## Spatial Relations on Lines

There are number of useful problems that can be solved with the basic construction techniques developed thus far. We now look at certain problems, which involve spatial relationships between lines, and between lines and planes.

### 5.1 PARALLEL LINES AND PLANES

We know that a line is parallel to a plane if it has no common point with the plane. This fact can be used to test if a given line and plane are parallel: Simply construct an edge view (see Construction 4-2 on page 130) of the plane and project the line into the same view; if the line appears in point view or parallel to the edge view, it cannot meet the plane in a point and is therefore parallel to the plane. The same fact can be used to construct a plane parallel to a given line or a line parallel to a given plane. In both cases, there exist infinitely many solutions to the problem. The construction below demonstrates this for the second case.

## Construction 5-1

Line parallel through a given point to a given plane

Suppose we are given $\triangle A B C$ and a point $O$ in two adjacent views 1 and 2 , we are required to find a line through $O$ parallel to plane $\triangle A B C$. See Figure 5-1. There are four steps:

1. Construct an edge view of the plane in an auxiliary view, 3 , and project $O$ into 3 .
2. Draw a line through $A$ parallel to the edge view of the plane and select two arbitrary points, $M$ and $N$, on this line.
3. Draw projection lines through $M$ and $N$ perpendicular to folding line $1 \mid 3$.
4. Draw a convenient line through $O_{p}$ that intersects both projection lines in points $M_{p}$ and $N_{p}$; this line is parallel to plane $A B C$. Project $M_{p}$ and $N_{p}$ into $q$ to obtain a view of the line in $q$.

The construction is illustrated below in Figure 5-1.


5-1
Constructing a line through a given point parallel to a given plane (Top Right) Problem configuration (Bottom) Solution construction

When two planes are parallel, any view showing one plane in edge view must show the other plane in a parallel edge view. This fact can be used to test whether two planes are in fact parallel. Recall that Construction 4-2 (on page 130) serves to find the edge view. We show the construction below in Figure 5-2.


5-2
Parallel edge views indicate parallel planes

### 5.2 PERPENDICULAR LINES AND PLANES

A line is perpendicular to a plane, referred to as normal to the plane, if every line in the plane that passes through the point of intersection of the given line and the plane makes a right angle with the given line.


We can apply the same principles as before to solving problems involving perpendicular lines and planes. For example, a line normal to a given plane can be found in an auxiliary showing the plane in edge view; in this view, the perpendicular appears in TL perpendicular to the edge view and can be projected into the given views. This method also yields the point where the line intersects the plane, which is called the piercing point (see the Chapter 6 on intersections).


5-4
A line perpendicular to plane in views showing the plane in edge view

We can also find the normal without constructing an auxiliary view by the following two-view method. The normal must be perpendicular to every line in the plane. If the plane is given in two auxiliary views, we find a line in the plane shown in TL in one of the views. The normal must be perpendicular to that line in the same view. However, this method does not automatically yield the piercing point. See Figure 5-5.


5-5
Two-view method for finding the direction of the normal to a plane

Similarly, we can find a plane perpendicular to a given plane. How?

### 5.3 PERPENDICULAR PLANES

Determining whether two planes are perpendicular generally requires finding their intersection and this is dealt with in another chapter and the construction is omitted for now.

### 5.4 SHORTEST DISTANCE CONSTRUCTIONS

We next consider various shortest distance constructions, namely, from point to plane, from point to line and between lines.

### 5.1.1 Shortest perpendicular distance to a plane

We can use the principles underlying Figure 5-5 to determine the shortest between a point and plane. For any given point, we construct a line perpendicular to the given plane. In edge view, this perpendicular line will be seen in true length and at right angles to the plane. The construction is given next.


5-6
Shortest distance to a plane from a point

## Construction 5-2

Shortest distance from a point to a plane

Suppose $A B C$ is the given plane and $M$ is the given point.

1. Construct the edge view of plane (see Figure 4.6) by constructing an auxiliary view \#3.
2. In this view, construct a perpendicular from $M$ to the edge view of the plane. Let $P$ be the foot of the perpendicular. $M P$ is the required distance.
3. To determine where $P$ is located in the views \#1 and \#2, see Figure 5-7.



### 5.1.2 Shortest distance between a point and a line

The Problem -
Given adjacent views of point $X$ and line $A B$, find the true length of the shortest distance from $X$ to the line $A B$


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We consider two methods: the line method and the plane method.

## Line method

We solved this problem in a previous chapter by constructing the line in point view and measuring the distance between the PV of the line and the view of the point. See Construction 3-7 (on page 108). We may call this the line method as we deal directly with the given line. The method is illustrated again below in Figure 5-9.


5-9
The line method for determining the shortest distance between a point and a line

## Plane method

A second method is based on a plane constructed from the point and the given line. We construct a view showing the plane in true shape (see Construction 4-3 on page 134) and the perpendicular from the point to the given line in this view gives the shortest distance. This method is illustrated in Figure 5-10.


## 5-10

The plane method for determining the shortest distance between a point and a line

### 5.1.3 Shortest distance between two skew lines

Skew lines are non-intersecting non-parallel lines. Skew lines are parallel to the same plan, but there is no plane in which the two lines are coplanar. For any two skew lines, there exists exactly one line perpendicular to both lines; this line is called the common perpendicular. The shortest distance between two skew lines is the distance between their intersection points with their common perpendicular. This distance can be determined by finding a view in which one of the lines is shown in point view.

5-11
Two skew lines and their common perpendicular


We highlight below, two constructions to determine this shortest distance, one is a line method and the other, a plane method.

In both cases, we are given two skew lines in two adjacent views, 1 and 2, where the lines are described by the segments, $A B$ and $C D$, the construction below finds the shortest distance between the lines.

5-12
Two skew lines


## Construction 5-3

Shortest distance between two skew lines (line method)

The construction is shown Figure 5-13.


5-13
Line method for constructing the shortest distance between two skew lines

There are five basic steps:

1. Construct a first auxiliary view 3 showing one of the segments, say $A B$, in true length (TL) and a second auxiliary view 4 showing the same segment in point view (PV).
2. Project the other segment into both auxiliary views.
3. In view 4, the common perpendicular appears in TL as a line through the point view of $A B, X$, and perpendicular to $C D$. Find the intersection of this perpendicular and $C D, \mathrm{Y} . X Y$ is the required shortest distance.
4. Project Y into view 3. In view 3, the common perpendicular is the line through Y parallel to folding line $3 \mid 4$ meeting the line $A B$ at X
5. Project $X$ and $Y$ into 1 and 2 to obtain all views of the common perpendicular.

## Construction 5-4

Shortest distance between two skew lines (plane method)

Again, as before, we are given two skew lines in two adjacent views, 1 and 2 , where the lines are described by the segments, $A B$ and $C D$. The construction below finds the shortest distance between the lines by constructing planes first. We consider four steps, although the stated problem is solvable in two.

1. Construct a plane parallel to segment $A B$ containing the segment $C D$ in both views. We do this by drawing a line $X Y$ parallel to $A B$ through a selected assumed point $W$ on $C D$. Project $W$ onto the other view and, in the other view, draw the line $X Y$ through this projected point $W$ parallel to $A B$. See step 2 for a good choice of X or Y. $C Y D X$ is a plane parallel to $A B$.

2. Construct an auxiliary view showing the plane CYDX in edge view. We do this by drawing a horizontal line $D X$ in one of the given views, say 2 , and project it into the other view. $D X$ is now shown in true length. A folding line $1 \mid 3$ is drawn perpendicular to $D X$ and the projection of the plane CYDX in that view will be seen in edge view. In this view $A B$ is parallel to CYDX. The distance between the two lines is the shortest distance between the two skew lines.

3. To find the common perpendicular we need one further step. Create another auxiliary view 4 in which $A B$ is shown in true length. The apparent point of intersection in view 4 shows the common perpendicular in point view. Incidentally, $C D$ is also shown in true length. This construction proves that for any two skew lines, there actually is a view that shows them both lines in TL.
4. This point is projected back into views 3,1 and 2 to give the actual location of the common perpendicular, $R S$.

The complete construction is illustrated in 5-14.


5-14
Plane method for constructing the shortest distance between two skew lines

### 5.1.4 Shortest horizontal distance between two skew lines

The shortest horizontal distance between two skew lines is the length of shortest horizontal line between them. See Figure 5-15 in which $X Y$ represents the shortest horizontal distance in true length. That is, XY must appear as a horizontal line in an (vertical) elevation view

5-15
Shortest horizontal distance between two skew lines


For this, we need to create an elevation view in which the required line appears in point view and project back into the original views. By the plane method it is possible to construct a view in which the two skew lines appear parallel as the planes they lie in are seen in edge view. We have the following construction given in Figure 5-16.

## Construction 5-5

Shortest horizontal distance between two skew lines

As before, we are given two skew lines in two adjacent views, 1 and 2 , where the lines are described by the segments, $A B$ and $C D$.

We employ the plane method.
There are five steps.

1. In the elevation view, construct a line $L M$ parallel to $C D$ passing through an assumed point $M$ on $A B$. Construct $L B$ to be parallel to the folding line $1 \mid 2$, that is, horizontal.


|  |  |  |  | 1 |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | 2 |

2. Project $L$ and $M$ back into the top view and again draw $L M$ parallel to $C D$ in this view. $L B$ is now shown in true length.



5-16
Shortest horizontal distance between two skew lines

### 5.1.5 Shortest grade distance between two skew lines

The grade is an alternative way of measuring the slope of a line favored by civil engineers, particularly ion reference to the incline of a road. Percent grade is defined as the number of vertical rise for each hundred units of horizontal distance. For any two skew lines, for a given grade (slope), there is a unique line that provides this shortest distance. To determine this we employ a construction similar to 4.11 using the plane method to obtain a view in which the skew lines appear parallel to each other from which we can determine the view that shows the required distance (line) in point view.

## Construction 5-6

Shortest grade distance between two skew lines.

Suppose that we want to find the shortest distance with a slope of $15^{\circ}$ from $A B$ to $C D$.
We consider two possibilities - an upward slope from $A B$ to $C D$, or a downward slope. The first three steps are the same as above. The remainder of the construction is given below and illustrated in Figures 5-17 and 5-18.


5-17
Shortest grade (upward slope) distance between two skew lines

The last two steps are:
4. We construct a line $P Q$ in view 3 on the folding line $3 \mid 1$ with the required slope. This line determines the direction of the shortest distance. Construct a view 4 perpendicular to this direction. In this view the shortest line XY with the specified grade between $A B$ and $C D$ will appear in point view, that is, as a point of intersection between $A B$ and $C D$ in view 4 .
5. Project $X$ and $Y$ back to views 3,1 and 2. $X Y$, the shortest distance at the given grade between the skew lines, is seen in true length in view 3 and 1.


5-18
Shortest grade (downward slope) distance between two skew lines

### 5.1.6 Visibility of skew lines revisited

As skew lines do not intersect, it becomes important to consider the visibility of lines, particularly when the lines represent solid objects. We have considered visibility before in Construction 2-2 on page 73. We show the construction again without explanation in Figure 5-20.


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### 5.5 WORKED EXAMPLES

### 5.5.1 A point on a line equidistant to two given points

You are given two points $A$ and $B$ and a line $l$. You are asked to find the point on $l$ that is equidistant to $A$ and $B$.
(Right) The problem

The set of all points that are equidistant between two points is a plane, which passes through the midpoint of the line formed by the two points and is perpendicular to the line. Alternatively, consider a true isosceles triangle with $A B$ as base and the desired point on $l$ as the vertex of the triangle.


Consequently, we construct a view showing $A B$ in true length. Draw a line perpendicular to $A B$ at its mid-point. This line will meet $l$ at the desired point. Back project to the top and front views. See Figure 5-21.


5-21
Finding a point on a line equidistant to two points

### 5.5.2 A line through a given point and intersecting two skew lines

Such a problem occurs in engineering situations, for instance, when connecting two skew braces by a stiffening brace anchored at a specific point. See figure to the right.
(Right) The problem

To solve the problem, consider a view in which one of the skew lines is shown in point view. We simply connect the point view with the specified point by a line and intersect the other skew line at a definite point. Back project to the original views to complete the line.

The construction is shown in Figure 5-22.
In view \#2, we extend the point view of $A B$ through $X$ to meet $C D$ at $Y$. This line gives the trace of the required line. Projecting back into view \#1, the line through $Y X$ intersecting $A B$ gives the desired line. This line can now be projected back into the top and front views.


5-22
Constructing a line through a specified point between two skew lines

### 5.5.3 A line at a certain grade between two skew lines

A typical practical problem might be the following.
Lines $A B$ and $C D$ specify centerlines of two existing sewers as shown in the figure below.


The sewer pipes are to be connected by a branch pipe having a downward grade of 2:7 from the higher to the lower pipe.

Given that point $C$ is $20^{\prime}$ North of point $A$, the problem is to determine the true length and bearing of the branch pipe and show this pipe in all views. Line $A C$ (in plan) measures 20'. The construction is shown in Figure 5-23. The scale is defined within the drawing. We follow the steps in Construction 5-6 (on page 159).


5-23
A line at a certain grade between two skew lines

### 5.5.4 How far from the nearest face?

Consider the views shown on the right of a transition piece shape in which a cable is passed through the opening of the piece from point $X$.

What is the shortest distance from $X$ to the nearest face?
The first step of the construction is given in Figures 5-24. The nearest face is $p$. We construct the perpendicular from $X$ to $p$. The perpendicular from $X$ does not meet the face but meets the plane represented by the face at a point off the face. We extend the projection line from view 1 so that it meets the edge of face $p$. This is the nearest distance.

We still have to determine its true length (see Figures 5-25 and 5-26).


5-24
Nearest distance to a face (step 1)


5-25
Nearest distance to a face (step 2)

In Figure 5-25 we determine the point view of the perpendicular to face $p$ shown in view 2. In this view the line $A B$ is shown in true length. The nearest point from $X$ to the face is the hypotenuse of the right-angled triangle $X Y Z$ shown in view 2 as the perpendicular from $X Y$ to the line $A B$.

The last step shown in Figure 5-26 determines the true length of the line $X Z$, which is given in view 3 .


Nearest distance to a face (step 3)

### 5.5.5 Clearance between a spherical tank and pipeline

In this problem, we consider the possible interference between an existing pipeline $A B$ of a given diameter (12") and a spherical tank also of given diameter (10'). Points $A$ and $B$ are located as shown in Figure 5-27, which has been drawn to scale ( $1^{\prime \prime}=8^{\prime}$ ). We are required determine the clearance, if any, between the pipe and the tank and show all constructed views.

5-27
Clearance between a spherical tank and a pipeline (indicated by it centerline) - problem definition


It is important to note that the pipeline and the center of the spherical tank form a plane, which means we can construct the edge view of this plane and then its true shape fro $m$ which we can determine, if any, the clearance between the tank and pipeline after accounting for its diameter. For this we construct the perpendicular from the center of the tank to the centerline of the pipeline. The construction is shown in Figure 5-28. Notice that the spherical tank appears as a circle in every view. The plane ABC is shown only for purposes of illustration. The true shape of the pipeline is shown in view \#2.


### 5.5.6 Adding a new pipeline

We are given two existing sewer lines $A B$ and $B C$ that intersect at a manhole $B$, with $A$ and $C$ located with respect to $B$ as follows: A is $35^{\prime}$ North 10 East of $B$ and $30^{\prime}$ above $B$; $C$ is $20^{\prime}$ North $60^{\prime}$ West of $B$ and $15^{\prime}$ above $B$. A new line $D B$ is to be located in the plane of $A B C$ at a point $30^{\prime}$ due west of $A$.

See configuration to the right.
We are required to determine the true lengths of each sewer line and the angle of the plane $A B C D$.

As $D$ lies in the plane of $A B C$, we can readily determine its location in front view. The remaining construction to determine true lengths is straightforward.


See Figure 5-29.


### 5.5.7 Shortest distance between non-intersecting diagonals of adjacent faces of a cube

Consider the top and front view of a cube with non-intersecting diagonals $A B$ and $C D$ of adjacent faces as shown on the right.

As $A B$ is parallel to the folding line in front view, we can easily construct the point view of line $A B$ in an auxiliary view taken from the top view. The perpendicular $E F$ to $C D$ in this view gives the shortest distance.
The construction is given in Figure 5-30.



5-30
Shortest distance between non-intersecting diagonals of adjacent faces of a cube

### 5.5.8 Completing the view of a plane parallel to another

Imagine we are given an incomplete view of two planes as shown in the figure below.


We are required to complete the view of plane $D E F$ parallel to $A B C$ without constructing any additional views. Since parallel lines are seen as parallel in adjacent views we can use this property to complete the construction. The construction is given in Figure 5-31. Here $A 1$ is parallel to $E F$ and $C 2$ parallel $D F$.


5-31
Completing the view of a plane parallel to a given plane


[^0]:    5-20
    Visibility test for lines

