Lines in 3-D Descriptive Geometry

3.1 DEPICTING A LINE SEGMENT

We have previously seen that to depict a line segment in an orthographic view we simply project its end points in the view and join the projected end points to form a segment. In two adjacent views the end points of the segment lie on projection lines that are perpendicular to the folding line.



3.1.1 Auxiliary view of a line segment

On occasions, it is useful to consider an auxiliary view of a line segment. The following illustrates how the construction shown in the last chapter (see Figure 2.38) can be used to solve certain basic problems, which, in turn, may be part of a larger problem. The

construction takes advantage of the fact that a parallel projection between planes maps segments on segments and preserves endpoints. Observe the transfer distance "rule" being applied to construct the auxiliary view.





3.2 POINTS ON A LINE SEGMENT

The basic problem considered here is to visualize a point on a segment in two adjacent views. There are two cases to consider: when the views of the segments are perpendicular to the folding line and when they are not, as illustrated by the following construction.

Construction 3-1 Adjacent views of a point on a segment

Given a segment in two adjacent views, top and front, and the view of a point, X, on the segment in one view, say top view, construct the view of X in front view, X_f .

There are two cases to consider.

If the views of the segment are not perpendicular to the folding line $t | f, X_f$ can immediately be projected from X_t (see Figure 3-3a).

Otherwise, if the views are perpendicular, we go through the following steps:

1. Use the auxiliary view construction (Figure 3-2) to project the end-points of the segment into a view, a, adjacent to t (see Figure 3-3b) and connect them to find the view of the segment.

- 2. Project X_t on the segment.
- 3. The distance of X_a from folding line $t \mid a, dX$, is also the distance of X_f from folding line $t \mid f$ and serves to locate that point in f.

Try to visualize why this construction works in space!



Constructing adjacent views of a point on a segment

3.3 TRUE LENGTH OF A SEGMENT

The true length (TL) of a segment is the distance between its end-points.

Finding the true length of a segment is a basic problem in descriptive geometry. The constructions that can be used to solve this problem are based—among other things—on the property that a parallel projection between two coplanar lines preserves distances.

There are some facts that we can rely upon. When a line segment in space is oriented so that it is parallel to a given projection plane, it is seen in its *true length* in the projection on to that projection plane.



Line segments parallel to projection planes are seen in true length

Segments parallel to one of the projection planes is seen in an adjacent view as a line parallel to the folding line.





Based on this observation, when applied to orthographic projections, this means that in order to find the TL of a segment, we must find its view in a picture plane containing the segment parallel to it (because the projection lines through the segment define a plane containing the segment and its image on parallel lines).

Given the standard assumption that a segment is given in two adjacent views, *top* and *front*, the problem is easily solved if

- The segment appears parallel to the folding line in, at least, one view, say *top* (see Figure 3-6a). In this case, the segment is parallel to the picture plane of *front* and appears in TL in *front* (the segment can be parallel to both views).
- The segment appears as a point (*point view of a segment*, PV) in one view, say *top* (see Figure 3-6b). In this case, it is on a projection line, and any picture plane parallel to that line, including *front*, will show the segment in TL.





There is a special case which occurs when the line segment is perpendicular to the folding line, in which case it can be constructed to appear in TL in an auxiliary view, say a side elevation.



3-7 Line segment seen in TL in an auxiliary view

Are there any other special cases?

3.3.1 General case of the true length of a line segment

A segment that neither appears in point view nor is not parallel to the folding line in any view is called *oblique* or *inclined*. Its true length can be determined with the help of an auxiliary view as the following construction shows.



Construction 3-2 True length of an oblique segment - auxiliary view method

Given two adjacent views of an oblique segment, determine the TL of the segment.

Suppose the given adjacent views are numbered 1 and 2.

There are three steps.

- 1. Select a view, say 1, and draw a folding line, 1 | 3, parallel to the segment for an auxiliary view 3.
- 2. Project the endpoints of the segment into the auxiliary view (see Figure 3-2).
- 3. Connect the projected endpoints. The resulting view shows the segment in TL.

The construction is illustrated in Figure 3-9(left). Figure 3-9(right) shows the same construction using an auxiliary 4 and the folding line $2 \mid 4$.





3.3.2 Worked example – True length of a chimney tie

We show below a practical application of Construction 3-2. Figure 3-10 shows the top and front views, T and F, of a roof with a chimney and two ties, s and t, that connect the chimney to the ridge of the roof. Both views are easy to construct once we select the points where the ties meet the ridge in the top view and the points where they meet the chimney in the front view. But neither view shows either tie in true length (why?).



Figure 3-11 shows how an auxiliary view, A, can be used to show one tie, s, in TL.





The figure shows the entire auxiliary view of the roof assembly, which can be constructed entirely by the constructions introduced so far with the exception of the bottom side edge of the chimney. Its front end point, P, is easy to construct from the given views. But its rear end point, Q, is not that obvious because it is not given in the front view, and we can consequently not find immediately the appropriate transfer distance. Standard constructions to solve this problem will be introduced in subsequent chapters. But a little thought will solve this problem based on what we know so far about parallel projections. I leave this as an exercise to the reader. Use the construction hint illustrated in the figure.

3.3.3 Distance between two points

Construction 3-2 can also be used to determine the distance between two points in space: it is the true length of the segment connecting the two points.

3.4 SUCCESSIVE AUXILIARY VIEWS

To solve certain basic problems in descriptive geometry, a second auxiliary view may be needed. This kind of view is constructed from a preceding auxiliary view by repeating the construction in Figure 3-2 (on page 88) for all points of interest. Successive auxiliary views are normally identified by indices 1, 2, ... that show the order of construction.

3.4.1 Point view of a line



Point view of a line segment

The next construction demonstrates an important application of successive auxiliary views that generates the point view of a line or segment; that is, it finds a picture plane to which the segment is perpendicular so that it belongs to the line family that generates an orthographic projection into the picture plane. See Figure 3-12.

Construction 3-3 Point view of a line

Given an oblique segment in two adjacent views, find a point view of the segment.

Suppose the adjacent views are numbered 1 and 2.

There are three steps:

- 1. Apply Construction 3-2 (on page 93) to obtain a primary auxiliary view 3 showing the segment in TL.
- 2. Place folding line 3 | 4 in view 3 perpendicular to the segment to define an auxiliary view 4.
- 3. Project any point of the segment into view 4. This is the point view of the entire segment.



3-13

Constructing the point view of a segment

Note that if the segment is shown to be parallel to folding line $1 \mid 2$ in one of the given views, the construction can be shortened because the segment appears in TL in the other view. Here, only one auxiliary view is needed to obtain the point view.

A point view of a segment is also the point view of the line to which it belongs. Therefore Construction 3-3 can also be used to obtain the point view of a line; any convenient distinct two points on the line can serve as the basis for the construction.

3.5 **ORTHOGONAL LINES**

The basic constructions introduced thus far can serve to solve seemingly more intricate problems in descriptive geometry. One type of problem considers two lines that are orthogonal; that is, they are either parallel or perpendicular. Orthogonality is important in many applications, as an attribute to be either tested for or desired; it also plays a crucial role in finding various types of distances. The following sections introduce constructions that address these issues.

3.5.1 Parallel Lines

We know (from Property 2-5 on page 58) that a parallel projection between planes preserves parallelism between lines. Since two parallel lines define a plane, the orthographic projection of the lines into a picture plane can be considered a parallel projection between planes and thus preserves their parallelism except when the lines are projection lines themselves, in which case they project into points, or when they are projected by the same lines, in which case their images coincide. That is:

- ٠ If two lines are truly parallel, they are parallel in every view, except when they coincide or appear in point view.
- Furthermore, if two lines are not parallel in a particular view and neither coincide nor appear as points in that view, they cannot be truly parallel.

Choosing views wisely

It should be noted that single pairs of adjacent views do not always readily help in interpreting an object.

In many cases, additional views are needed. Fortunately, these additional views can, almost always, be produced by purely two -dimensional constructions, from the two adjacent views that are given.



What am I looking at?



The converse unfortunately is not always true: two lines that are parallel in a particular view or coincide might not be truly parallel. Figure 3-14 illustrates this with an assembly of two wedge-shaped objects that contain, among others, two segments, l and m, that are not parallel, but appear parallel in the top and front views of the assembly. Such situations are not infrequent in architectural and engineering applications.



3-14 Non-parallel lines that appear in adjacent views as parallel

3.5.2 Testing for parallel lines

True parallelism between two lines that appear parallel in two adjacent views can always be established by the following construction.

Construction 3-4 Testing for parallel lines

Given two lines shown parallel in two adjacent views, determine whether the lines are truly parallel.

There are two cases to consider.

• The lines are not perpendicular to the folding line.

See Figures 3-15a) through 3-15c). In this case, the lines are truly parallel.

• The lines are perpendicular to the folding line.

This case is also illustrated in Figure 3-15. Construct an auxiliary view not parallel to the folding line. The lines are truly parallel if and only if they are also parallel in this view. In the example shown in Figure 3-15d) the lines are not parallel, and in Figure 3-15e) the lines are parallel.



Testing for parallel lines Case (a), (b), (c) and (e): lines are parallel; Case (d): lines not parallel

When two lines appear both in point view, they belong to the family of projection lines and thus are truly parallel. This leaves the case of two lines that coincide in a view. If they coincide also in the other view, they must be identical or be perpendicular to the folding line, in which case an auxiliary view as in Figure 3-15 decides the case. If the lines do not coincide in the adjacent view, they are truly parallel if and only if they appear as points or parallel in that view. This is illustrated in Figure 3-16.



3-16 Lines seen simultaneously in point view are parallel

3.5.3 Distance between parallel lines

The distance between two parallel lines is the distance between the intersection points of the lines with any line perpendicular to them. The following construction allows you to find this distance.

Construction 3-5 Distance between two parallel lines

Given two adjacent views of two parallel lines, find the distance between the two lines.

There are two steps:

1. Use two successive auxiliary views as in Figure 3-13 to show one of the lines in point view. This will also show the other line in point view. (This step can be shortened or omitted if the lines are parallel to the folding line in one or both of the given views; see the following example.)

2. Measure the distance between the two point views, which is also the distance between the lines (see Figure 3-17).





3.5.4 Worked example – A practical application

Figure 3-18 illustrates a practical application of Construction 3-5. It shows the top and front view of a small polygonal balcony as it may emerge during the design of a building. The architect plans to use three pairs of parallel tubes as railing and has drawn them in both views. For further detailing, the architect is interested in the distance between the tubes in any pair, measured for example between their center lines.

Neither view shows this distance in TL for any pair because no pair is perpendicular to the folding line, and their center line consequently do not appear in point view in any of the views. Note that the right-most pair appears in TL in both views. We thus can skip generating the first auxiliary in step 1 and proceed immediately with the construction of an auxiliary view using a folding line perpendicular to the pair in the front view; this view shows the desired distance as illustrated in the figure. Note also that the top view

shows *every pair in TL* (why?) and therefore, it could also be used in different auxiliary views to construct point views for each pair of center lines.



3-18

Distance between (center-lines) of parallel railings-a practical example

This example is meant not only to demonstrate the application of Construction 3-5, but also to make a more general point. The constructions based on auxiliary views introduced in this and the preceding chapter can be used flexibly to answer questions about the geometry of an evolving design as the design process unfolds.

It is often sufficient to produce **auxiliary views only of a portion of the design**, which can often be done on-the-fly in some convenient region of the drawing sheet.

The starting point in every case is the following:

Select an appropriate folding line (or picture plane).

Furthermore, you are urged:

To pay particular attention to the way in which the constructions depend on properly selected folding lines.

The general principles that guide these selections are the most important aspects to assimilate.

3.6 PERPENDICULAR LINES

Two intersecting lines are *perpendicular* whenever rays pointing away from their point of intersection form a right angle. We call two lines perpendicular *in a particular view* if both appear as lines that form a right angle, or if one appears in point view and the other one in line view. Given this notion, two perpendicular lines appear perpendicular in any view *showing at least one line in TL*.





Note that in this view, the other line might appear in point view. Figure 3-20 illustrates this by showing two perpendicular rods in space one in TL and the other in PV. Notice that in the view in which one rod is shown in TL, the other rod is seen at right angles to it. Observe that we have ignored the thickness of the rods and treat them as lines.



3-20

Two perpendicular rods with at least one shown in TL and the other in PV





The converse is also true:

If in a view two intersecting lines are perpendicular and at least one line appears in TL, the lines are truly perpendicular.

Again, the other line might be shown in point view.

It should be easy to test whether two lines given in two adjacent views are perpendicular, by using Construction 3-2 (on page 93) to construct an auxiliary view in which at least one of the lines appears in TL. *The lines are truly perpendicular if and only if they are perpendicular in this view.* See Figure 3-21.





However as Figure 3-22 shows, the same conditions also hold for two non-intersecting (skew) lines that are in directions at right angles to one another.





Figure 3-21 motivates the following construction.

Construction 3-6 A line perpendicular to given line

Given a line and a point in two adjacent views, find the line through the perpendicular to the segment through the point.

Let *l* and *O* be the given line and point respectively in adjacent views, 1 and 2.





Constructing a line through a point perpendicular to a given line

There are three steps:

- 1. Use Construction 3-2 (on page 93) to show *l* in TL in an auxiliary view *a*.
- 2. In *a*, draw a line through *O* perpendicular to *l*. Call the intersection point *X*. This segment defines the desired line in *a*.
- 3. Project back into the other views.

The construction is illustrated in Figure 3-24.

3.6.1 Shortest distance between a point and a line

The shortest distance between a point and a line lies on a line perpendicular to the given line passing through the point. If we have a point, O, line l, and line m, through O perpendicular to l, the shortest distance between O and l is the distance between O and the intersection point between m and l. Construction 3-6 can be extended to find the shortest distance between a point and a line.

Construction 3-7 Shortest distance between a point and a line

Let l and O be the given line and point respectively in adjacent views, 1 and 2. We look to find the true distance between O and l.





3-25

Shortest distance between a point and a line

There are two steps:

- 1. Construct in a second auxiliary view, 4, the point view of *l*.
- 2. Project *O* into view 4. The distance between O_4 and the point view shows the true distance between *O* and *l*.

The construction is illustrated in Figure 3-25.

3.7 SPECIFYING LINE SEGMENTS IN SPACE

Line segments can be specified in space whenever we know or can establish the position of either a) two points on the segment, or b) a point on the segment and the angular position of the line with respect to the frame or system of reference.

3.7.1 Two points specifying a segment

Mostly, for this course, and for descriptive geometry, in general, *folding or reference lines serve as the frames of reference*. Therefore, when applying orthographic projection concepts, we locate lines by perpendicular distances from horizontal and vertical projection planes. That is, distances of points *below* the horizontal plane are seen in the vertical projection view and the distances *behind* the vertical plane are in the seen the horizontal projection view.





3.7.2 One point and an angle specifying a segment

Alternatively, a line segment is specified by a point, an angle of inclination with respect to the horizontal plane, and a direction, termed *bearing*, relative to a compass reading.

BEARING

The bearing is always seen in a horizontal projection plane, typically the top view, relative to the compass North.

Observe that the bearing of a line has no relationship to the angle of incline.

Bearing always measured from a compass direction (typically north or south) to a compass direction through a certain angle.





3-27 The bearing of a line is seen in the top view

ANGLE OF INCLINATION OR SLOPE ANGLE

The angle of inclination of a line segment is the angle it makes to any horizontal plane. It is the slope angle between the line and the horizontal projection plane and is seen only when —

The line is in true length and the horizontal plane is seen in edge view.

By applying Construction 3-2 (on page 93) to the line in top view, we obtain an auxiliary view in which the line is seen in TL and the horizontal plane is seen in edge view.



GRADE

The slope angle also described as a percentage grade is given as:



WORKED EXAMPLE - Adjacent views from a line specification

Given a point, the bearing, angle of inclination and true length of a line, construct the top and front views of the line

The data supplied includes the top and front projections of the given point, A, bearing N30°E, a downward slope 45° and true length = 1.5" of the line.

The following are the steps in the construction.

 Establish a top view 1 by drawing a line from A, with the supplied bearing. Assume North points upwards.
Choose the point A in front view 2 arbitrarily
A

- Construct an auxiliary view 3 using a folding line 3|1 parallel to the top view of the given line.
- 4. Project A_1 to A_3 using the transfer distance from the front view 2.
- 5. Draw a line from A_3 with given downward slope and measure off the supplied true length to construct point B_3



- 6. Project B_3 to meet the line in top view at B_1 . A_1B_1 is the required top view.
- 7. Project B_1 to the front view and measure off the transfer distance from the auxiliary view 3 to get B_2 . A_2B_2 is the required front view.



3-30

Top and front views of a line given a point, its bearing, downward slope and true length

3.7.3 Traces of a line

These are points in which a line, extended if necessary, intersects the horizontal and vertical planes. The trace on the horizontal plane is called the *horizontal trace*, *HT*, and that on the vertical plane, the *vertical trace*, *VT*. See Figure 3-31.





It should be clear that:

- A line may have zero, one or two traces.
- The traces of a line specify its direction without specifying its position or length

• A trace of a line can be used to determine the true length and angle of inclination of the line to both the horizontal and vertical planes as the construction in Figure 3-32.





3.7.4 Specifying lines on quad coordinate paper

Practical problems in descriptive geometry require methods for specifying or laying out lines with a degree of precision. It is impractical to draw lines that are, say 20 feet long. Clearly, lines are represented according to a *scale*.

It is convenient to use coordinate dimensions. Conventionally coordinate dimensions are specified in inches and drawings are made to full scale irrespective of the scale of the problem. It is common to use coordinate paper, also known as *quad paper*, which is $8\frac{1}{2}$ " x 11" divided into $\frac{1}{4}$ " squares, and is very convenient for solving practical problems using coordinate dimensions. Sometimes a *working area* of 8" x 10" is employed to allow for the axis to be shown.



3-33 Using quad coordinate paper: *A*(2, *X*, 9), *P*(2, 3, 7¹/₂) and *B* (3, 3, *X*)

The origin is always assumed to be the lower left hand corner of the working area.

The top and front views of a point are plotted on the sheet by three coordinate dimensions, *which are always given in the same order*.

For some problems some of the coordinate values are unknown and may be omitted in the any accompanying data. If a coordinate is unknown and the complete location of a point is part of the problem, the letter X is introduced in the data

The *first* is the distance from the left border of the working area to the line joining the top and front views.

The second is the distance from the lower border of the working area to the front view.

The *third* is the distance from the lower border of the working area to the *top* view.

Thus, P: 2, 3, 7 $\frac{1}{2}$ - also written $P(2, 3, 7 \frac{1}{2})$ - represents a point 2 inches to the right of the left border. The front view is 3 inches above the lower border and the top view is 7 $\frac{1}{2}$ inches above the lower border.

For some problems some of these coordinate values are unknown and are omitted in the any accompanying data. If a coordinate is unknown or if the complete location of a point is part of the problem, the letter X is introduced in the data. That is, when a position is not specified because it is either not needed or must be found during the course of a solution this is indicated by an "X." Thus, A: 2, X, 9 specifies a point in top view only and B: 3, 3, X specifies a point in front view only.

Hence, a point is represented as the distance in inches (from the left border, in front view, in top view). See Figure 3-33.

Additionally, on rare occasions, we may want to increase precision in which case we will also use the center of the ¹/₄" square or the mid-point of a grid square. In the drawing shown in Figure 3-34 three kinds of points are specified: i) at a grid point; ii) at the center of a grid cell; or iii) mid-way between two grid lines. The diagram also show how these points might be reproduced using simple construction lines between grid points. It is usual to ensure that the construction lines are not made visible.



3-34 More on using quad coordinate paper

WORKED EXAMPLE - Views of a truncated pyramid

On quad paper, line A: (2, 2, 6), D: (2, 2, 9) is a diagonal of a horizontal hexagonal base of a right pyramid. The vertex is 3" above the base. The pyramid is truncated by a plane that passes through points P: $(1, 4 \ 1/2, X)$ and Q: $(4, 1 \ 1/2, X)$ and projects edgewise in the front view. Draw the top and front views of the truncated pyramid.

We use the construction shown here to produce a hexagon given its diameter (that is, two opposite points).

The construction of the truncated pyramid is shown in Figure 3-35. Notice that we need the sides of the pyramid to determine where the truncating plane meets them.





3.7.5 Typical problems involving lines

Problem (Structural Framework)

Figure 3-36 shows the top and front elevation views of a structural framework between two neighboring buildings to scale. The problem is to *determine the true length of structural members AB and CD and the percentage grade of member BC*.





To solve this problem we construct an auxiliary view by drawing a folding line $T \mid 1$ parallel to *AB* and *CD* in plan view and projecting onto view 1.

The auxiliary view gives the true length and grade as required.





Problem (Mine reclamation - locating a new tunnel [line])

Pittsburgh has a number of old mines and consequently, mine tunnels, which are being reclaimed to farm mushrooms. Consider two such mine tunnels AB and AC, which start at a common point A. Tunnel AB is 110' long bearing N 40° E on a downward slope of 18°. Tunnel AC is 160' long bearing S 42° E on a downward slope of 24°. Suppose a new tunnel is dug between points B and C. What would be its length, bearing, and percent grade?



We can solve this problem from plan view drawing it according to some scale (which may be provided).

We start by locating A in plan and draw indefinite lines with the given bearings.

Next, we construct an auxiliary view 1 showing AB in true length and locate B by projecting back into the plan view. See figure below.



We repeat this step for C by creating an auxiliary view 2 in which AC is seen in true length and then locating C by projecting back into the top view.

To determine the true length of BC we construct another auxiliary view 3 and using the transfer distances for B and C in views 1 and 2, we obtain the true length of BC. The bearing and percent grade are then easily determined. The construction is shown in Figure 3-38.



Problem (Locating a point)

Sometimes a problem may appear to contain more information than seems necessary. Here is an example. Let ABC be a triangular planar surface with B 25' west 20' south of A and at the same elevation. C is 12' west 20' south and 15' above A. Locate a point X on the triangle 5' above and 10' south of A. Determine the true distance from A to X.

The construction is given in Figure 3-39 originally drawn to a scale, 1'' = 5', on quad paper. We draw the triangular plane *ABC* in both top and front elevation views as shown.

In front elevation, draw a line 5' above A. This meets sides BC and AC at points 1 and 2. X must lie on this line. Project 1 and 2 to top view.

In top view, X lies on the line 12. In top view, draw a line 10' south of A. X must lie on this line. Therefore, X is the intersection of this line and 12. Project X into the elevation.

The last step is to take an auxiliary view parallel to AX and project A and X to determine the true distance between them.



