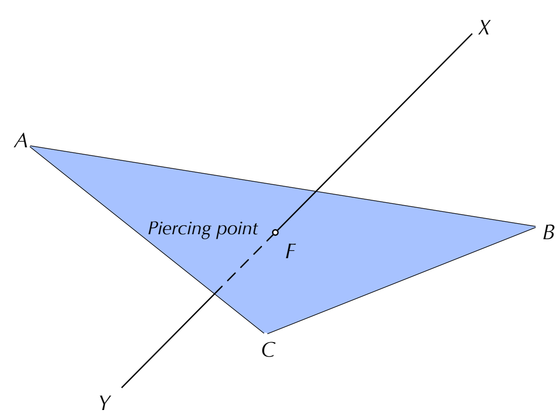
6
Piercing Points and Plane Intersections

We now consider problems that occur frequently in connection with the design of objects composed of various intersecting parts. One of the more frequent examples in architectural design is 'roofscapes,' which consist of several intersecting planes meeting at possibly odd angles. The problem is to find accurate projections of these configurations in various views. Such problems are solved using the basic techniques and constructions previously introduced. Intersection constructions also form the basis for depicting shades and shadows in orthographic views.

6.1 PIERCING POINT - INTERSECTIONS BETWEEN LINES AND PLANES

When a line neither on nor parallel to a plane intersects that plane, it does so at a point called the *piercing point*. See Figure 6-1.

6-1
Piercing point



The piercing point for a given plane and line can be found by two alternative constructions. The first of these finds an edge view of the plane using an auxiliary view familiar from previous constructions.

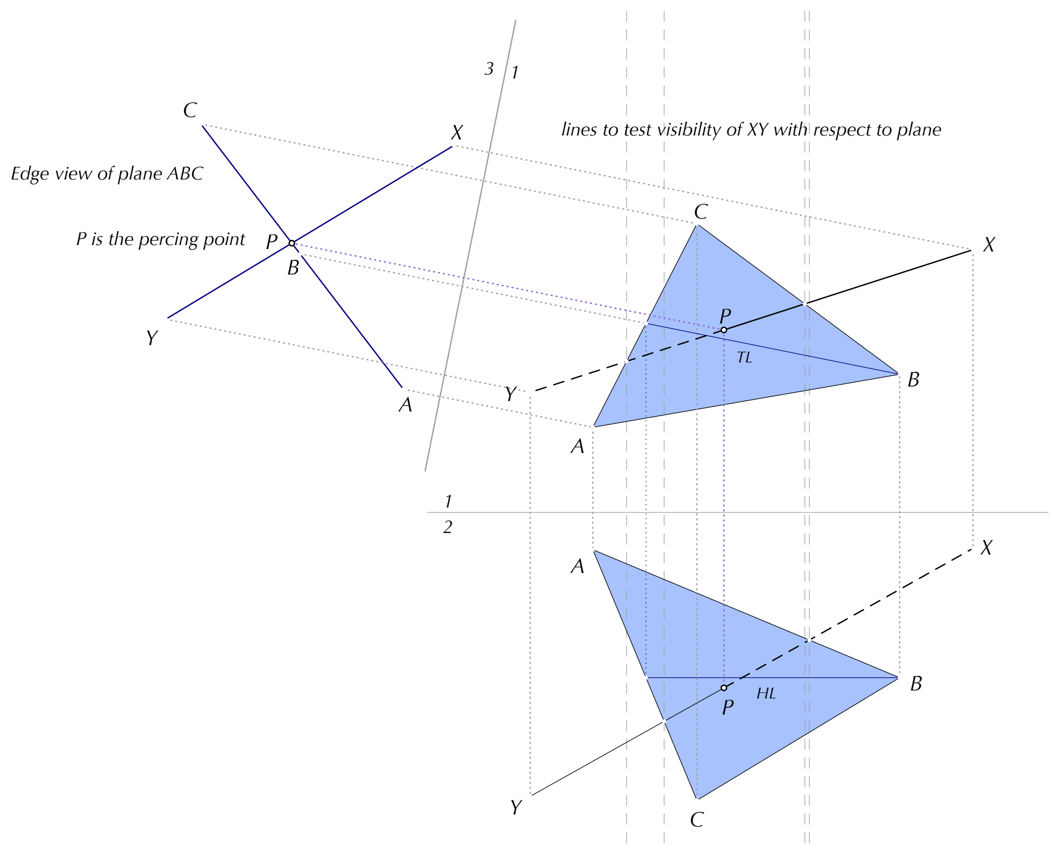
Construction 6-1
Piercing point (edge view method)

Given a line and a plane in two adjacent views, 1 and 2, where the line is defined by segment XY, and the plane by ΔABC, find the piercing point by the edge view method.

There are three steps:

1. Use Construction 4-2 (on page 126) to find an edge view of Δ*ABC* in an auxiliary view, 3.
2. Project Δ*ABC* into view 3. The point of intersection, *P*, of the segment *XY* and the edge view of Δ*ABC* is the piercing point (Note that if either segment is too short, extend them sufficiently to produce the intersection).
3. Project *P* into the other views; determine the visibility of the line with respect to the plane at the piercing point by Construction 2-2 (on page 73). See also Figure 5-19.

The construction is illustrated in Figure 6-2.

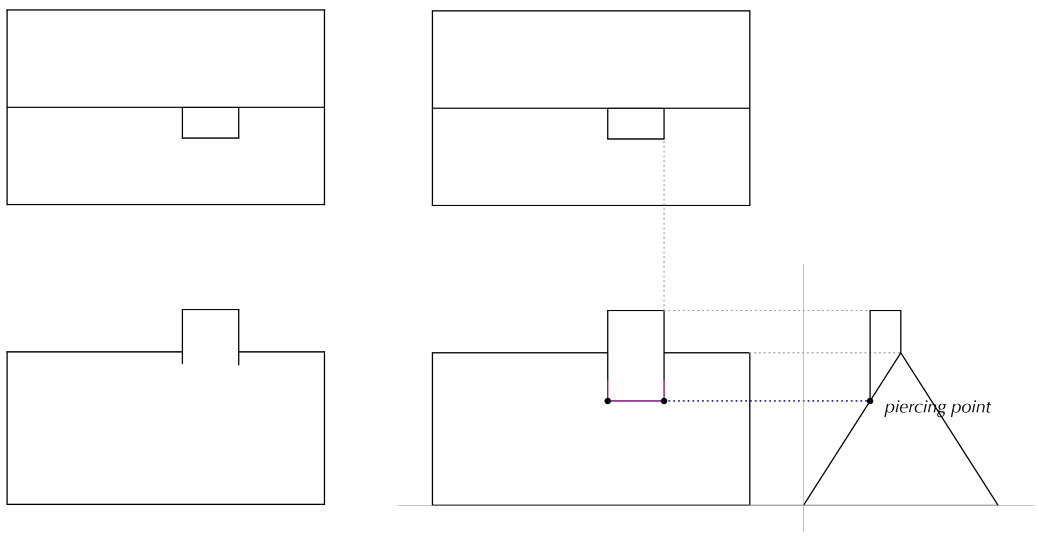


6-2
Finding the piercing point by the edge view method

6.1.1 Cutting plane

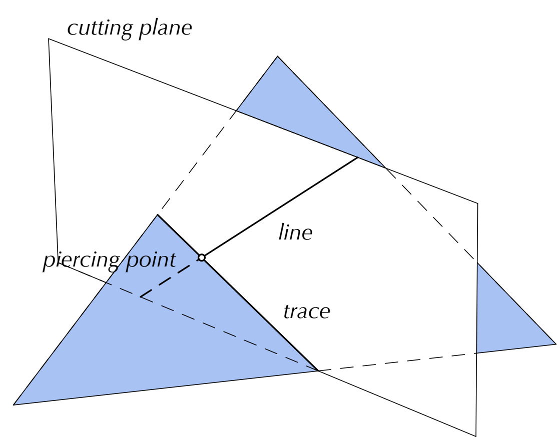
Take, as an example, the familiar problem of finding the intersection between the vertical edges of a chimney, given in top view in a roof plan and partially in front view, with a sloping roof (see the left side of Figure 6-3).

The obvious approach is to take an auxiliary view as the figure below shows.



6-3
Where does the chimney meet the roof?

However, there is a second method for finding a piercing point is particularly elegant because it does not need an auxiliary view. It is based on the following observation. Whenever we have a line, *l*, piercing a plane, *p*, and a plane, *c*, that contains *l*, *c* intersects *p* at a line, *t*, that contains the piercing point. *c* is called a *cutting plane* and *t* its *trace* on *p* (see Figure 6-4).



6-4
A cutting plane and its trace

6.2 PIERCING POINT – CUTTING PLANE METHOD

The following construction illustrates in very generally how one might solve such problems.

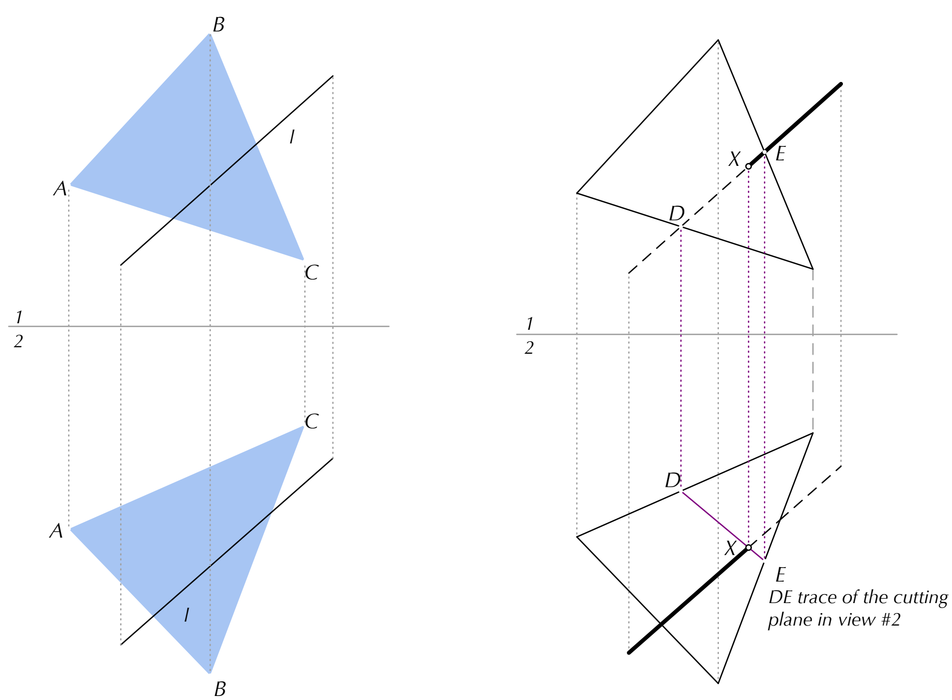
Construction 6-2
Piercing point – cutting plane method

Given a line, l, and a plane in two adjacent views, 1 and 2, where the plane is defined by ΔABC, find the piercing point by the cutting plane method.

There are three steps:

1. Select a view, say #1, so that a cutting plane perpendicular to view #1 appears in edge view in #1 and coincides with the view of l in #1. This line must intersect with two sides of ΔABC in #1. Call the intersection points D and E.
2. Project D and E into view #2. DE is the trace of the cutting plane in that view. Its intersection with l is the piercing point, X.
3. Project X into view #1, and determine the visibility between line and plane at the piercing points using Construction 2-2 (on page 70). See also Figure 5.19.

The construction is illustrated in 6-5.



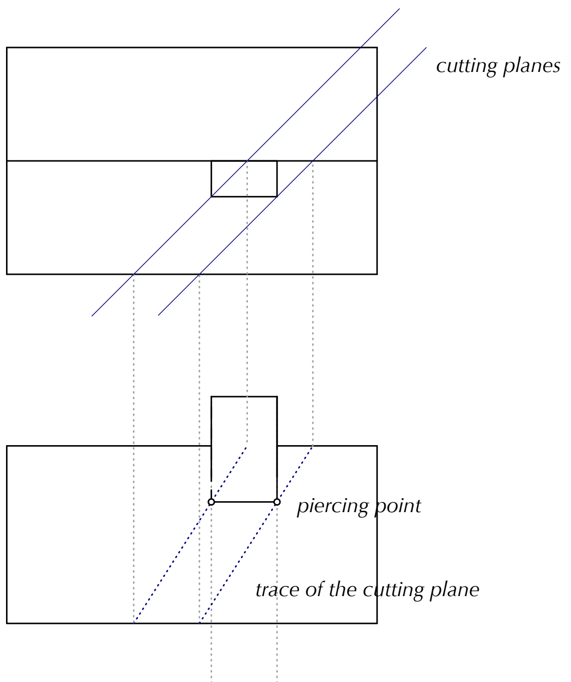
Problem: *Where is the piercing point?*

Construction

6-5
Piercing point – cutting plane method

6.2.1 Worked example – Back to the chimney problem

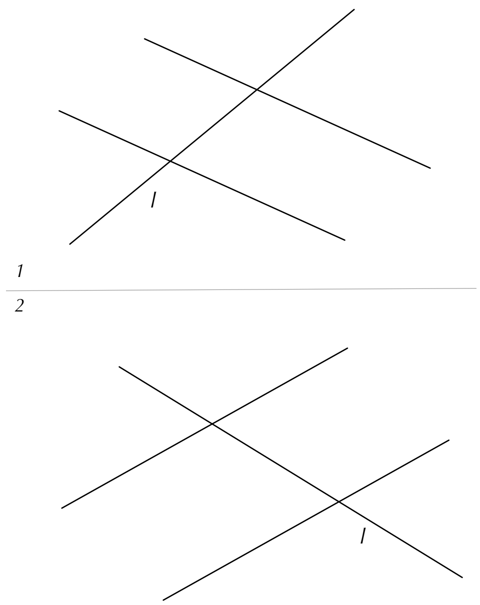
Lets get back to the chimney problem in Figure 6-3. We can use a simplified version of the above construction to solve this. In the top view, every vertical edge appears in point view, and every vertical plane is perpendicular to the picture plane of that view and will appear in edge view in the top view. If we draw a line through the point view of a front edge, *e*, that intersects the eave and ridge lines of the roof at points *A* and *B*, respectively, we can immediately project points *A* and *B* into the front view. The line through *A* and *B* in the front view is the trace of a vertical plane through *e* on the roof. The point of intersection of the trace with the front view of *e* is the piercing point of the edge with the roof plane. The construction is shown in Figure 6-6.



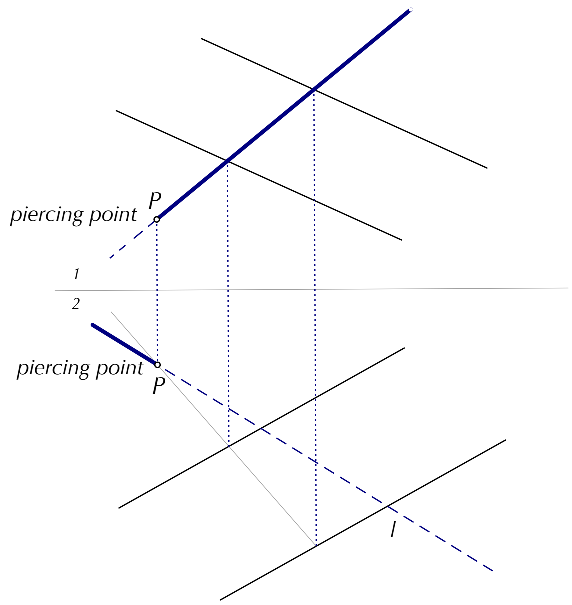
6-6
Completing the chimney-roof intersection

6.2.2 Piercing point of a line with a plane not specified by a triangle

We can apply Construction 6-2 in a straightforward manner when planes are specified by parallel lines or intersecting lines. The two cases are illustrated in Figures 6-7 and 6-8. *P* marks the piercing point in both views. In both cases, the visibility construction is applied to the line *l*. In the lower diagram, note that the point of intersection of the lines that specify the plane must correspond to the same point in both views. This can be used as a check to verify that the lines truly intersect. The reader is urged to study this example since the visibility of *l* with respect to the plane is not apparent from a 'first sight' of the adjacent views.

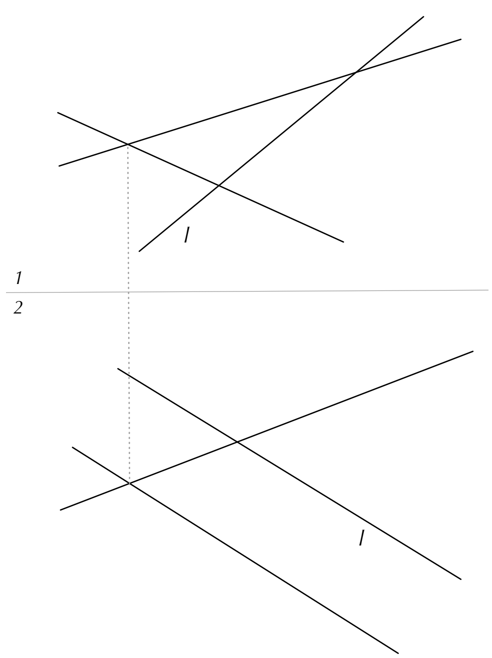


Problem: W*here is the piercing point?*

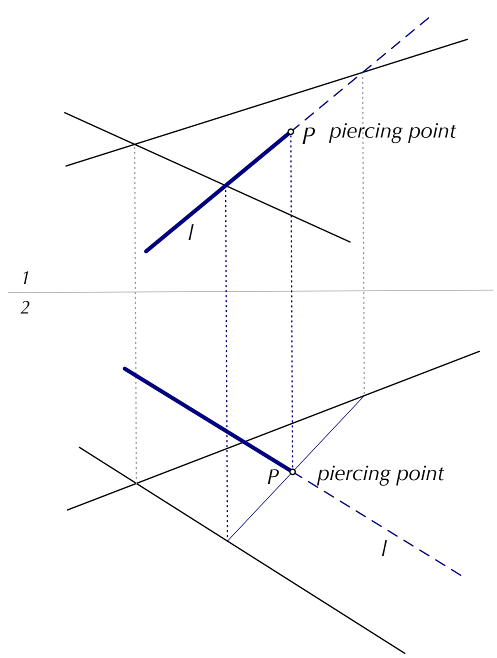


Construction

6-7
Piercing point when a plane is specified by two parallel lines



Problem: W*here is the piercing point?*



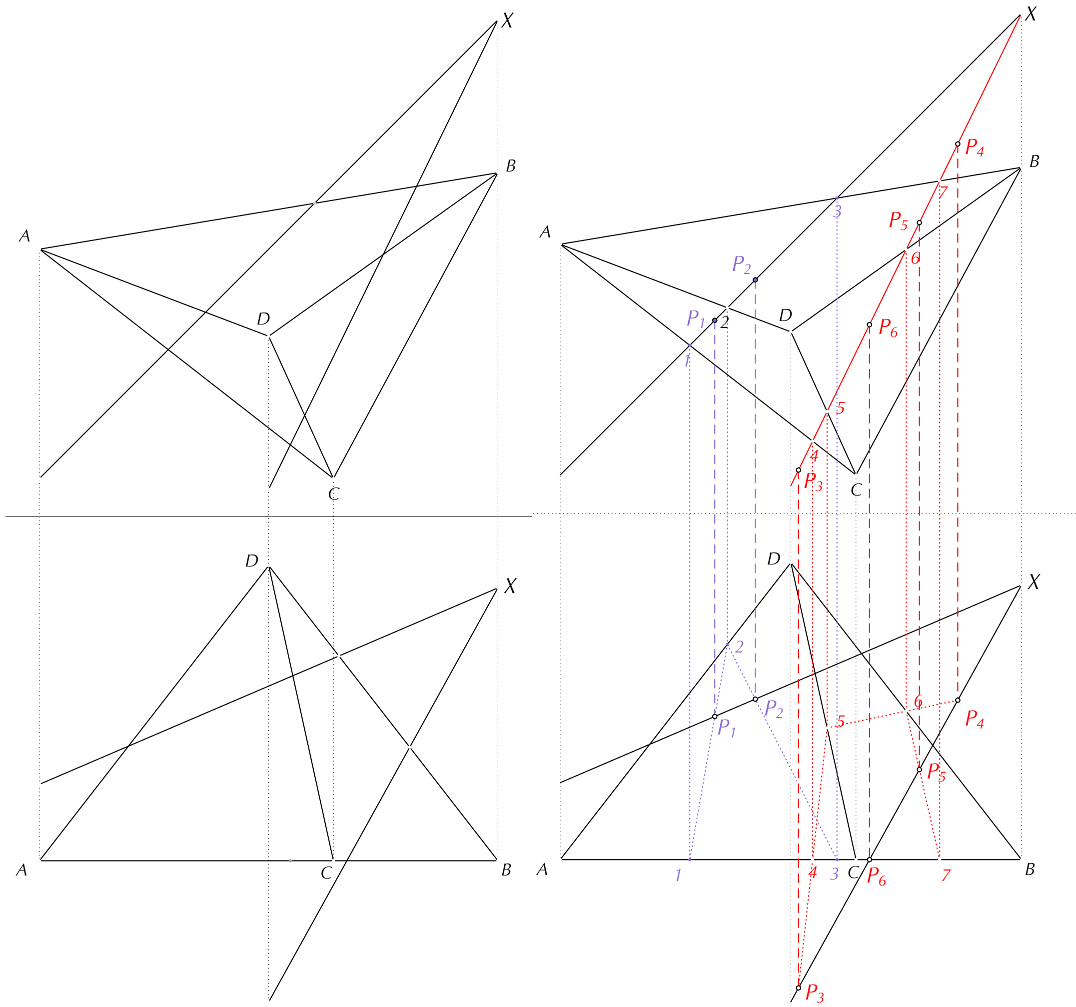
Construction

6-8
Piercing point when the plane is specified by two intersecting lines

6.2.3 Worked example – Lines intersecting a pyramid

Consider the following problem. We are given a pyramid and two lines emanating from the same point *X*. *Determine whether the two lines intersect the pyramid and if so, where.*

The steps to the problem are straightforward. For each line from *X* we determine the piercing points by constructing the traces of the cutting plane. The traces 12 and 23 determine the piercings points P_1 and P_2. Likewise, traces 45 and 56 determine (apparent) piercing points P_3 and P_4, which lie outside the pyramid. Trace 67 determines the piercing point point P_5. Trace 47 determines the piercing point P_6, which intersects the base of the pyramid.

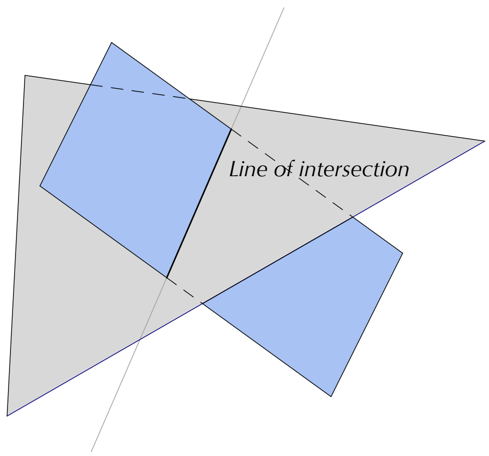


Problem: *Where do the lines meet the pyramid?* Construction

6-9
Intersection between two intersecting lines and pyramid

6.3 INTERSECTIONS BETWEEN PLANES AND PLANES

The preceding problem suggests the following. Two planes either intersect at a common line or are parallel. When planes intersect, the problem of finding the intersection of two planes reduces to finding two lines in a plane and then the piercing points for each of these lines with respect to the other plane; the piercing points define the line where the planes intersect.



6-10
Line of intersection of two intersecting planes

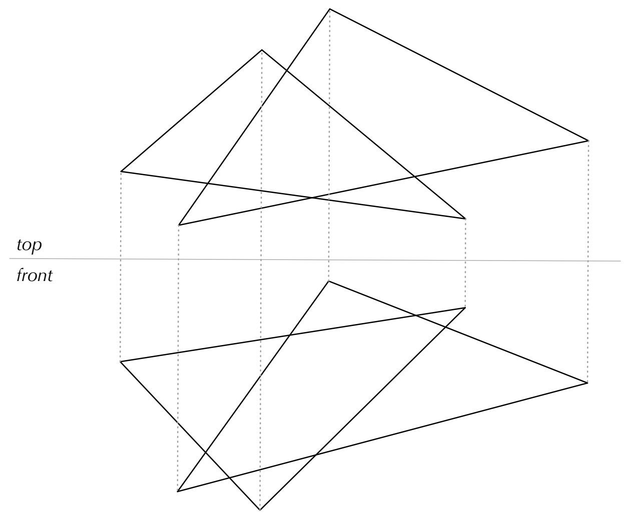
However, the intersection may fall actually outside a particular portion of the plane as given, for example, by three non-collinear points. But, no matter how planes are specified, it is always possible to find two distinct lines in each plane. The piercing points for these lines can then be determined by applying either Construction 6-1 or Construction 6-2 twice. The construction below demonstrates this for the cutting plane method.

Construction 6-3
Intersection between two planes – cutting plane method

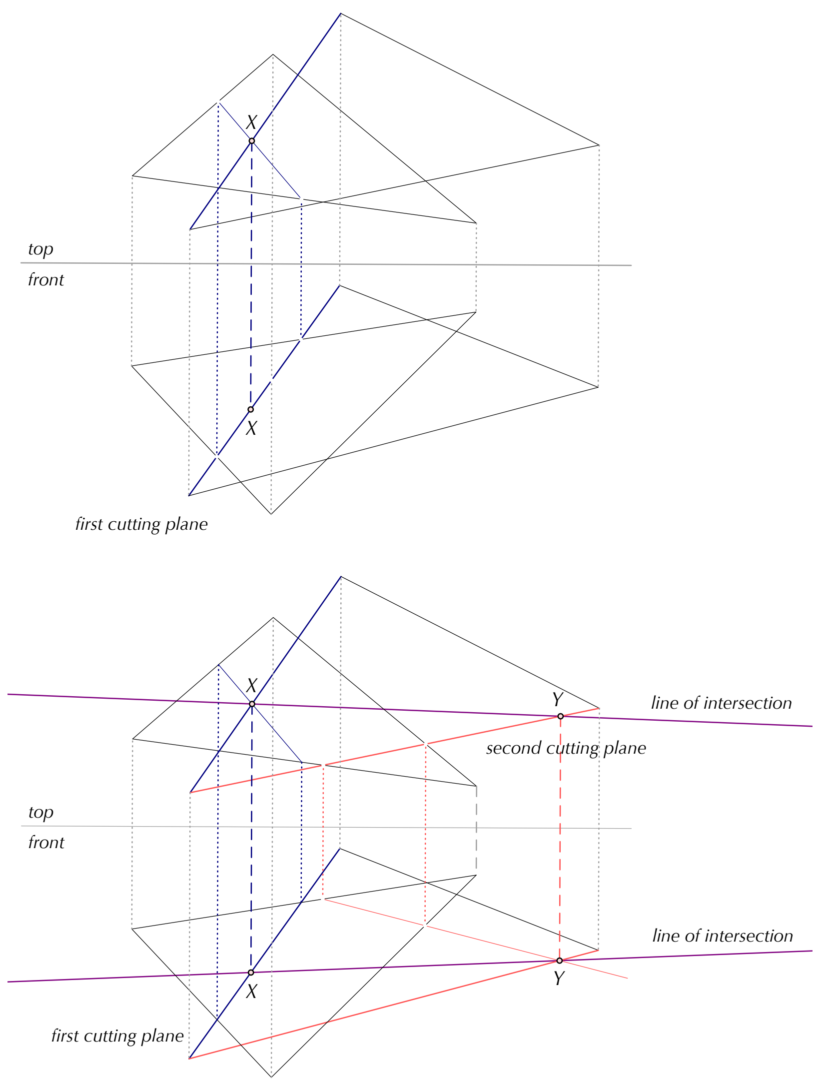
Given two planes in two adjacent views, where the planes are defined by ΔABC and ΔDEF, find the intersection line by the two-view method.

The two planes are shown in Figure 6-11a. There are two steps in the procedure.

1. Apply Construction 6-2 twice for two different cutting planes to find two intersection points, X and Y. In Figure 6-11b, the first cutting plane was selected in the front view, yielding piercing point X in both views. The second was selected in the top view yielding piercing point Y in both views. The line through X and Y is the common intersection.
2. Determine the visibility between lines in each plane using by Construction 2-2 (on page 70). See also Figure 5.19. See Figure 6-11c.

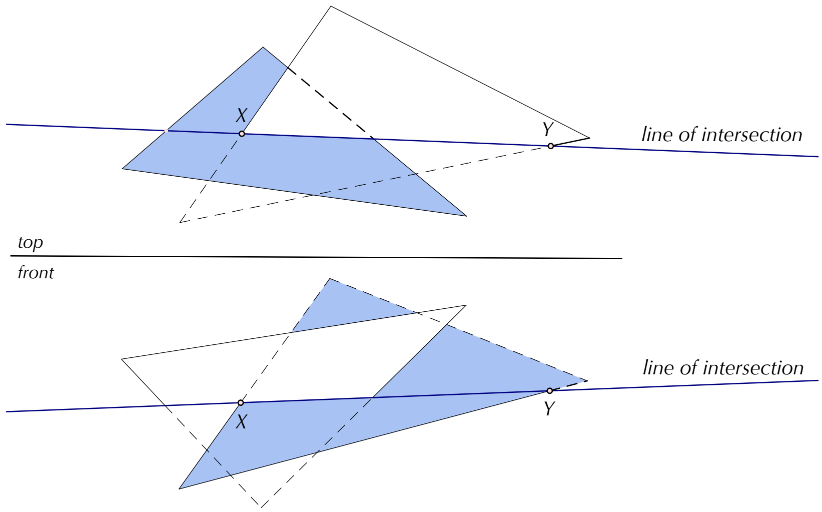


a.



b.

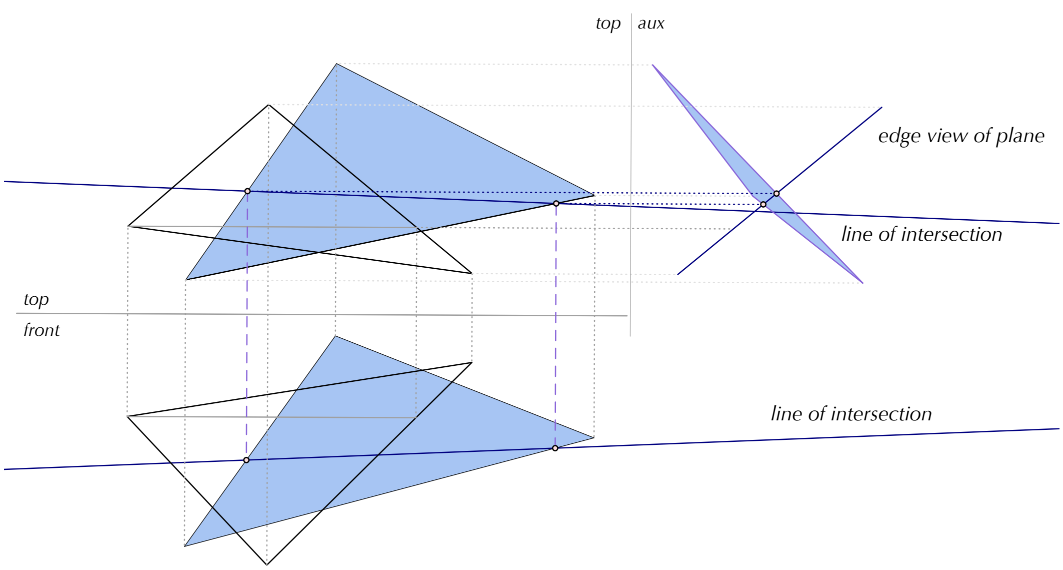
6-11
Constructing the intersection of two planes: line of intersection



c.

6-11(continued)
Constructing the intersection of two planes: visibility

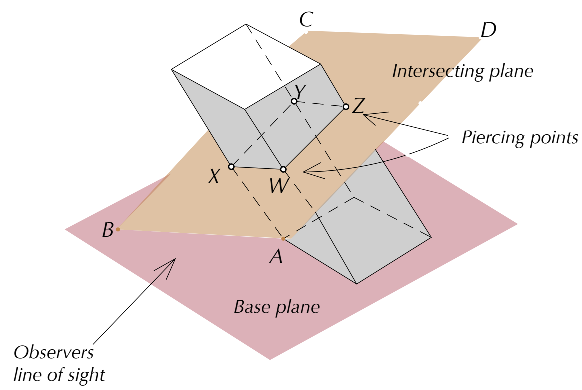
For reasons of completeness, Figure 6-12 illustrates the construction using the line method. Here we find the edge view of one of the planes from which the line of intersection of the two planes can be easily determined.



6-12
Constructing the intersection of two planes by constructing an auxiliary view

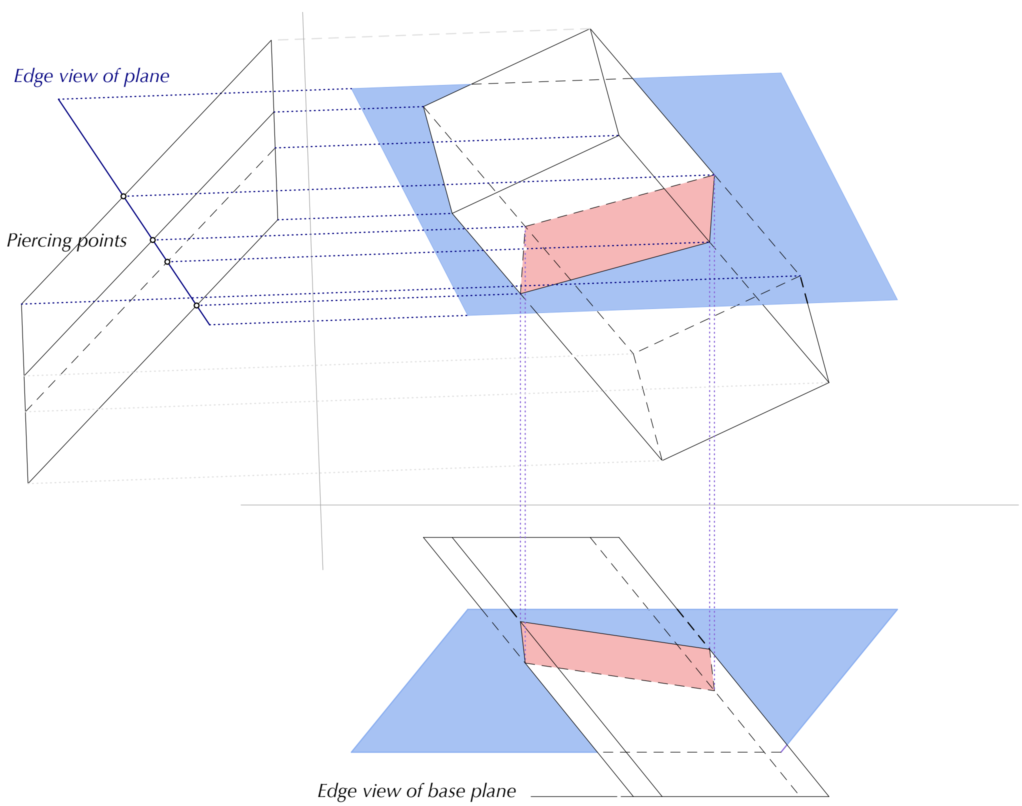
6.3.1 Lines of intersection of a plane surface and the faces of a prism

The intersection of two flat surfaces is a line. Therefore, when a plane surface intersects the face of a prism it does so in a line. The individual lines of intersection between the plane and faces of the prism form the *complete* lines of intersection between the plane and the prism. In Figure 6-13 plane *ABCD* intersects the prism along lines *WXYZ*.

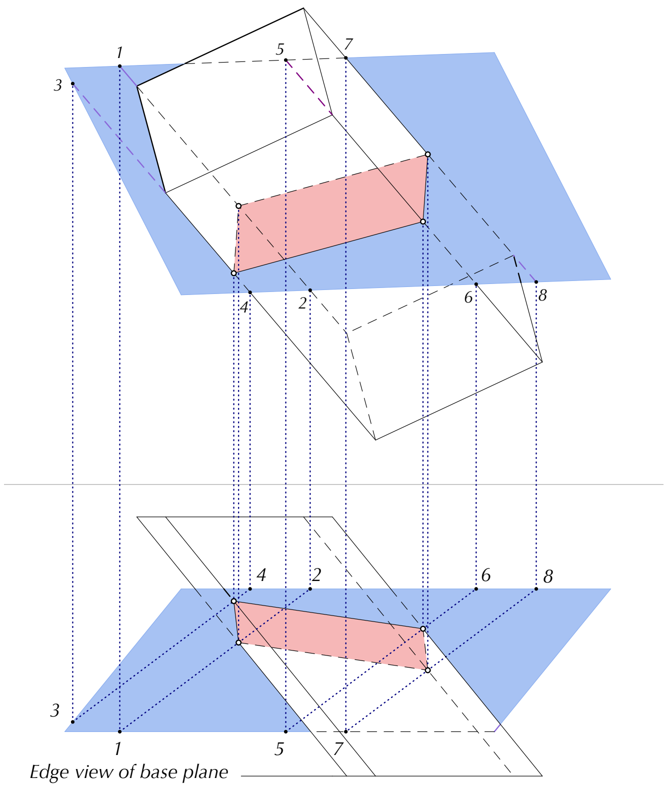


6-13
Intersection of plane and a prism

We can determine the piercing points either by taking an edge view of the intersecting plane or by using the cutting plane method. Both methods are illustrated below in Figures 6-14 and 6-15 respectively.



6-14
Intersection of a plane surface with a prism (Line method)



6-15
Intersection of a plane surface with a prism (Cutting plane method)

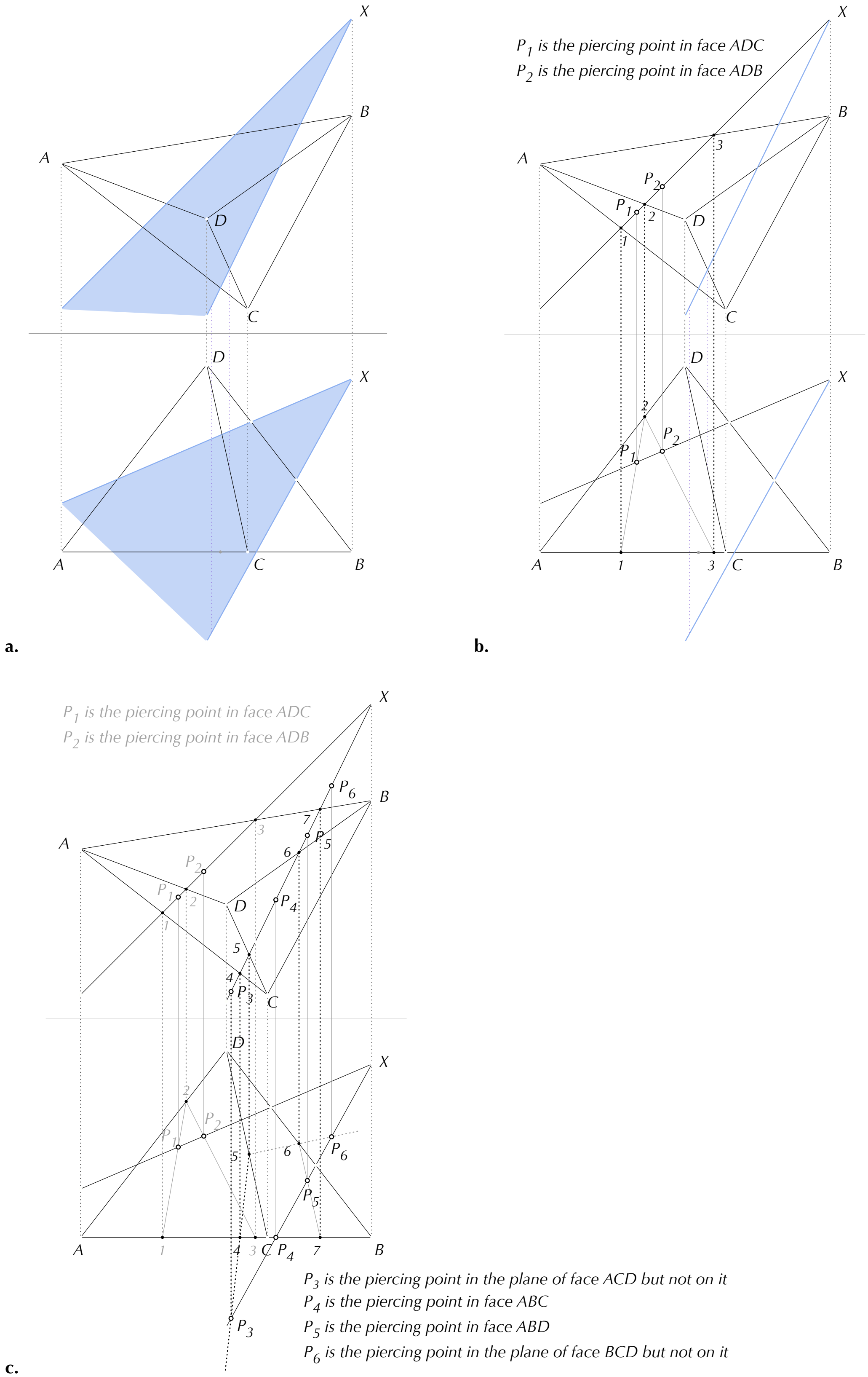
Other the intersection of a plane surface with any polyhedron can be similarly constructed. We revisit this construction in Chapter 8.

6.3.2 Problem revisited

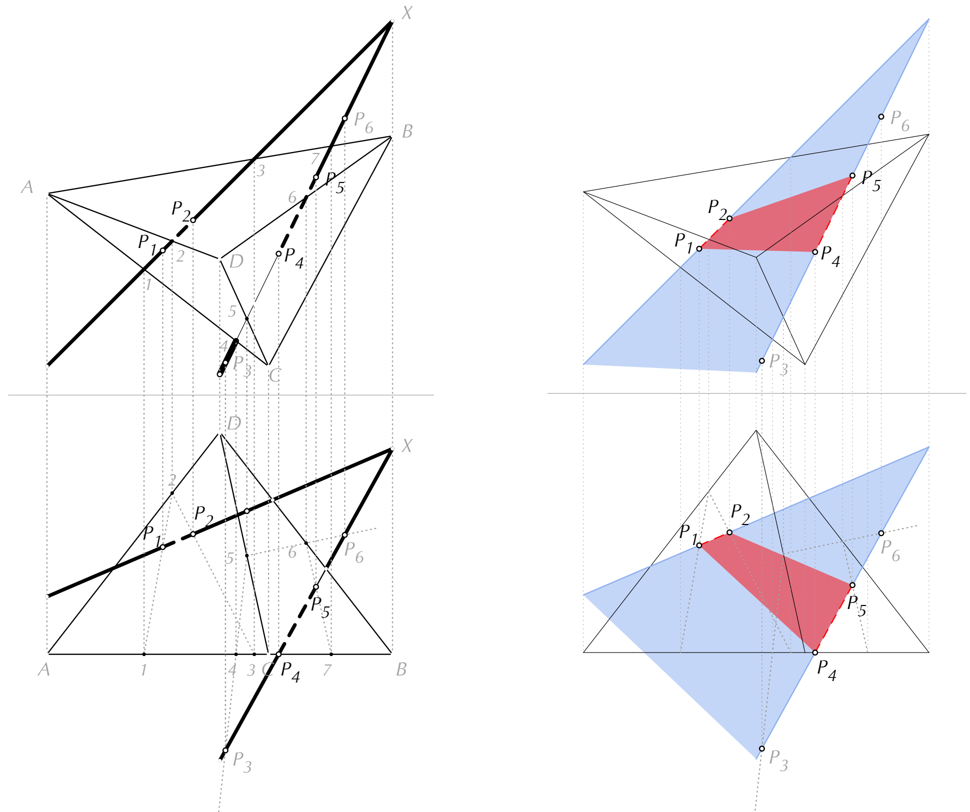
In a previous problem (see Figure 6-9), we had considered the intersection of two intersecting lines with a pyramid.

The construction follows the same steps as before except here the two intersecting lines specify a plane, in which case the pairs of points, (P_1, P_2) specify a line of intersection. Likewise pairs (P_2, P_5), (P_5, P_4) and (P_4, P_1) specify lines of intersection. In fact, the points $P_1 P_2 P_5 P_4$ define a quadrilateral plane of intersection.

The steps in the construction using the cutting plane method are shown below in Figure 6-16. The construction using the line method (constructing the edge view) is left to the reader.



6-16
Intersection of a plane with a pyramid: Five steps



d.

e.

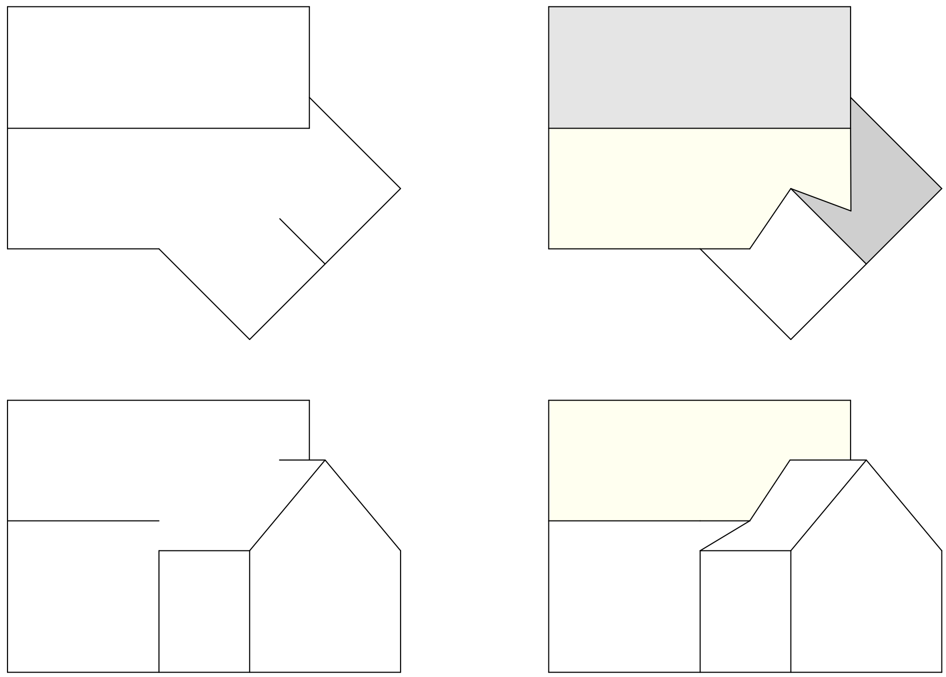
6-16(continued)
Intersection of a plane with a pyramid: Five steps

6.3.3 Worked example – Construction of a roof view

Intersections between planes have to be determined frequently in architectural design and require usually that the techniques of descriptive geometry be used explicitly because intuition fails to provide an efficient solution. The following example is typical.

Given are an incomplete roof plan and front elevation of a house with a sloped roof and an addition at a 45° angle with lower ridge and eaves as shown on the left in Figure 6.17; such incomplete drawings are easy to produce and result immediately from some basic decisions about heights and other dimensions of the objects under consideration.

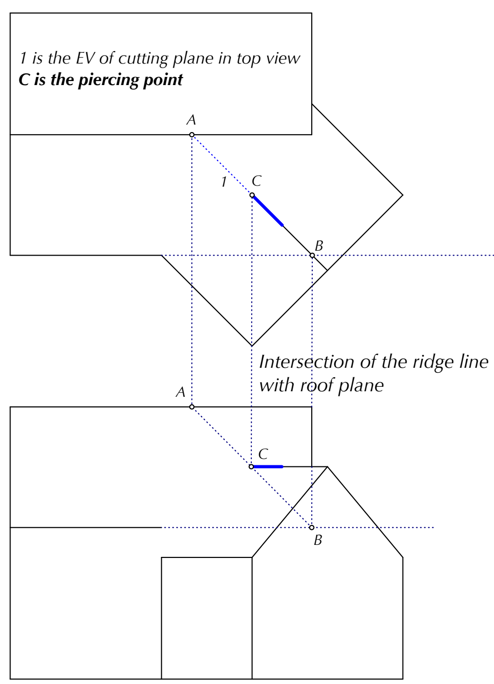
The task is to complete both views, which will lead eventually to the projections shown on the right in Figure 6-17.



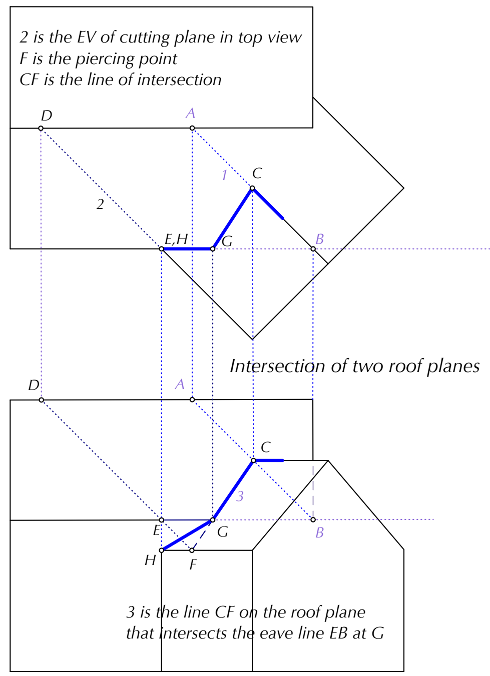
6-17
(*Left*) Incomplete and (*Right*) completed roof views of a house with addition

The figures below show the steps in the construction using the two-view method for finding the correct view.

The first cutting plane (1) was selected in the top view through the ridge of the addition; its trace in the front view is the segment AB, which intersects with the ridge to give piercing point C between the ridge and the main roof.

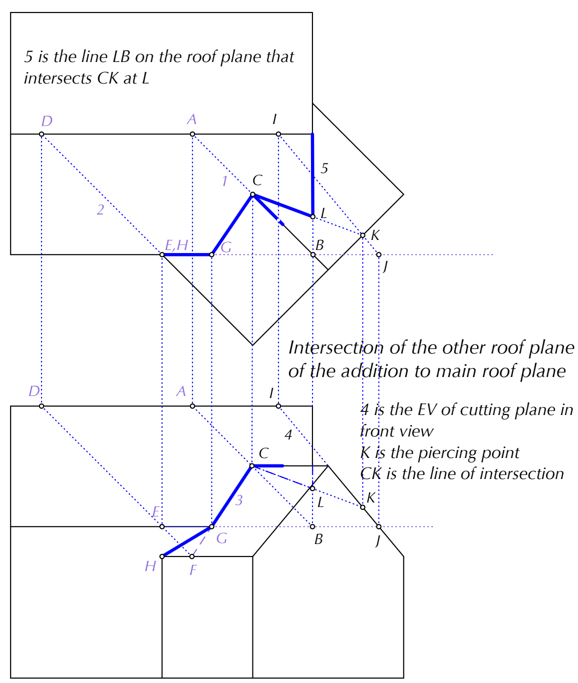


A second piercing point, *F*, is found by selecting a cutting plane (2) through the left-hand eve of the addition. The two piercing points define the intersection between the front part of the main roof and the left-hand part of the addition roof. Note that because the eaves of the two roofs are at different heights, the roofs intersect only between points *G* and *C* (segment 3). This completes the front view because the right-hand side of the addition is not visible in that view.

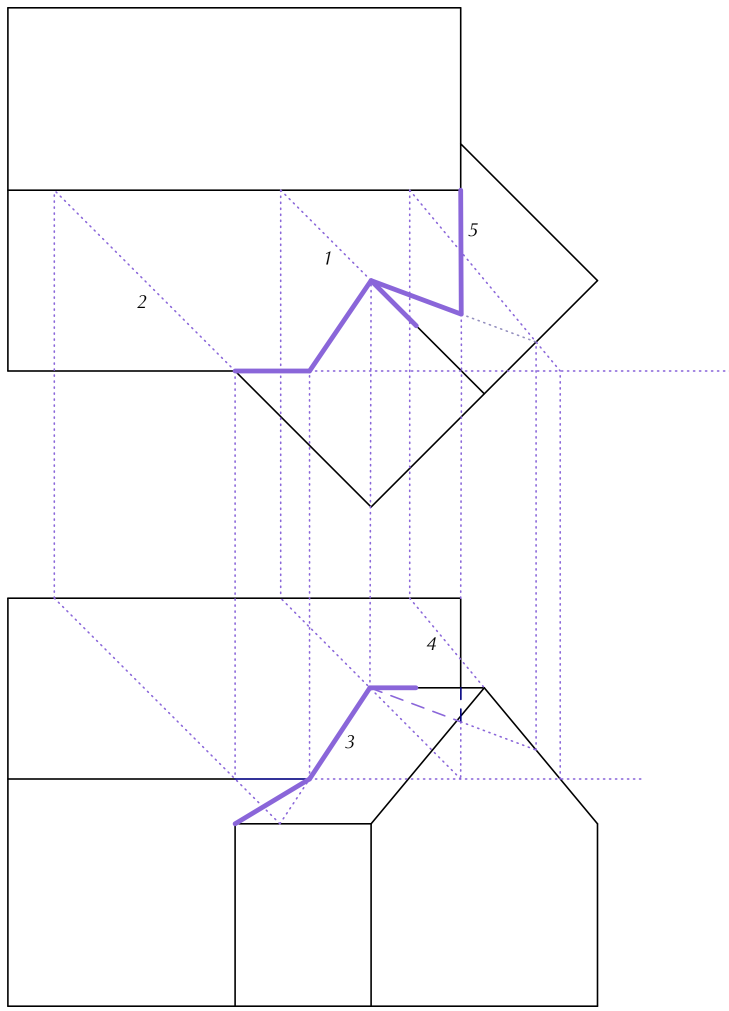


To complete the top view, we must find the intersection between the main roof and the right-hand addition roof. Cutting plane (4) is selected in the front view. This gives the trace *IJ*, and piercing point *K* in the top view.

The second piercing point for the two planes is point *C*. Thus, *CJ* is the intersection between the two planes, which is again shortened because of the different eve lines and ends at *L*. It is not visible in the front view.



The complete construction is shown in Figure 6-18.



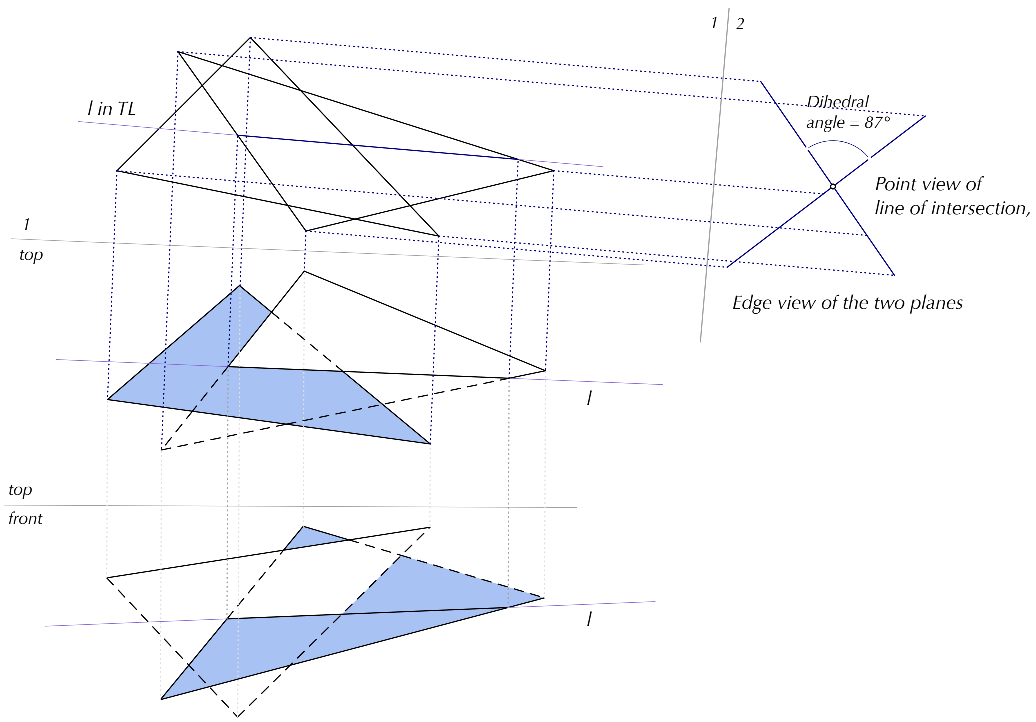
6-18
Constructing the roof intersection

6.4 TRUE ANGLE BETWEEN PLANES

Finding the intersection between two planes is also the basis for determining the true acute angle between the planes (provided the planes do in fact meet at a line); this angle is called the *dihedral angle* between the planes.

The true angle appears in any view showing both planes in edge view.

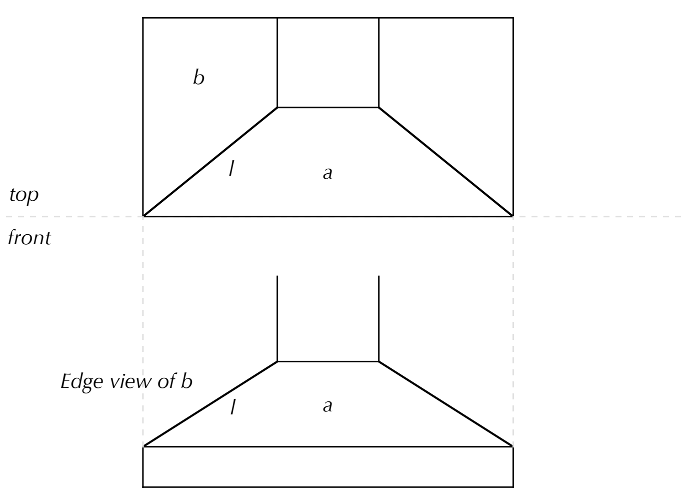
This view can always be found if the intersection line, *l*, is known. Continue by constructing a first auxiliary showing *l* in TL and a second auxiliary with folding line perpendicular to *l*. In this second view, both planes appear in edge view and *l* in point view. The true angle between the planes can be measured in this view. This method might be needed to check whether two planes are perpendicular. Figure 6-19 demonstrates this construction for the two planes used in Construction 6-3.



6-19
Finding the dihedral angle between two planes by showing them in edge view

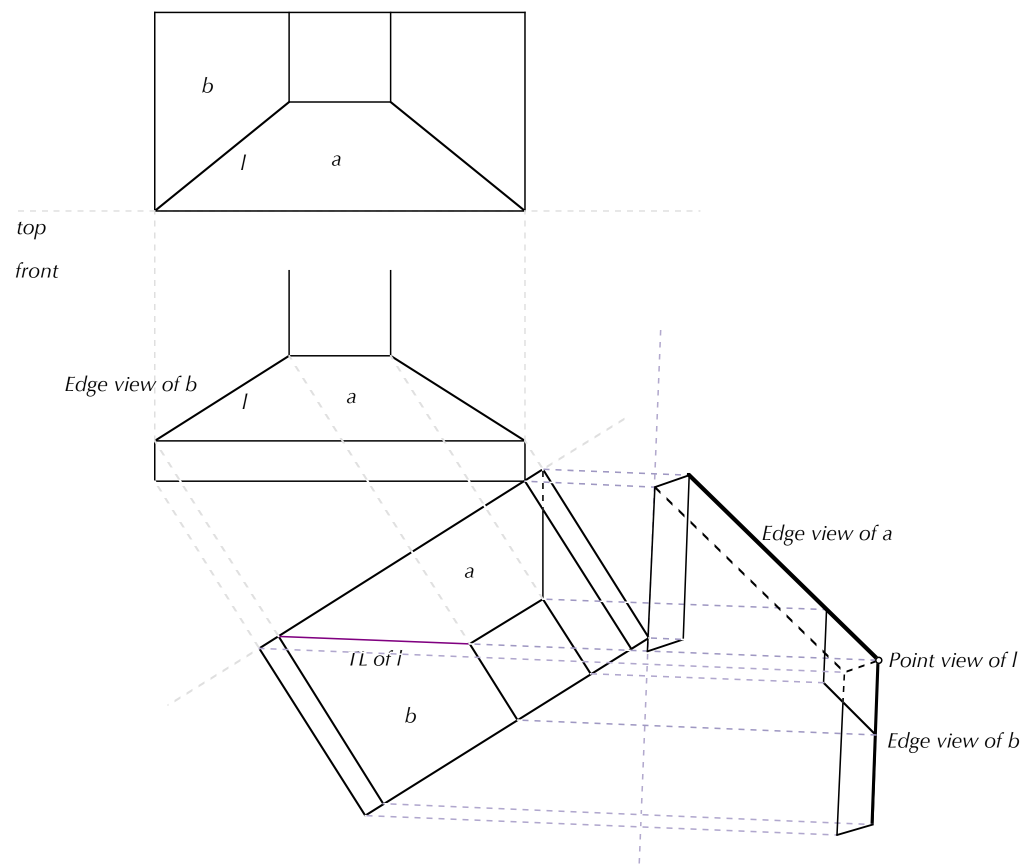
6.4.1 Worked example – True angle between the front and side views of a metal hood

A practical application of this construction is illustrated in Figure 6-20. The figure shows the top and front view of a metal hood. The problem is to find the true angle between the front and side panel, *a* and *b*, respectively.



6-20
A metal hood

In this case, the construction is simplified because we know already the intersection, *l*, between these planes. Figure 6-21 demonstrates how to solve this problem through two successive views that generate first a view of *l* in TL and then edge views of the two panels, which also depicts the true angle between them.



6-21
Solving the metal hood problem

It is worth studying the auxiliary views carefully. Clearly, view 1 is a 'tilted' bottom view (this can be gleaned from the position of the folding line relative to the front view). View 2 is a side view perpendicular to the bottom view and therefore appears to show the hood upside-down. In both views, we show hidden portions of segments by dashed lines to aid the viewer's perception of the object (this was done by using construction 1.31 or 4.15). Readers are encouraged to try to visualize the fully three-dimensional object in these views.

Alternatives to these views exist. For example, we could have used a tilted top view as view 1 by placing the folding line on the other side of *l* in the front view. This would have led to an oblique side view in view 2 that may be easier to understand because the hood would appear in a more natural orientation. That is to say, the selection of auxiliary views may influence how easy they are to understand. But this selection should make no difference for the primary purpose of the auxiliaries: to show the true angle between planes *a* and *b*.

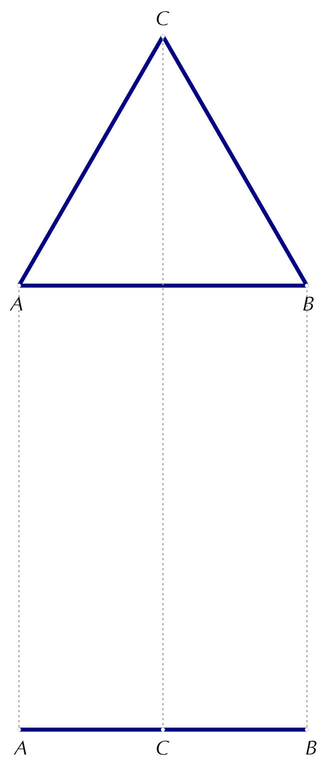
6.4.2 Worked example (slightly harder) – Constructing a polyhedron given its base and lateral angles of its sides

We can use the ideas developed in this chapter to construct a polyhedron given its base and the lateral angles that its sides make with the base. In particular, we consider the following problem:

Construct a plan and elevation for a pyramid with an equilateral triangular base, say 3" with sides at lateral angles 45º, 60º and 75º to the base.

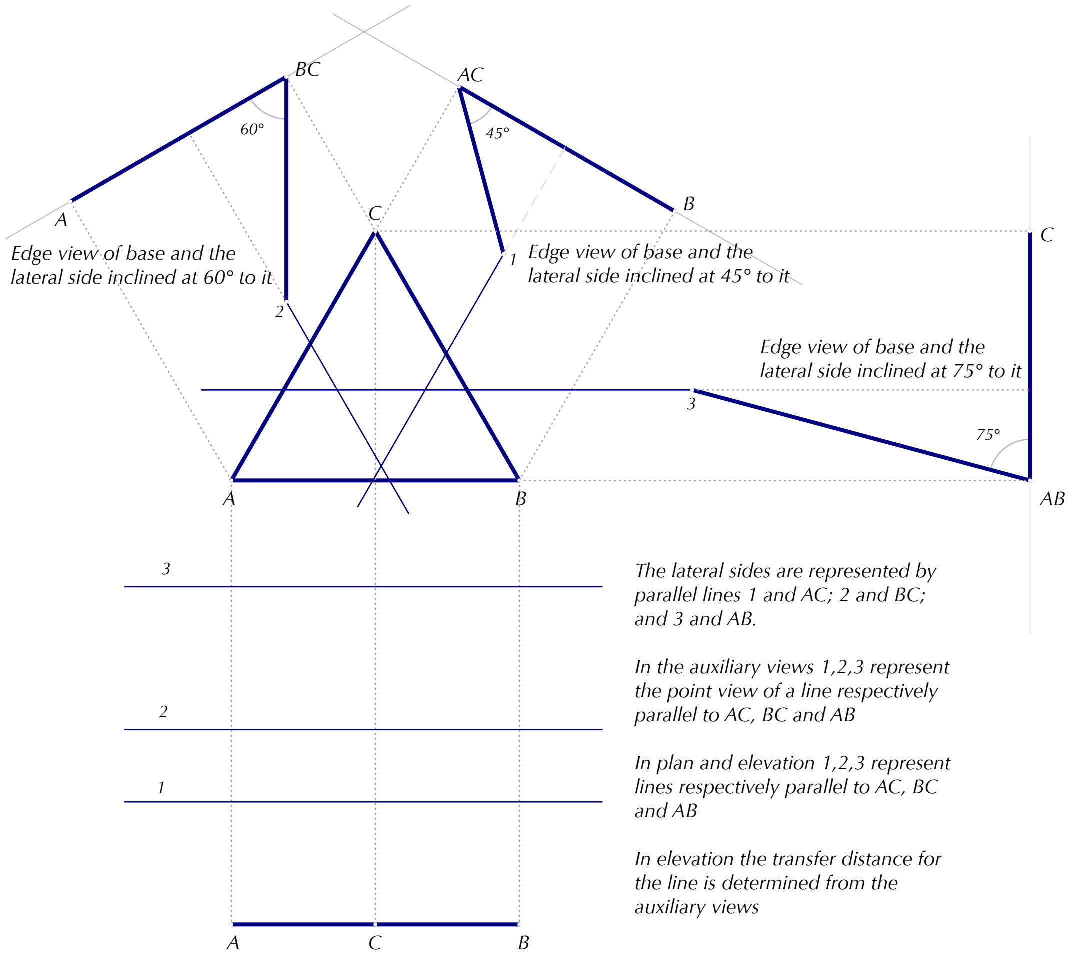
We describe the construction in stages.

The first step is to construct the base in plan (in true shape) and elevation (edge view) as shown.



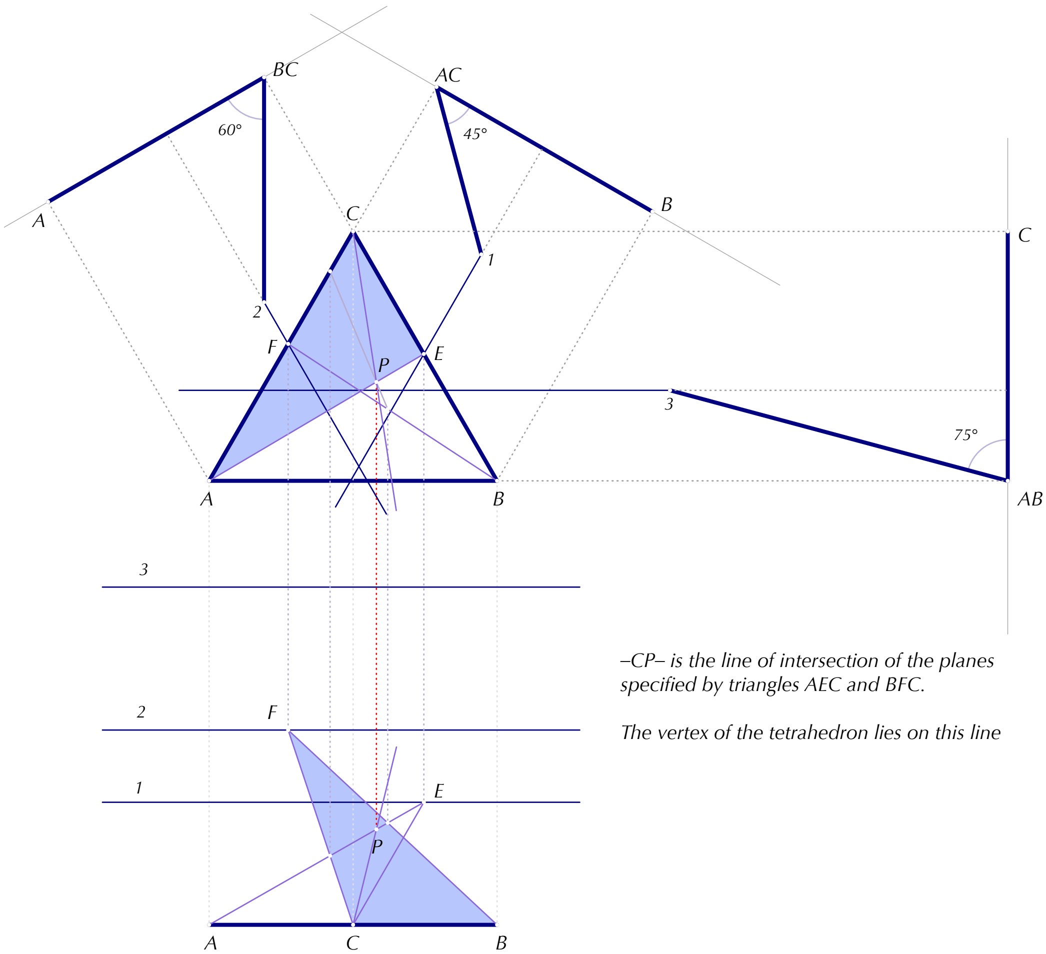
6-22
Constructing a tetrahedron given its base and lateral angles of its sides
Step 1: Construct the plan and elevation of the base

The second step is to construct showing the base in edge view a side of the base in point view and the edge view of a lateral side.



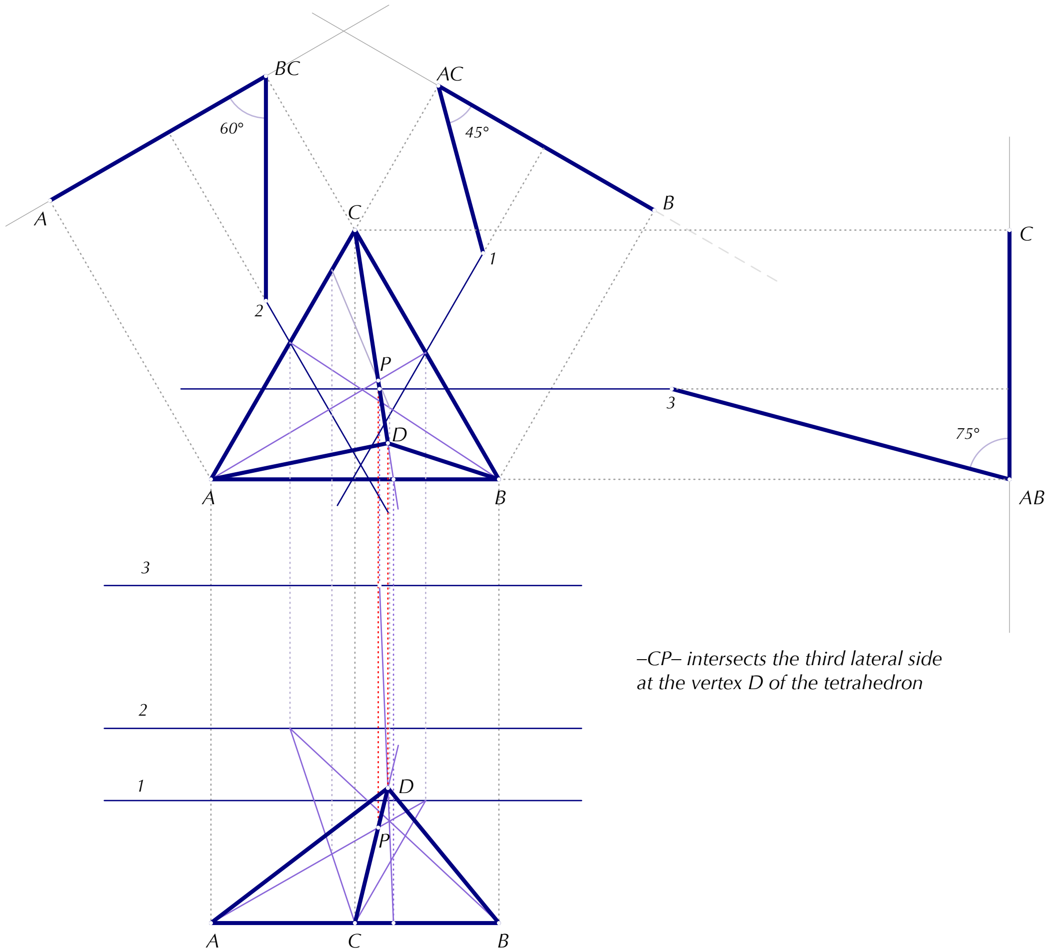
6-22(continued)
Constructing a tetrahedron given its base and lateral angles of its sides
Step 2: Construct edge views of the lateral sides and project back into plan and elevation

The next step is to determine the line of intersection of two of the lateral sides, say, bordering on *AC* and *BC*. For this we use a cutting plane to determine one of the piercing points *P*. The other piercing point is *C*. *CP* lies on the line of intersection of the two lateral sides. The vertex of the tetrahedron lies on this line.



6-22(continued)
Constructing a tetrahedron given its base and the lateral angles of its sides
Step 3: Find the line of intersection of two of the lateral sides

The last step is to find the vertex of the tetrahedron, which is similarly determined by using a cutting plane.



6-22(continued)
Constructing a tetrahedron given its base and lateral angles of its sides
Step 4: –*CP*– intersects the third lateral side at the vertex completing the construction

6.4.3 A note on the intersections between lines and solids

Intersections between a line and a solid can be constructed by the methods described above when the solid is bounded by planar surfaces: a piercing point has to be found for every surface pierced by the line. This is the problem most often found in architectural applications. The basic technique is related to the notion of cutting away a section of a solid by a plane, which is described in the sequel. But first, it would help to have some preliminary notions about solids, their surfaces and points on solids and surfaces in descriptive geometry, which are described in Chapter 8.