48-175
Descriptive Geometry

Basic Concepts of Descriptive Geometry

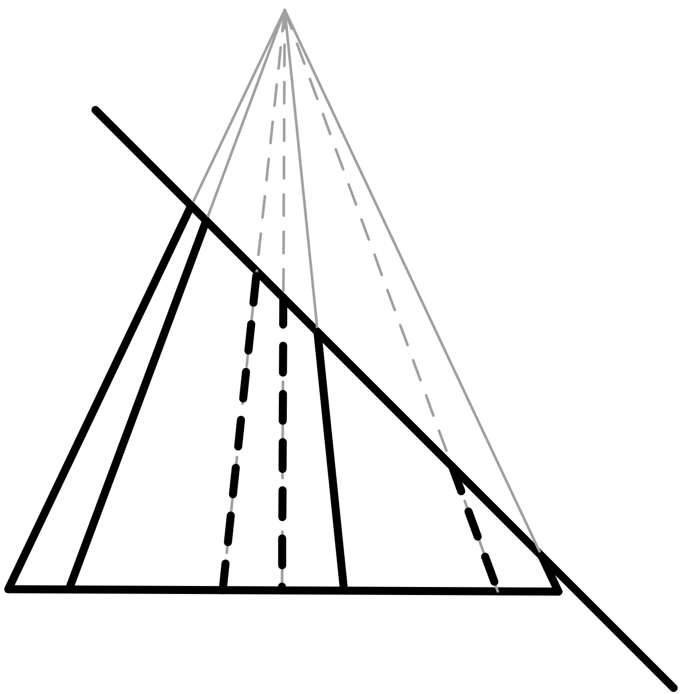
Descriptive geometry

is about

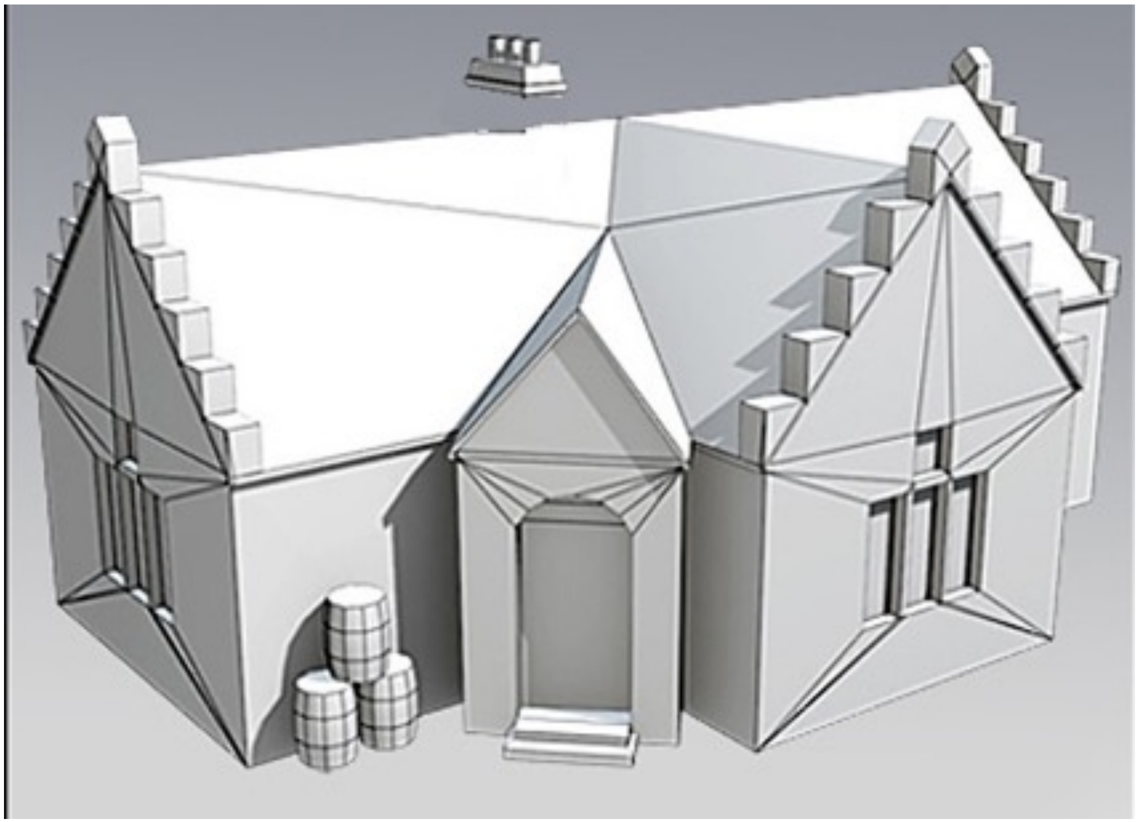
manually solving *problems* in three-dimensional geometry

by generating two-dimensional VIEWS

What do we see in these directions?



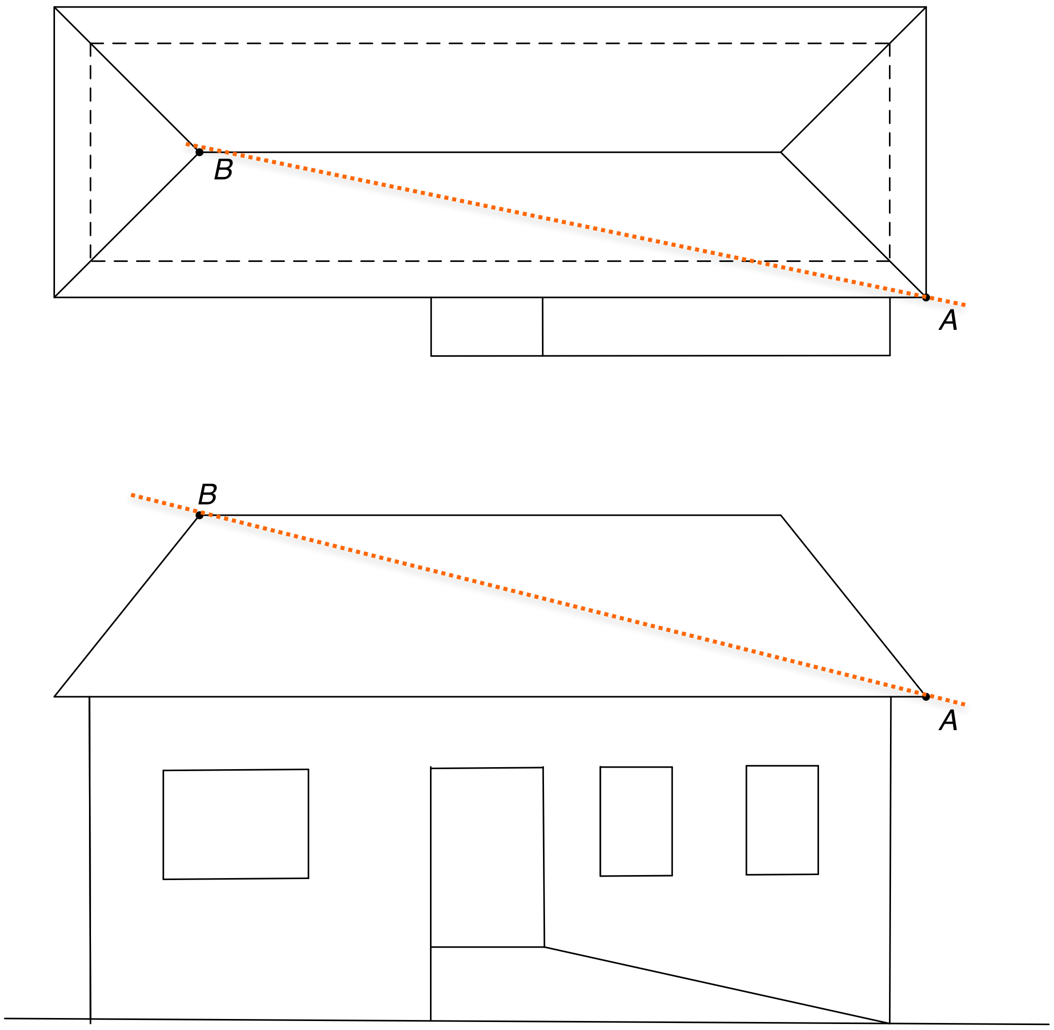
- truncated pyramid - what do we see?



where do the roof planes meet?

- where does the chimney meet the roof?

- lighthouse problems



aligning two points

Descriptive geometry

is about

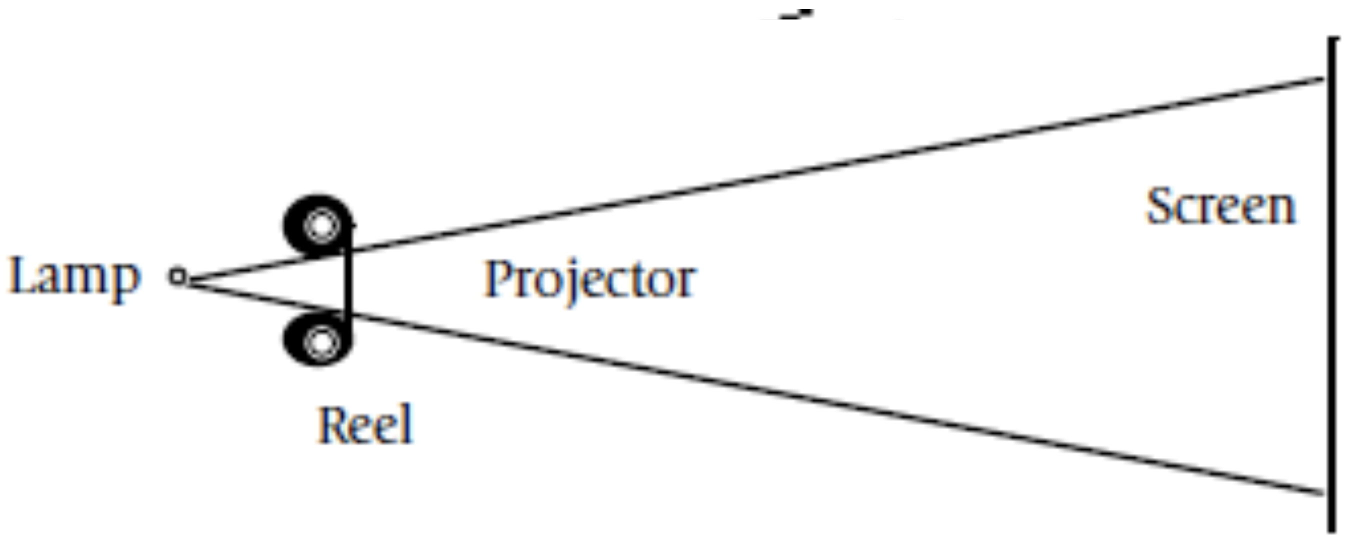
manually solving problems in three-dimensional geometry

by generating **two-dimensional VIEWS**

View

is a *two dimensional picture* of geometric objects.

not any old picture, but, more precisely, a 'PROJECTION' of geometrical objects onto a planar surface.



Projections

are MAPPINGS of 2- or 3-dimensional figures onto planes or 3-dimensional surfaces

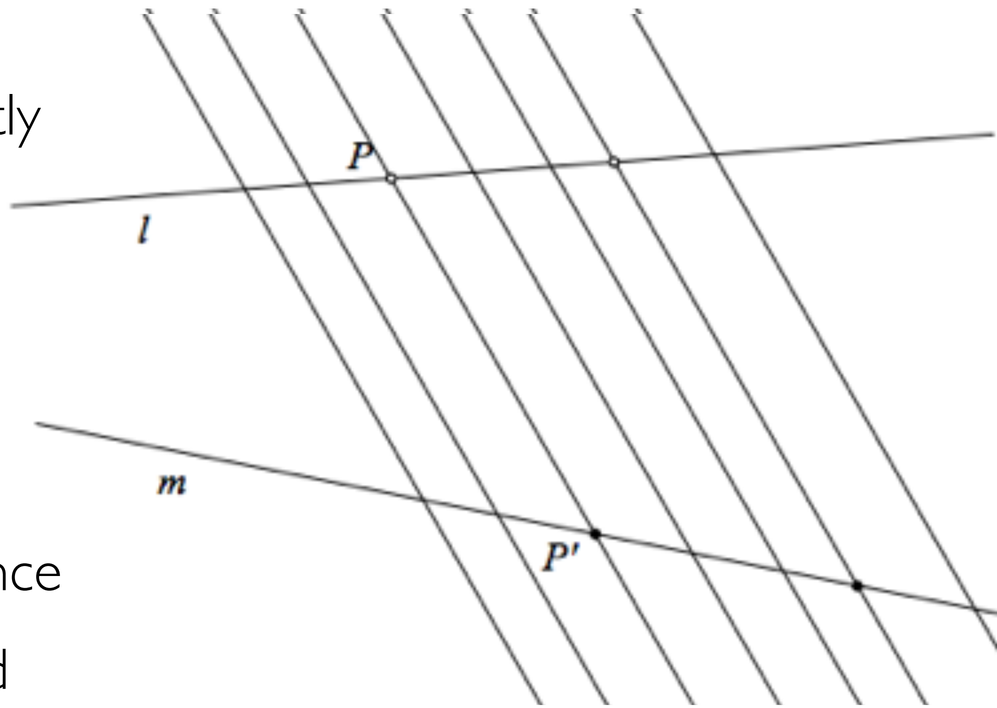
(for now we consider) an 'association' between points on an object and points on a plane, known as the PICTURE PLANE

this association— between a geometric figure and its IMAGE — is established by LINES from points on the figure to corresponding points on the image in the picture plane

these lines are referred to as PROJECTION LINES

Line Family – a set of parallel lines

UNIQUENESS – for any line family and any given point *P*, there is exactly one line in the family that passes through that point.

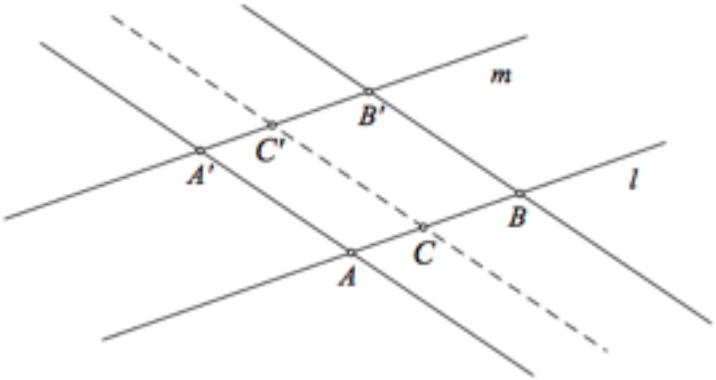


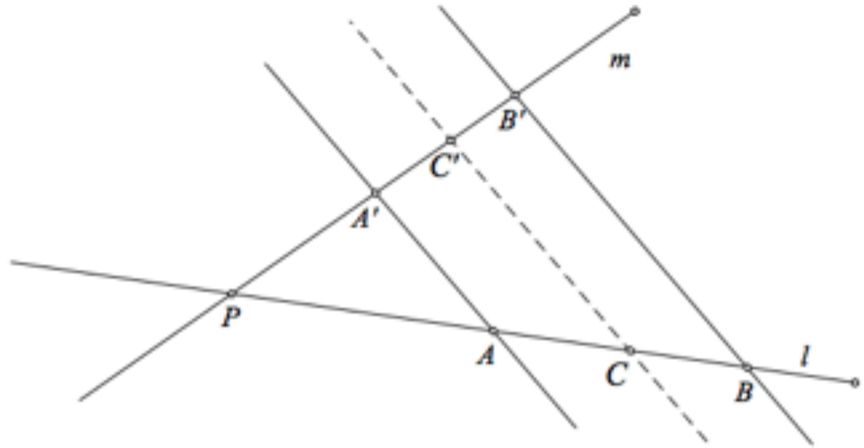
PROJECTION OF A LINE ONTO ANOTHER – is a 1-1 correspondence between the points on one line and the points on the other

P' is the ***image*** of *P* and vice versa

m is the ***projection*** of *l* and vice versa

- **more on projections**





A'B' = AB

ıngles formed

PA'/PA = PB'/PB

intersection points are identical in measure

parallel projections multiplies distances by a constant factor (could be 1)

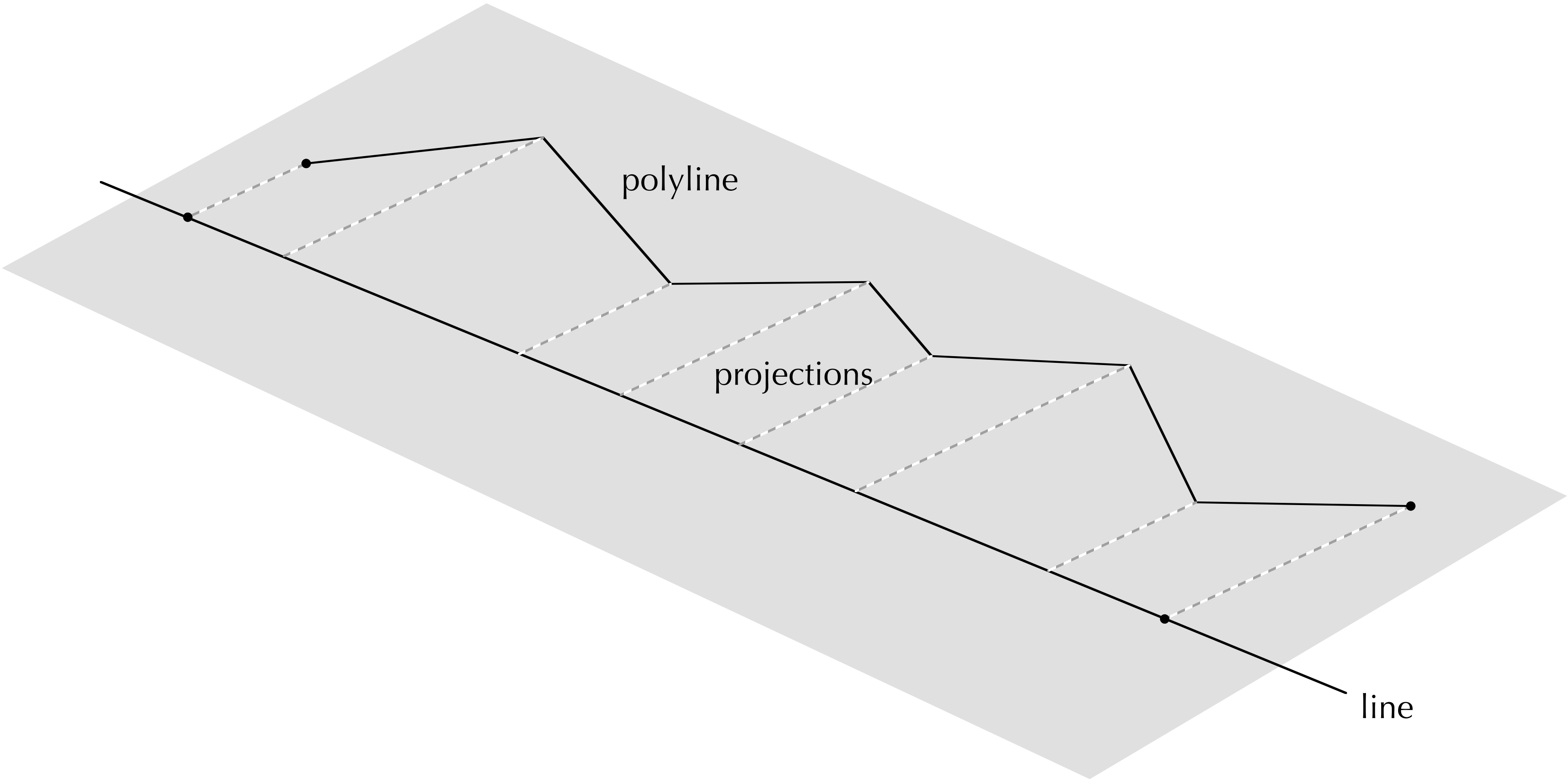
AC + CB = AB
A'C' + C'B' = A'B'

AC + CB = AB
A'C' + C'B' = A'B'

parallel projections preserves between-ness

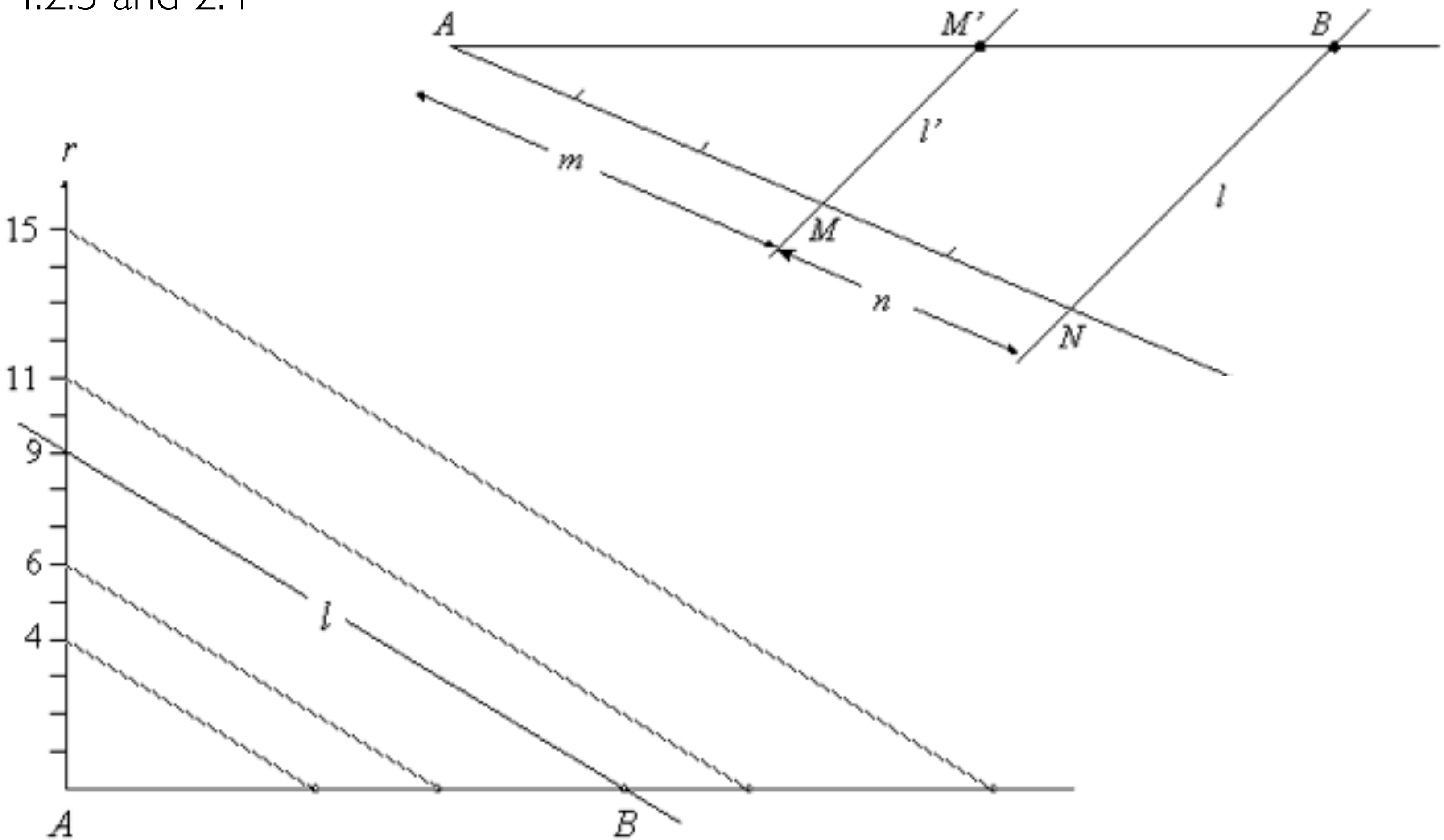
- **projection of a line onto another**

the ***sum of the projections*** of segments of a polyline onto a line equals the projection of the segment between ***the first and last end-points*** of the polyline



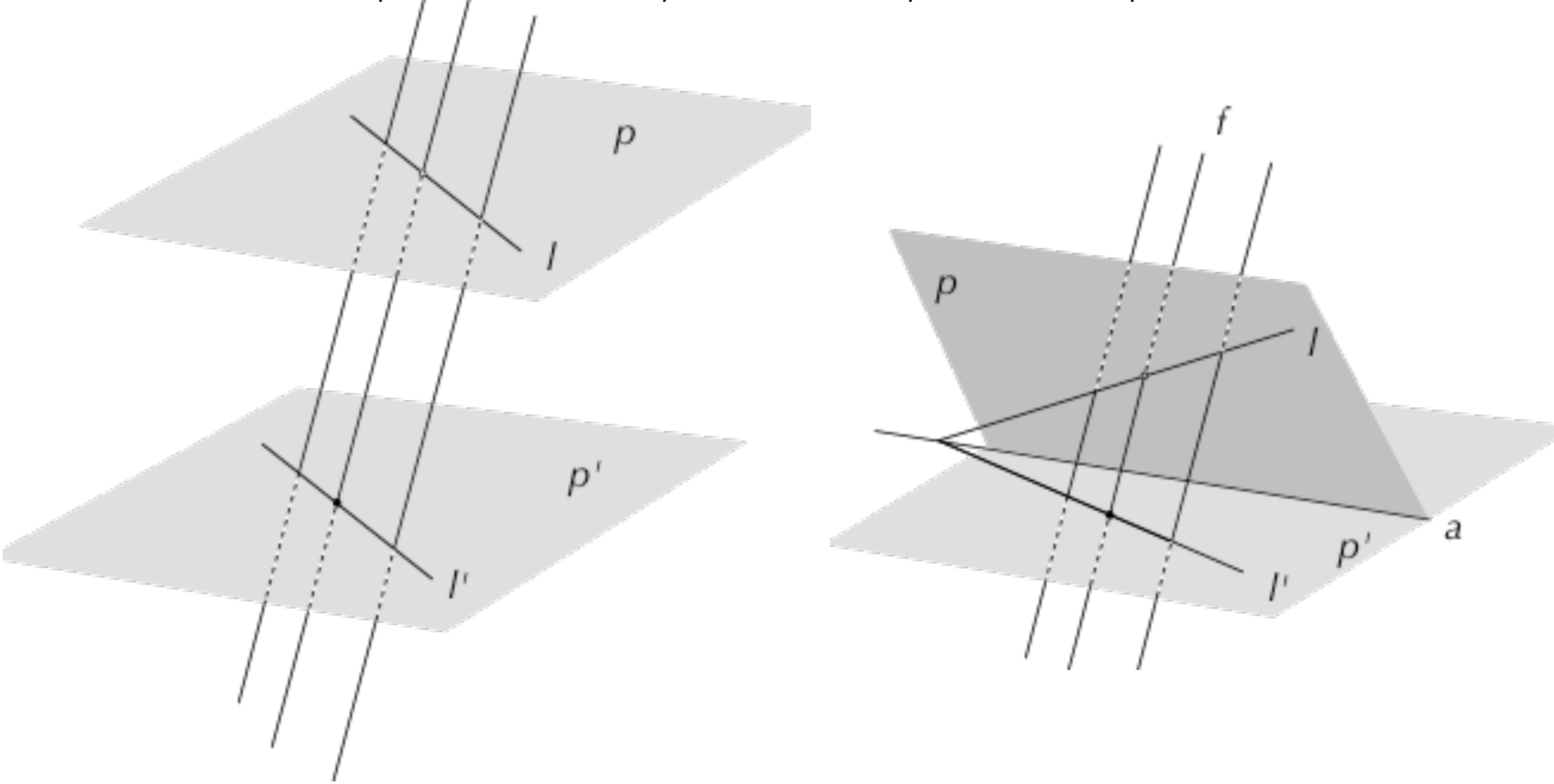
Dividing and extending a segment into an arbitrary number of given ratios

4:2:3 and 2:4

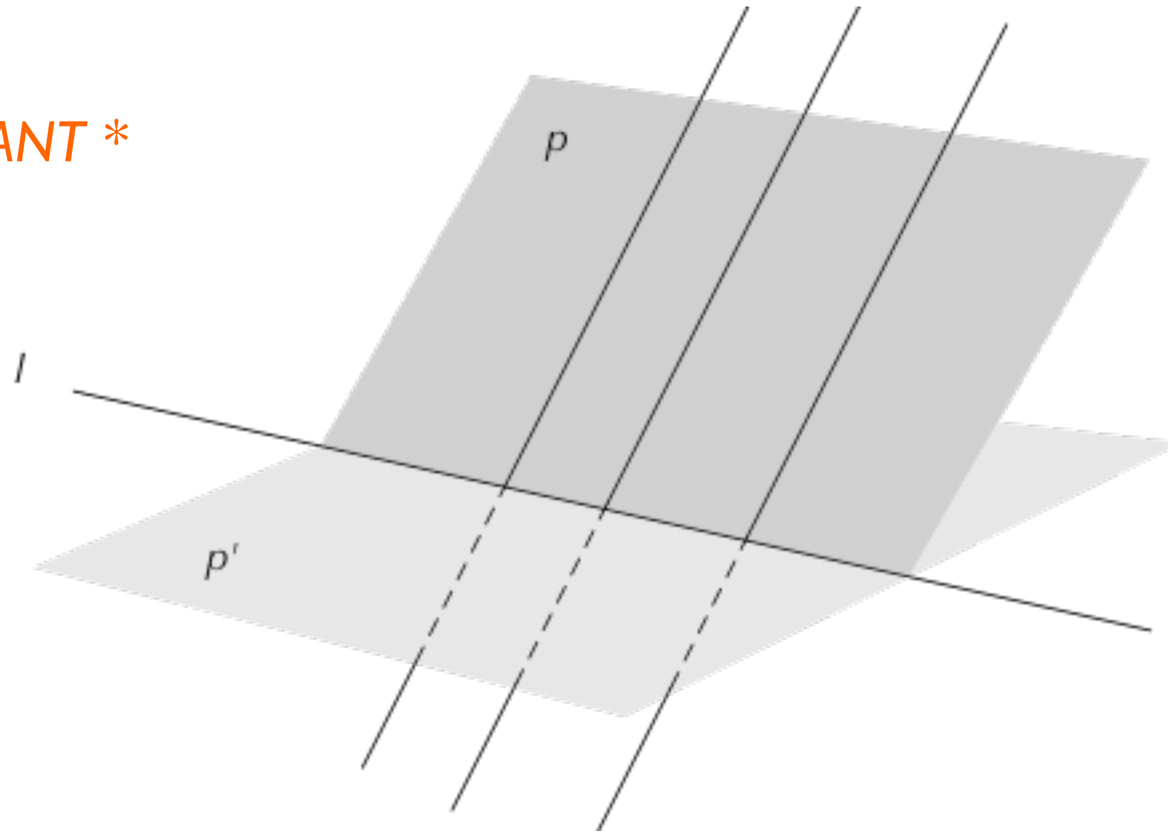


a 1-1 mapping between points on planes

- preserves between-ness between points and parallelism, concurrence and ratio of division between lines.
- distances are preserved only when the planes are parallel



* IMPORTANT *



The image of a parallel projection in a plane onto another plane is the line common to both planes

(THE LINE OF INTERSECTION OF THE PLANES)

Parallel projection between two planes maps

lines on lines, **segments on segments**, **rays on rays** etc.;

that is, it maps linear figures on linear figures of the same type AND it maps

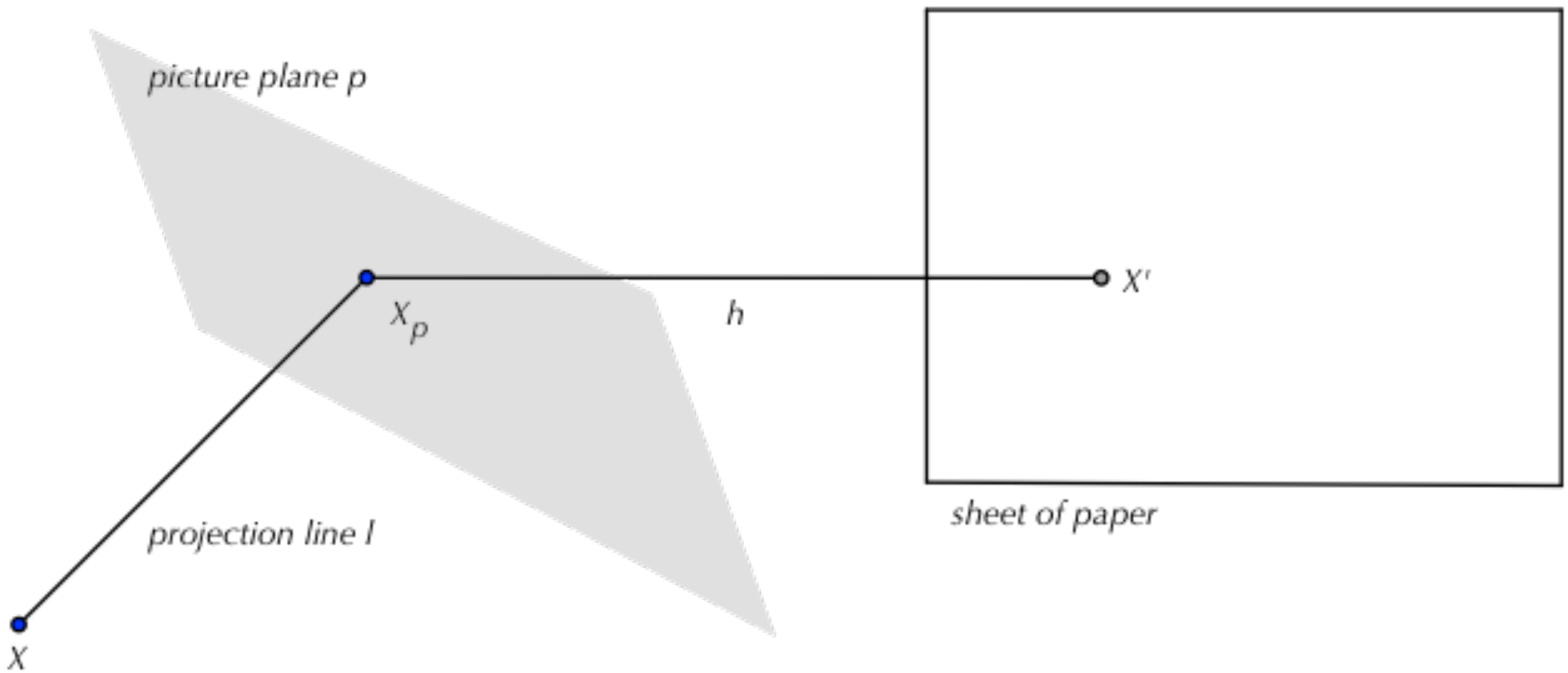
parabolas on **parabolas, hyperbolas** on **hyperbolas**,

circles or **ellipses** on **circles** or **ellipses**, and,

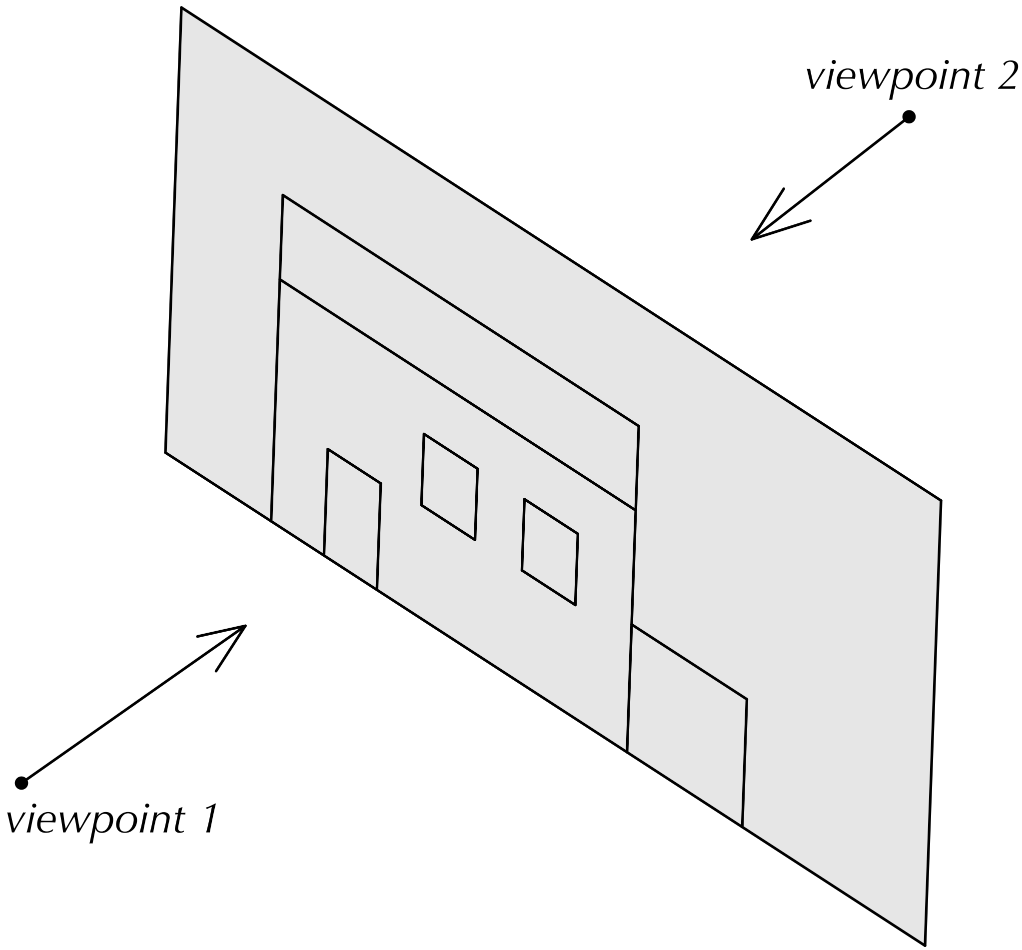
more generally,

curves of degree *n* on **curves of degree *n***

- **not all points have to be projected**

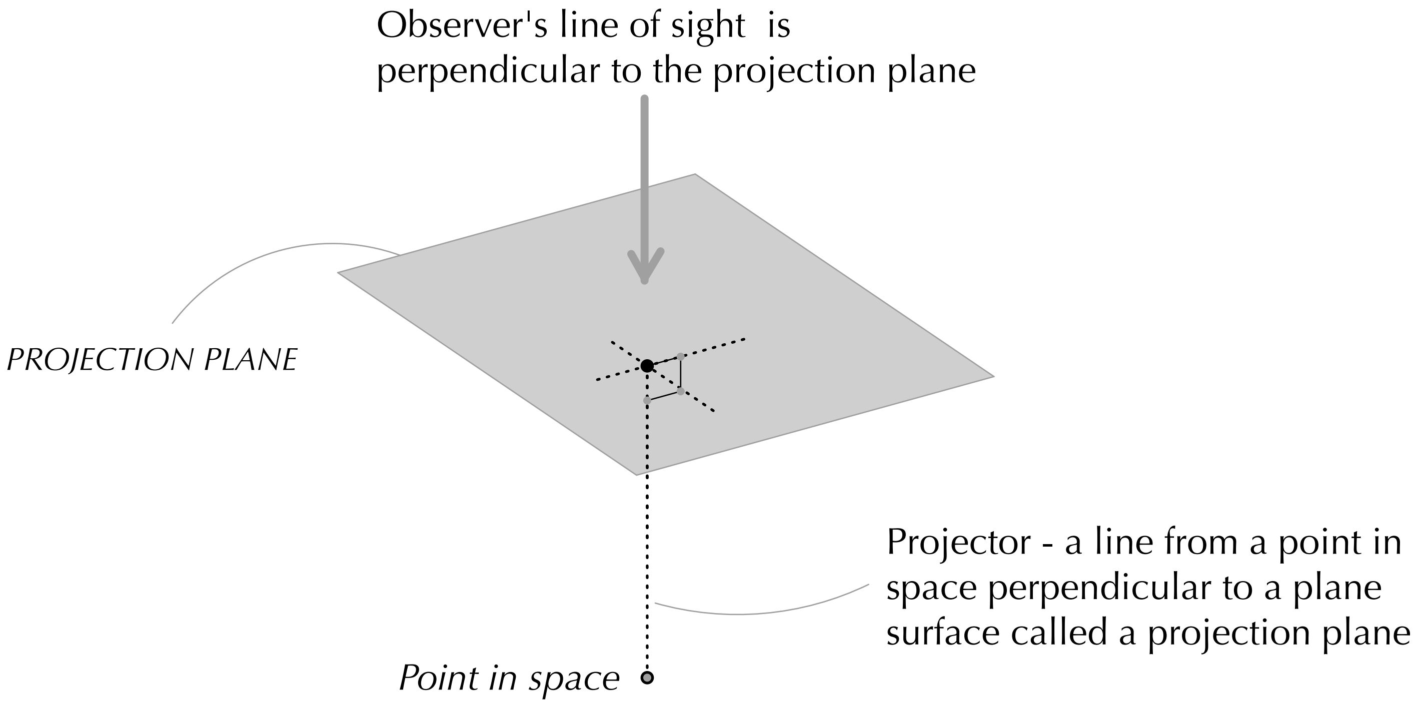


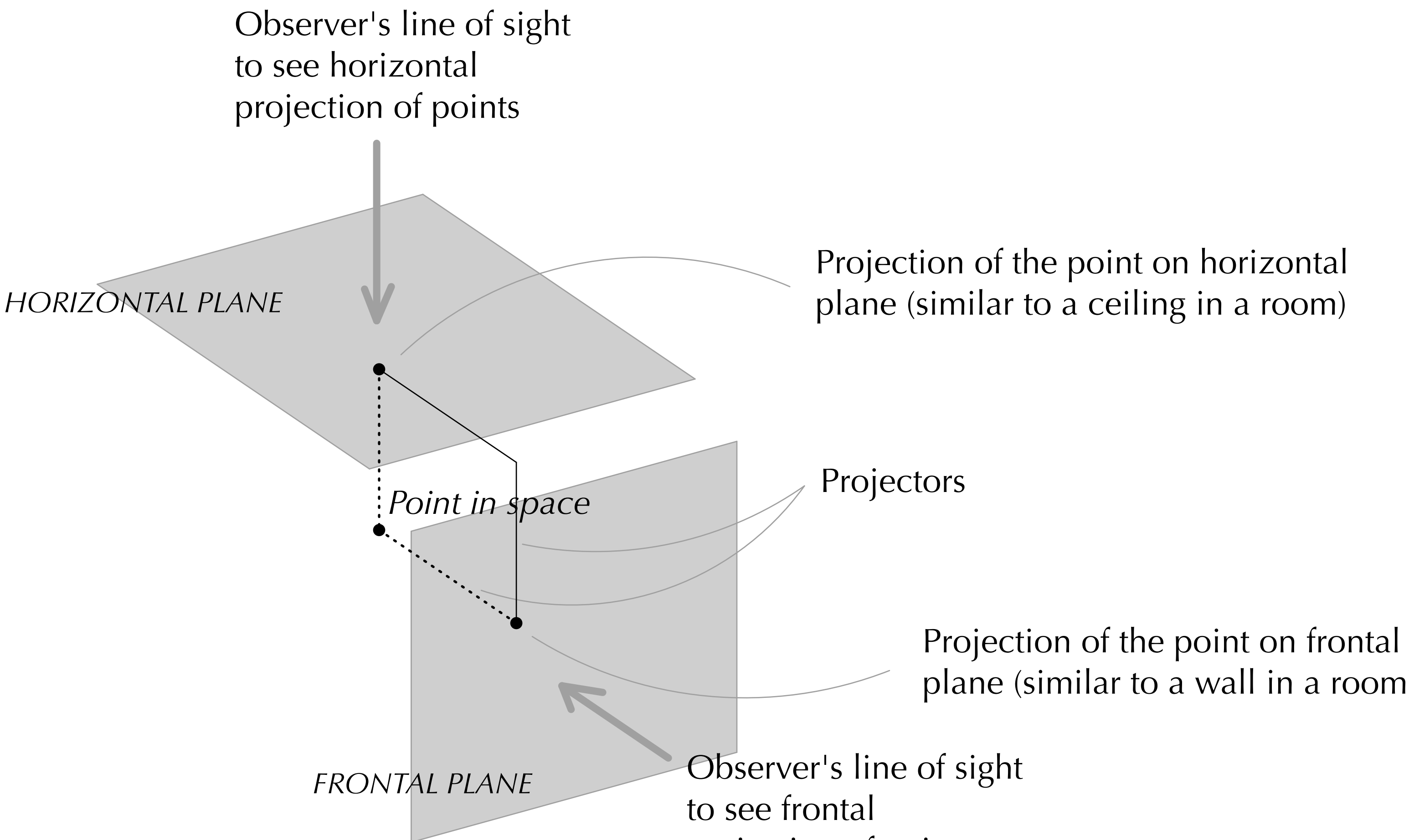
- connecting a projection to ‘paper’



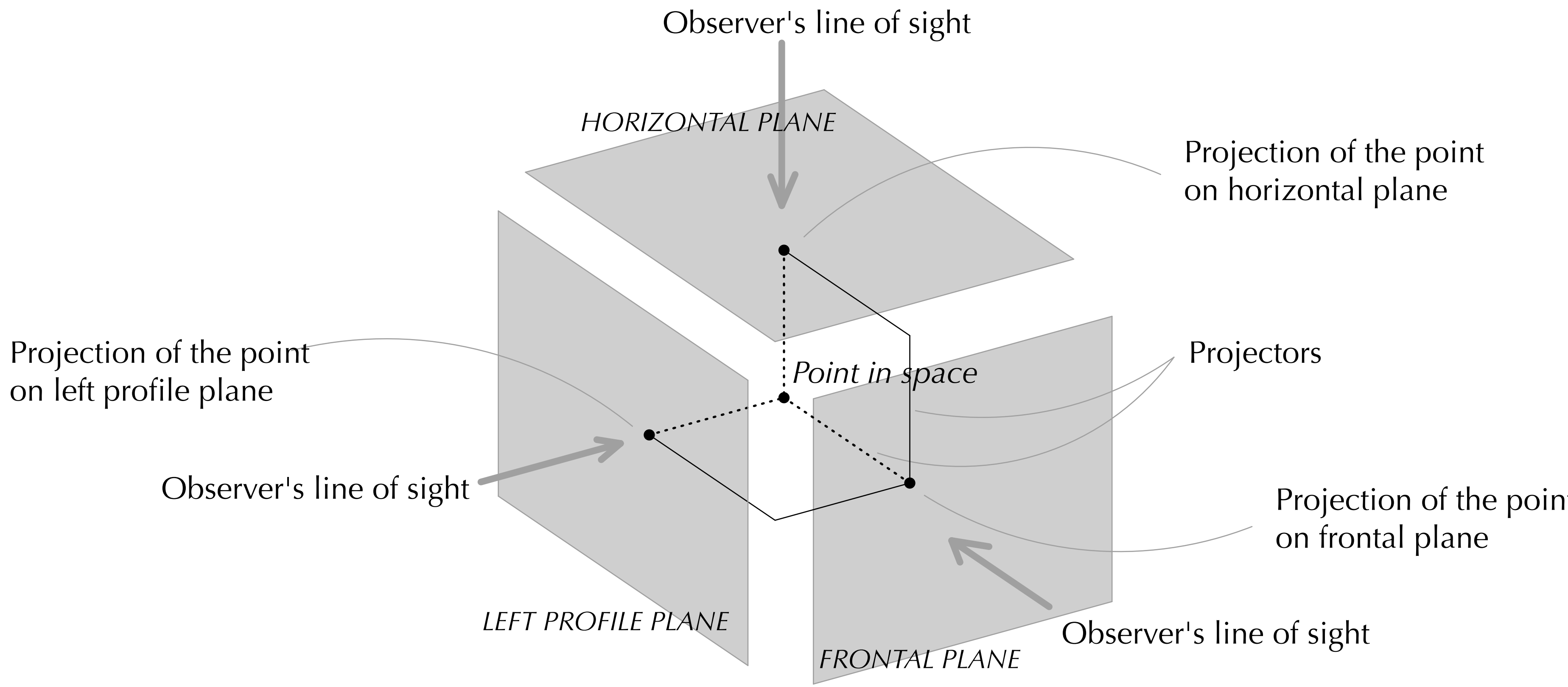
- two ways of viewing a picture plane

A parallel projection of a figure onto a plane is an ***ORTHOGRAPHIC PROJECTION*** if the projection lines are ***NORMAL*** (perpendicular) to the plane (also called the ***PICTURE PLANE***)

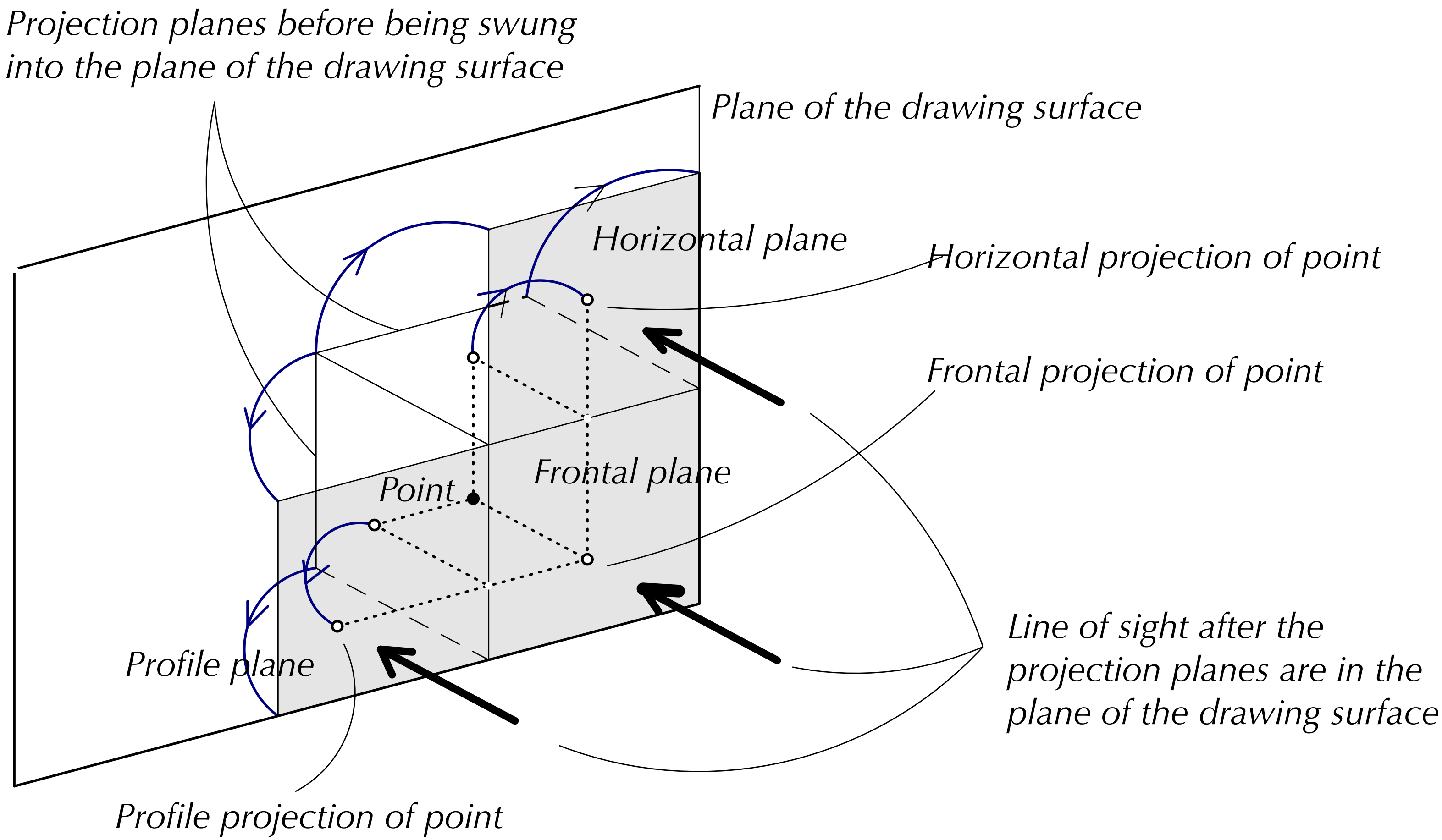




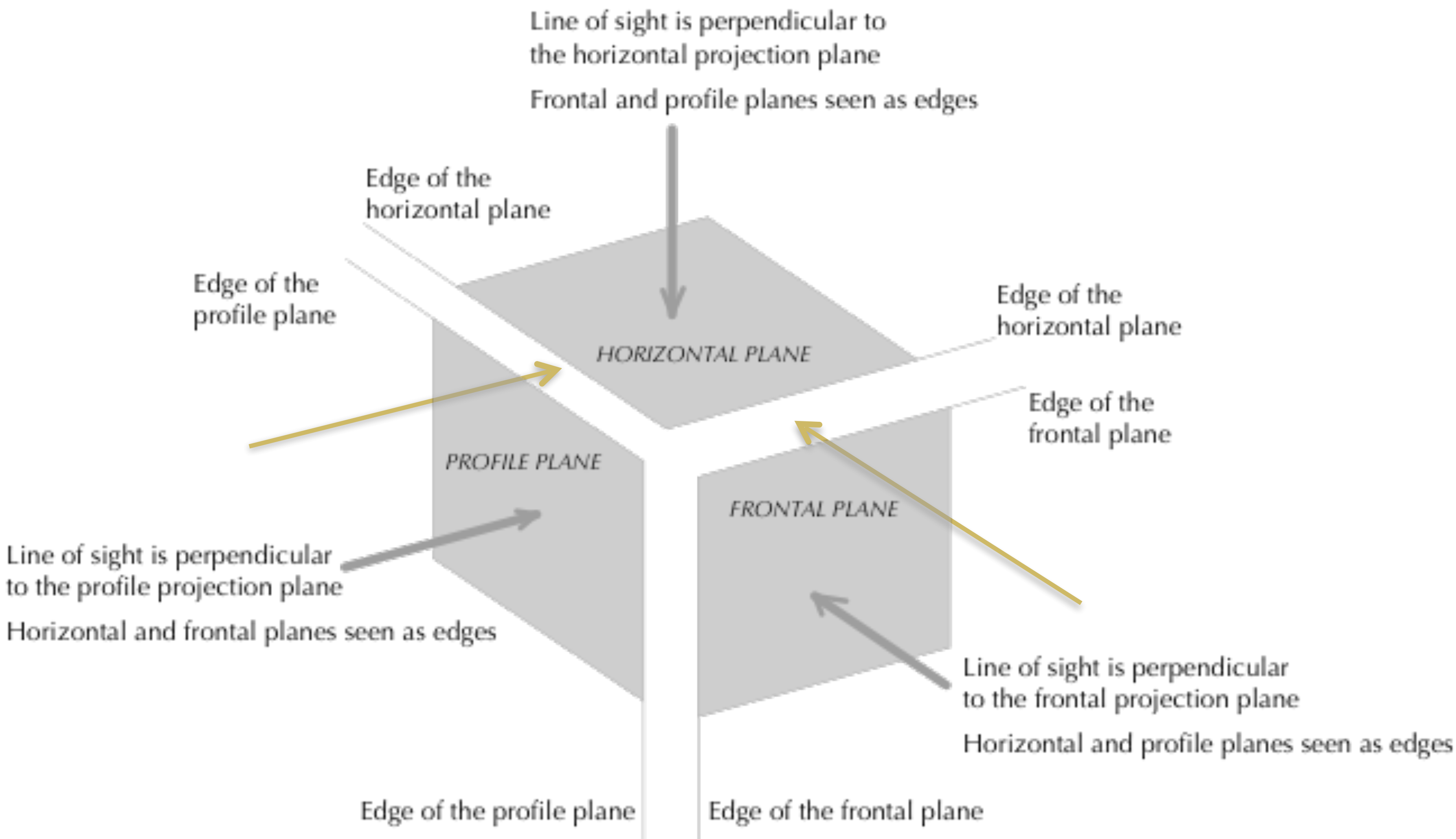
top and front view



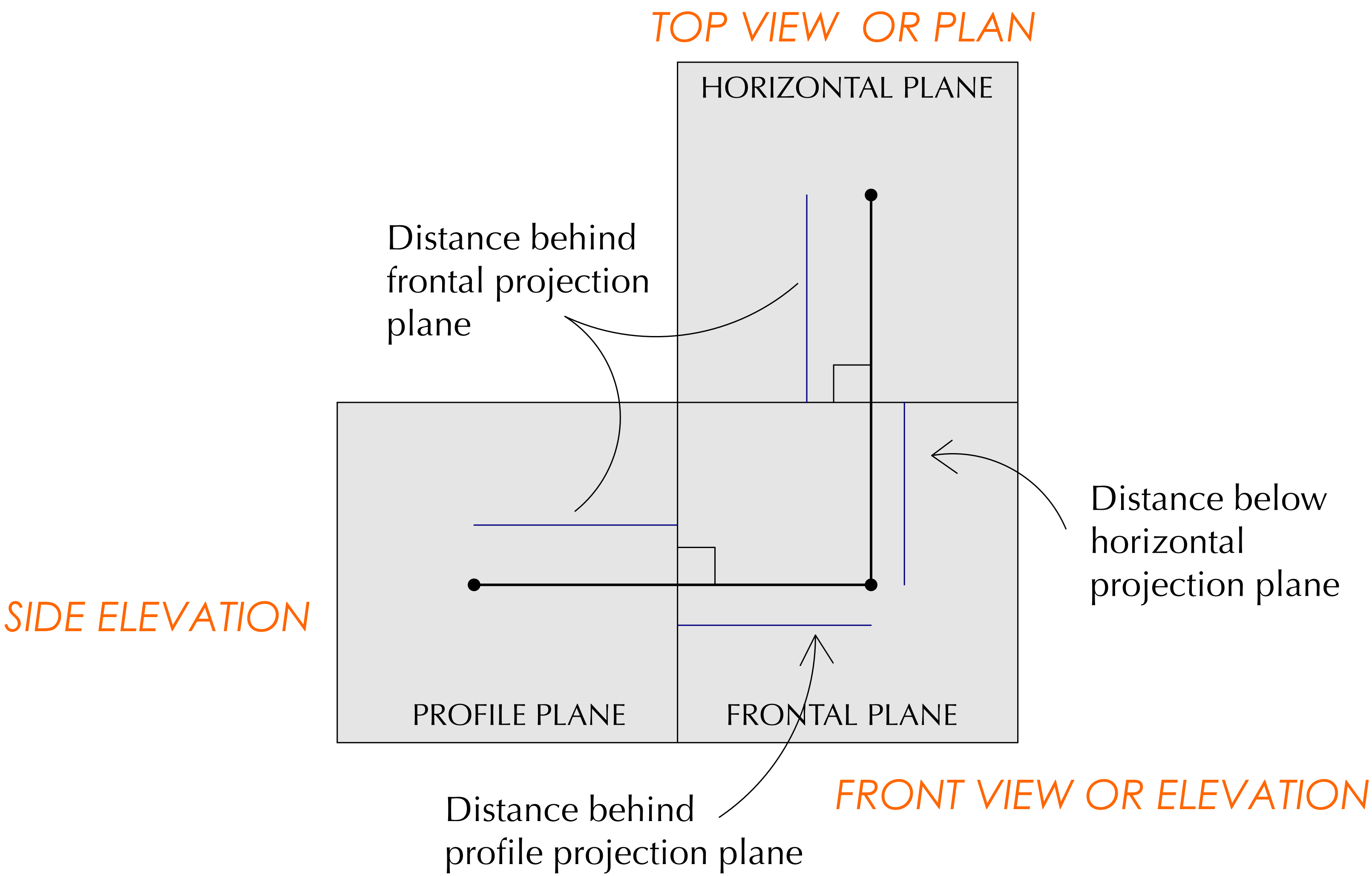
top, front and side view



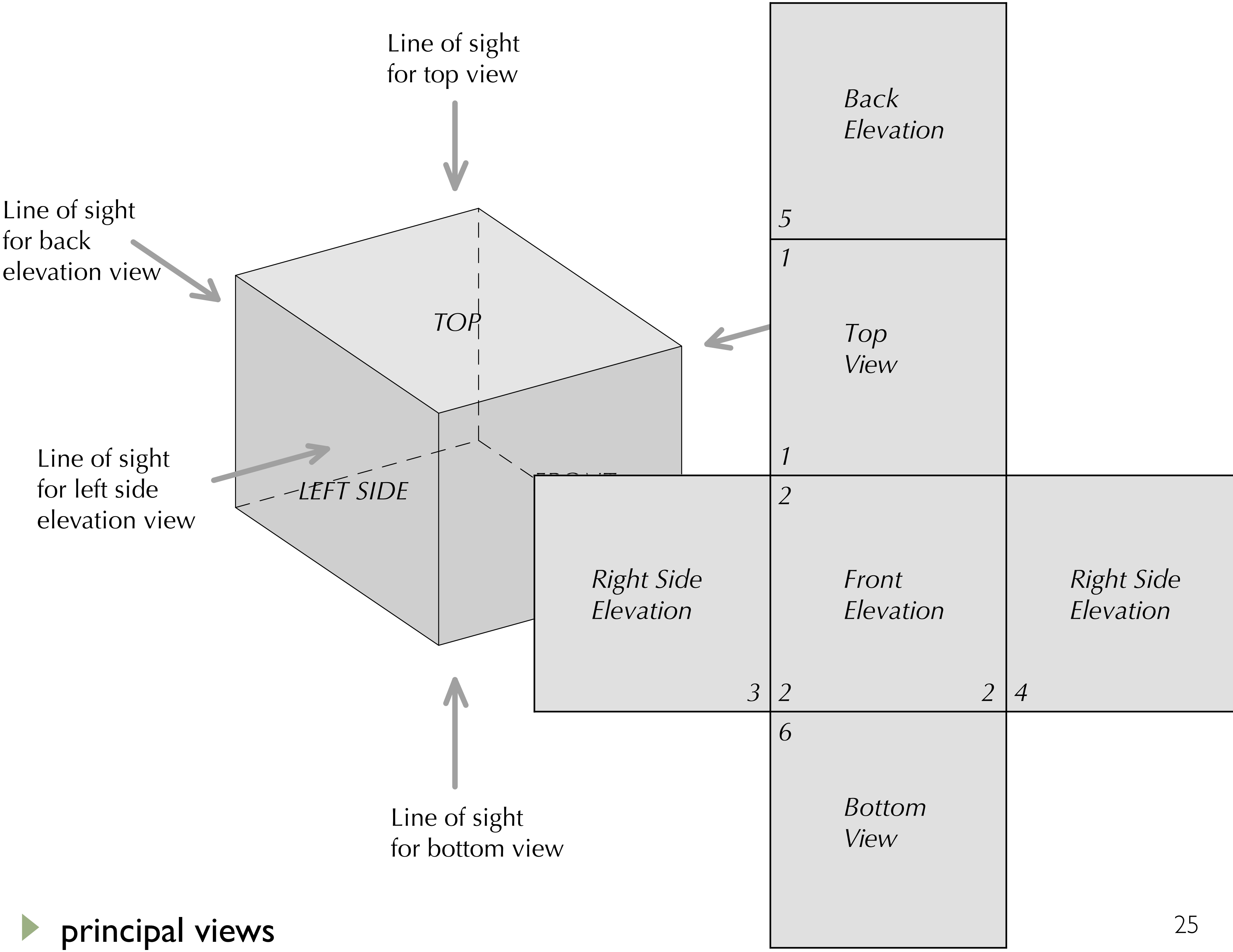
unfolding the views onto a single drawing surface



- visualizing picture planes in other views

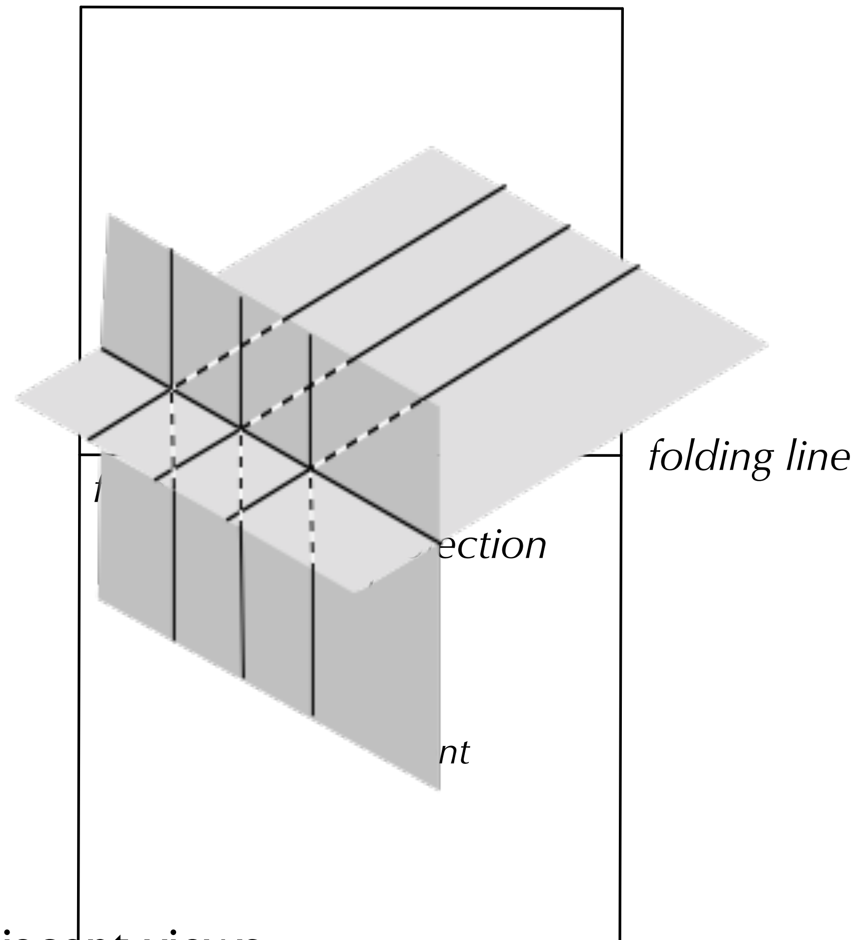


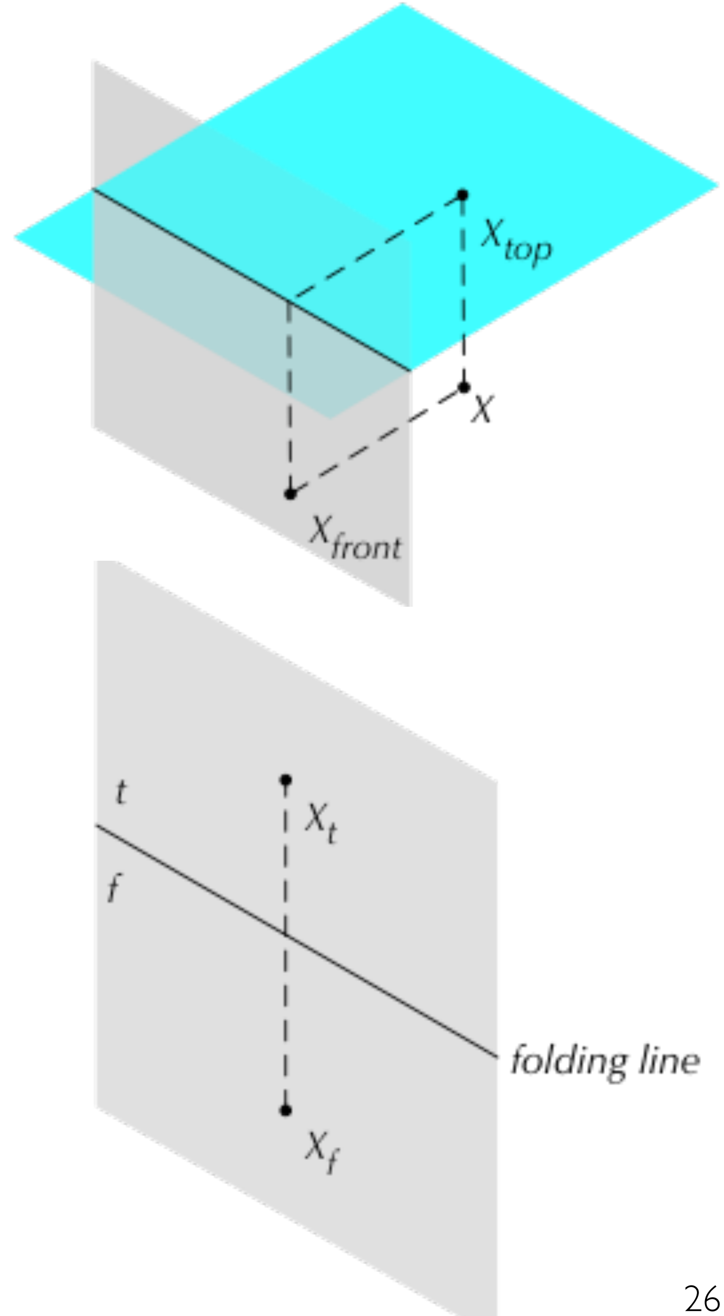
individual views

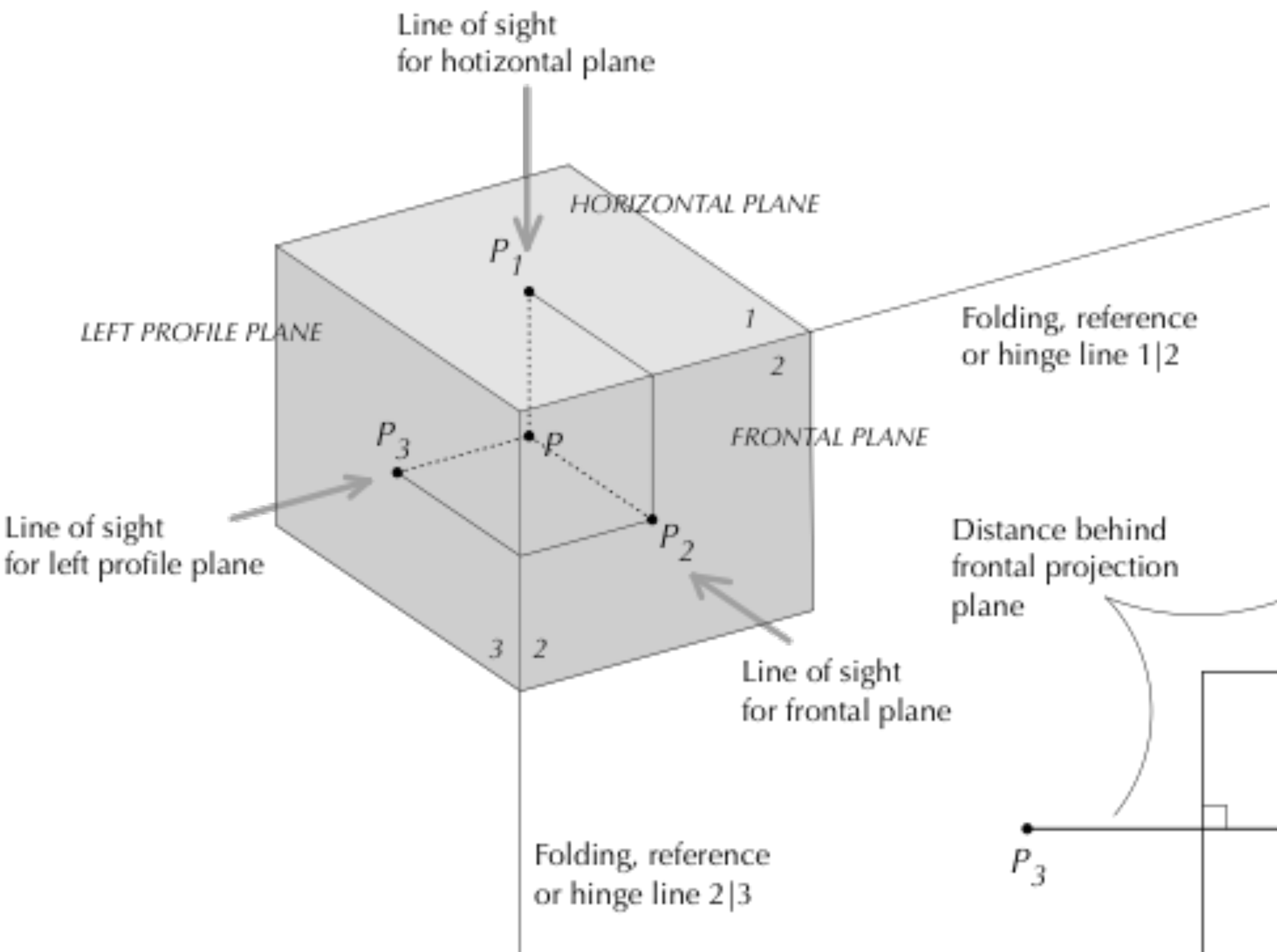


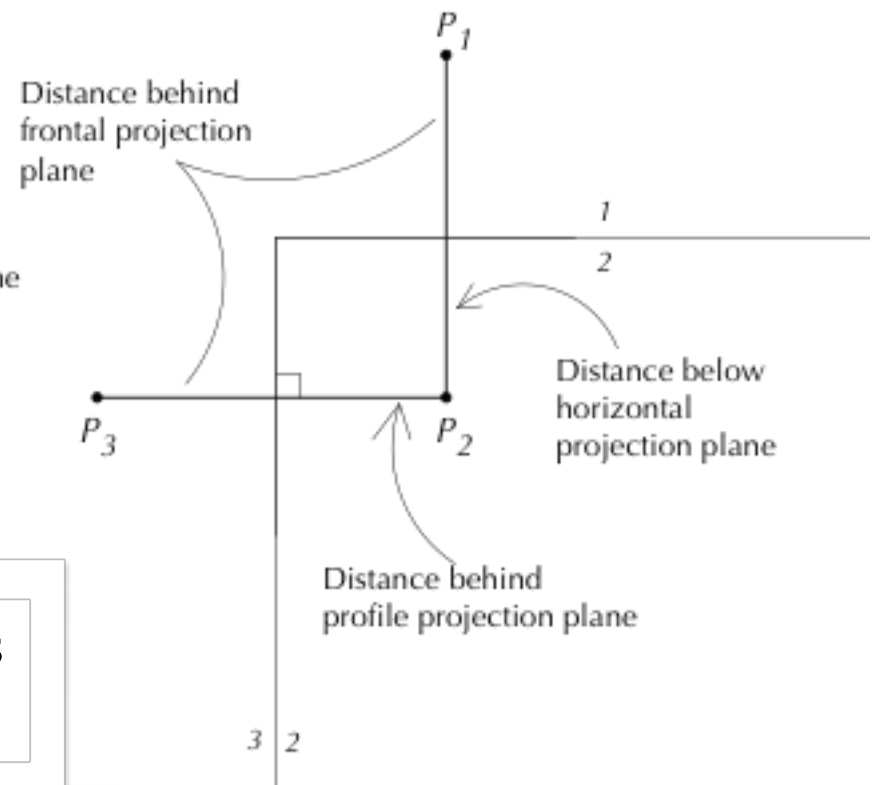
principal views

two orthographic views obtained from two perpendicular picture planes are called ADJACENT



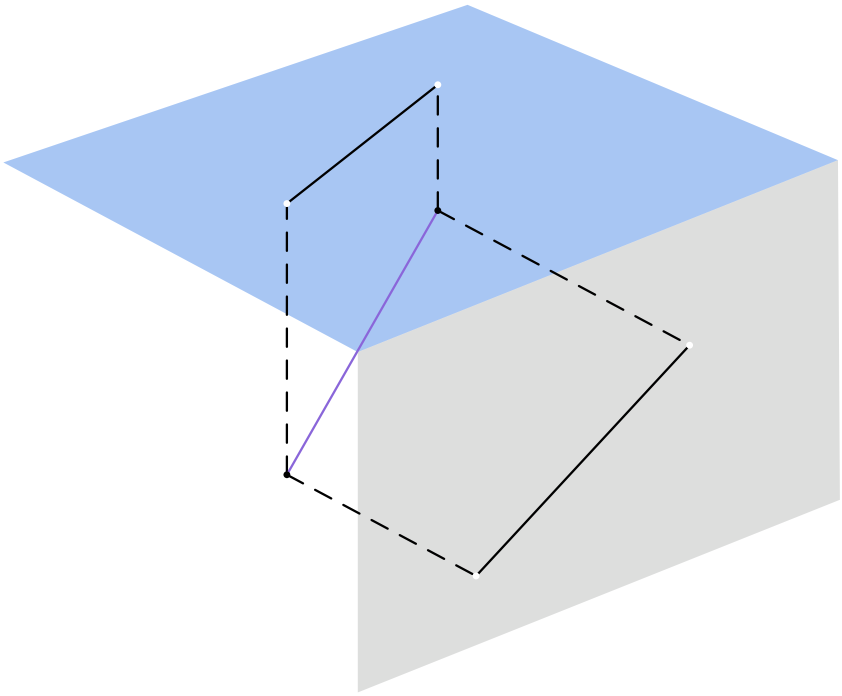


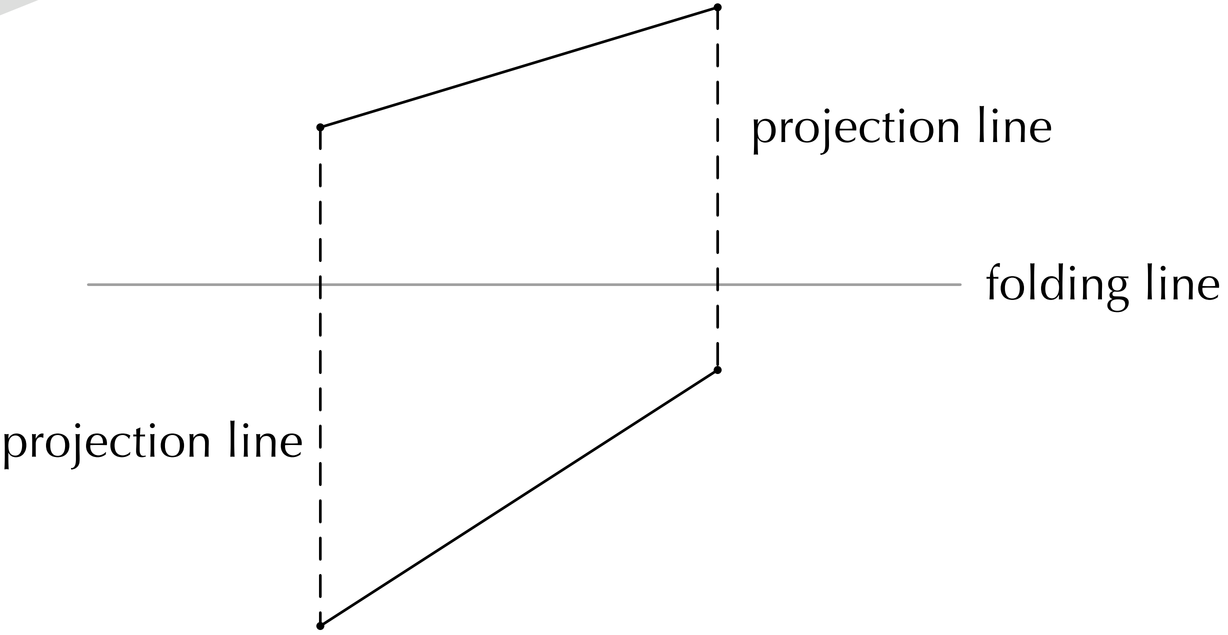




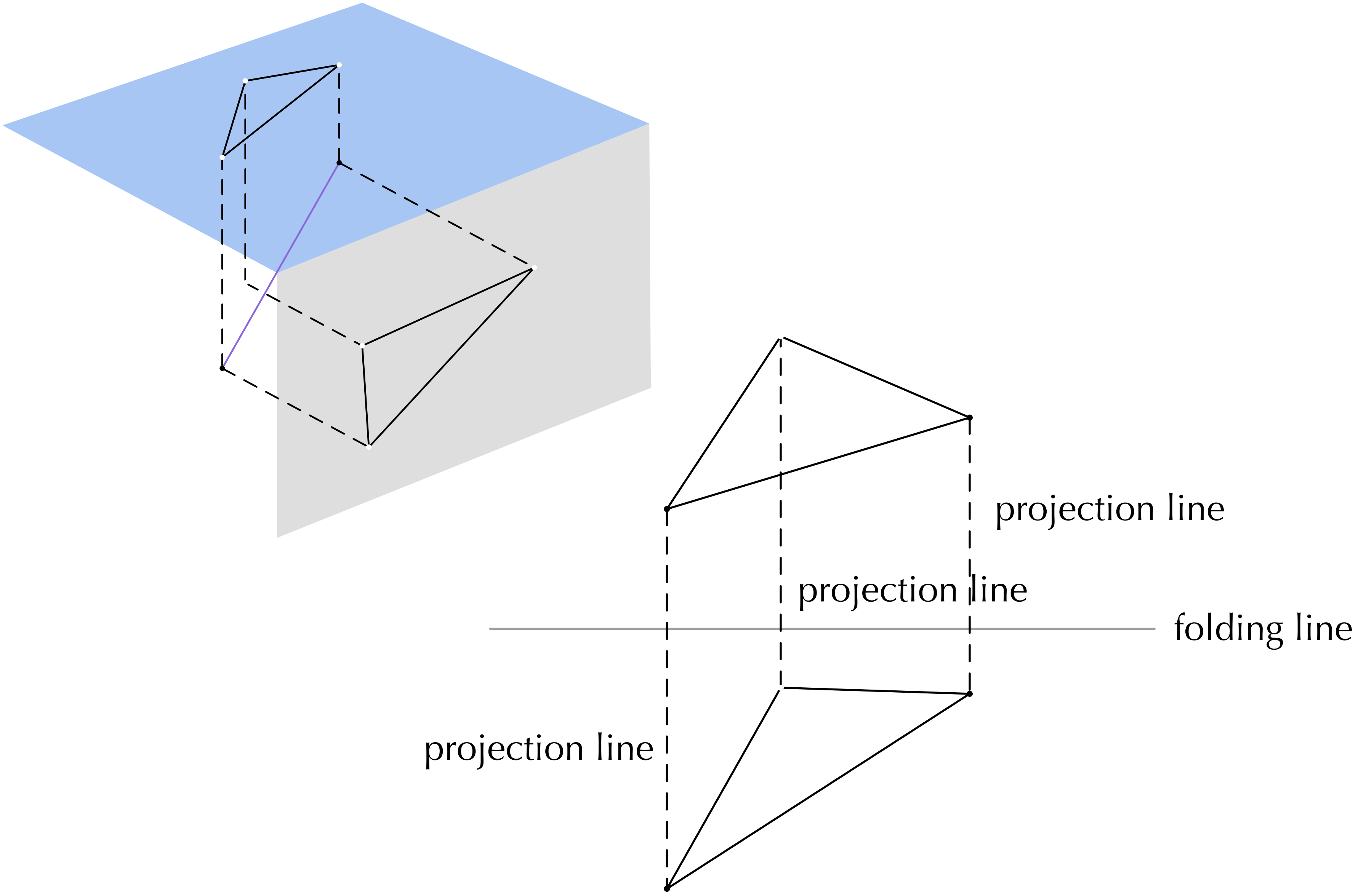
only the **reference** or **folding lines** are important

- **convention**



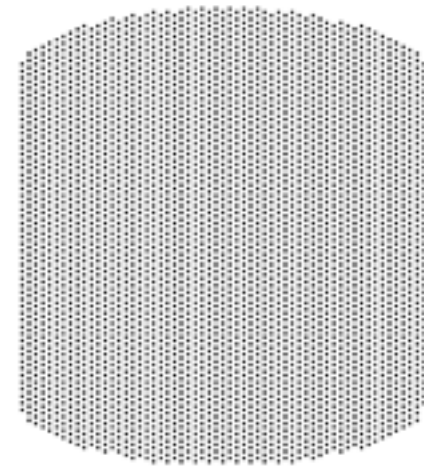


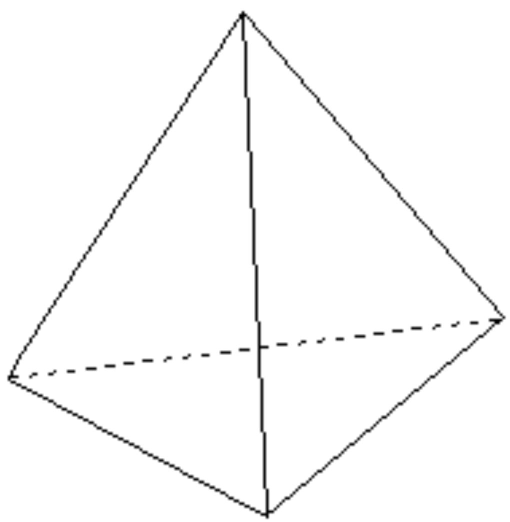
representing a line

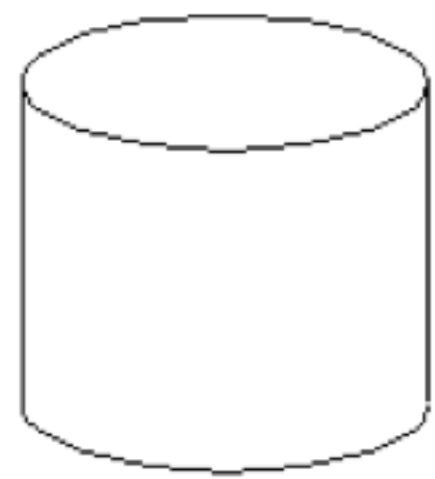


representing a figure

Orthographic views, in architecture or other fields, are **generated for a purpose**, and the selection of the features to be shown may vary with that purpose





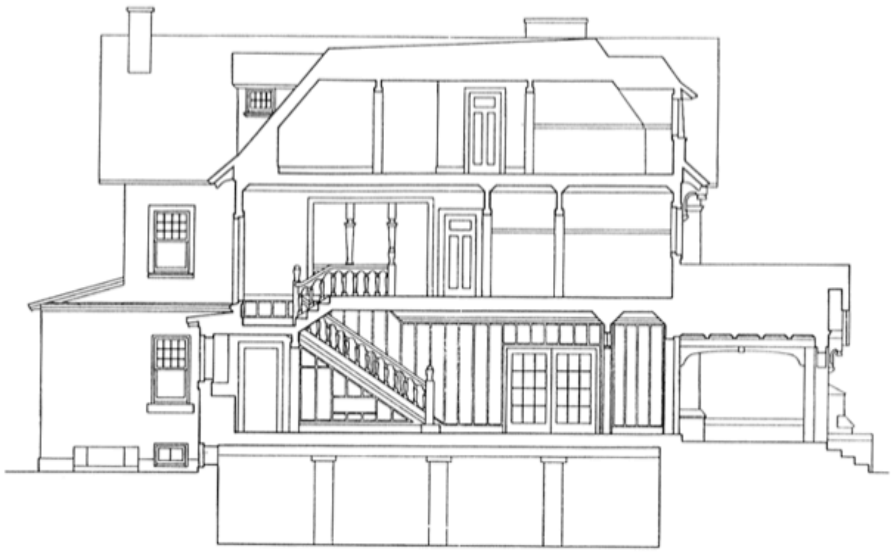


- **Literal versus normal renderings in orthographic views**

A FLOOR PLAN of a building is the top view of a portion of a building below a picture plane cutting horizontally through the building. It shows the parts of the building underneath this plane as seen when we view the picture plane from above.

All the projections that come into play in such a drawing use the same family of projection lines normal to the cutting plane (and this family is unique)

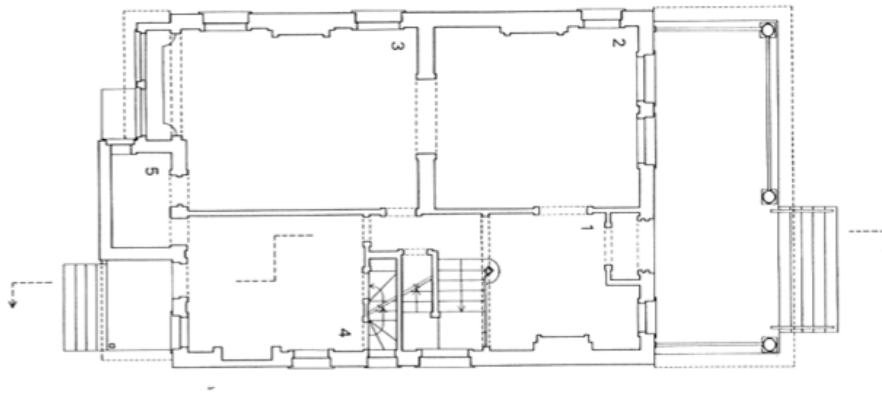
A SECTION is developed in the same way using a vertical cutting plane that cuts through the building



ELEVATIONS are developed with vertical picture planes that do not intersect the building

plan and side elevation

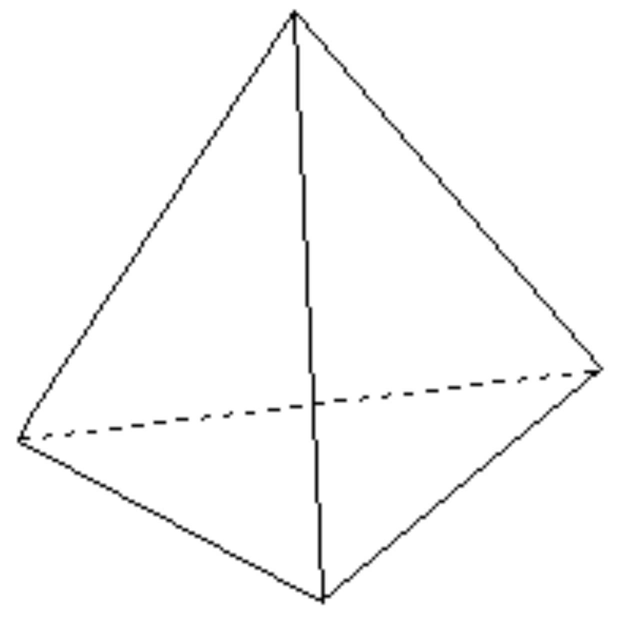
are *adjacent views*



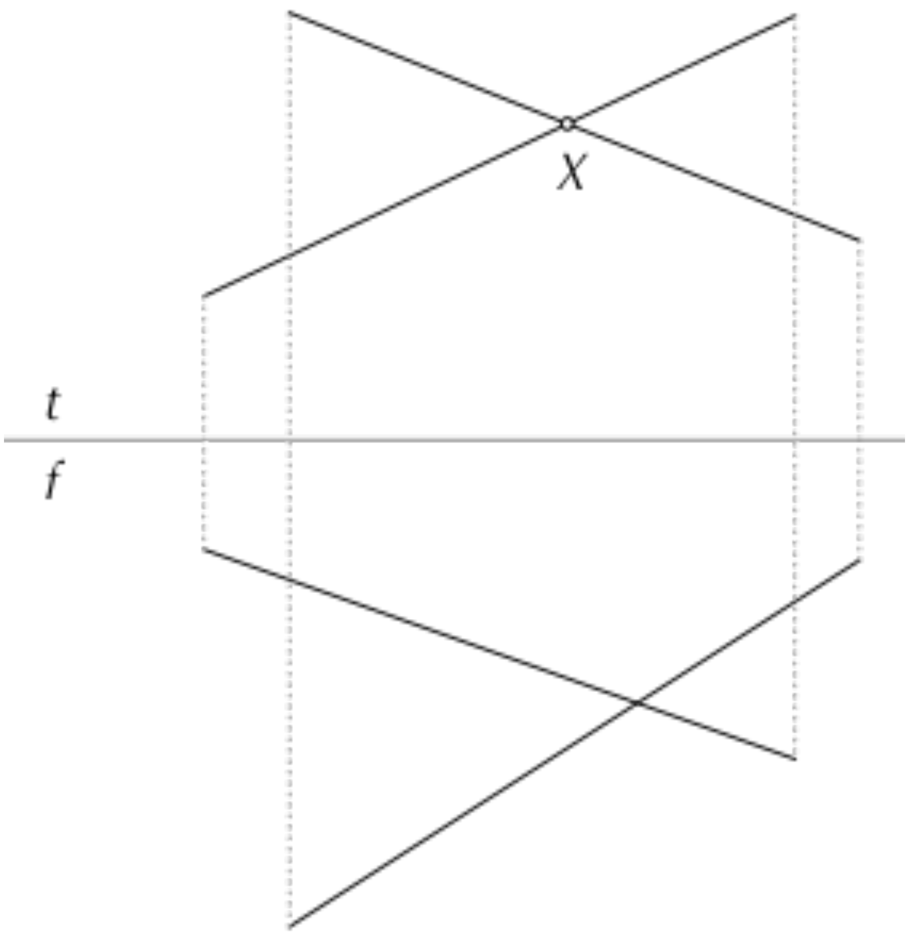
plan and side elevation

It is conventional to depict lines differently depending on their visibility

Here the hidden edge is shown dashed



Given two lines in two adjacent views, neither line perpendicular to the folding line, that meet at a point, X, in at least one view, t, determine which line is in front of the other (relative to t) at the intersection point.

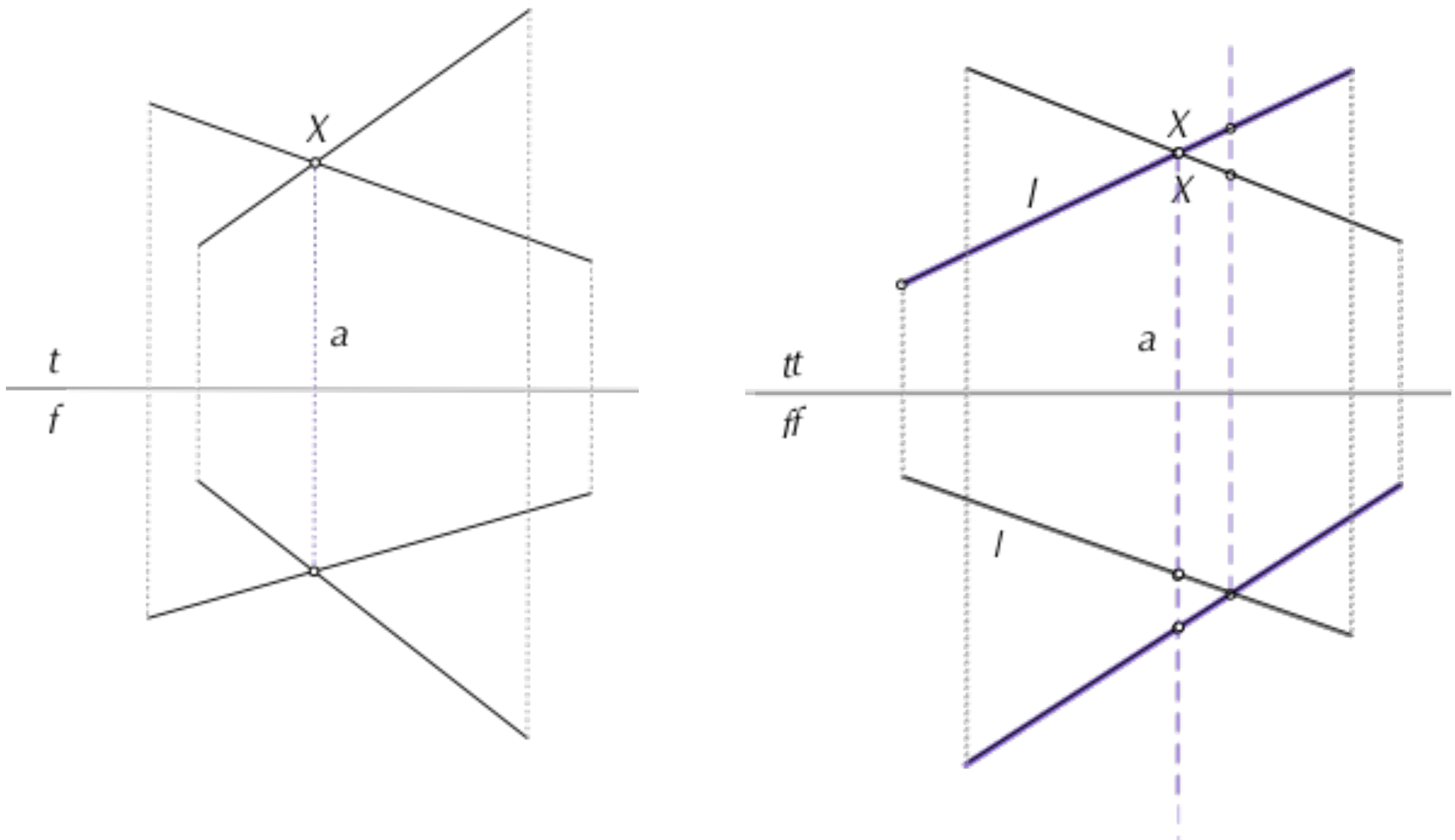


There are two steps.

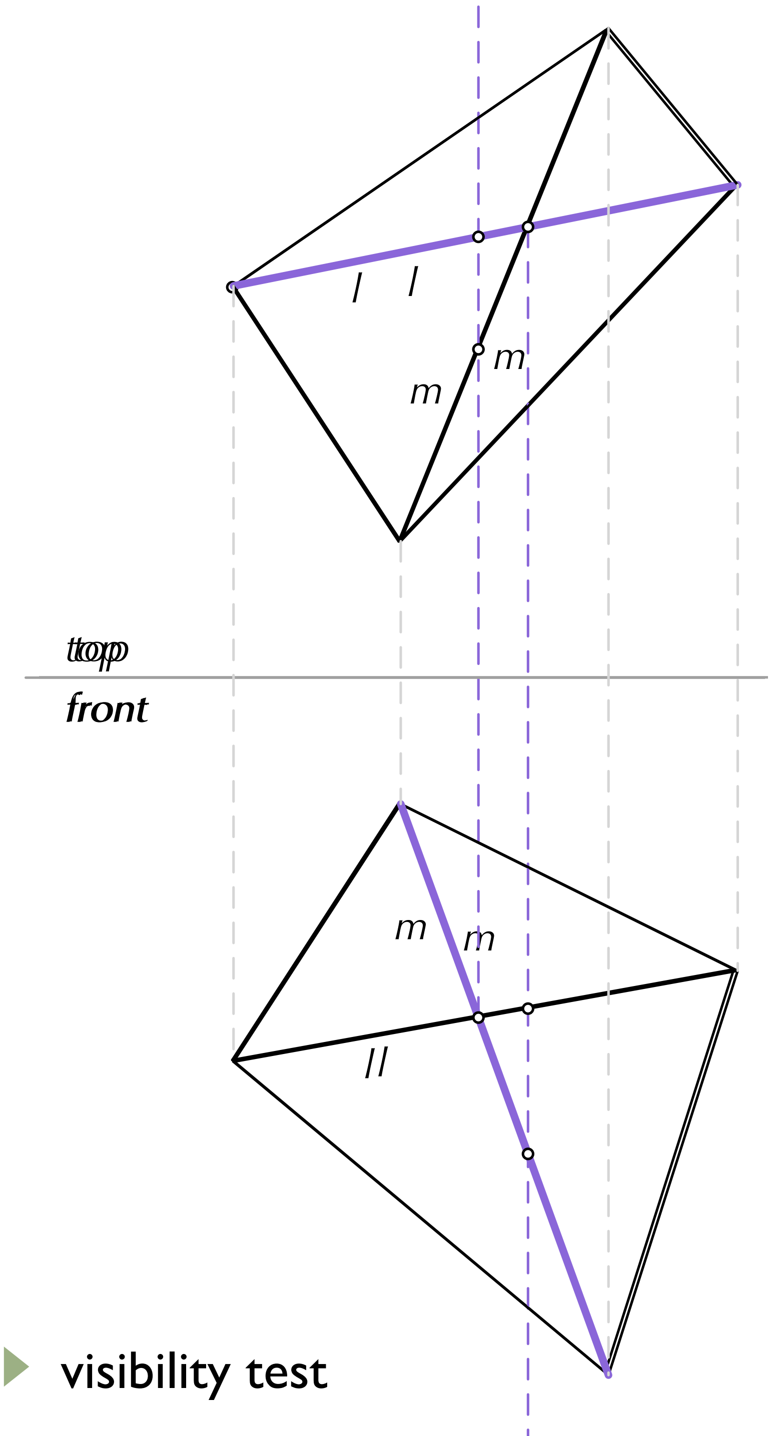
1. Draw the projection line through X_t into view *f*.

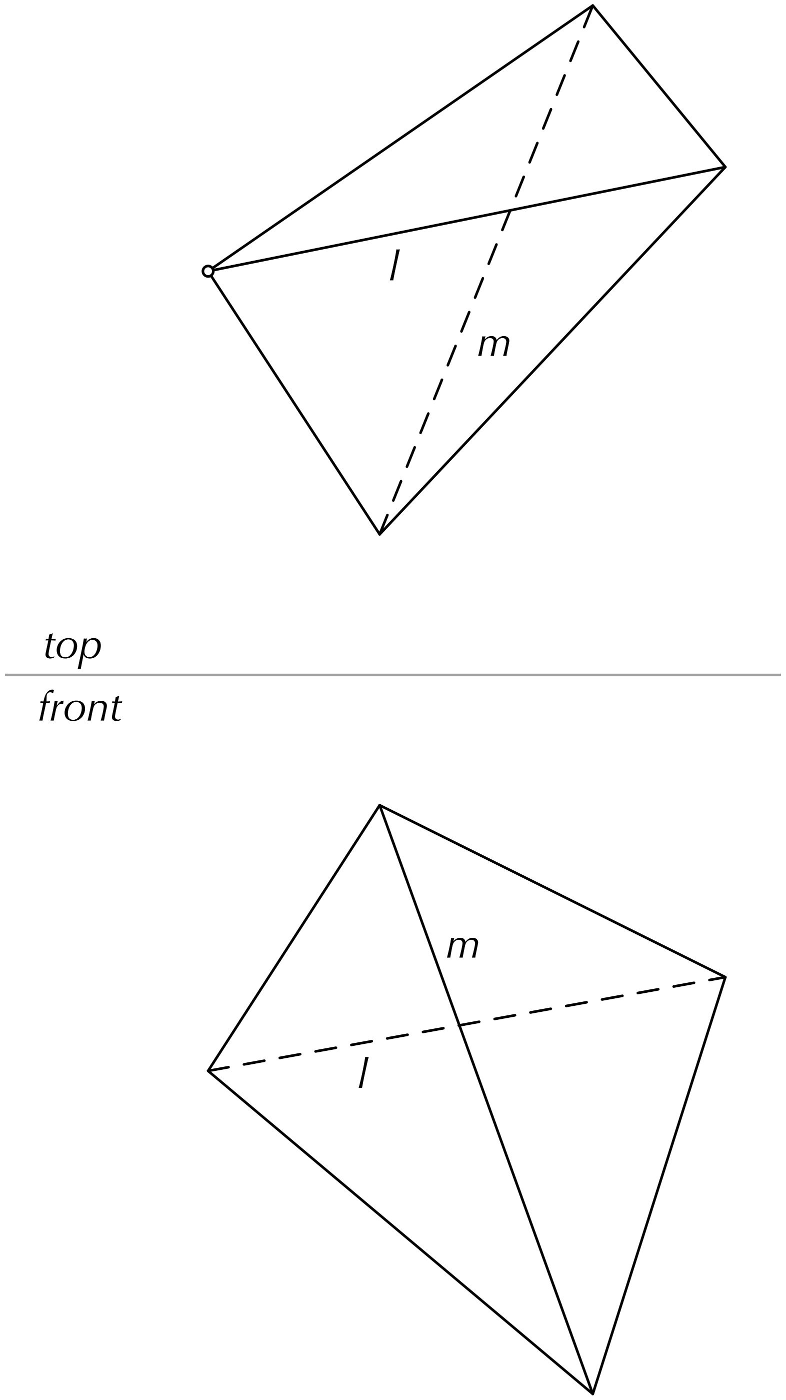
2. If the lines meet also at a point on *a* in *f*, the lines truly intersect.

   Otherwise, determine the spatial relation between the lines at X_t from the relative positions of their intersections with *a* in *f*: the line that intersects a at a point **closer to the folding line** than the other line is **closer to the picture plane** of *f* at that point; consequently, it is **in front of the other line** at X_f in *f*.



- **visibility test**

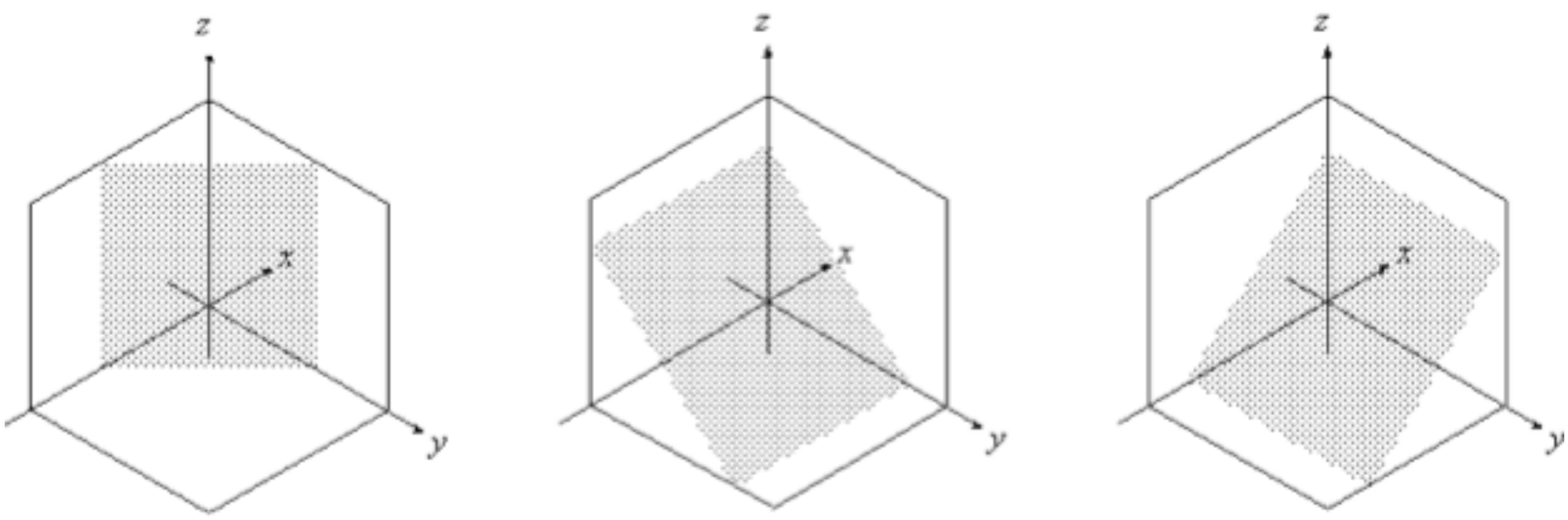




visibility test

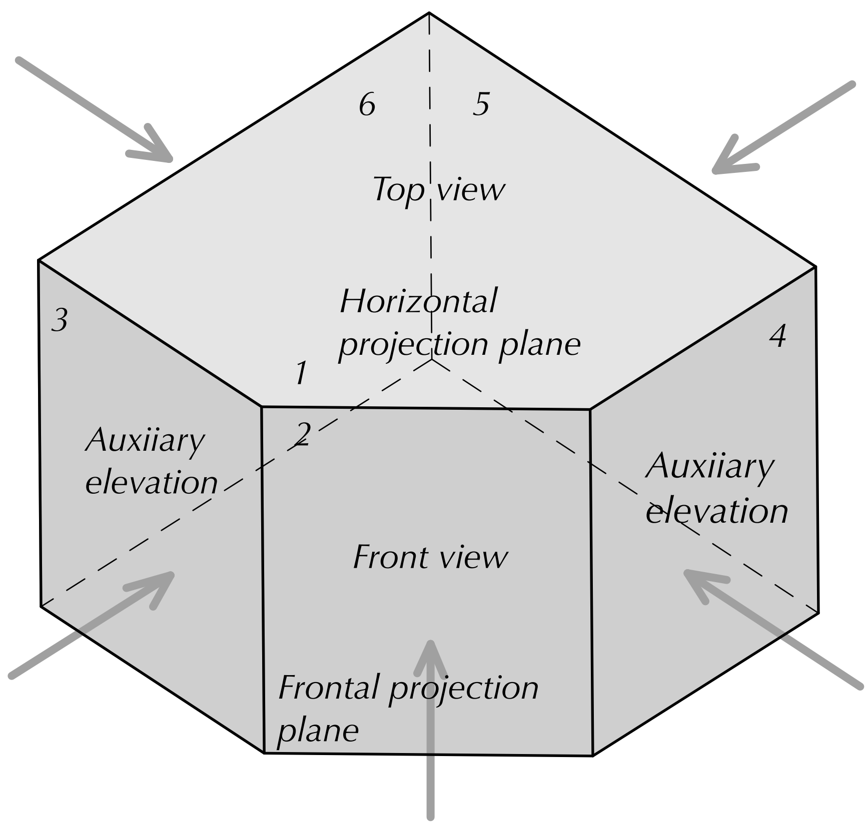
- A **primary auxiliary view** is a view using a picture plane perpendicular to one of the coordinate planes and inclined to the other two coordinate planes.

  A **secondary auxiliary view** is an auxiliary perpendicular to a primary auxiliary view.



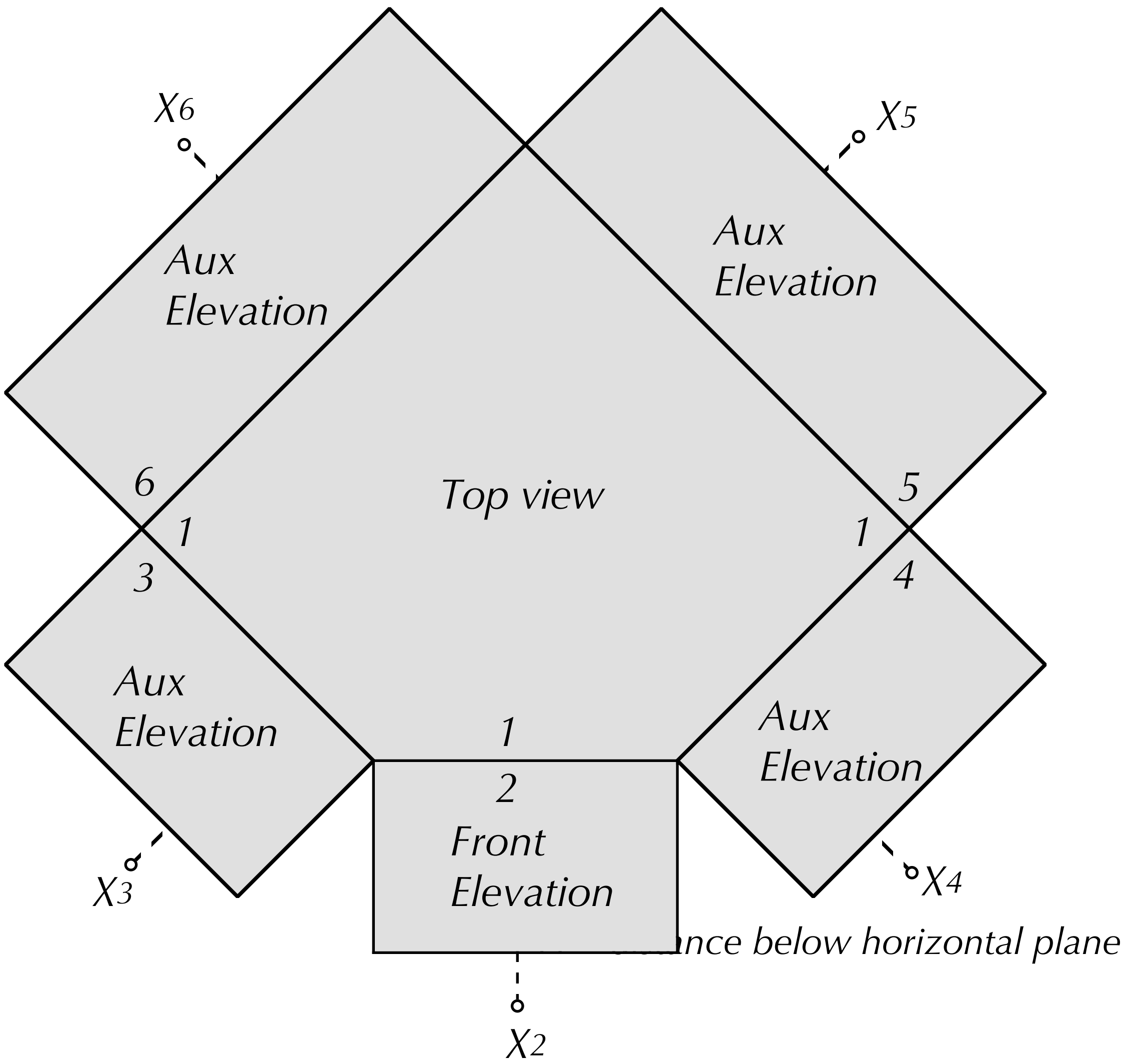
- principal auxiliary views

Given two adjacent views, a view using a picture plane perpendicular to the picture plane used in one or other view is an ***auxiliary view***

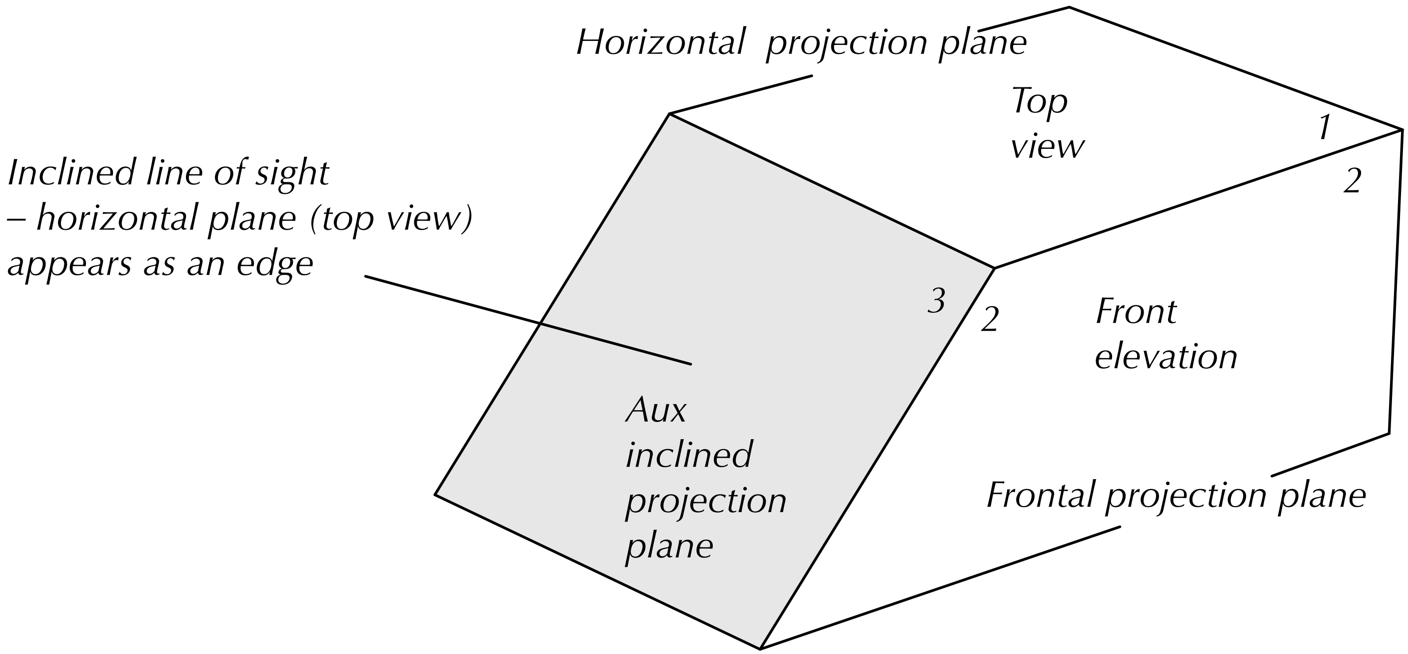


Planes 2,3,4,5 and 6 are all elevations as each is perpendicular to the horizontal projection planes

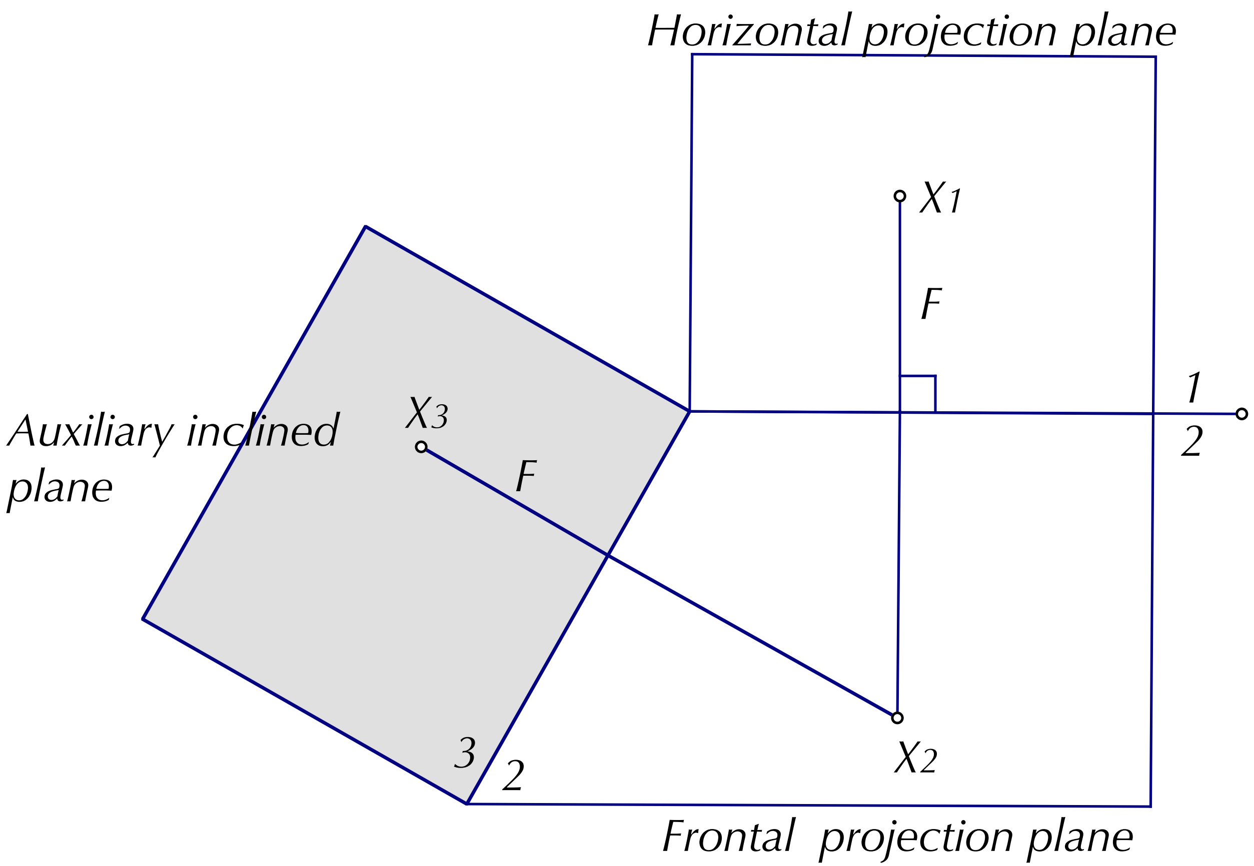
Observer's line of sight remains horizontal when viewing elevations



- **auxiliary views** and 'transfer' distance



inclined auxiliary view



inclined auxiliary view

Given a point, X, in two adjacent views, t and f, construct an auxiliary view of X using a picture plane perpendicular to the picture plane of t.

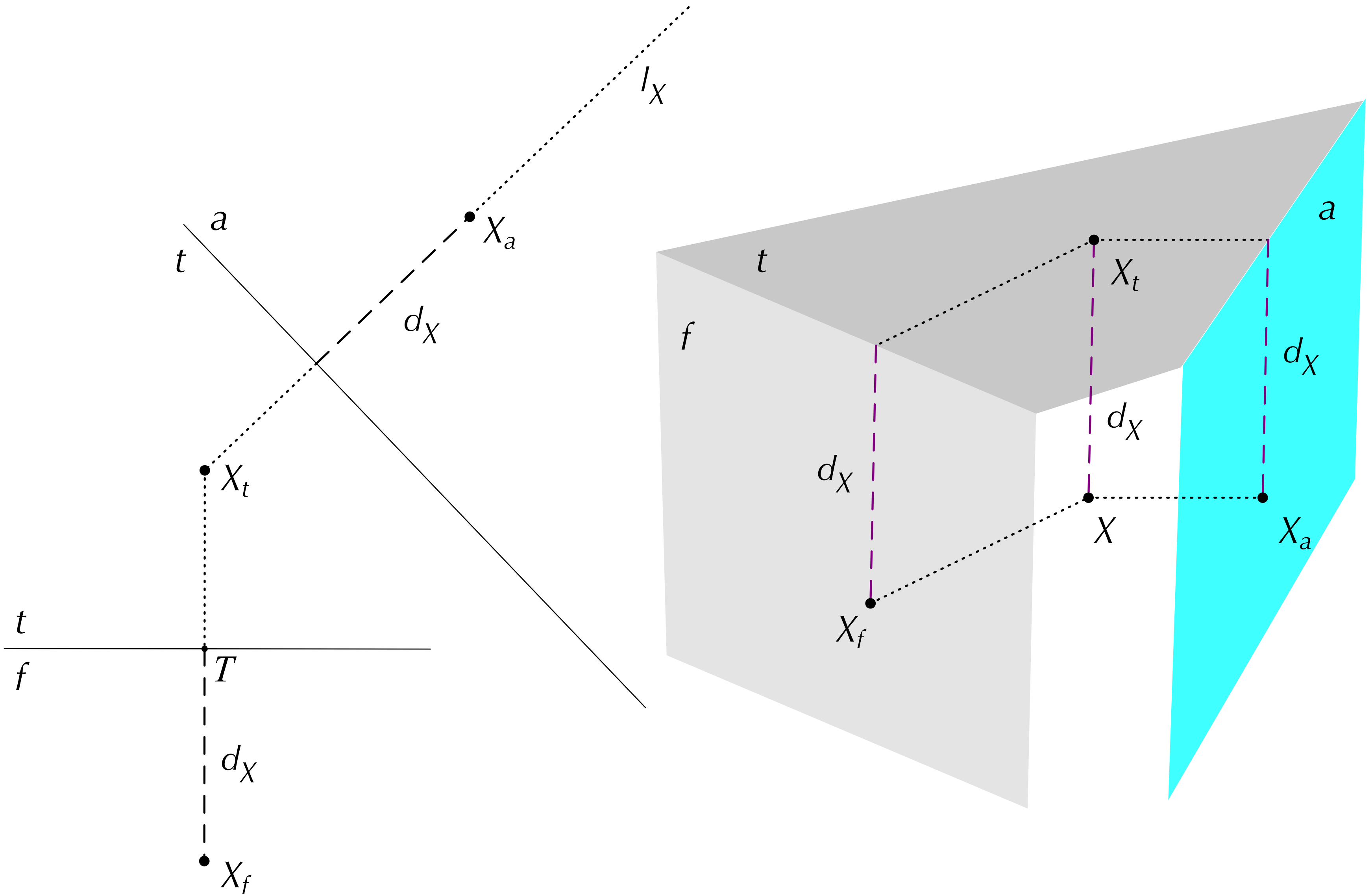
There are three steps.

1. Call the auxiliary view a, and select a folding line, $t \mid a$, in t (any convenient line other than $t \mid f$ will do).

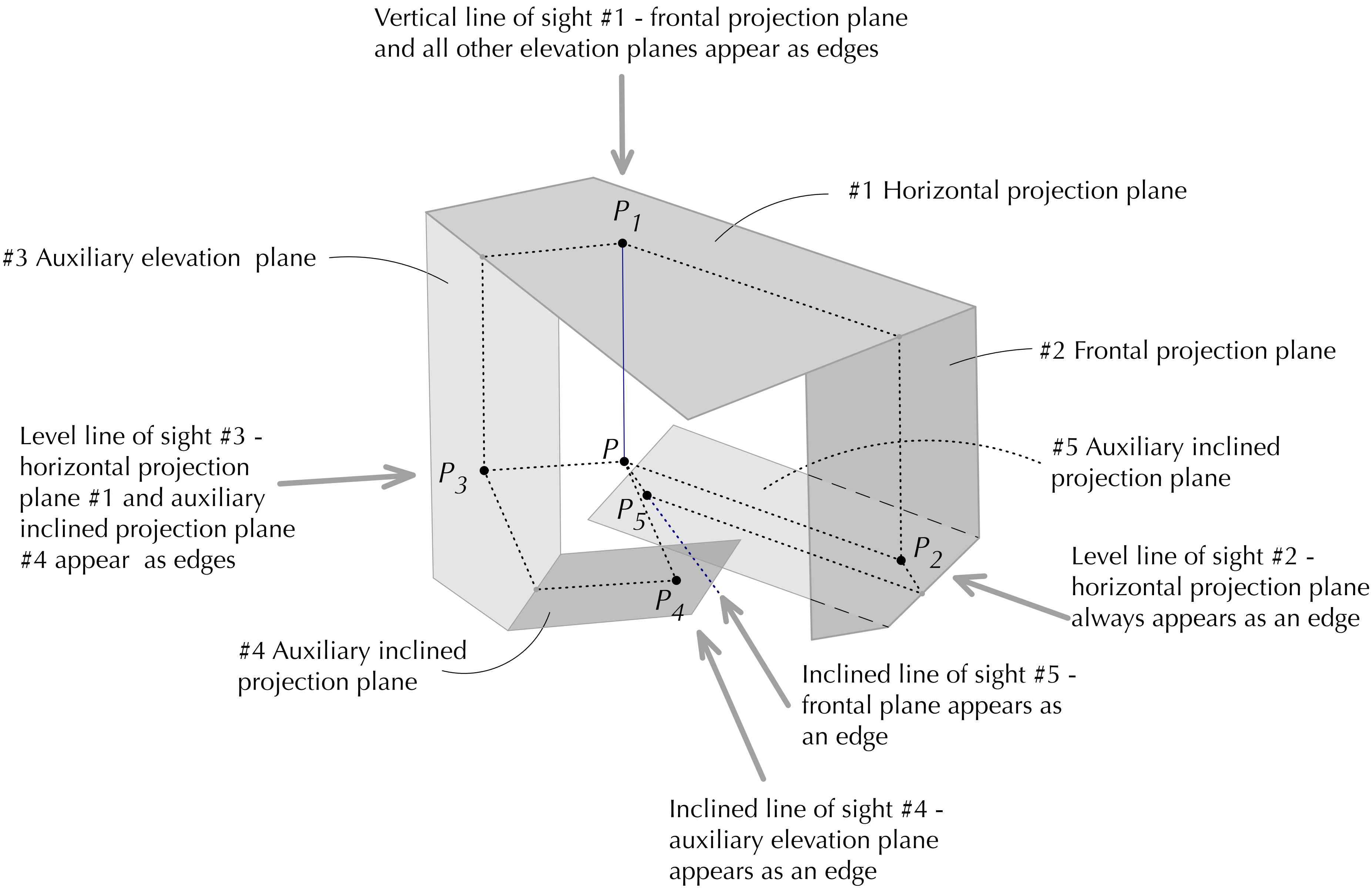
2. Draw the projection line, l_x, through X_t perpendicular to $t \mid a$.

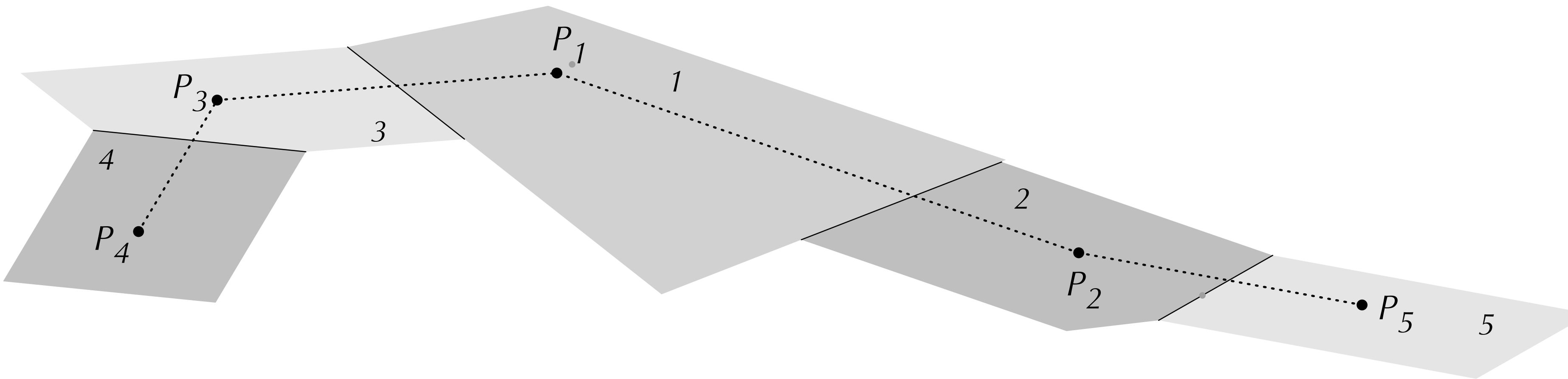
3. Let d_x be the distance of X_f from folding line $t \mid f$.

   X_a (that is, the view of X in a) is the point on l_x that has distance d_x from the folding line $t \mid a$. The distance d_x is called a **transfer distance**

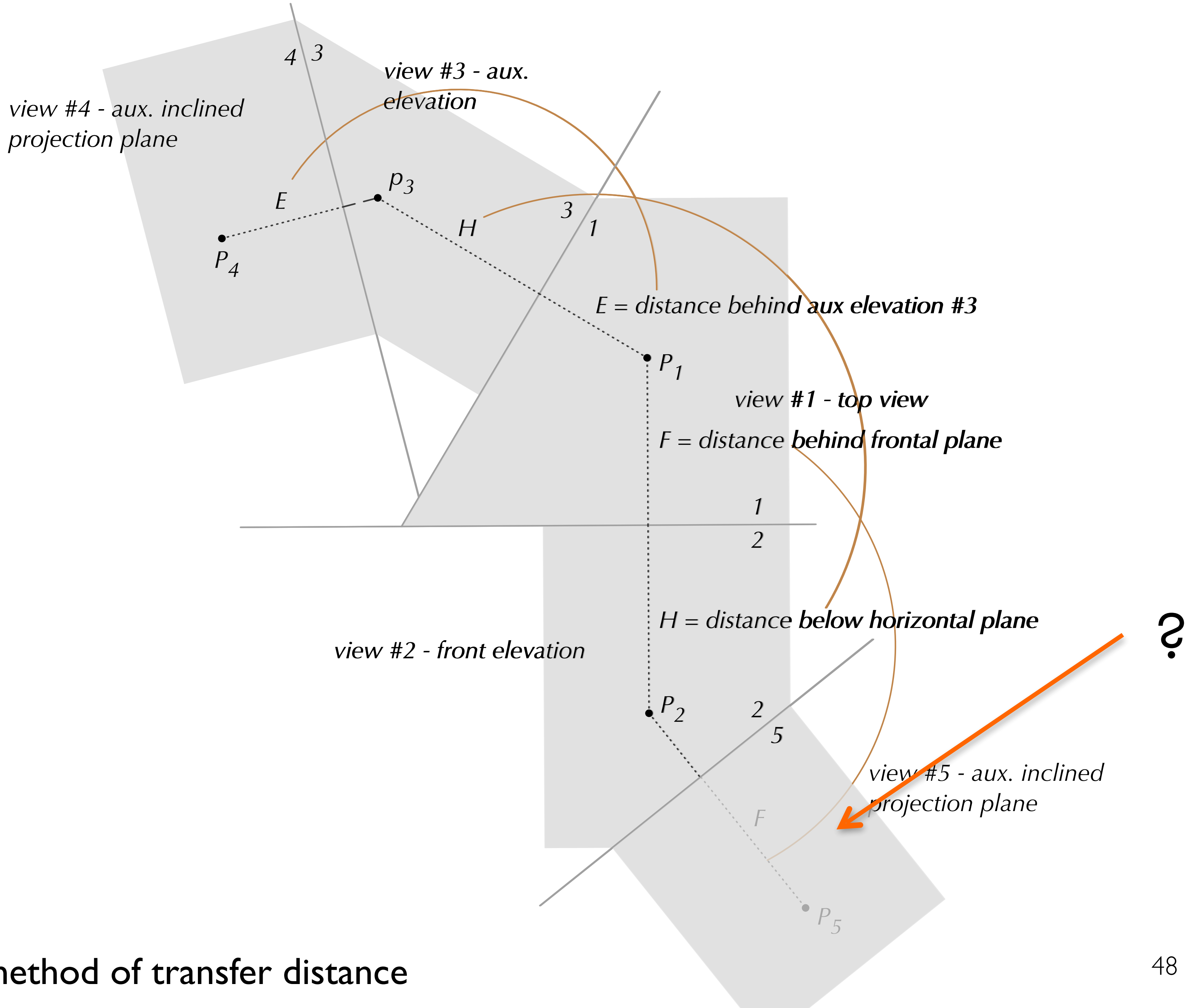


- constructing an auxiliary view - transfer distance

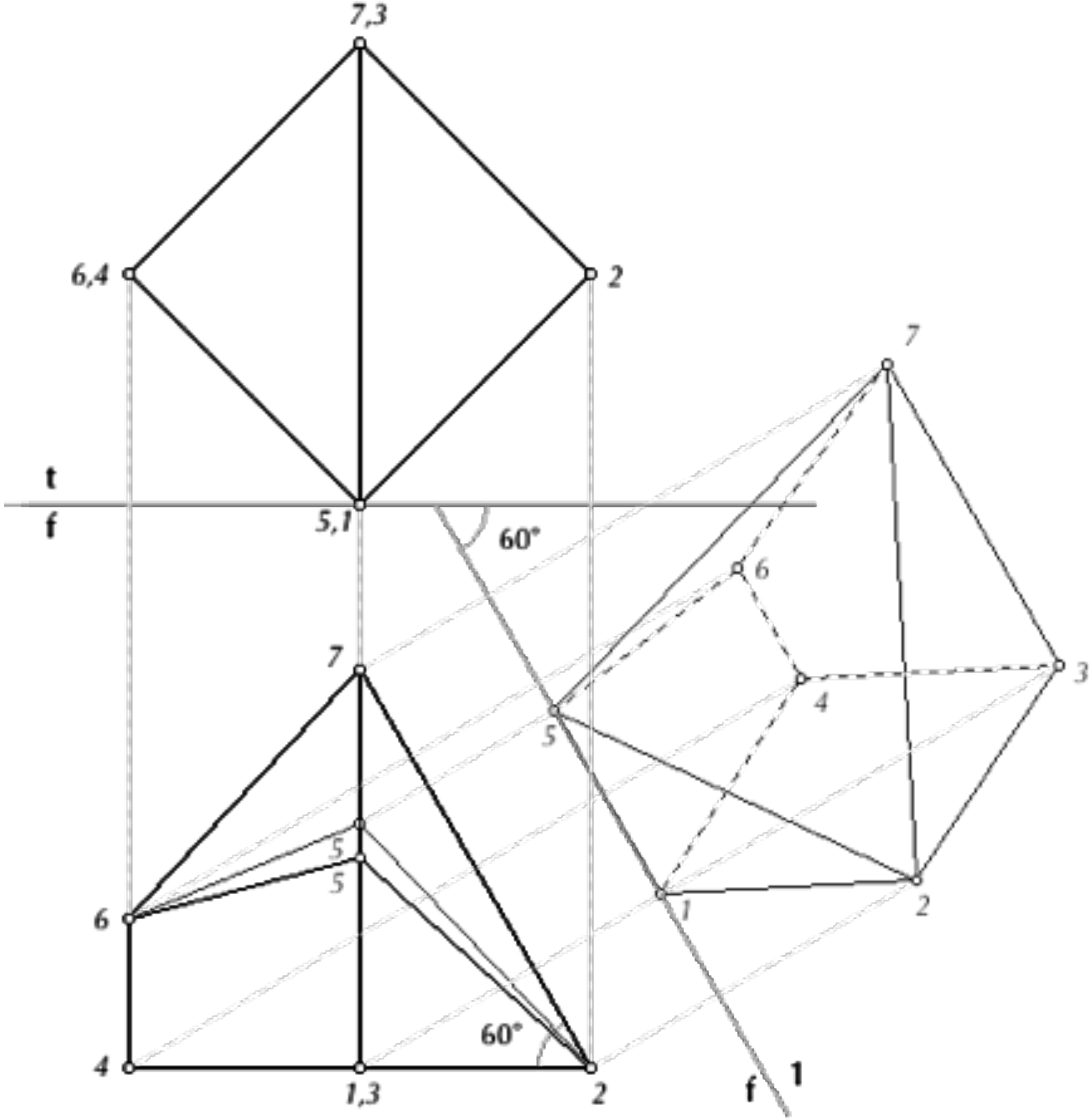

Vertical line of sight #1 - frontal projection plane
and all other elevation planes appear as edges
#1 Horizontal projection plane
#3 Auxiliary elevation plane
#2 Frontal projection plane
P 1
P
P 3
P 5
P 2
P 4
Level line of sight #3 -
horizontal projection
plane #1 and auxiliary
inclined projection plane
#4 appear as edges
#5 Auxiliary inclined
projection plane
Level line of sight #2 -
horizontal projection plane
always appears as an edge
#4 Auxiliary inclined
projection plane
Inclined line of sight #5 -
frontal plane appears as
an edge
Inclined line of sight #4 -
auxiliary elevation plane
appears as an edge

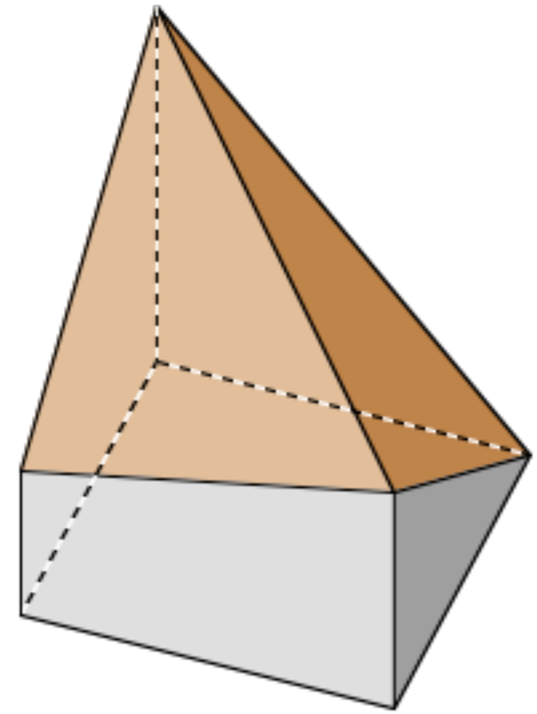


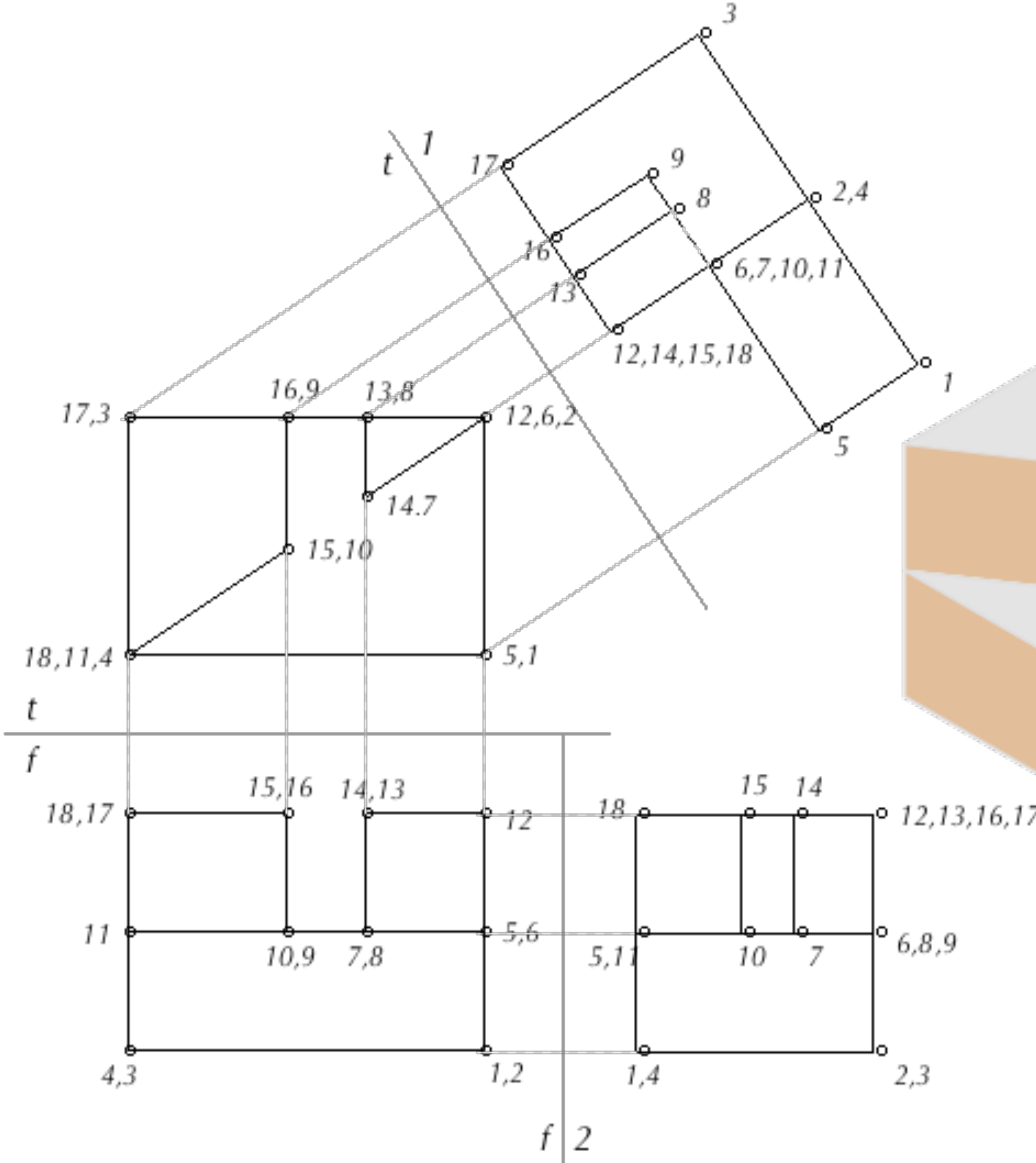
unfolded



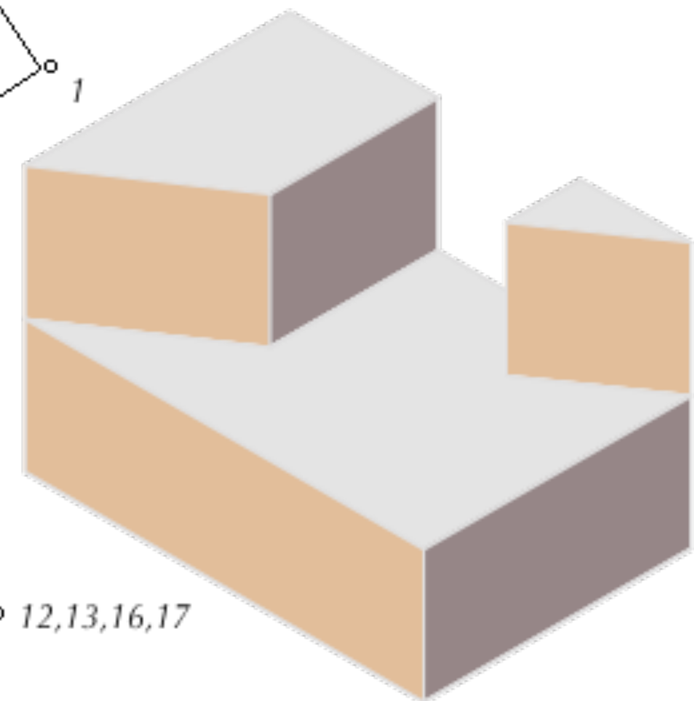
method of transfer distance

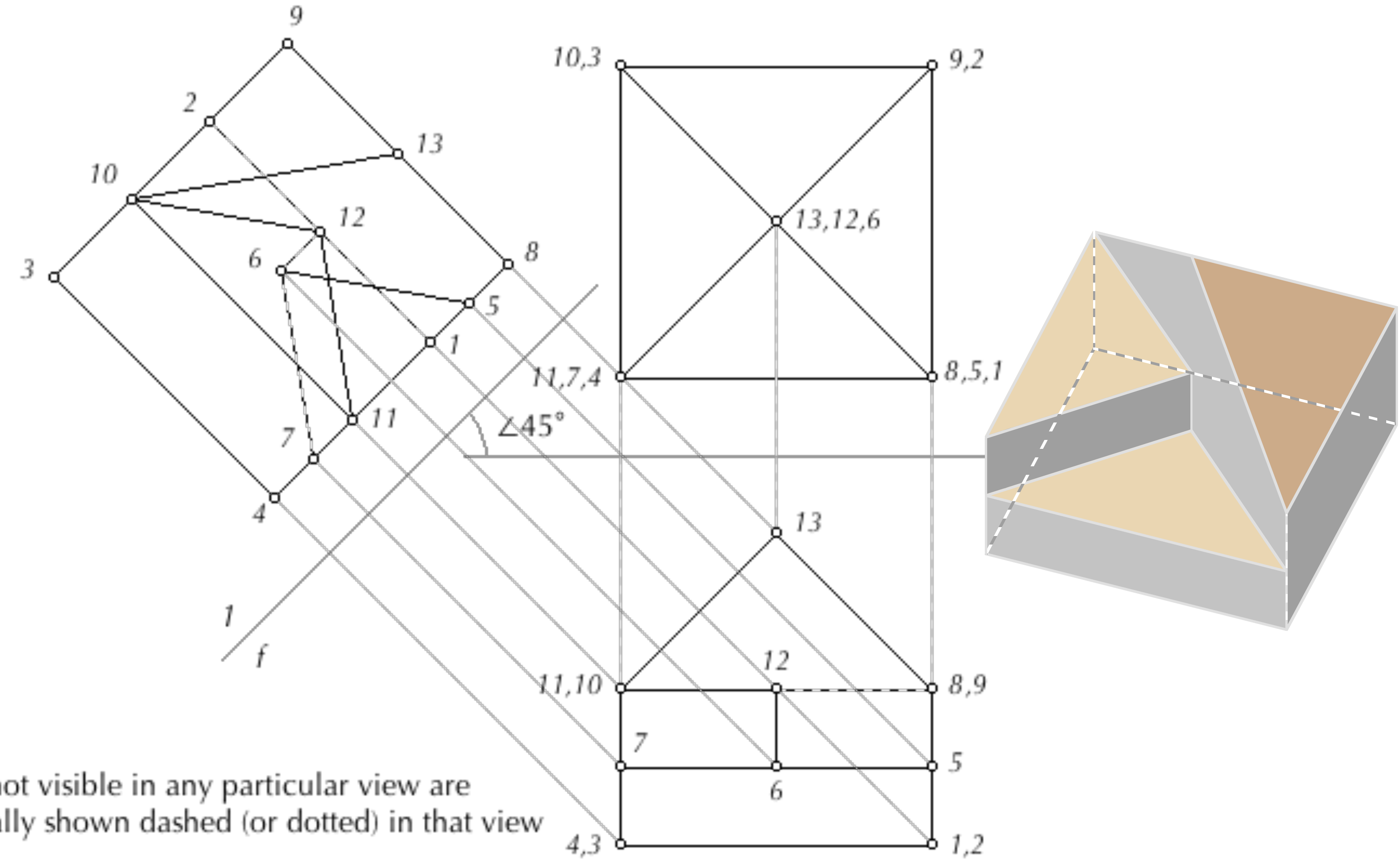

7,3
6,4
2
t
f
5,1
60°
7
6
5
5
4
1,3
2
60°
6
4
3
5
7
1
2
f
1





3
t
1
17
9
8
2,4
16
13
6,7,10,11
12,14,15,18
1
5
16,9
13,8
17,3
12,6,2
14.7
15,10
18,11,4
5,1
t
f
15,16
14,13
18,17
12
18
15
14
12,13,16,17
11
10,9
7,8
5,6
5,11
10
7
6,8,9
4,3
1,2
1,4
2,3
f
2





lines not visible in any particular view are
normally shown dashed (or dotted) in that view