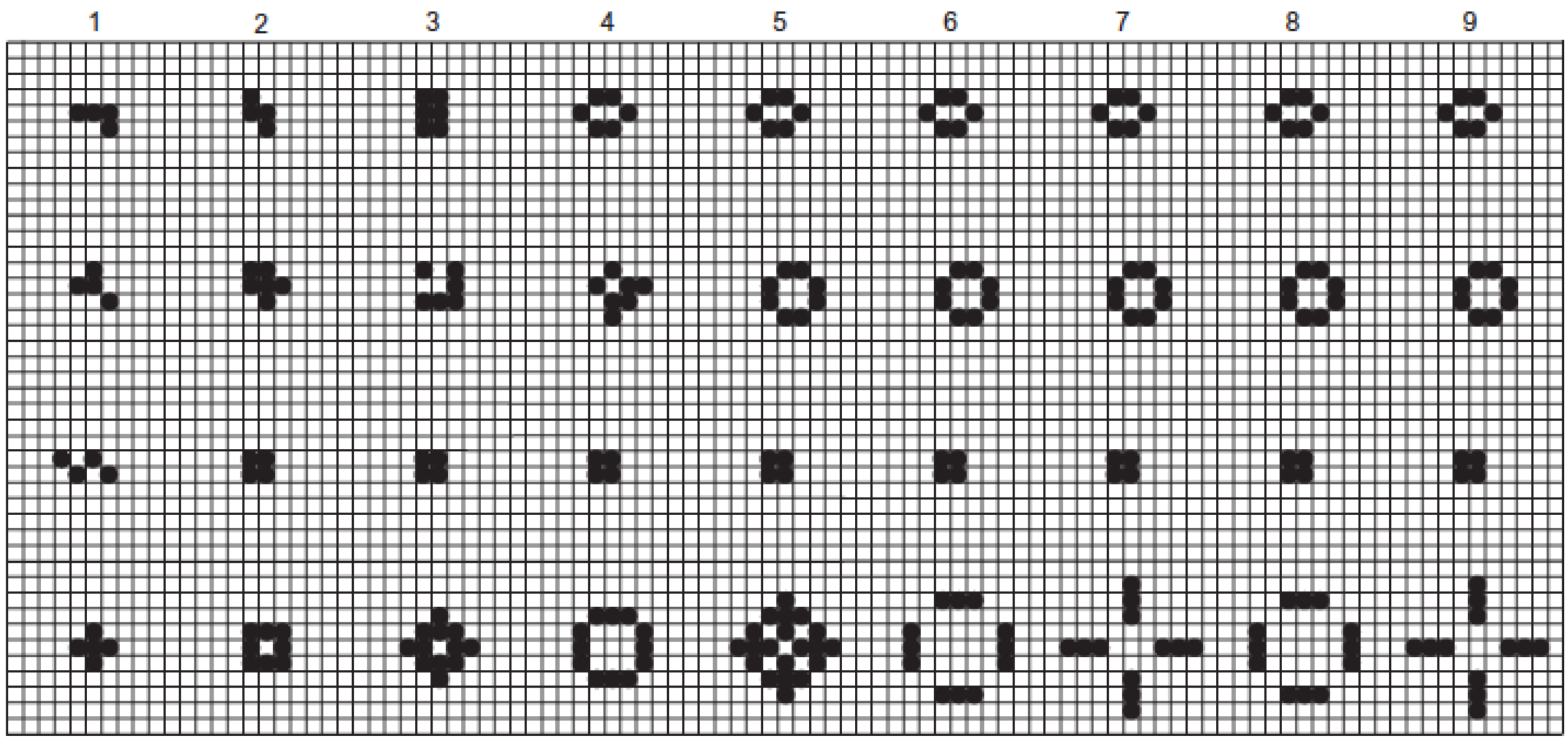
b, aba, aaba, aaabaaa, . . . or $a^n\ b\ a^n$





Game of Life

Survival	If an occupied cell has two or three neighbors, it survives.
Death	If an occupied cell has four or more neighbors, it dies from overcrowding. If an occupied cell has one or no neigbors, it dies from isolation.
Birth	If an unoccupied cell has exactly three neighbors, it becomes occupied.

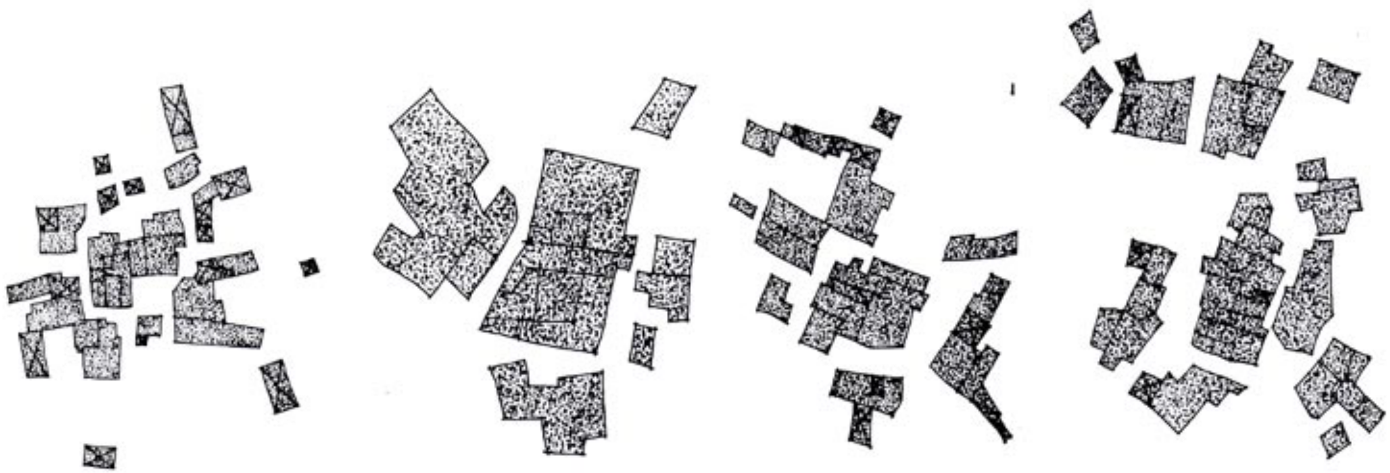


The evolution of four starting configurations

Some characteristics of **beady ring** settlements in southern France:

the open space is not in the form of a single central space with buildings grouped around it, but is rather like beads on a string: there are wider parts, and narrower parts, but all are linked together direct,

the beady ring is everywhere defined by an inner clump of buildings, and a set of outer clumps, the beady ring being defined between the two.



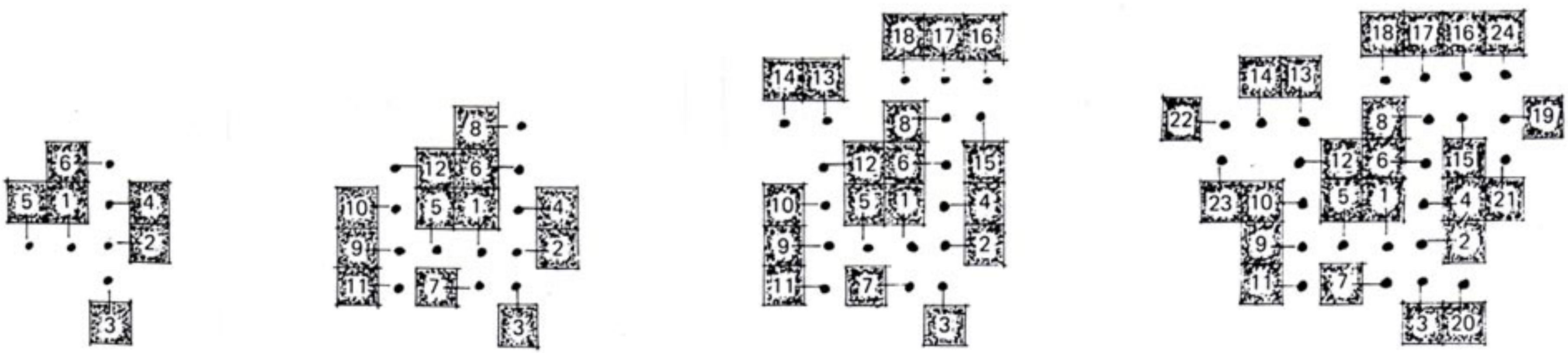
(*a*) Les Yves 1961 (*b*) Les Marchands 1968 (*c*) Les Redons 1968 (*d*) Les Huguets 1961

beady ring settlements – look at page 361

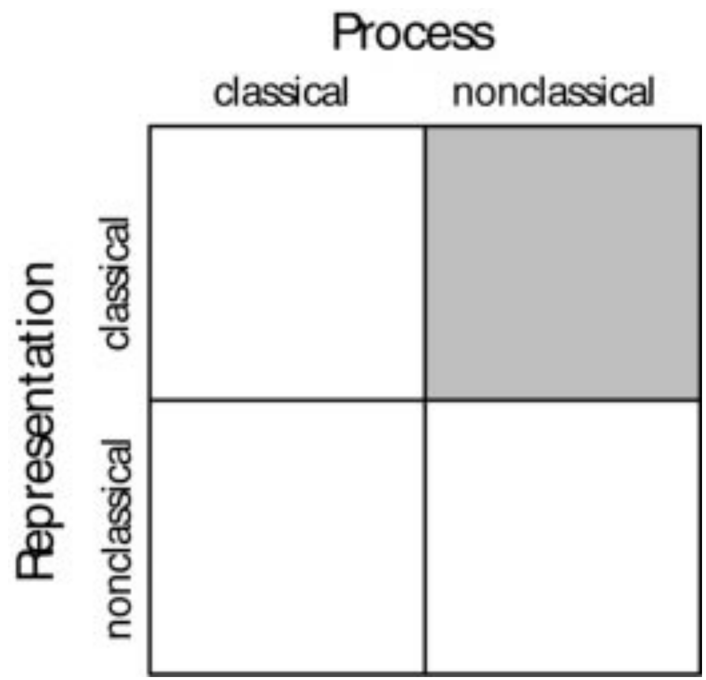
Let there be two kinds of objects, closed cells with an entrance  and open cells •

Join the two together by a full facewise join on the entrance face to form a doublet 

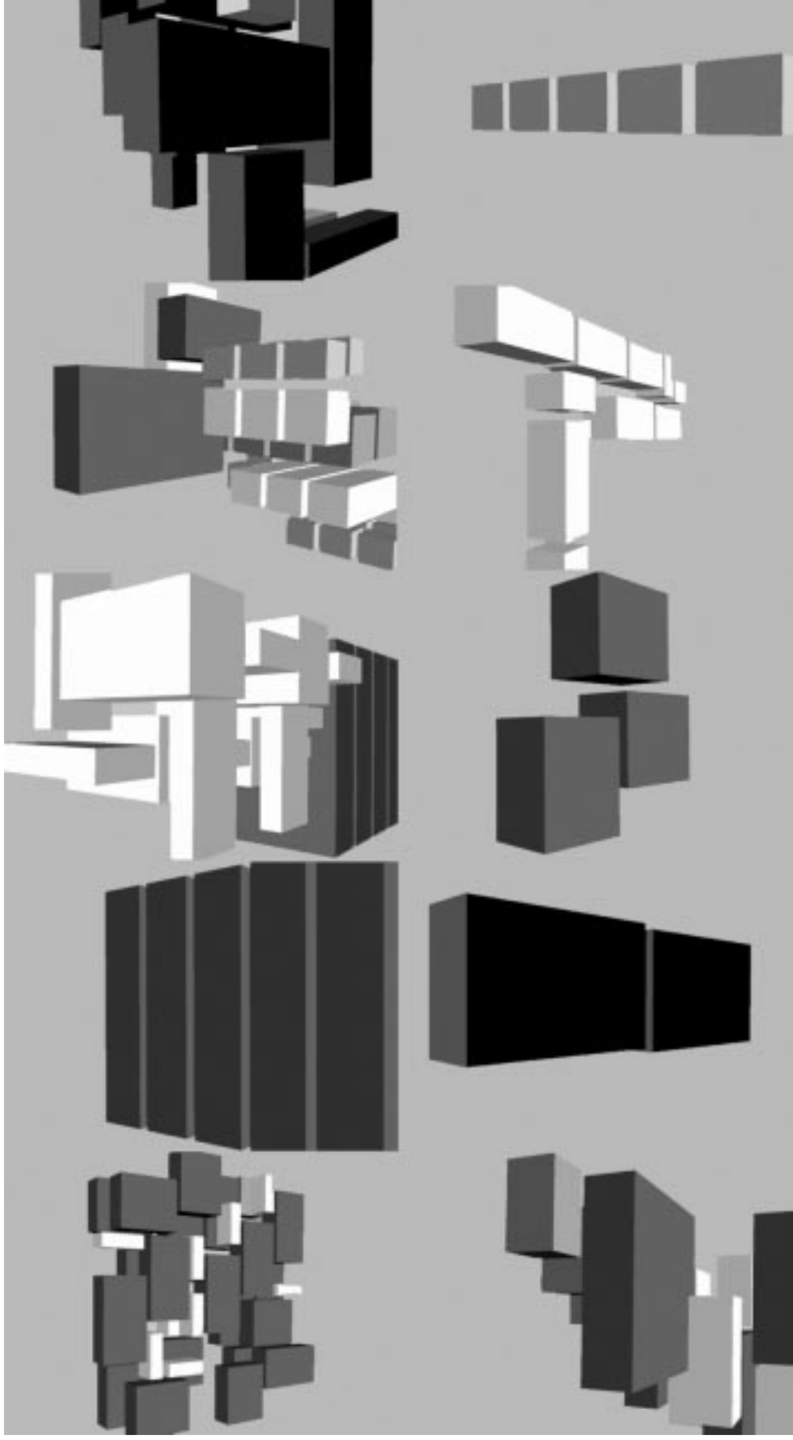
Allow these doublets to aggregate randomly, requiring only that each new object added to the surface joins its open cell full facewise onto at least one other open cell. The location of the closed cell is randomised, one closed cell joining another full facewise, but not vertex to vertex.



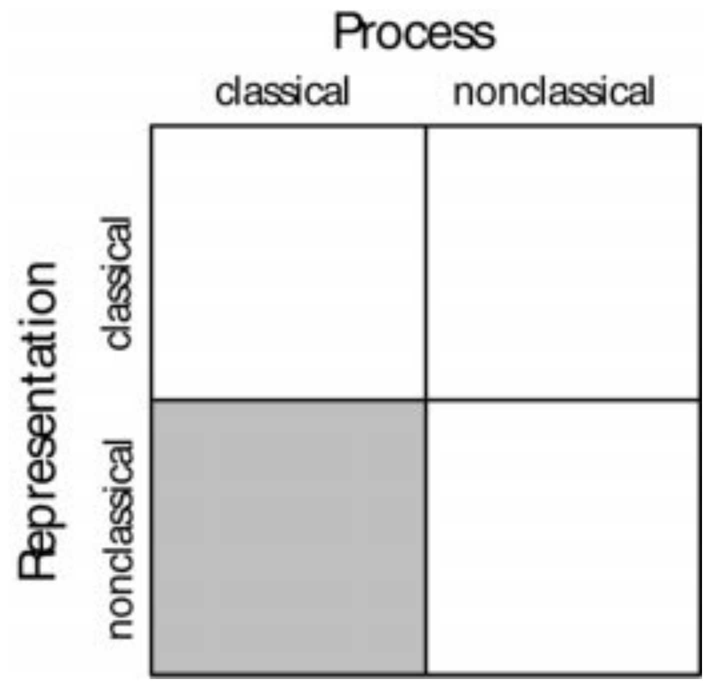
Four stages of a computer-generated beady ring structure.



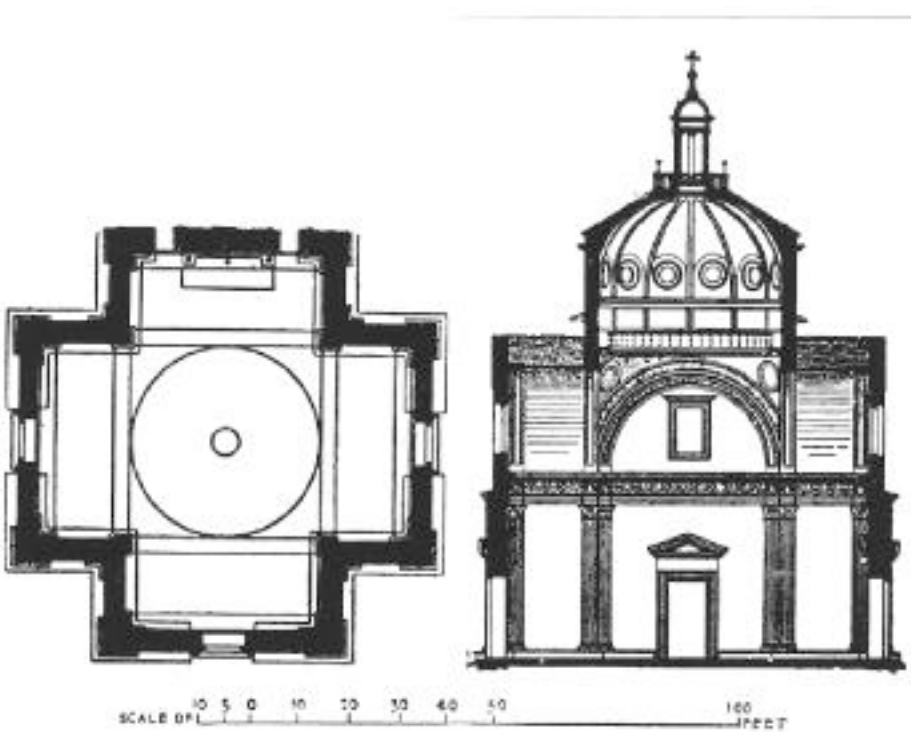
example – evolutionary or genetic algorithm



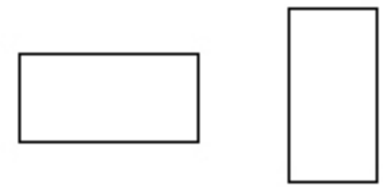
developed from a genetic algorithm
by Testa



shape grammars



Shape



Shape relation

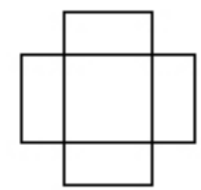
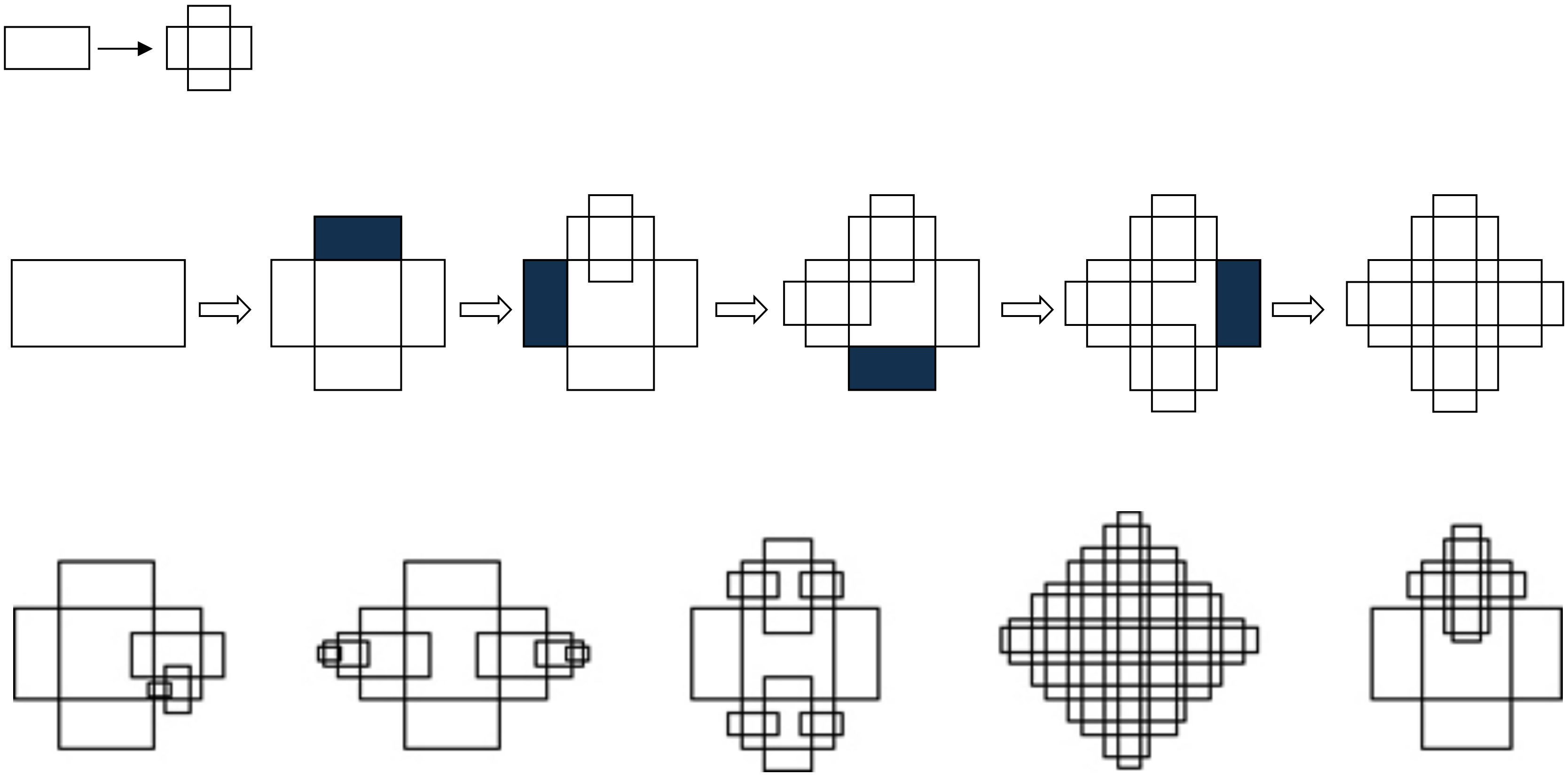


Illustration by Peter Murray, "the Artchitecture of the Italian Renaissance", Shocken Books Inc. 1963, Pp.96



derivation

Analytical

Original

two kinds of grammars

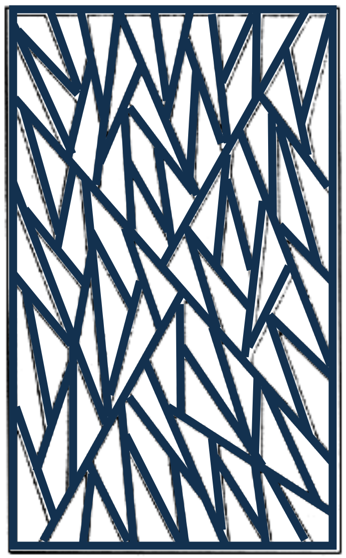
analyses

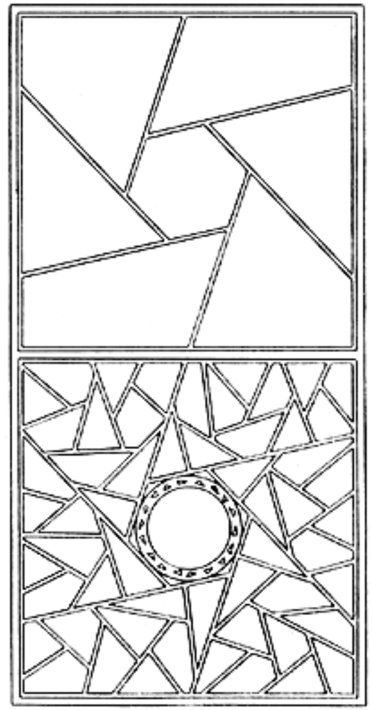
original designs

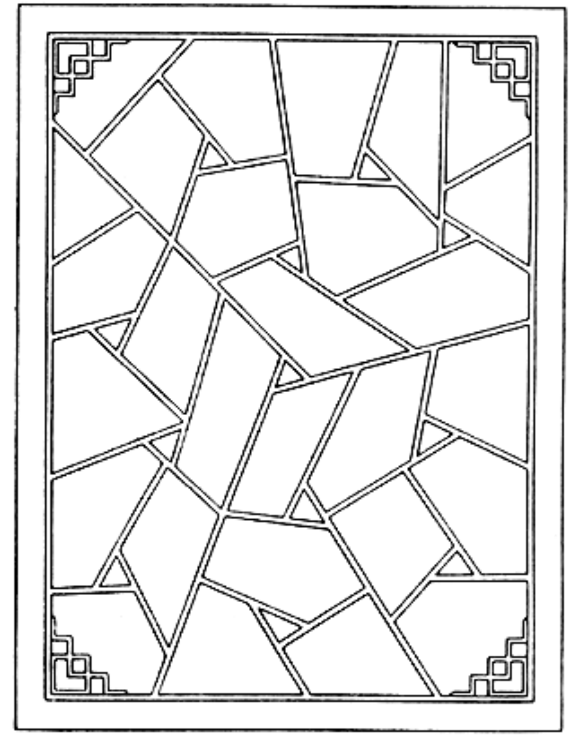
as explanatory devices

how are they used?

[demo]

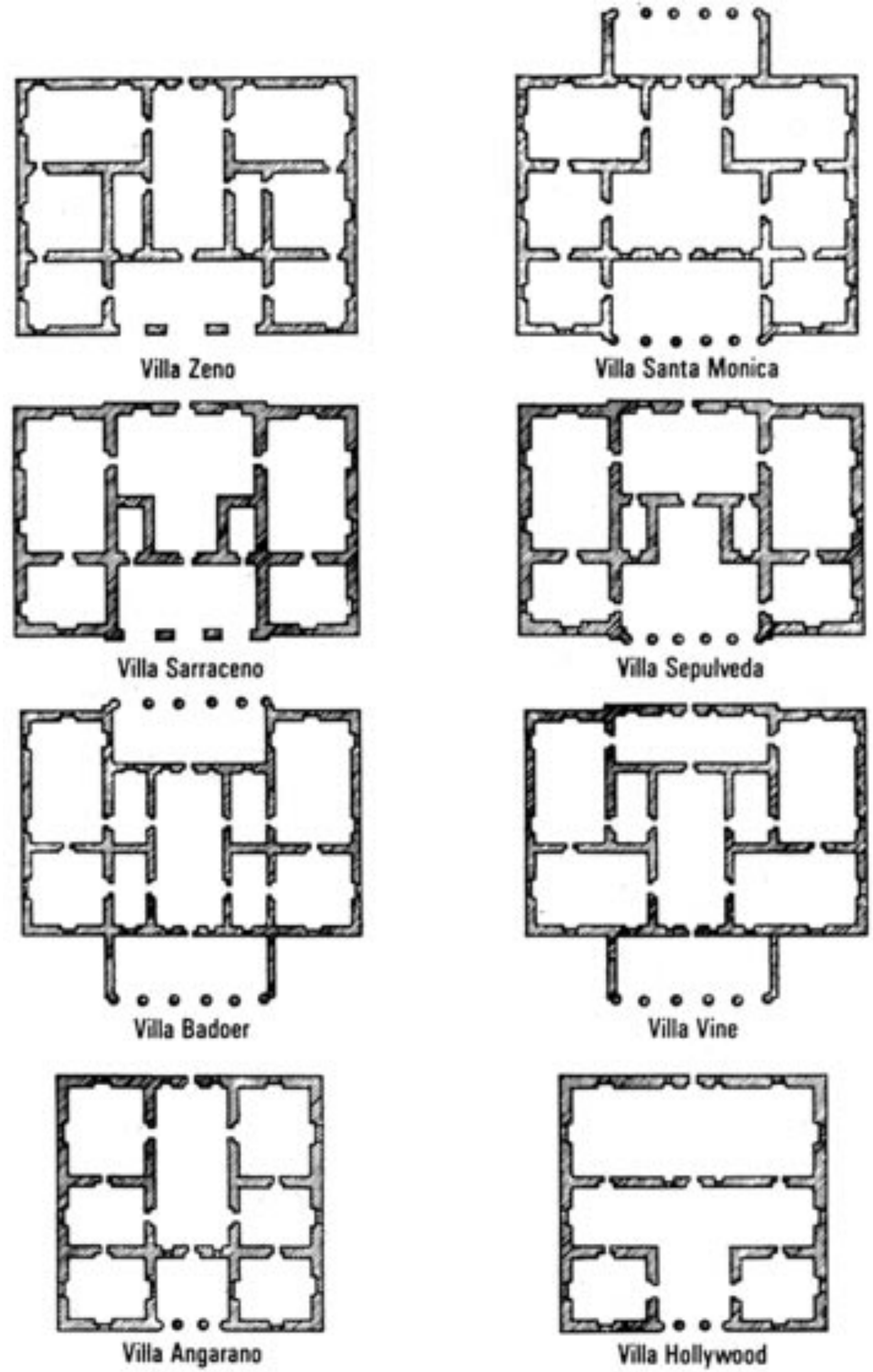




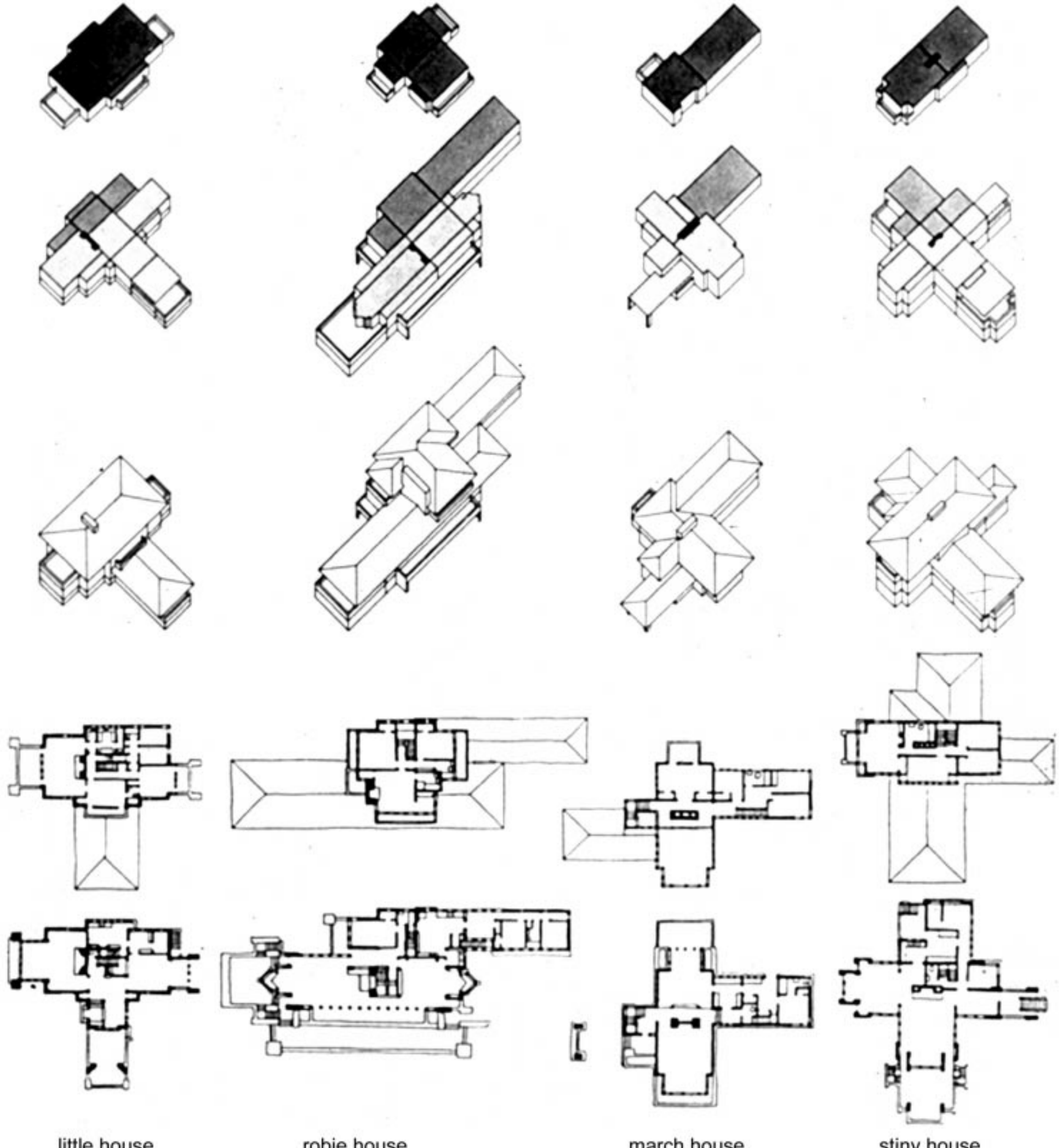


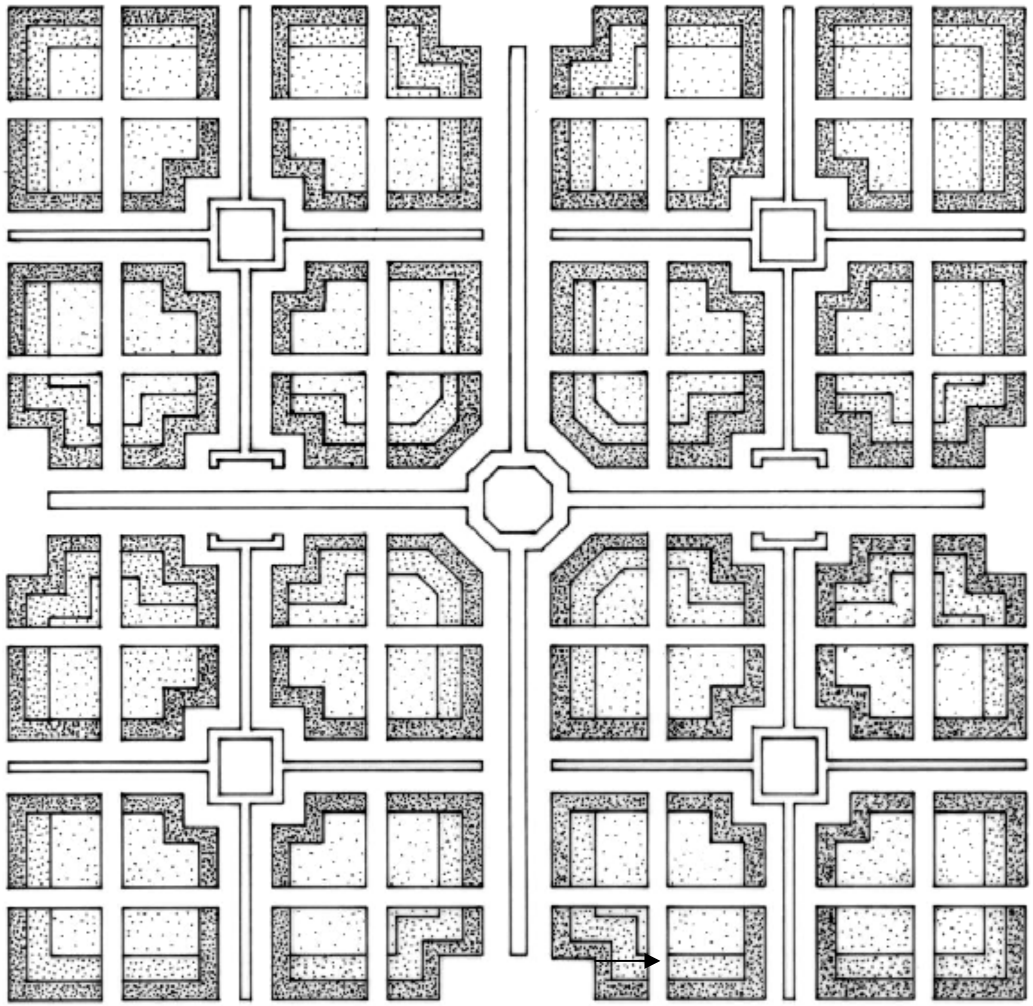
analytical – ice-ray designs

analytical – palladian villas

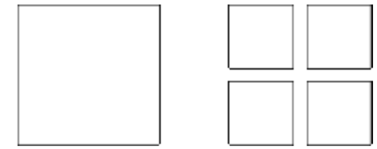


analytical – FLW houses

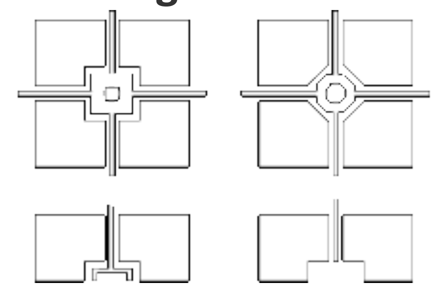




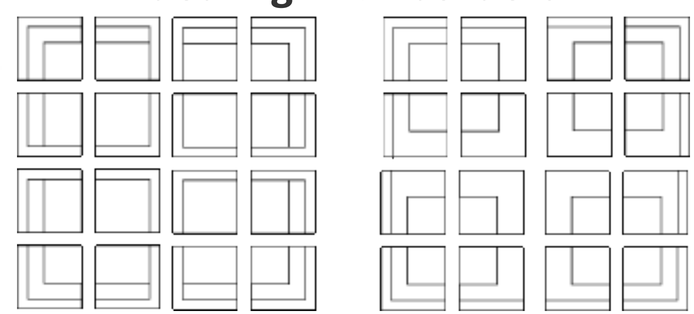
parti subdivision



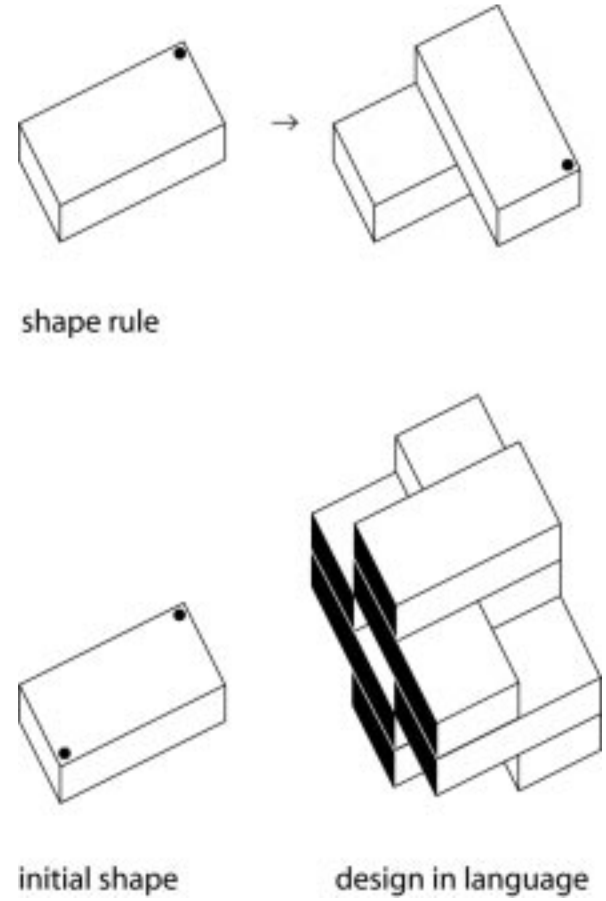
dealing with canals



dealing with borders

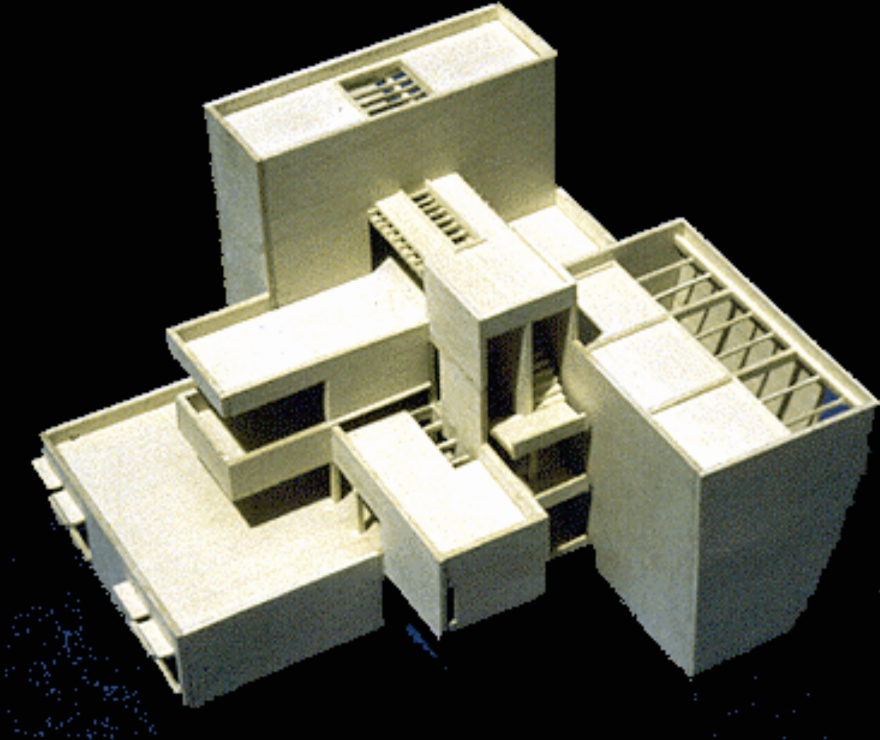
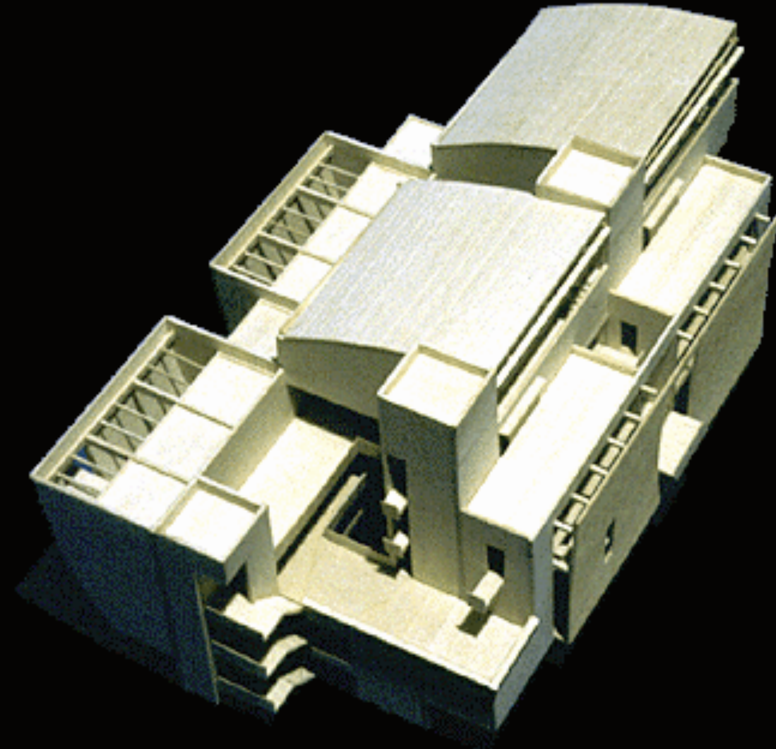


analytical – mughal gardens

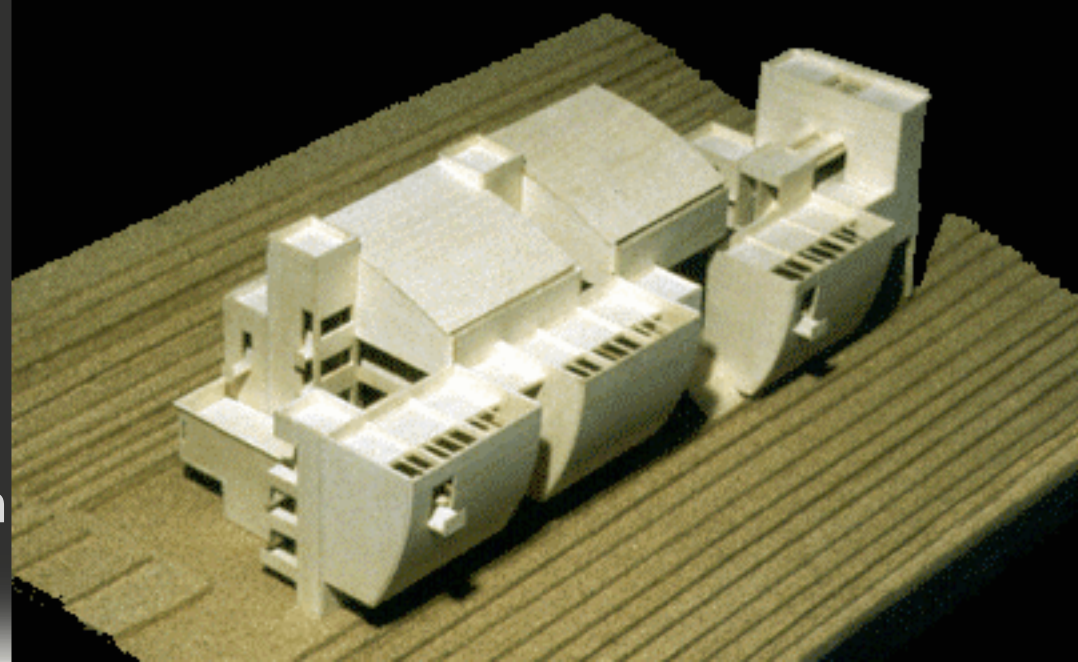


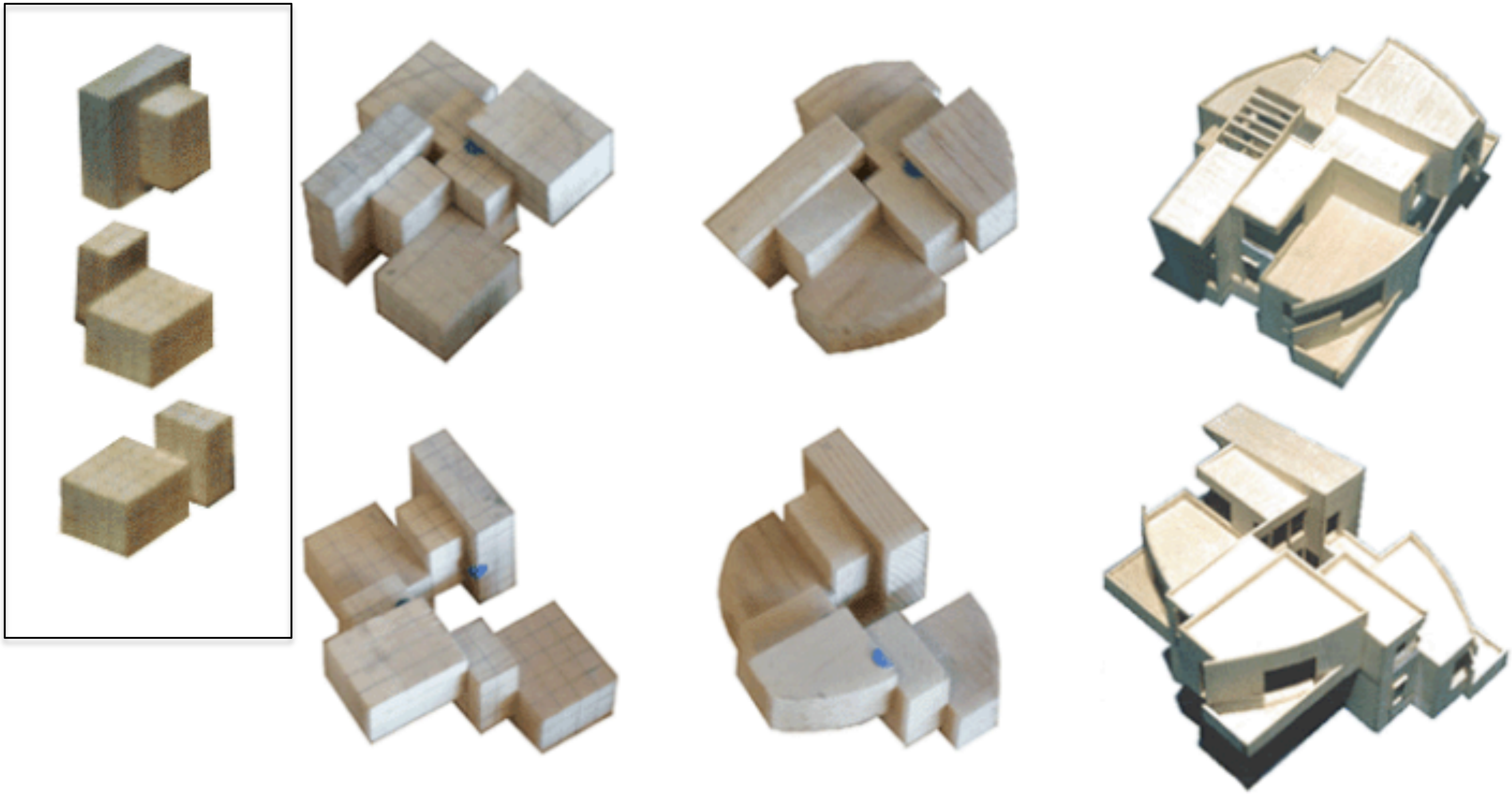
original – kindergarten grammar based on Froebel blocks

original – apartment building in manhattan

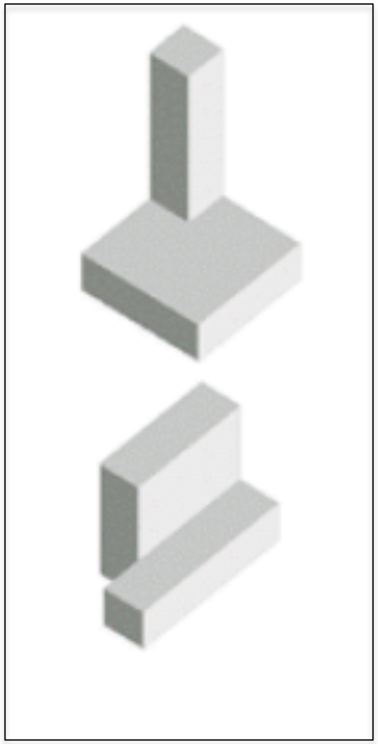
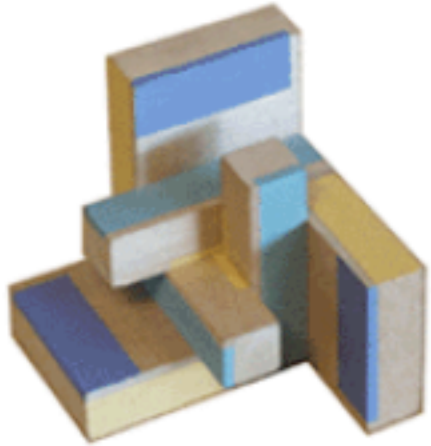
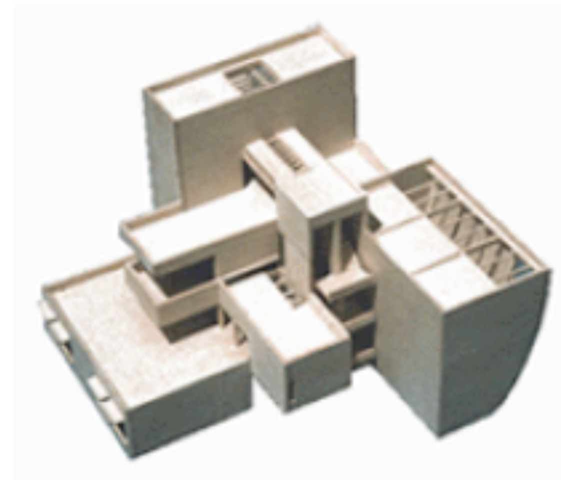



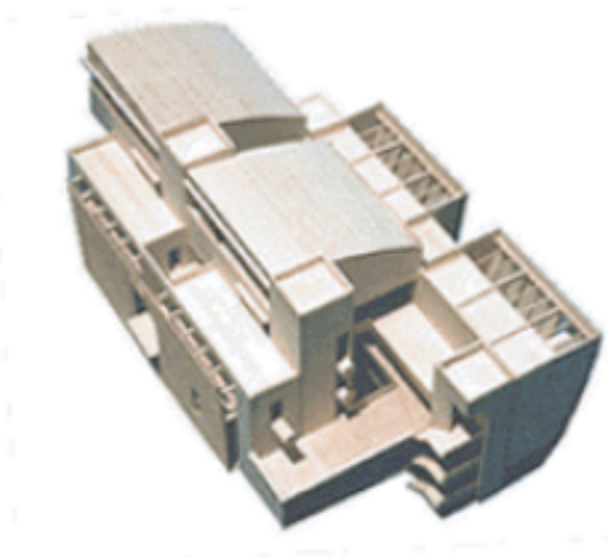
original – cultural museum





application in a real design context



developing a shape grammar

Vocabulary – basic shapes

Establish spatial relations

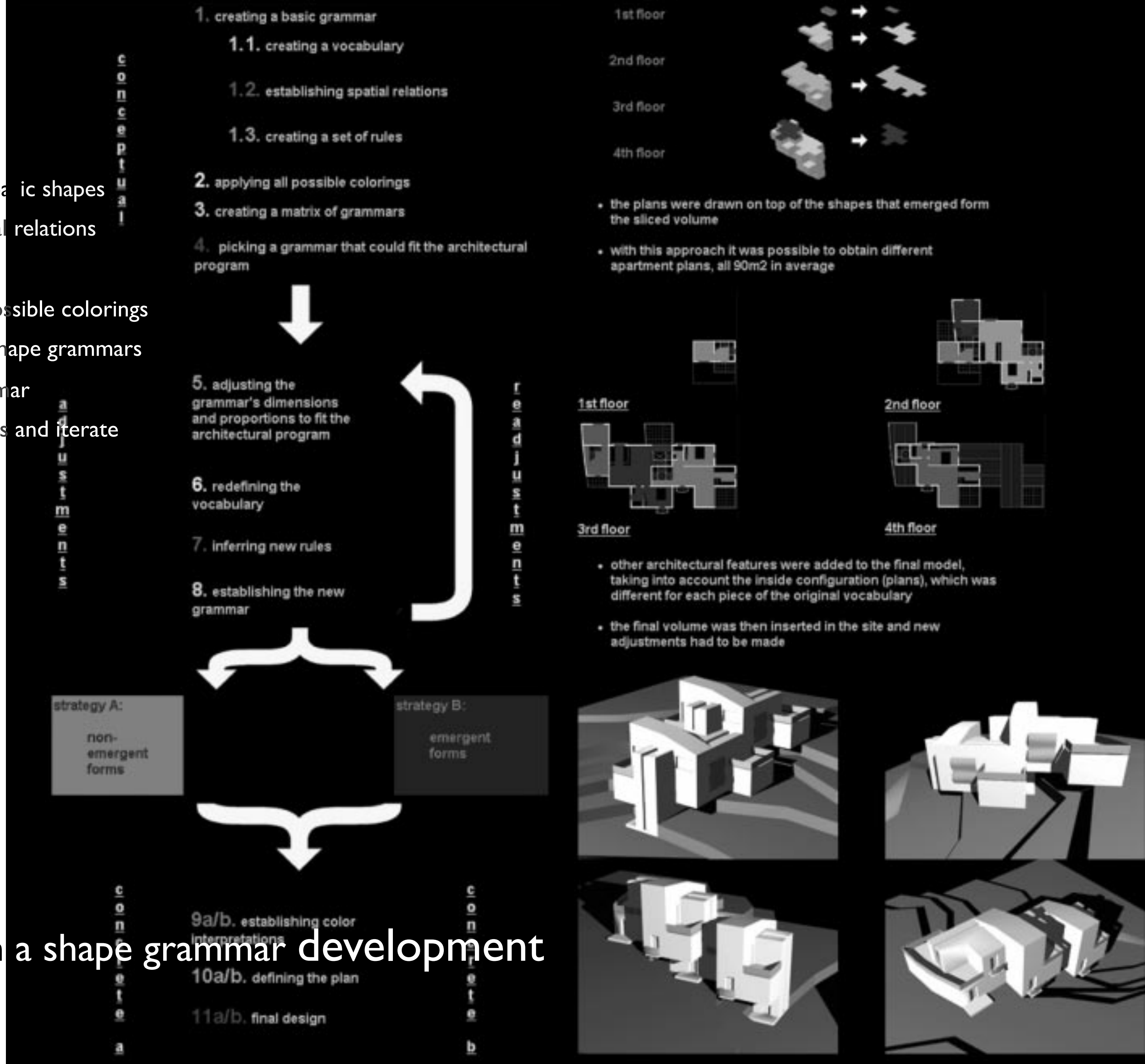
Create rules

Apply all possible colorings

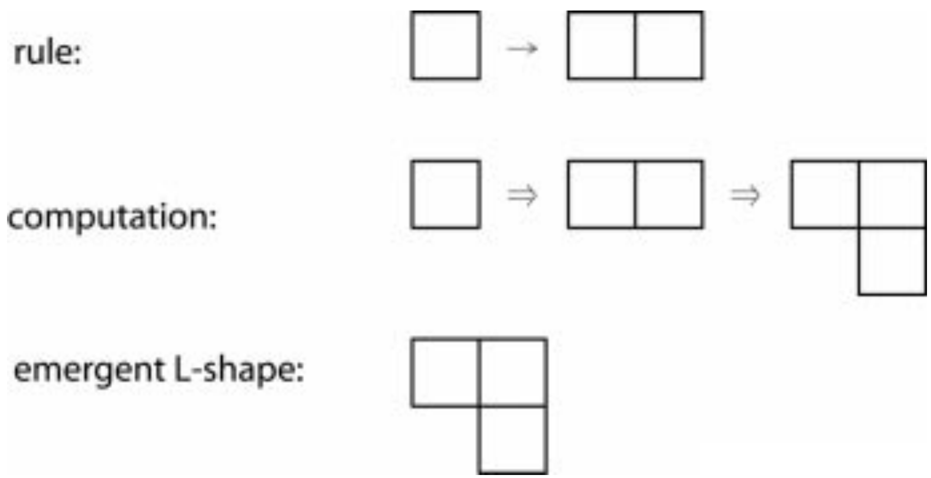
Matrix of shape grammars

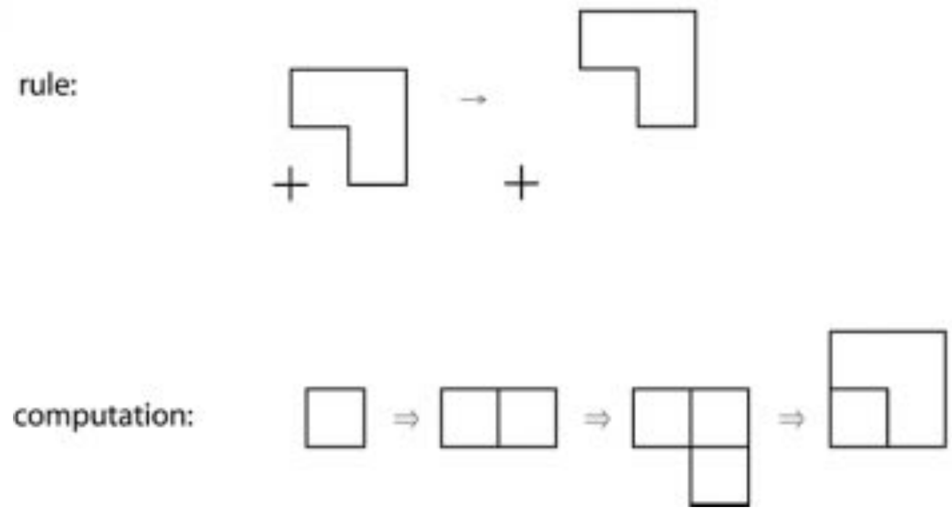
Pick grammar

Explore designs and iterate

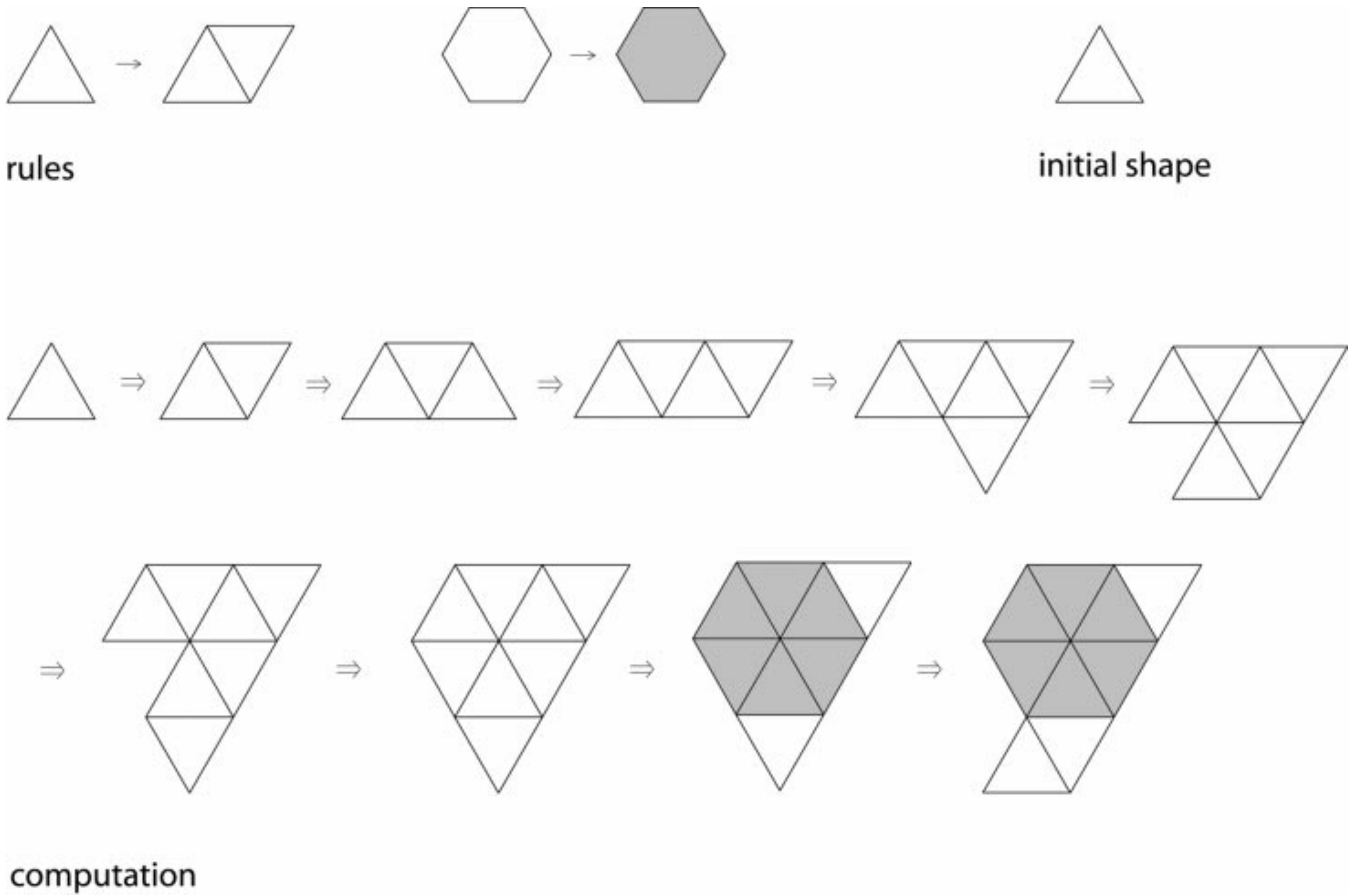


stages in a shape grammar development

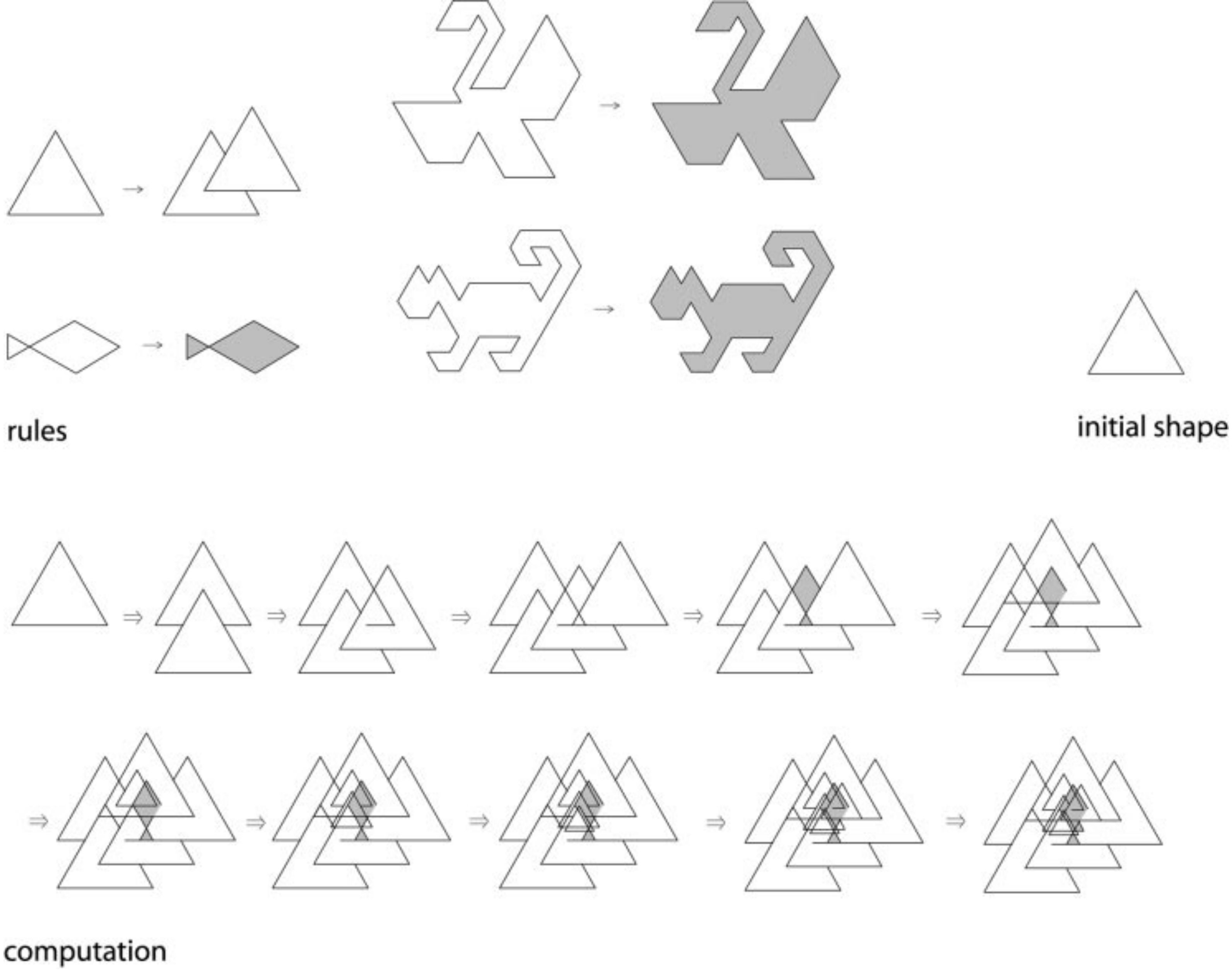




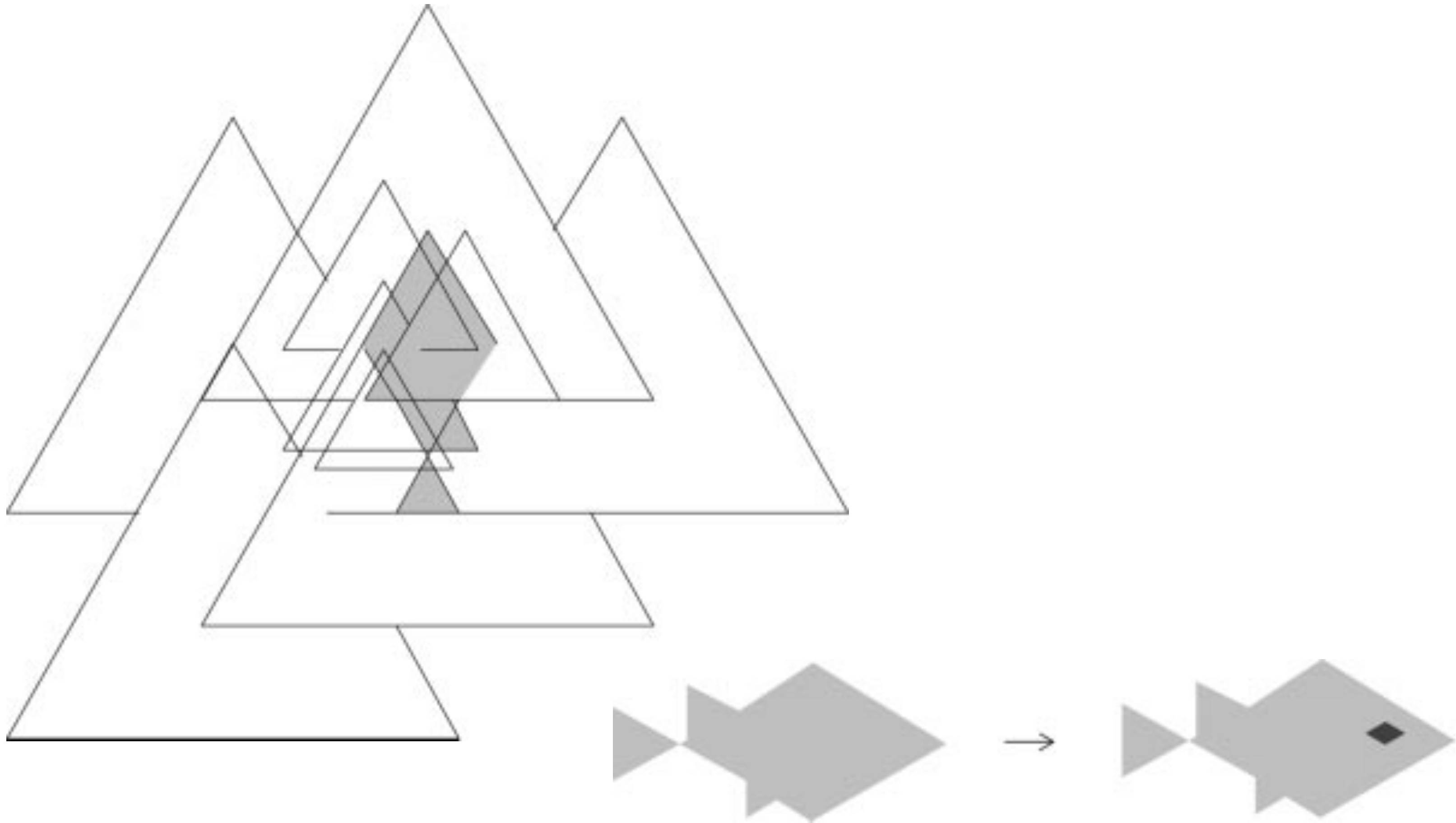
emergence - anticipated



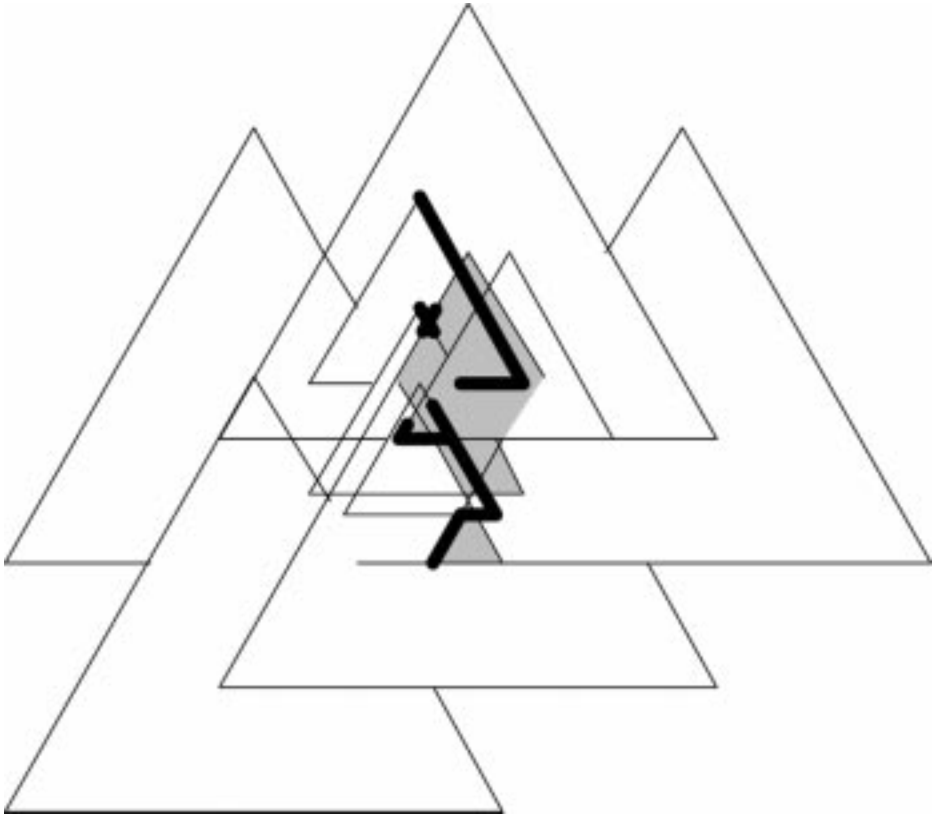
anticipated emergence

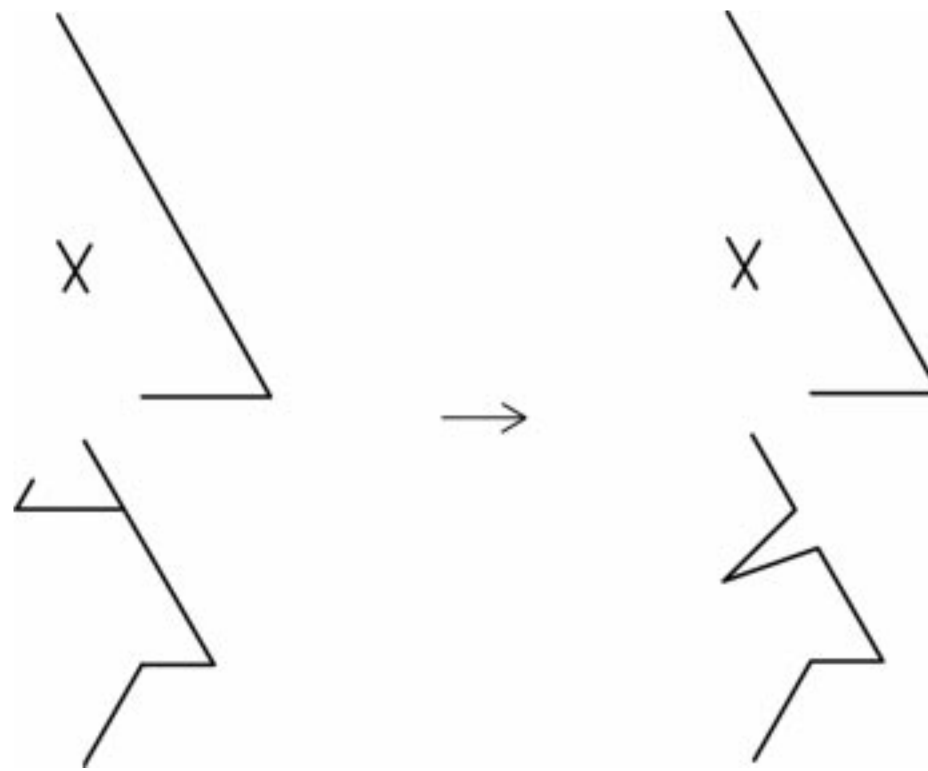


possible emergence



unanticipated emergence





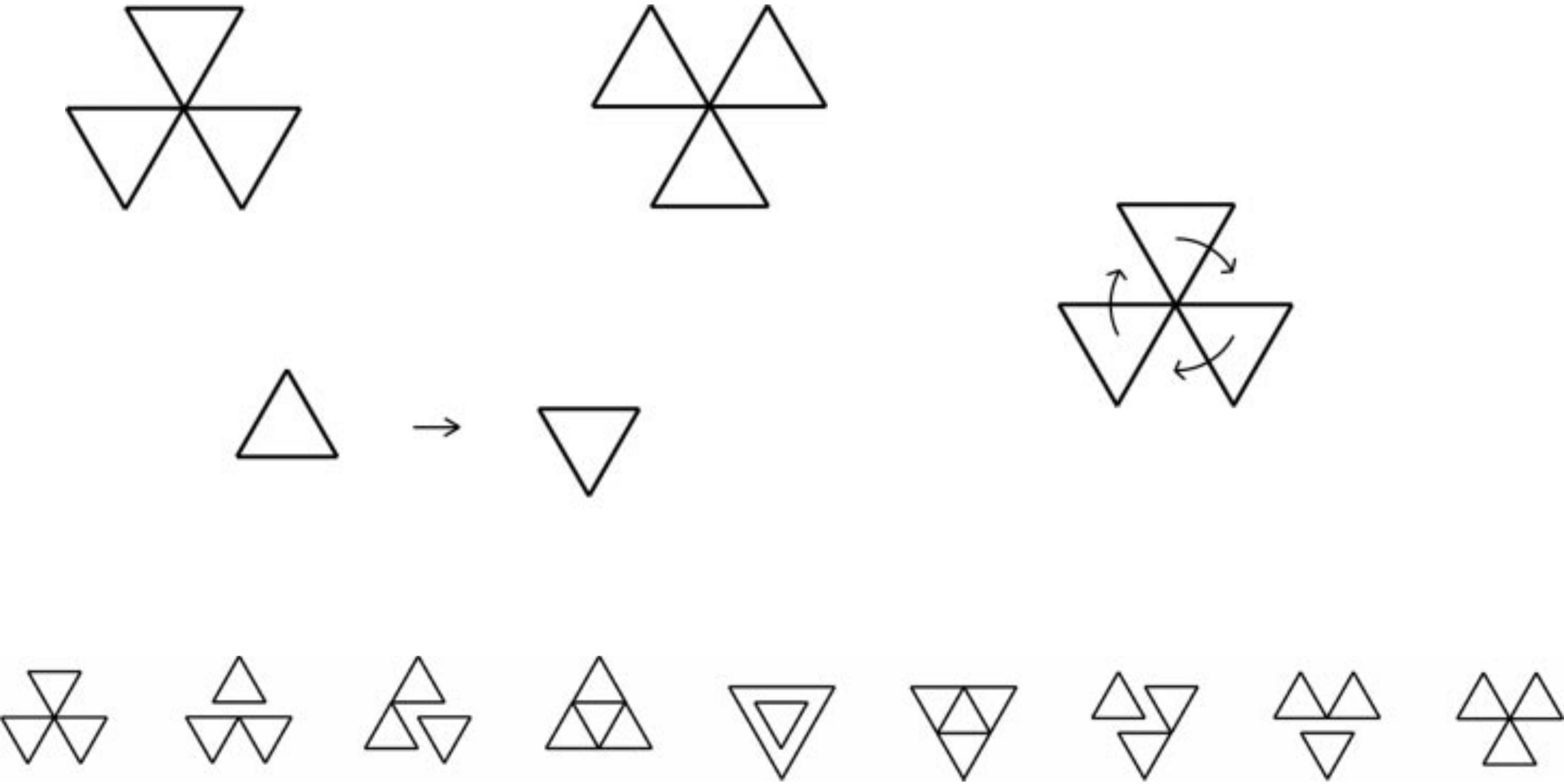
unanticipated emergence

What makes emergence possible? The answer is unambiguous ‹ ambiguity. Ambiguity is the special property of concrete things like shapes that lets you see them in different ways whenever you like. Ambiguity gets a bad press. One of the pioneers of cognitive science, George Miller (1983), thinks ambiguity is noise:

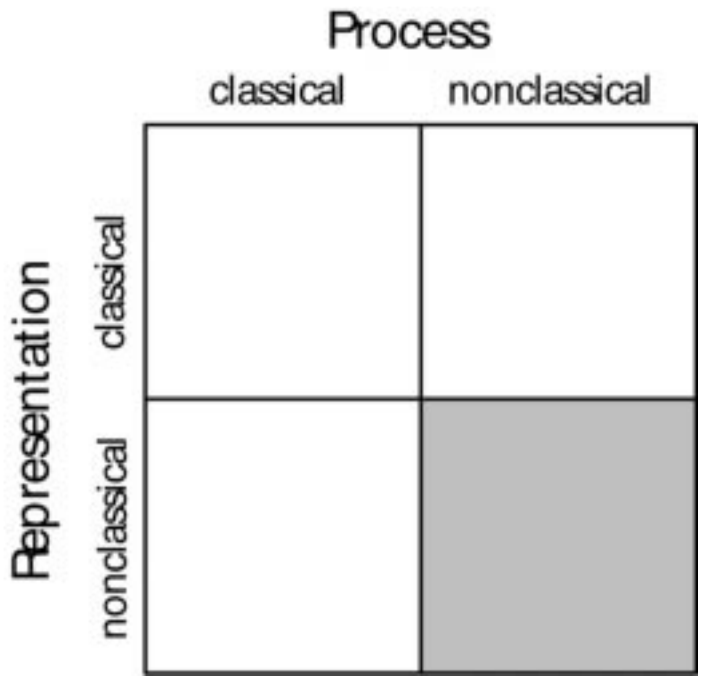
An interesting question for a theory of semantic information is whether there is any equivalent for the engineer's concept of noise. For example, if a statement can have more than one interpretation and if one meaning is understood by the hearer and another is intended by the speaker, then there is a kind of semantic noise in the communication even though the physical signals might have been transmitted perfectly. (pp.495‹496)

Perhaps Miller is right for cognition, at least if it is rationality, but there may be more to it than this. Ambiguity can be a designer's best friend.

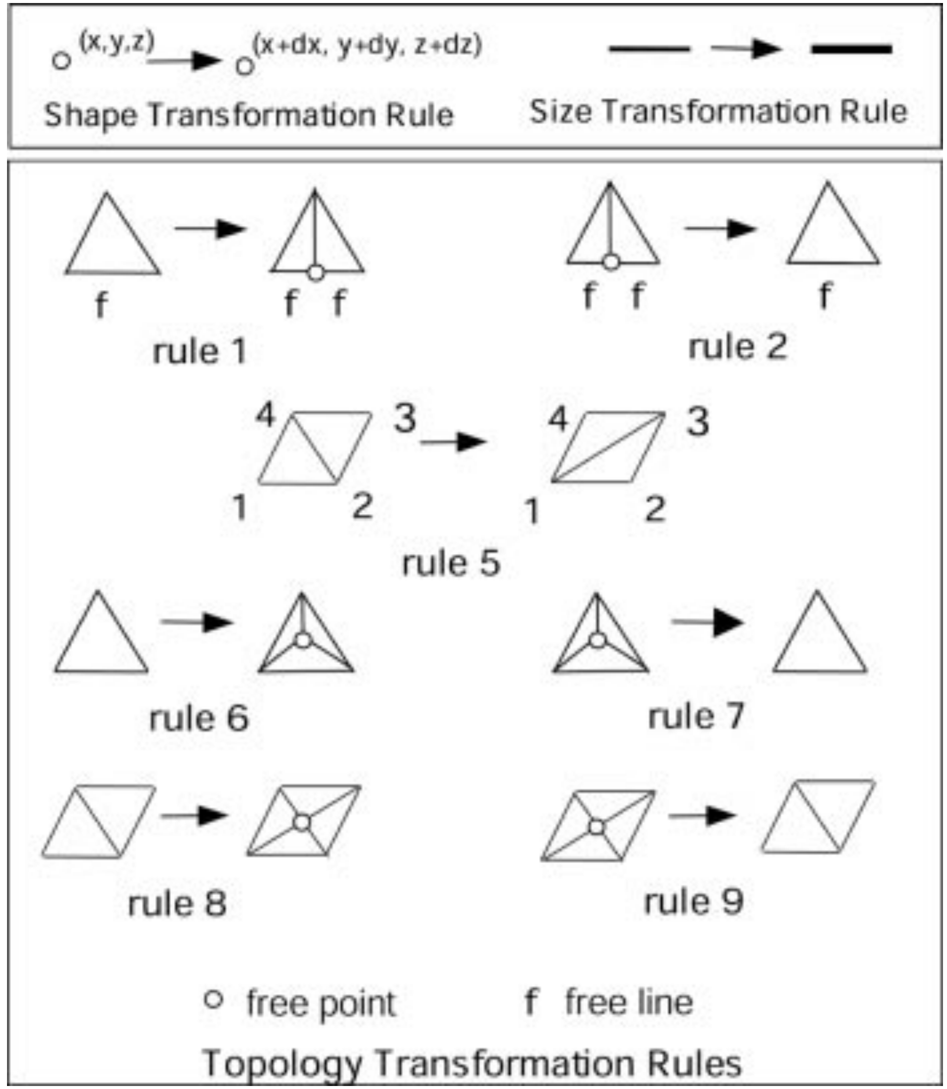
ambiguity – *friend or foe*



making emergence work



non-classical representation and computation



Eiform (Kristina Shea)

Modeling Structural Design Knowledge in Shape Annealing

Specifications (Syntax)

- material properties
- number of supports and locations
- symmetry
- joint angles

Constraints (Semantics)

- stress
- Euler buckling
- displacement
- geometric obstacles

Objectives (Semantics)

- <u>efficiency</u>

  minimum mass
- <u>economy</u>

  minimum number of distinct cross-sections

  minimum number of distinct lengths
- <u>utility</u>

  maximum enclosure space

  minimum surface area
- <u>aesthetics</u>

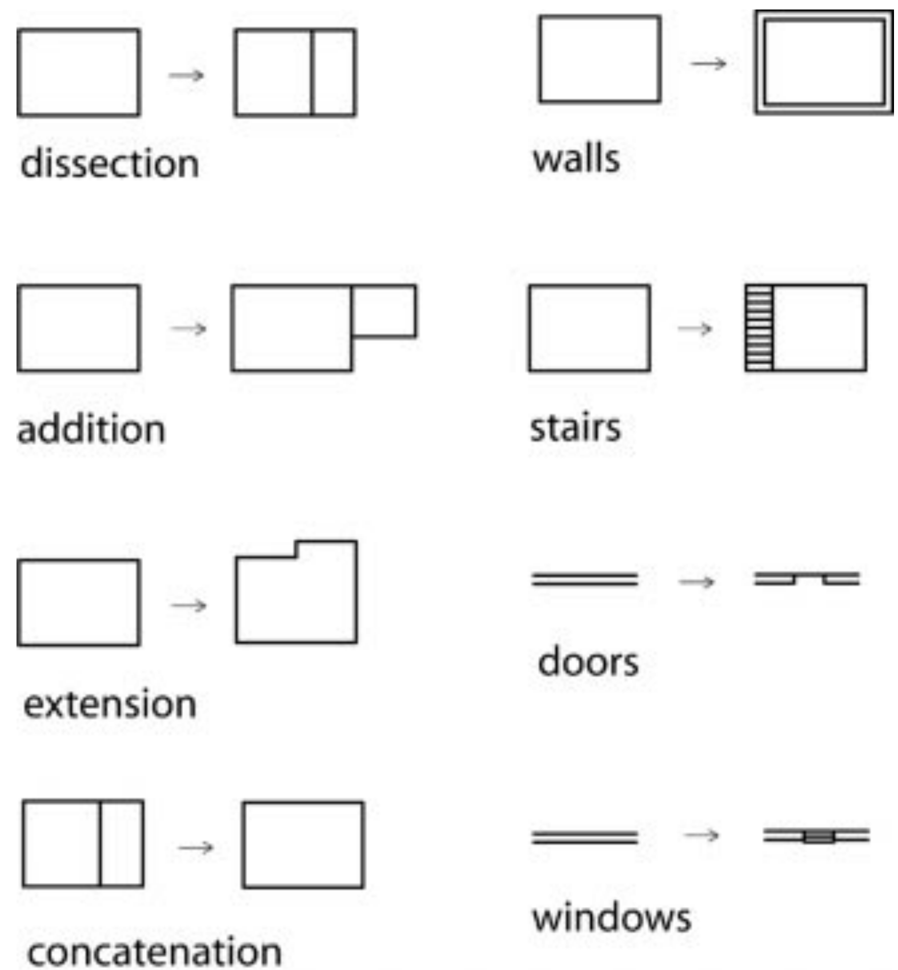
  uniformity metric = σ(member lengths)

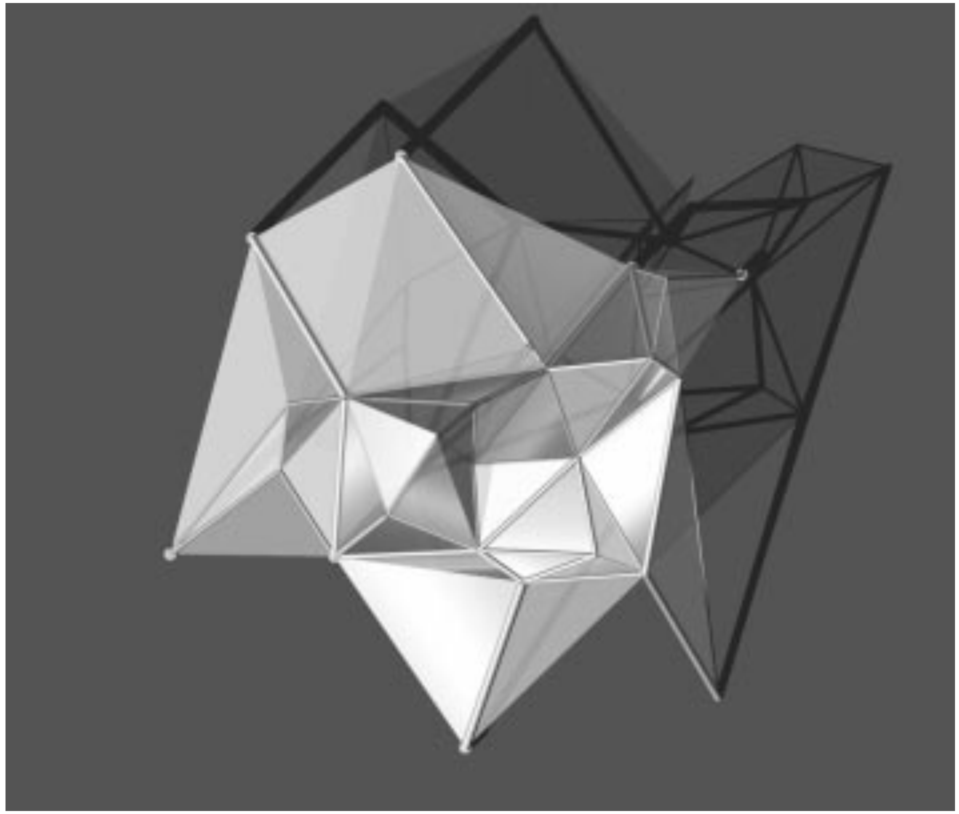
  golden ratio metric =

$$\sum_{numshapes} \left|\phi - \frac{b}{a}\right| + \left|\phi - \frac{b}{c}\right| + \left|\phi - \frac{a}{b}\right|$$

Eiform constraints for a simulated annealing solver

A collection of shape rules that could be developed in an evolutionary process to define grammars for floor plans





… that can lead to impressive, artistic, structurally sound forms

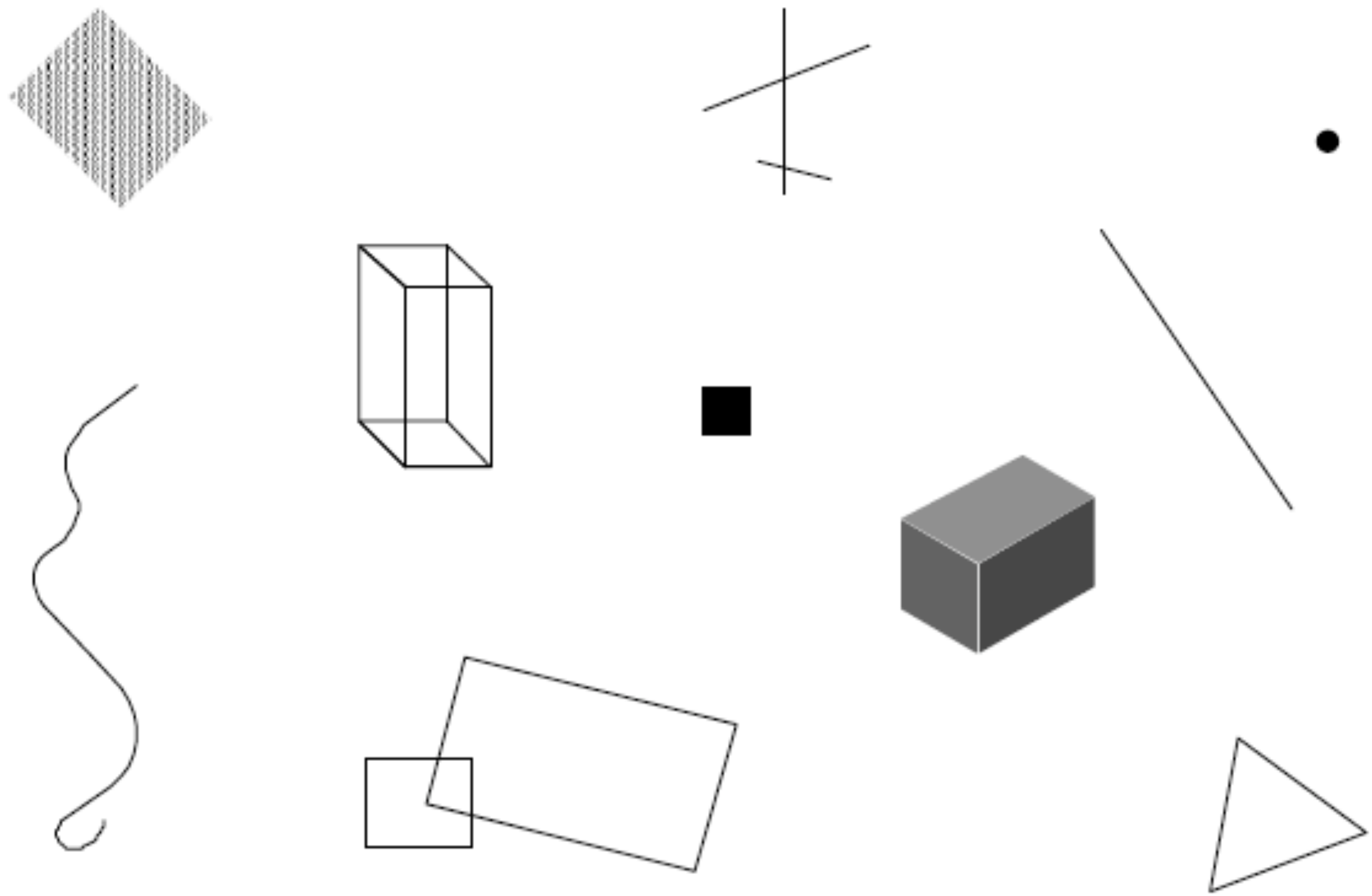
Computation has been described in this paper in terms of representation and process.

However, representation and process are just two aspects of computation among a host of many, many others.

In many ways, computation is much like shapes.

There is always some other way to describe it that may prove insightful.

conclusion



basic components of grammars and design

shapes

Shape is a finite arrangement of lines of non-zero length with respect to a coordinate system

Shapes can be formed by *addition* of shapes which consists of lines in both shapes.

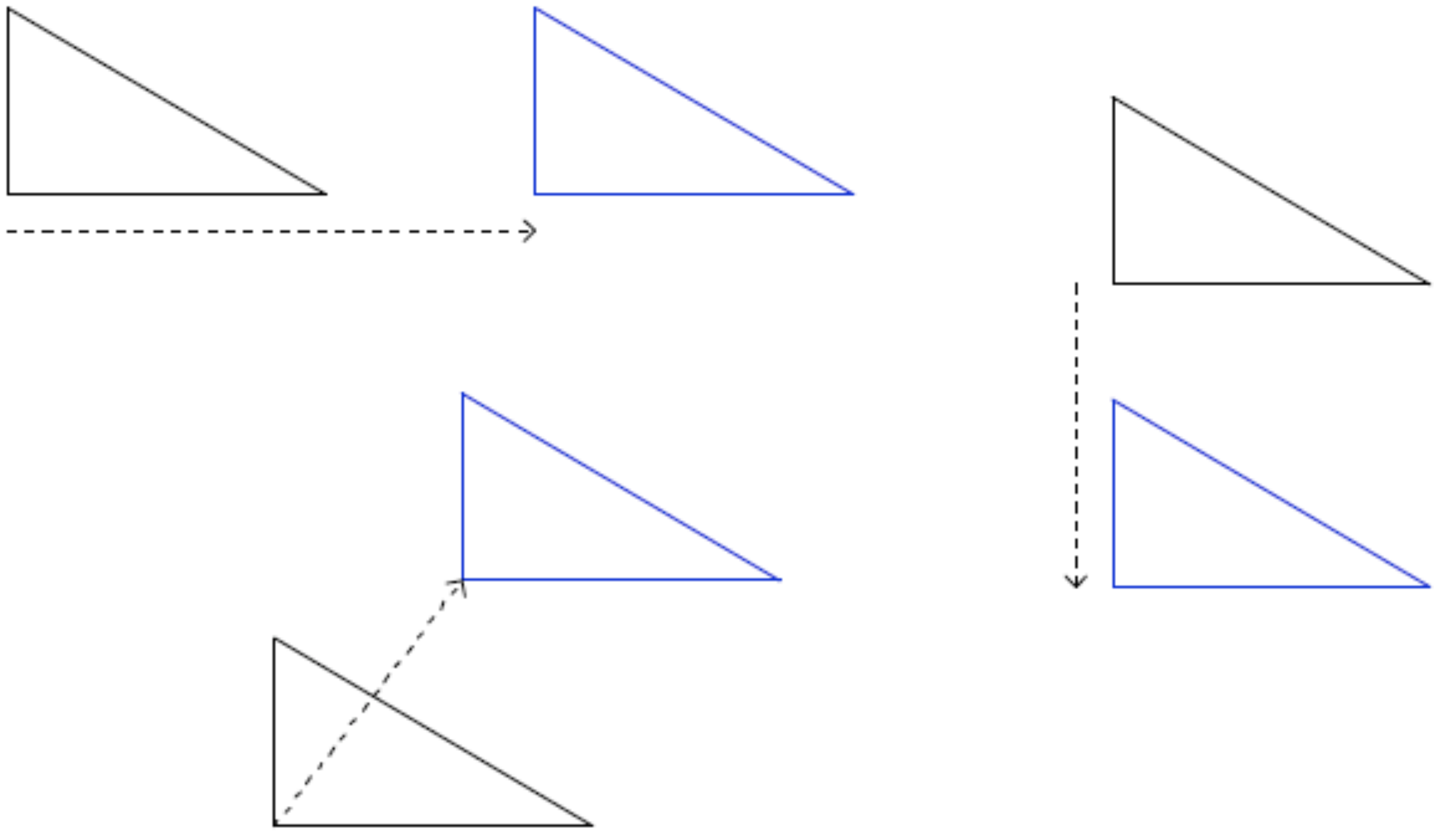
Shapes can be formed by *subtraction* of shapes which consists of the lines in the first shape that are not in the second.

Shapes can be formed by combinations of the two under an ***Euclidean transformation***

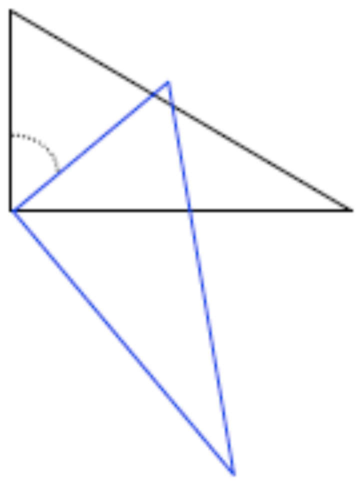
A central notion in this definition of shapes is ***pictorial equivalence***

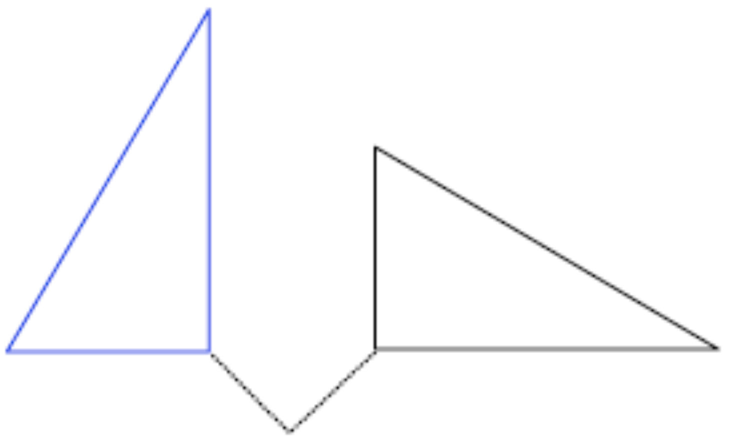
for our purposes, **shape is …**

euclidean transformations

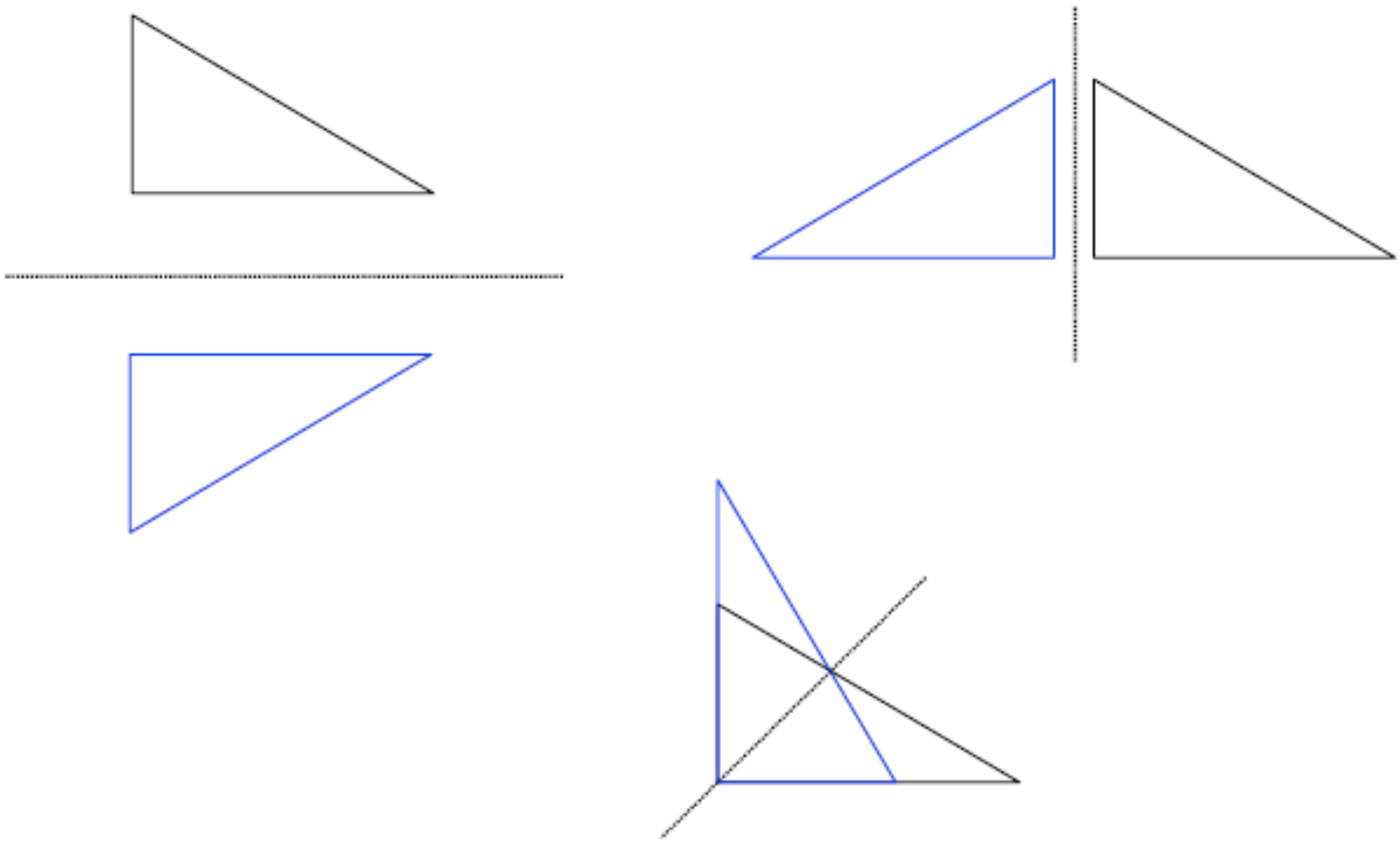


translation

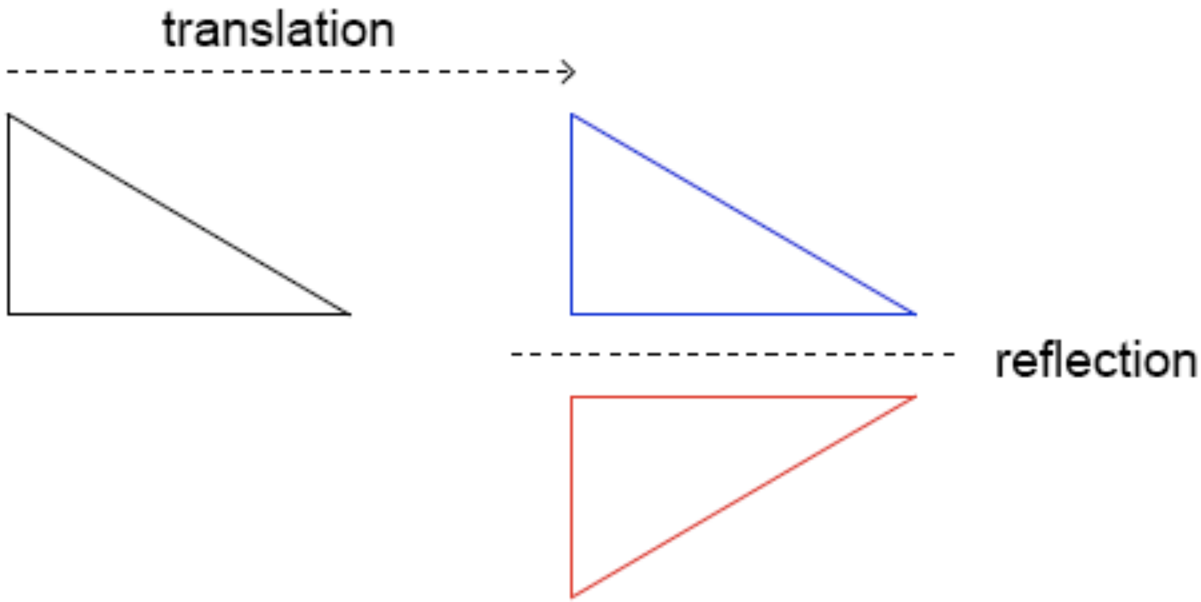




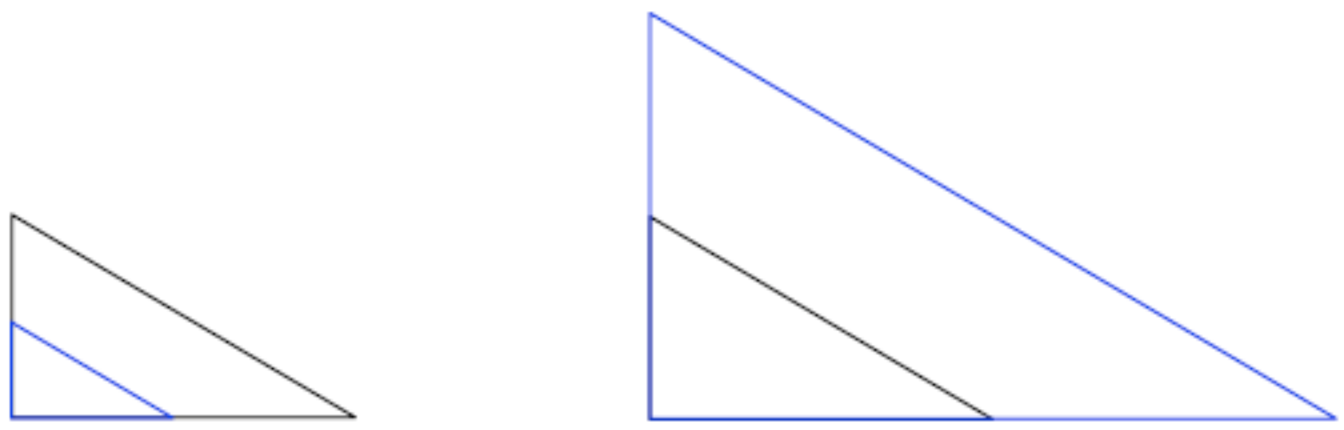
rotation



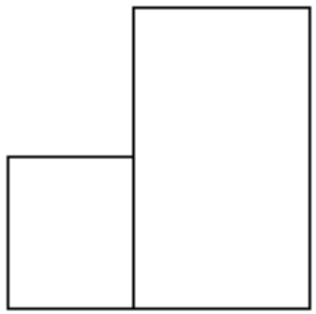
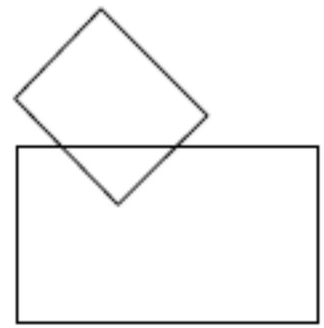
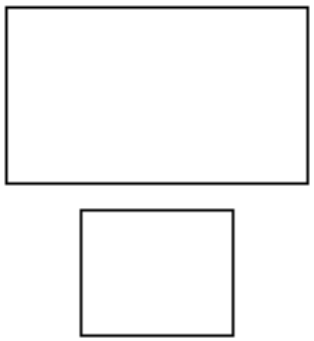
reflection



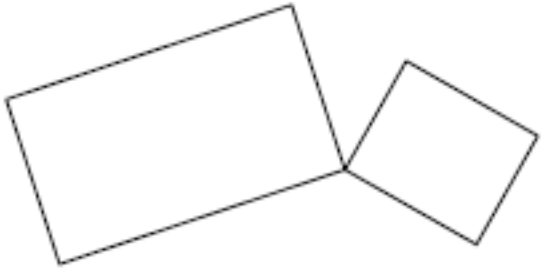
glide reflection – example of a combination



scale

When two or more shapes combine they form a *spatial relation*
That is, a set of shapes specifies a spatial relation

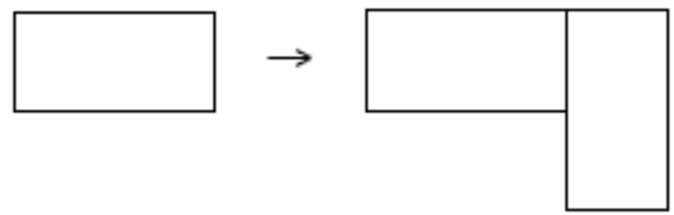



spatial relations

Shape	A, B
Relation	$A+B$
Rule	$A \rightarrow A + B$
	$B \rightarrow A + B$

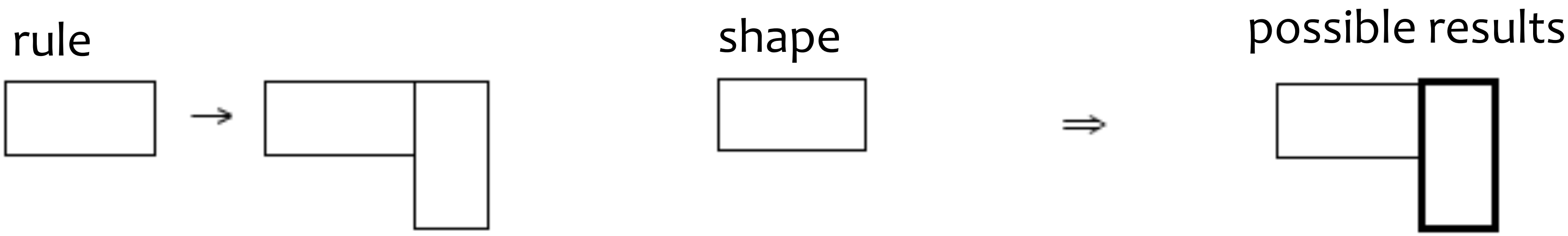
shape rules

Relation

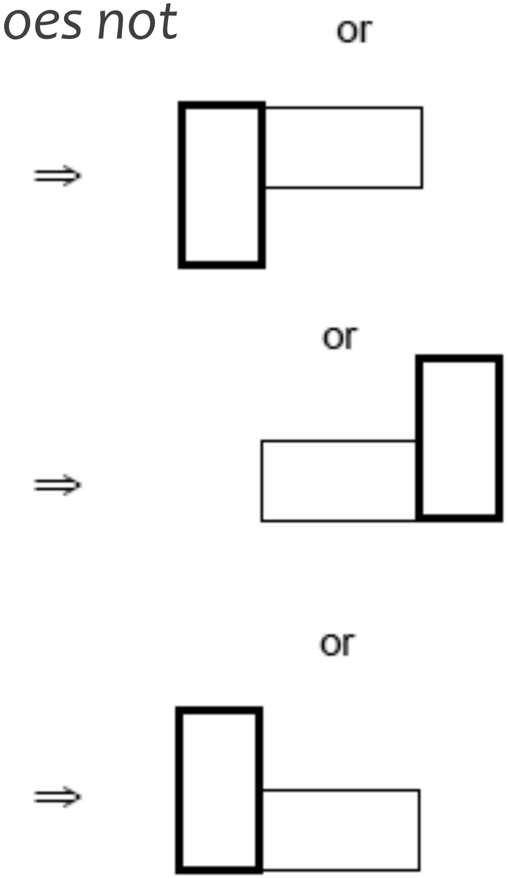


Rule

from **relation** to **rule**



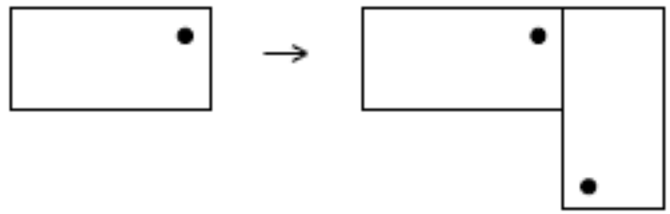
The application of Euclidean transformation does not alter the spatial relation



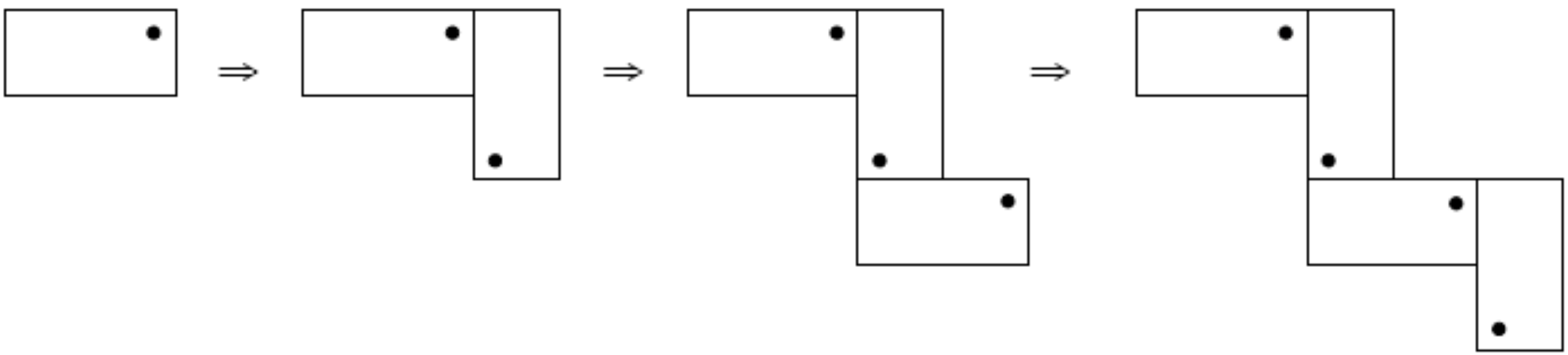
to **derivation**

labeling rules helps control rule application

Labeled rule

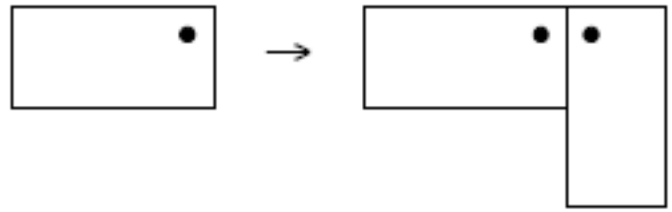


derivation

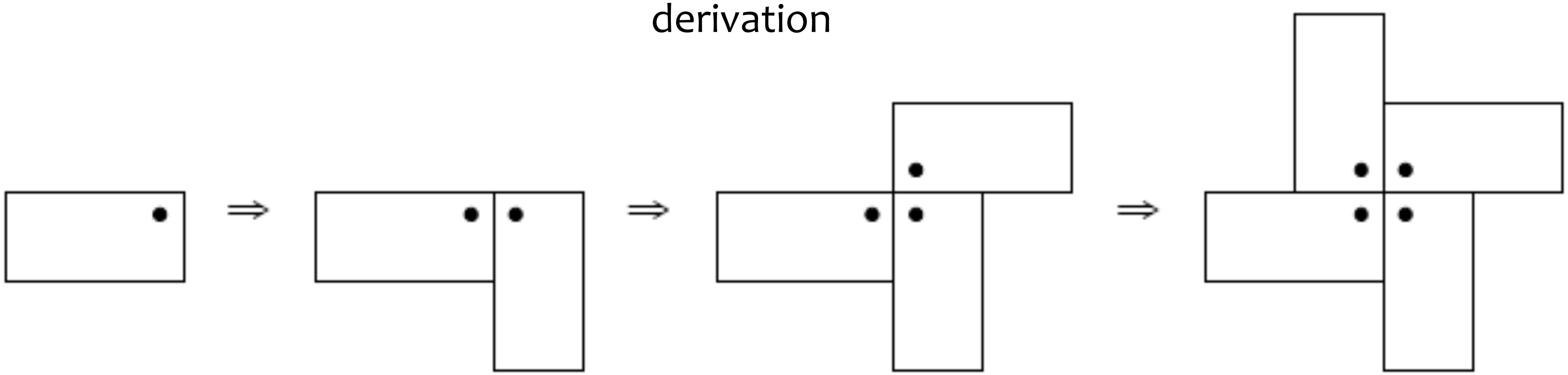


labeled rule based **derivation**

Labeled rule

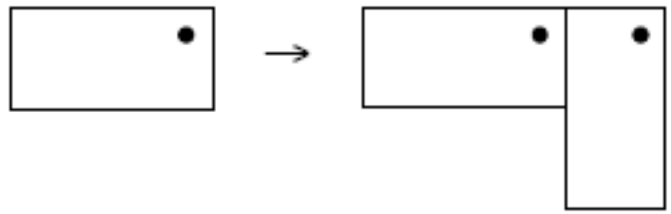


derivation

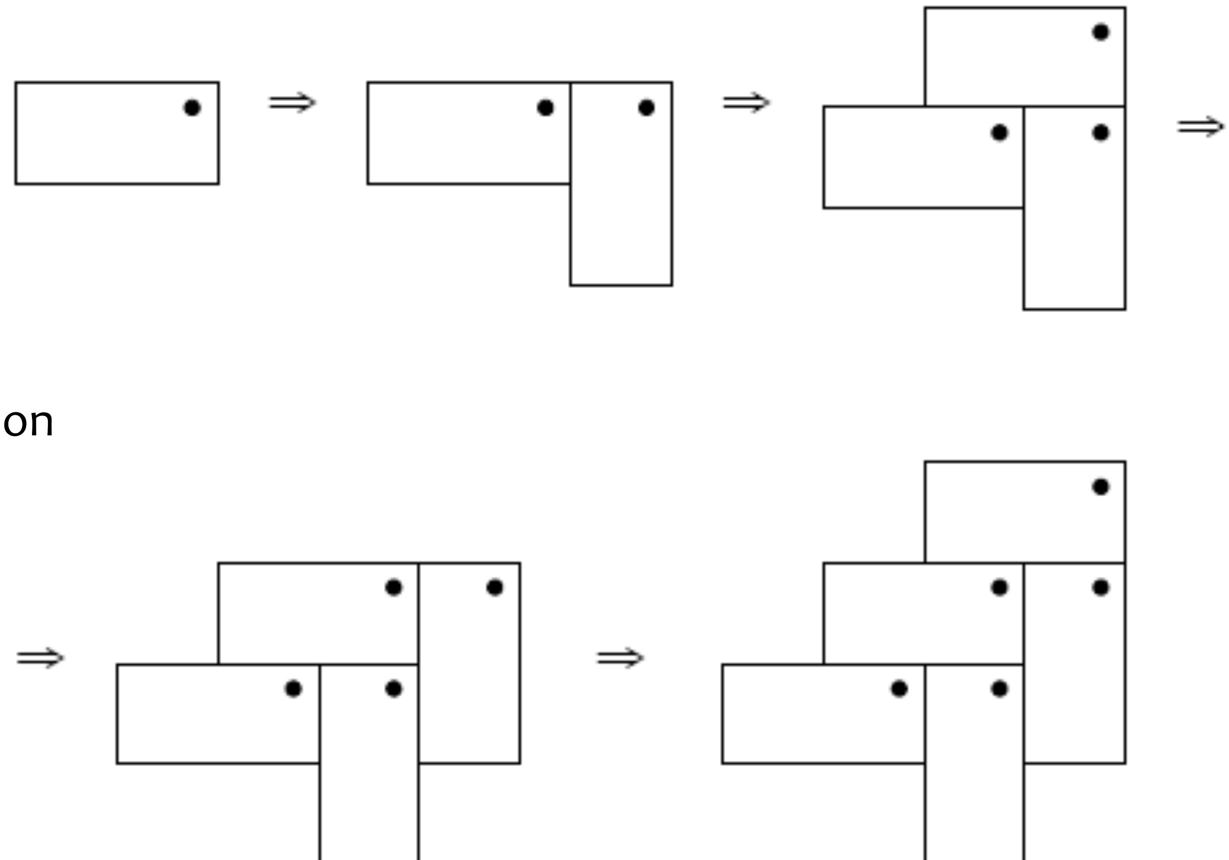


labeled rule based **derivation**

Labeled rule

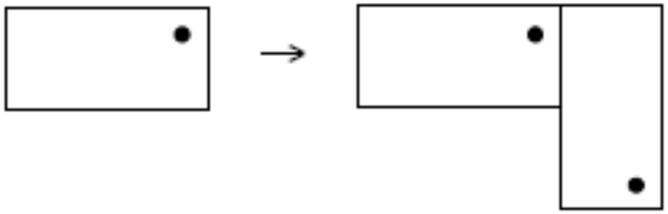


derivation

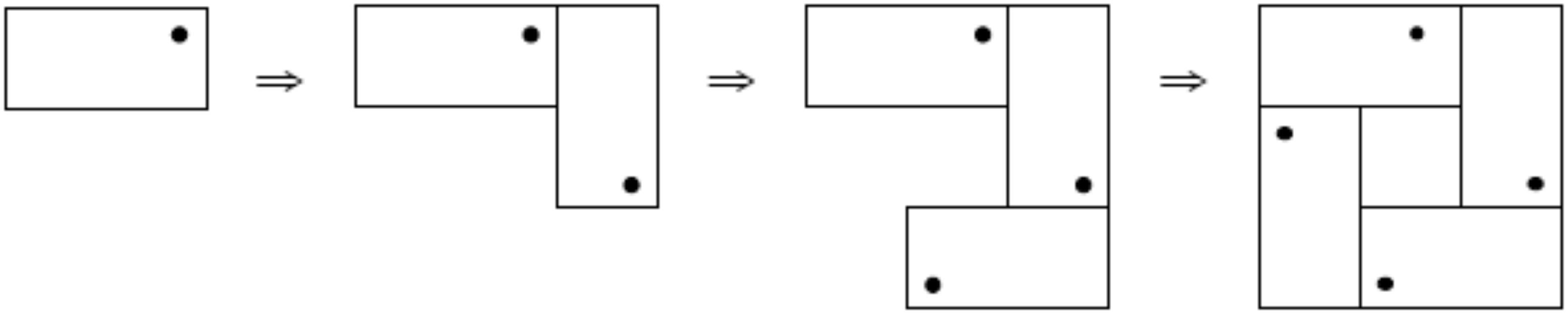


labeled rule based **derivation**

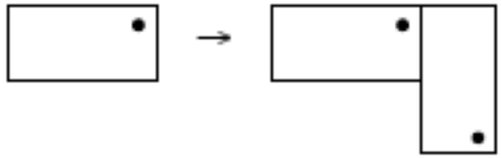
Labeled rule



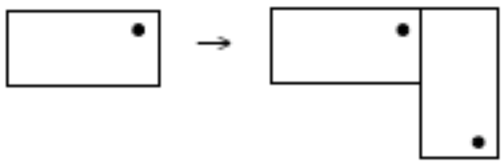
derivation

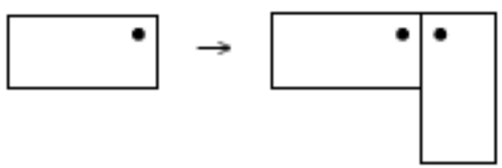


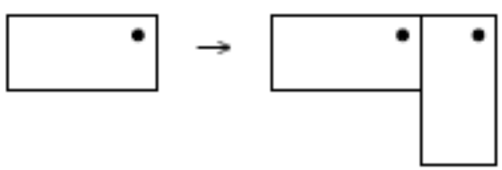
labeled rule based **derivation**



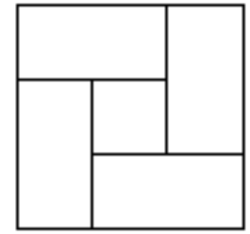
rules

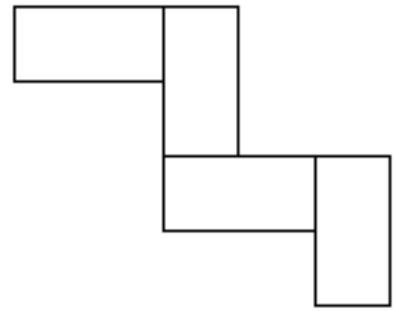


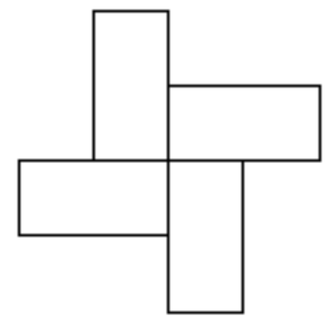


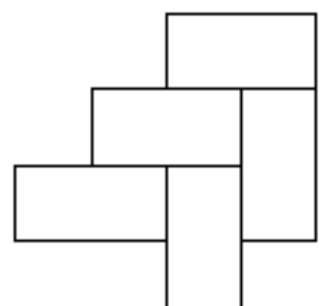


designs









shape grammar

shape grammar

vocabulary

shapes made up from these vocabulary

In the general case, shapes are parameterized schemes

production rules
(or rules of change, encapsulate a spatial relation)

seed shape (we have to start somewhere)

+ a “notion” of **rule application**

shape grammar embodies **change** implies **generation**

shape grammar G = (S, L, I, R)

shape rules

initial or seed shape

label set

vocabulary

initial (seed) shape
belongs to the *universe* of labeled shapes made up of shapes in S and labels in L

R contain rules of the form a → b where **a** and **b** belong to the universe of labeled shapes made of shapes in S and labels in L except **a** cannot be empty

formally: a shape grammar is

Vocabulary is a limited set of shapes no two of which are similar.

The vocabulary provides the basic building blocks by means of which shapes can be generated through shape arithmetic and geometric (euclidean) transformations.

vocabulary

If we are given a set of shapes *S*, then we can create a set *U* called the ***universe*** of *S* in the following manner:

The empty shape is in *U*

Every shape in *S* is in *U*

What can you say about the universe of the set of shapes consisting just one shape, a single line of unit length, {(0.0),(1,0)} ?

f and *g*, *f*(*s*)+g(*t*) is in *U*

U is thus closed under shape addition and the Euclidean transformations.

universe

A rule is *applicable* to the *current shape*
which is either the initial shape
or a shape produced from the initial shape
whenever the left hand side of the rule ‘*occurs*’ in the object
in which case
it is *replaced* by the right hand side of the rule under rule application

shape rule application

A rule $a \rightarrow b$ is applies only if a 'occurs' in the given shape u under some 'transformation' T in which case $T(a)$ is substituted by $T(b)$ in the current shape

Rule application

$$v \leftarrow u - T(a) + T(b) \text{ provided } T(a) \leq u$$

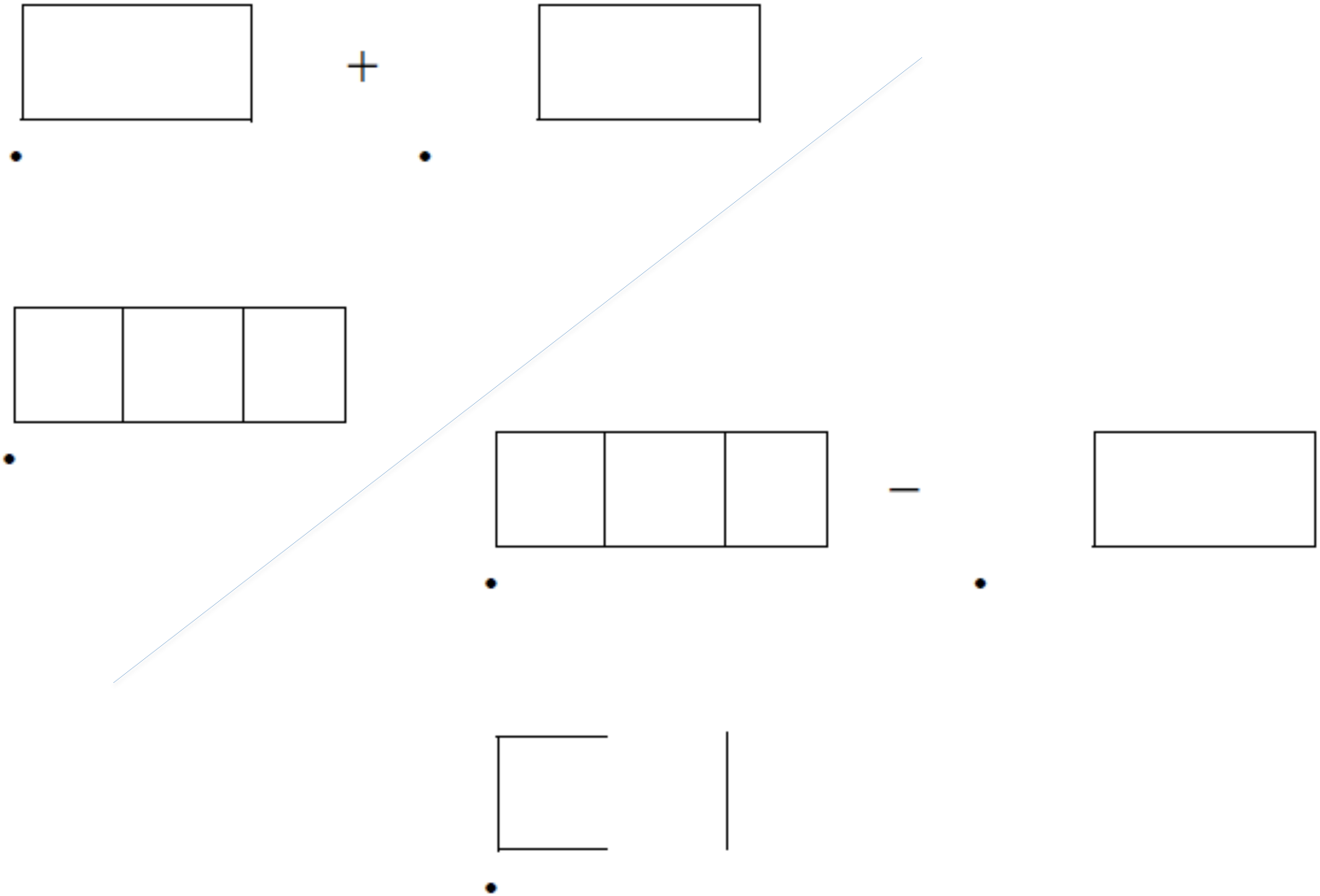
difference sum subshape

We describe this as $\mathrm{u} \Rightarrow \mathrm{v}$

rule application

Implicit in this definition is the fact that 'parts' of shapes are recognizable in ***arbitrary*** ways

shape



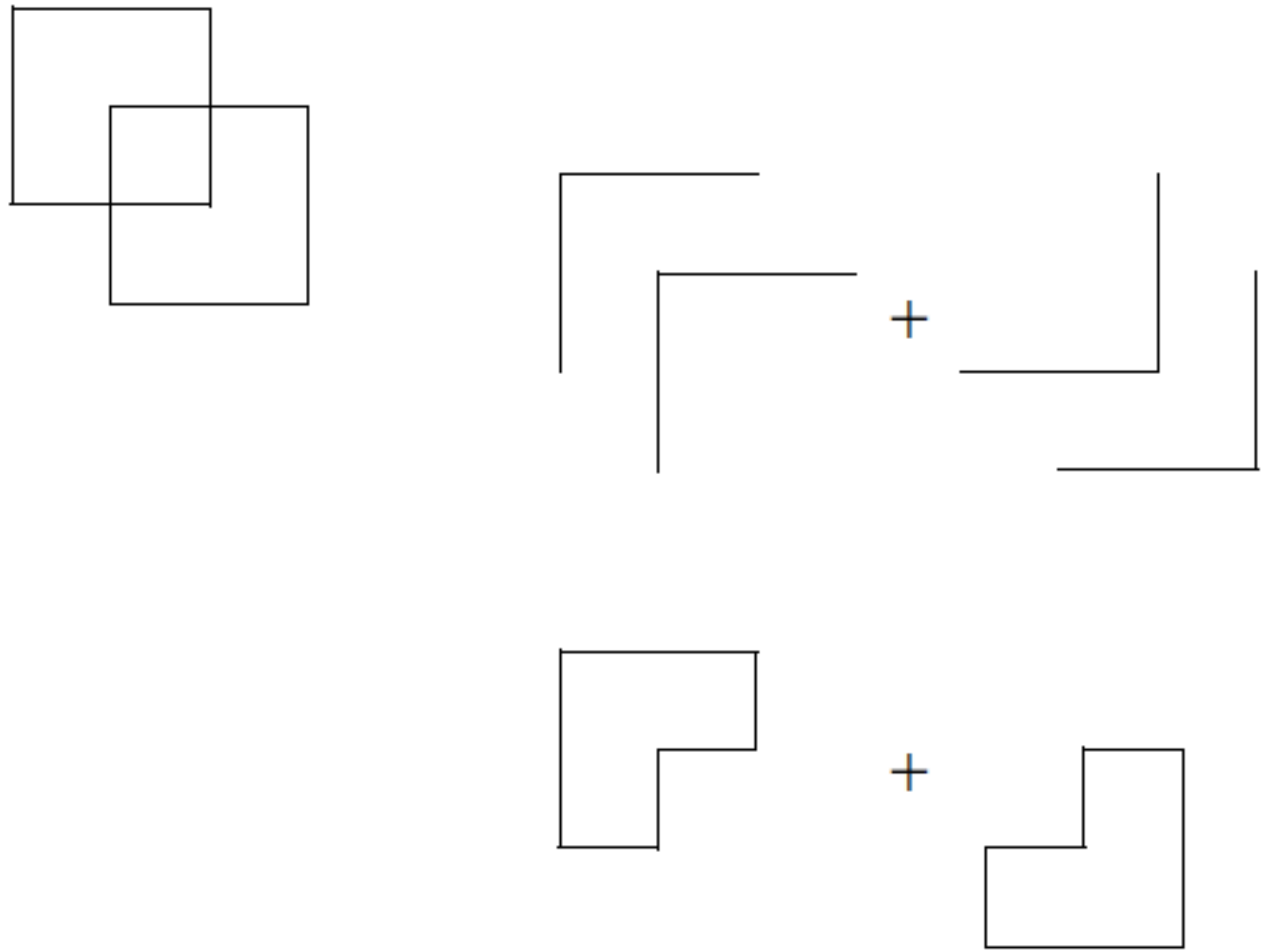
addition and **subtraction**

A shape is a *subshape* of another
if all the lines in the first shape are lines in the second

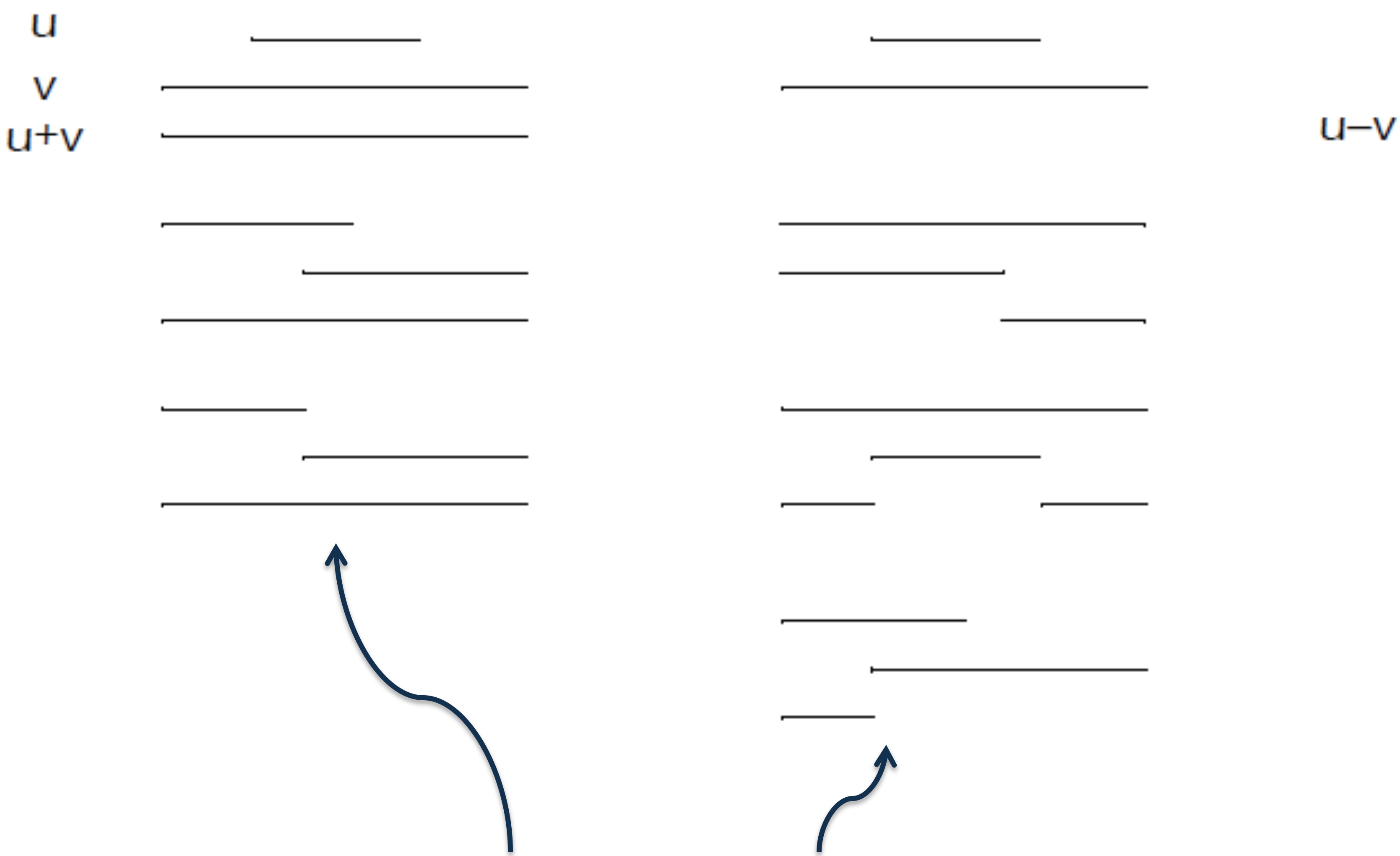
A subshape identifies a *part* of a shape

A shape has *indefinitely many* subshapes

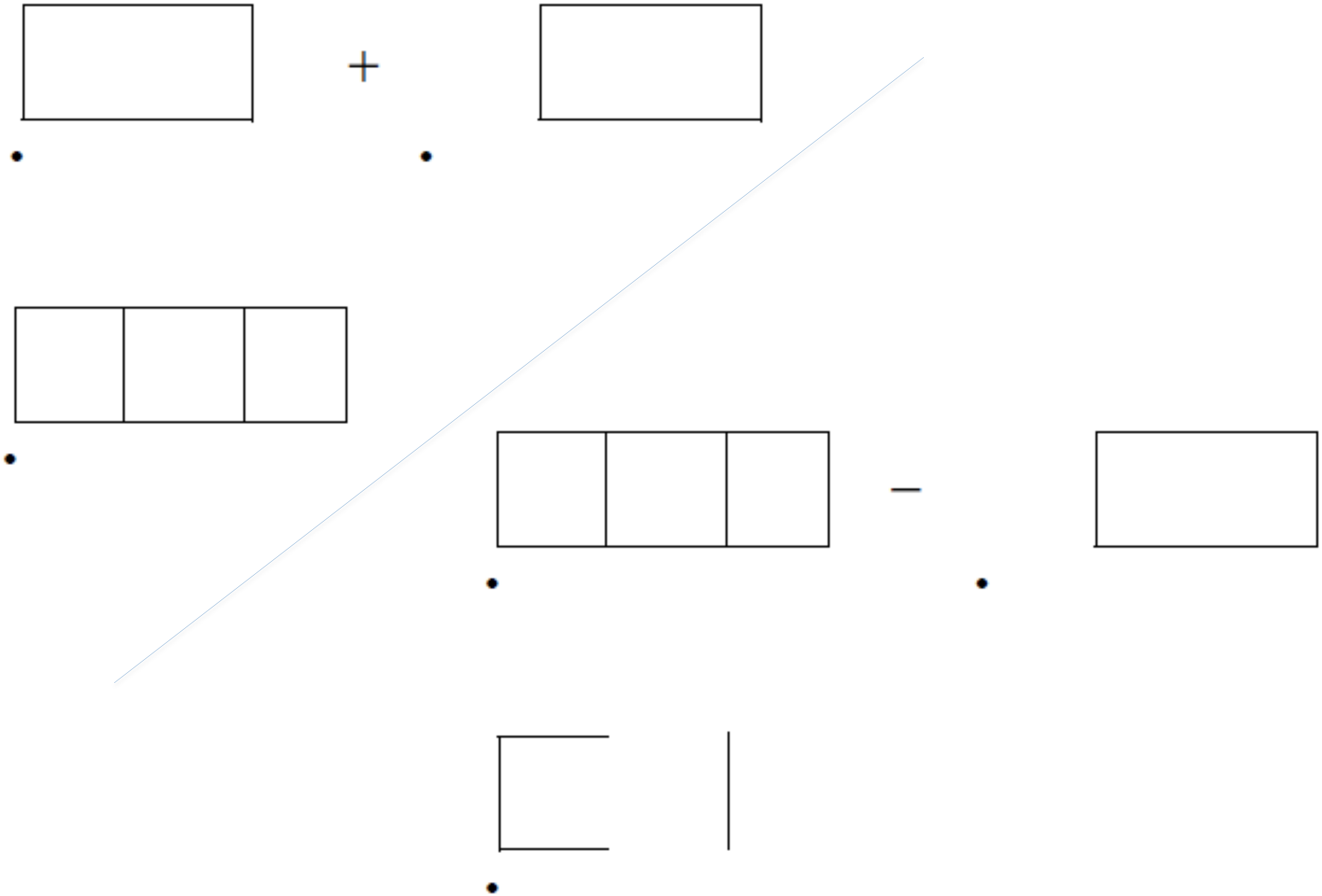
subshapes



decomposing a shape



reduction rules for add and **subtraction**



addition and **subtraction**

a metaphor for shape grammars

a pencil –

lead at one end to **add** marks

an eraser at the other to **subtract** marks

leaded side has a **shape**

eraser side has another **shape,** not necessarily the same

together they specify a **rule**

– that is, **see** something
take it away and its place do (**make/add**) something else

metaphor for shape grammars

deriving designs as a sequence of designs

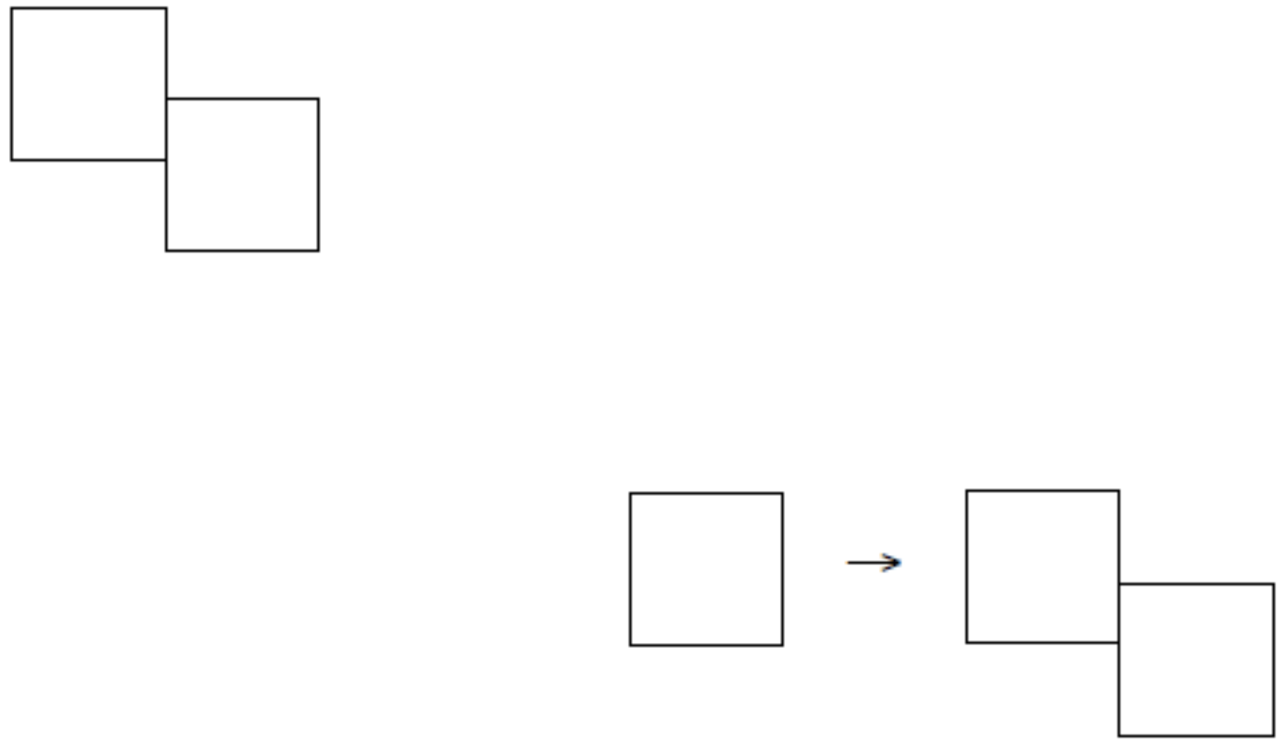
based on the work of Terry Knight http://www.mit.edu/~tknight/IJDC/

By labeling shapes we can "scaffold" the process of generating shapes

The *language* of a grammar G, is the set of all shapes (i.e., without non-terminals) that are produced from the initial shape through rule application

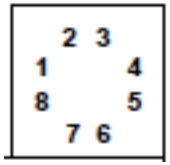
Language = { shape | initial-shape $\Rightarrow^*$ shape }

language

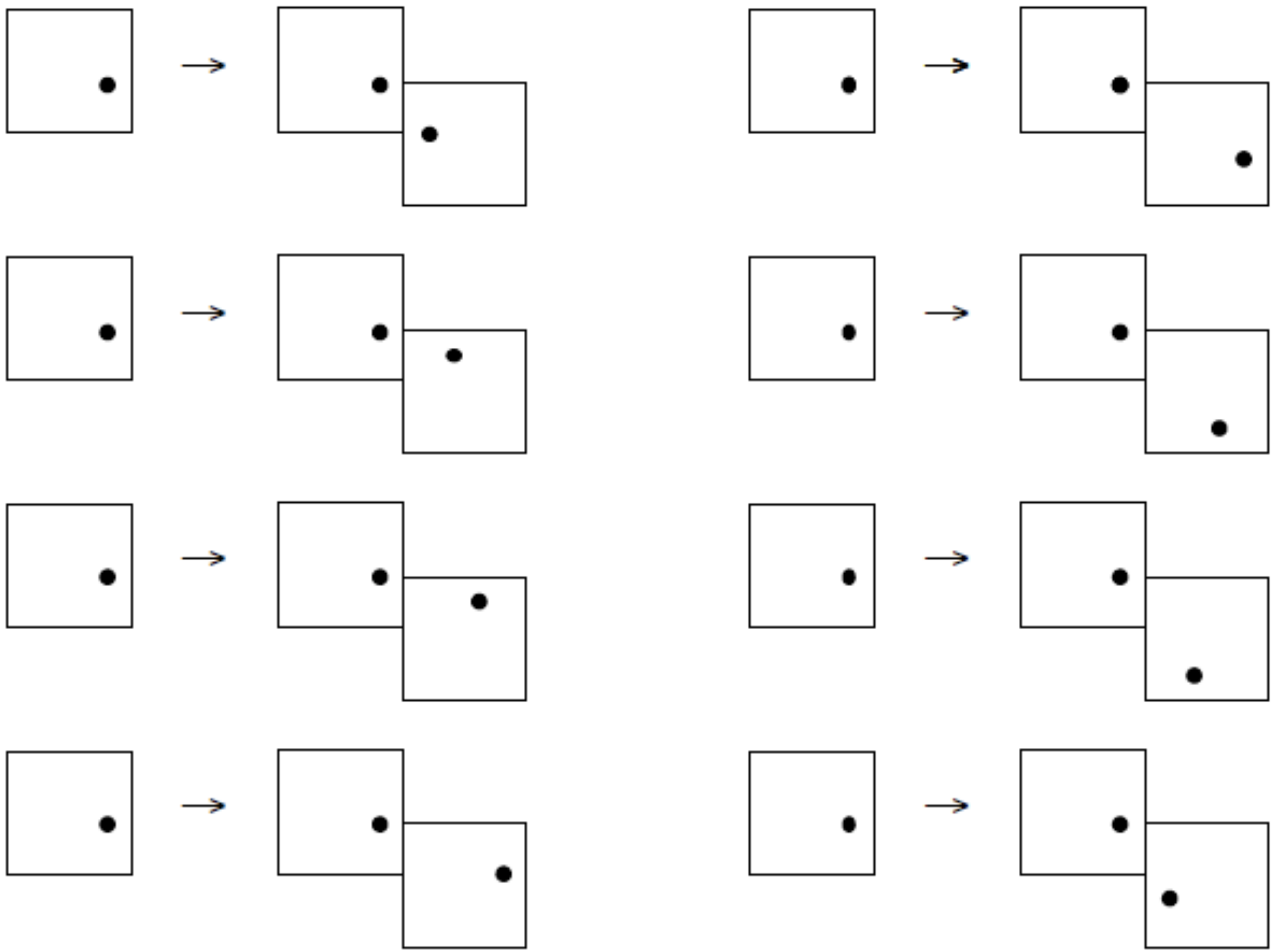


a shape relation and **a corresponding rule**

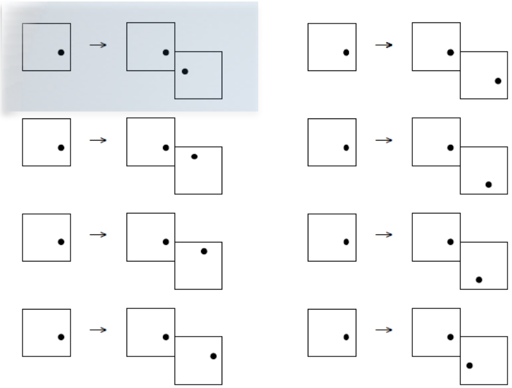
consider the symmetry of a square

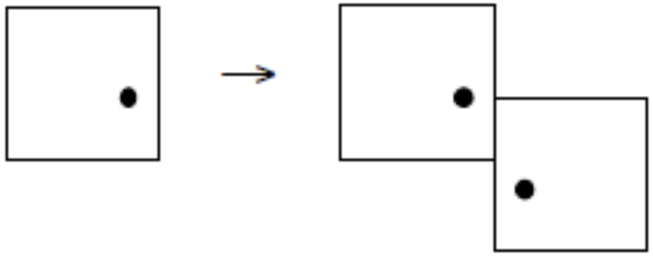


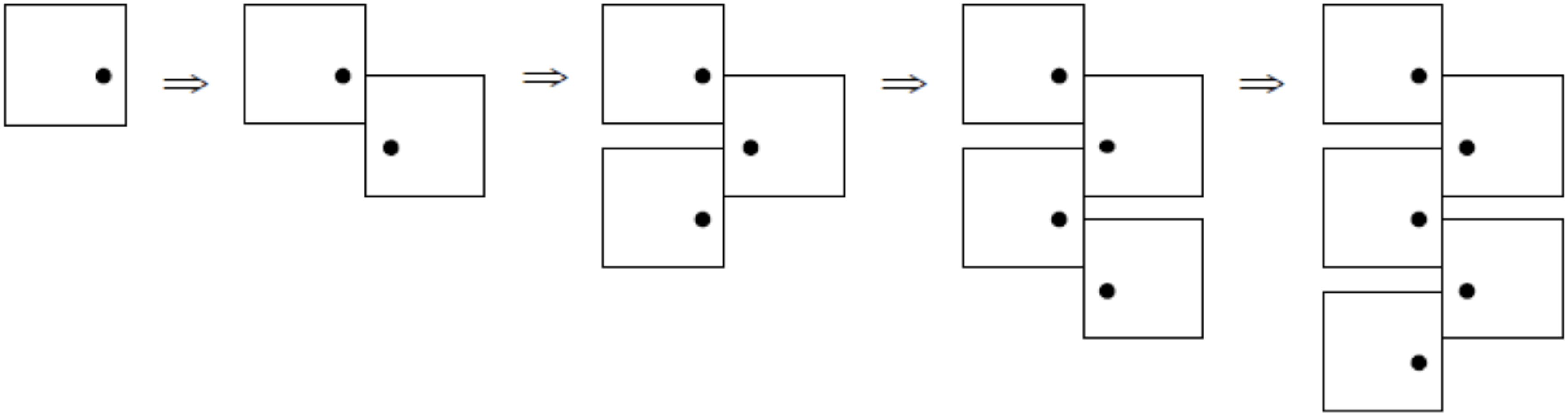
some possible labeling positions for a square



labeled rules

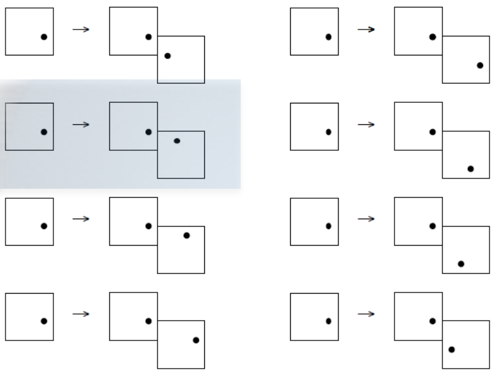


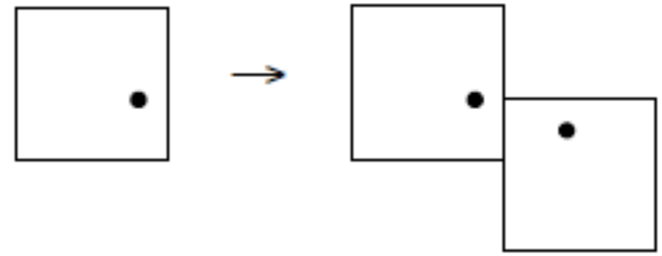


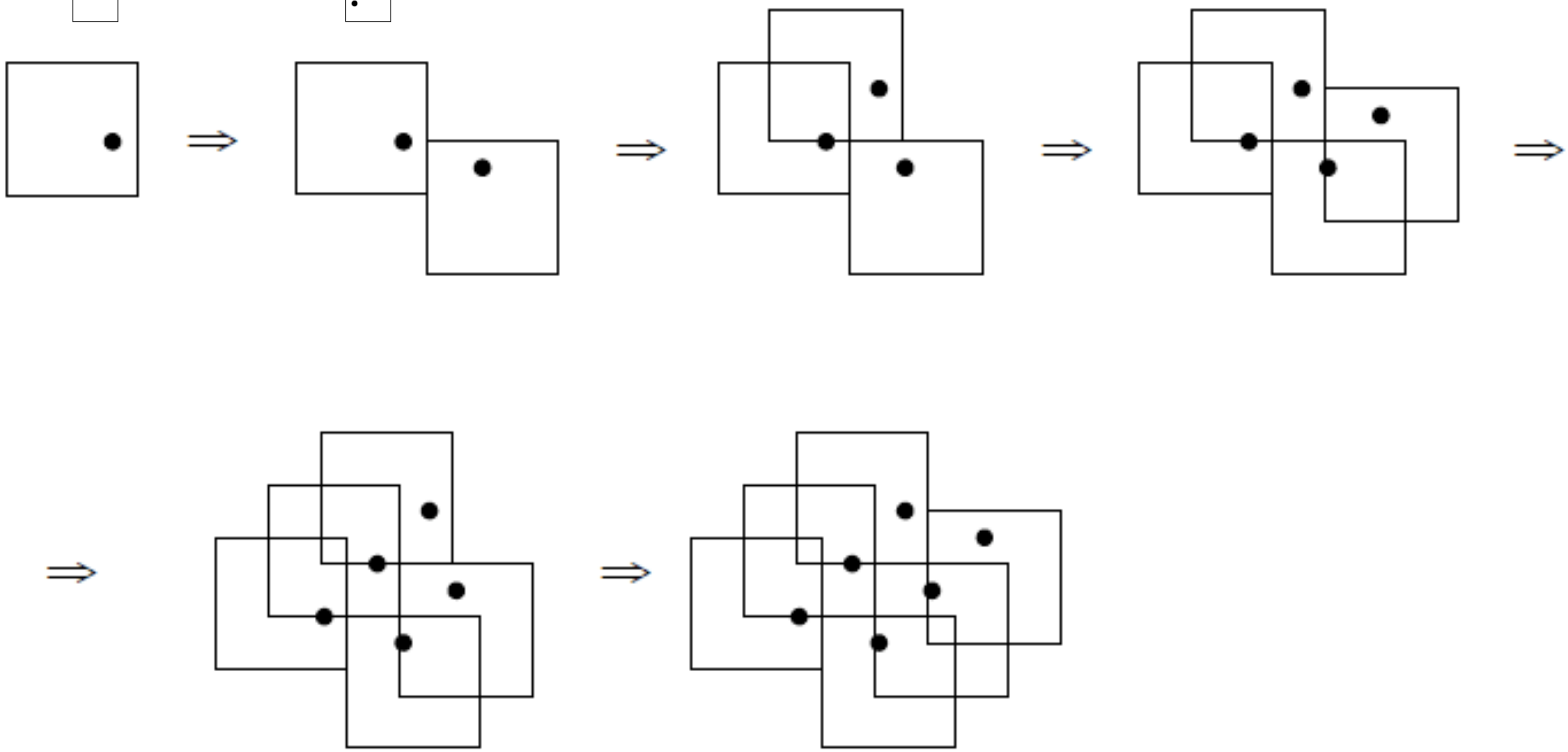


and **sample derivations**

a labeled rule

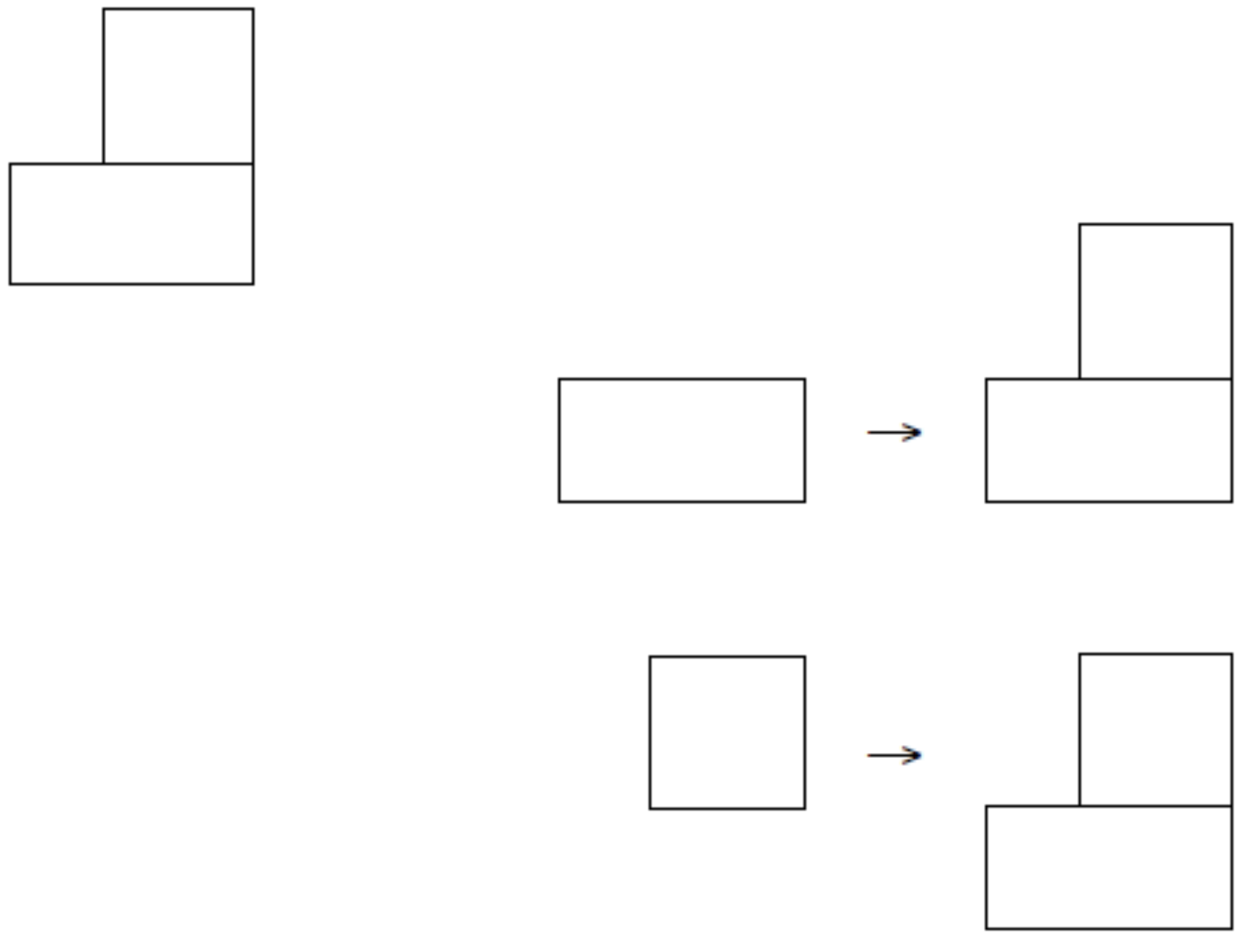




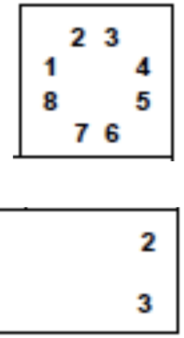


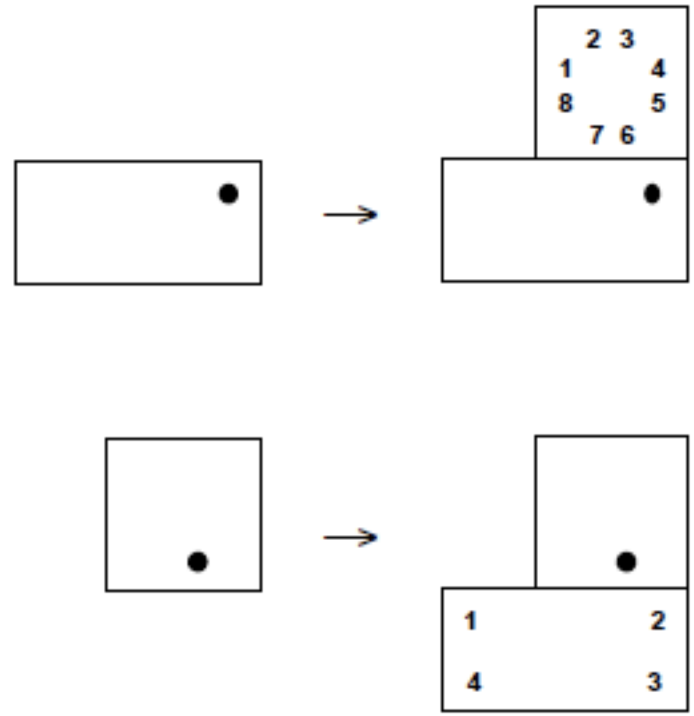
and **sample derivations**

another labeled rule



another shape relation and **two corresponding rules**



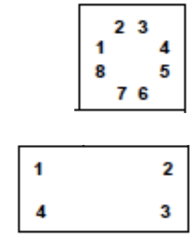


consider the symmetry of a square

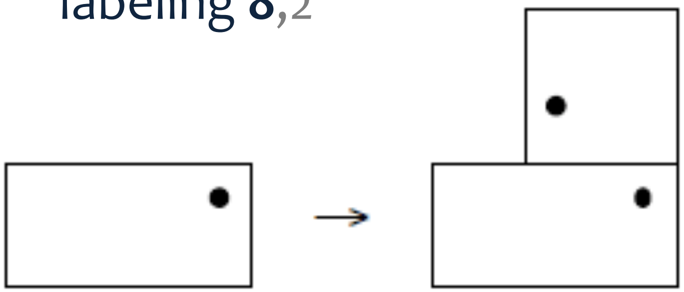
and the rectangle

some possible **labeling positions**

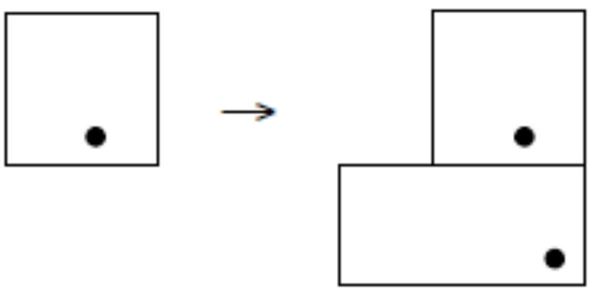
and possible **shape rules**



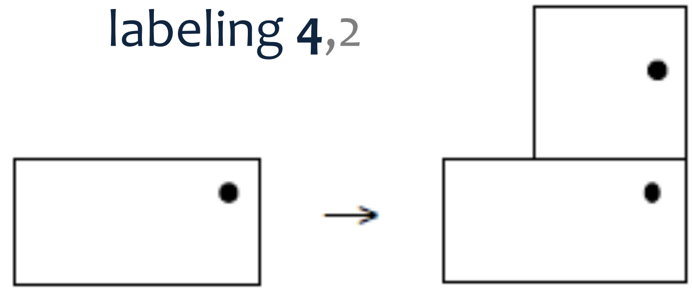
labeling **8**,2



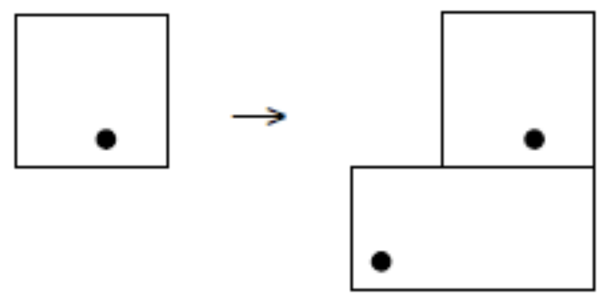
labeling 6,**3**



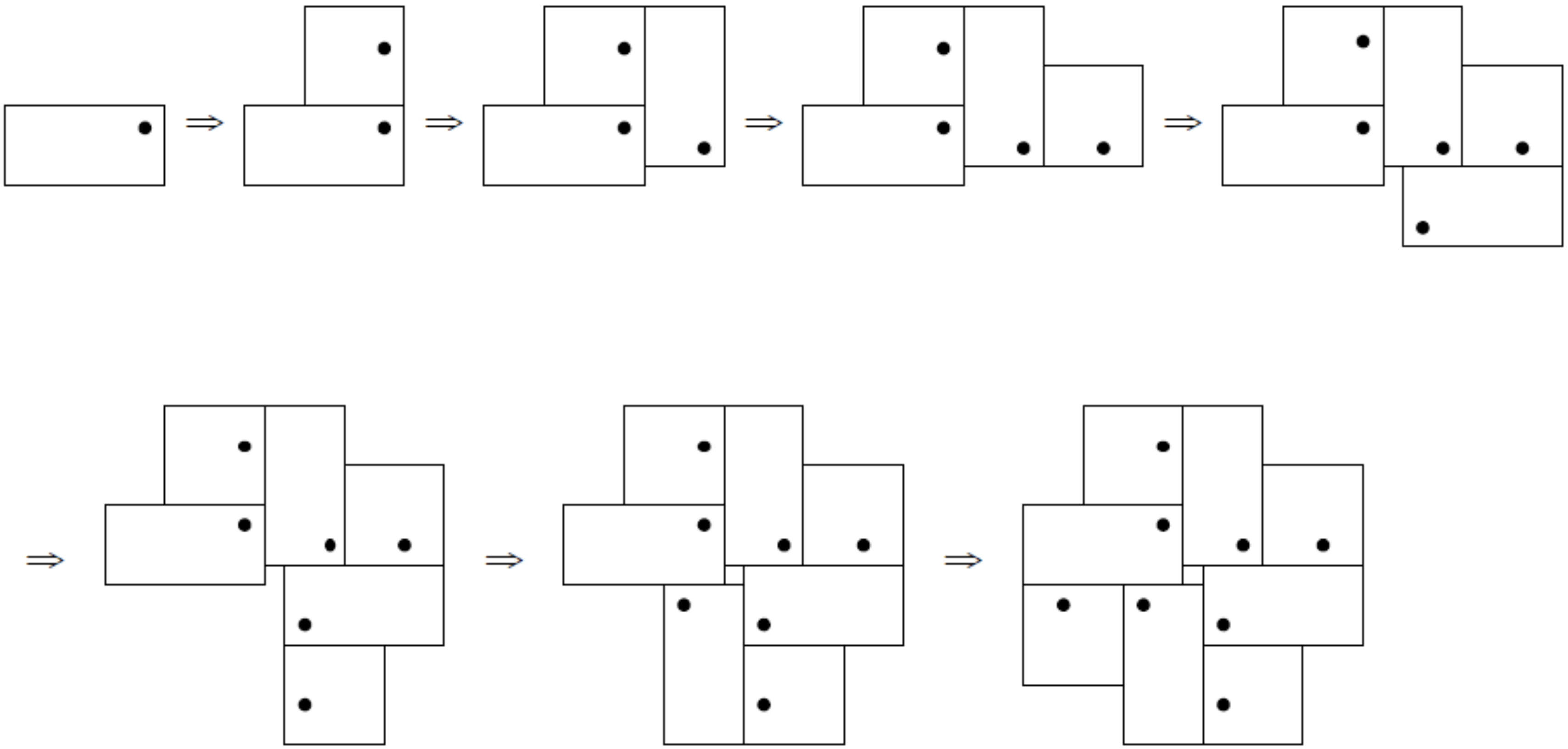
labeling **4**,2



labeling 6,**4**



example **labeling**



derivation using rule with 4,4 labeling inspiration

every house worth considering as a work of art
must have a grammar of its own

"grammar" in this sense, means the same thing in any
construction - whether it be of words or of stone or wood

the worlds we study can be understood by capturing underlying relationships

that enter into the construction of the thing

the "grammar" of the house is the manifest articulation of
its parts. this will be the "speech" it uses

to be achieved, *construction must be grammatical*

frank lloyd wright *The Natural House*