

# 11

## Central Projections

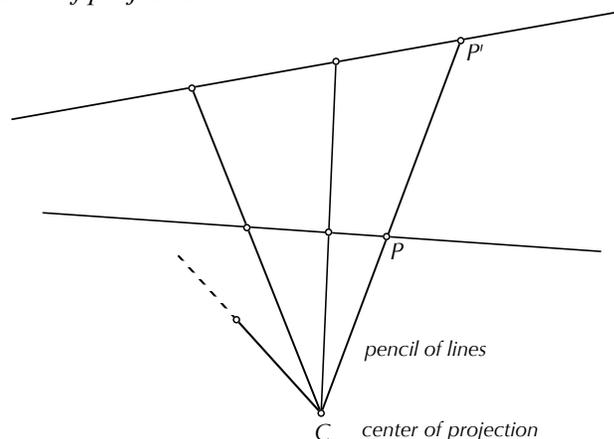
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*Central projections* are similar to parallel projections in that they also associate points in one set with points in another set by projection lines that pass through the associated points, with the difference that the projection lines all pass through a given point. Central projections, behave differently too, in ways that make them, at the same time, difficult to construct manually, and interesting as visual objects.

### 11.1 BASIC PROPERTIES

#### 11.1.1 Central projection of a line on a line

The set of all lines that pass through any point  $C$  is a *pencil of lines through  $C$* . For any two lines coplanar with a pencil of lines through a point  $C$  on neither line, for any point on one line there is a line in the pencil maps it to a point on the other. This is called a *central projection* and  $C$  is the *center of projection*.

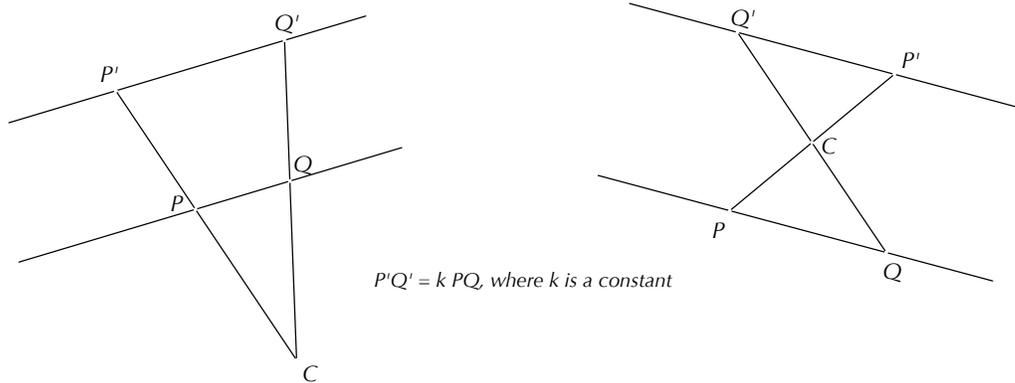


**11-1**  
Illustrating a central projection

As is the case with parallel projections, the lines may or may not be parallel, and these distinctions produce important differences in the resulting mappings.

**Case: Parallel lines**

Consider the case where the lines are parallel (see Figure 11-2).



**11-2**  
Central projections between parallel lines

As  $C$  is not on either line, every point  $P$  on one line defines, together with  $C$ , a line that passes through these two points; it is therefore not parallel to either line; moreover, it passes through a point on the other line, which is the unique image of  $P$  under the projection. By the same argument, every point on the second line is the image of a unique point on the first. That is, the projection maps the points one on one.

Using similar triangles, we can show that a central projection of a line on a parallel line multiplies distances by a constant factor,  $k$ . This factor is not normally 1, except in the trivial case where the lines coincide or when  $C$  is equidistant to both lines.

We summarize these observations in the following assertion:

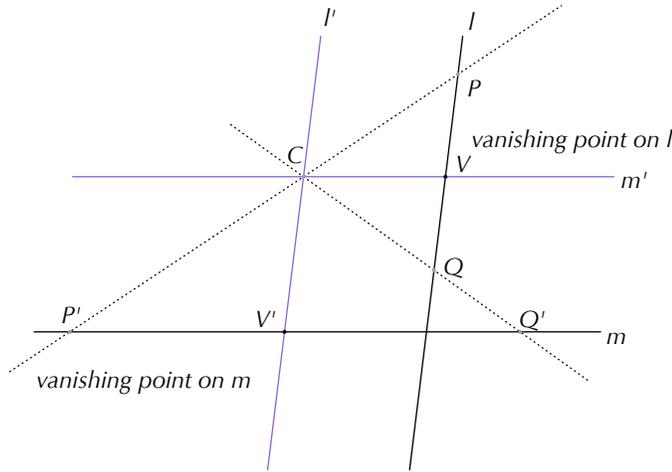
**Property 11-1**  
*A central projection of a line on a parallel line uniquely maps between the points on the lines and multiplies distances by a constant factor (generally not 1).*

It follows, as an immediate consequence, that this type of projection preserves betweenness, and therefore, maps line segments onto line segments, rays on rays and lines onto lines.

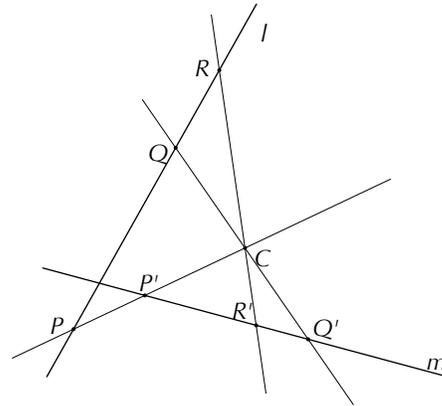
These are the same properties that are also preserved by a parallel projection between lines (which do not have to be parallel); a difference between the two types of projections is that a parallel projection between concurrent lines fixes a point (the point of intersection), while a central projection between parallel lines, of course, cannot fix a point.

### Case: Concurrent lines

The case becomes more intricate when we consider the central projection of a line on a *concurrent* line. See Figure 11-3.



(i) Vanishing points



(ii) Between-ness is not preserved

### 11-3

Central projection of a line onto a concurrent line

Let  $l$  and  $m$  be two concurrent lines and  $C$ , the center of projection. Consider the point of intersection,  $V$ , where line,  $m'$ , parallel to  $m$ , passing through  $C$ , meets  $l$  (see Figure 11-3i).  $m'$  does not intersect  $m$  at any point; consequently,  $V$  has no image under this projection. All other points of  $l$  have a unique image on  $m$ . If such a point  $P$  on  $l$  moves closer and closer to  $V$ , its image  $P'$  moves farther and farther away on  $m$ . For this reason,  $V$  is called a *vanishing point* on  $l$ .

Likewise, the point,  $V'$ , where the line  $l'$ , parallel to  $l$ , passing through  $C$ , meets  $m$  cannot be the image of any point on  $l$ . All other points of  $m$  are the unique image of a point on  $l$ .  $V'$  is called the *vanishing point* on  $m$ .

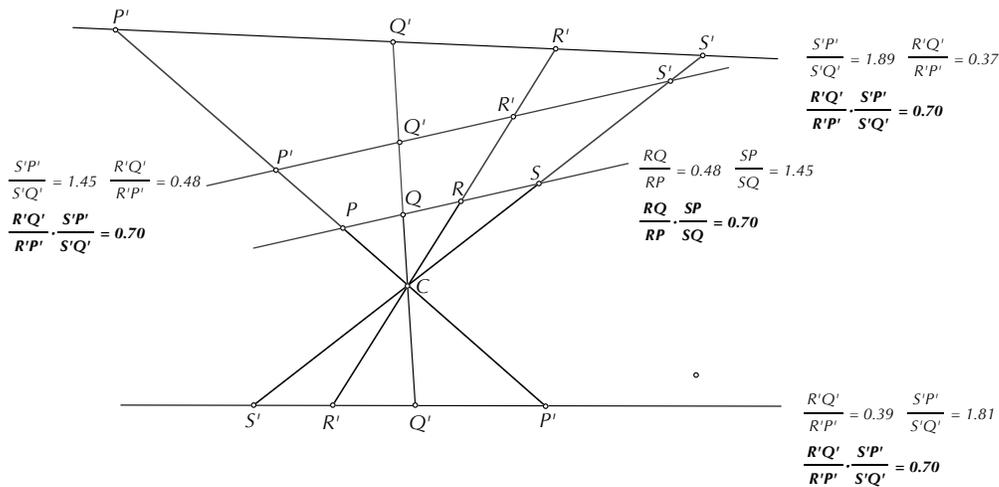
Figure 11-3ii illustrates the fact that a central projection of a line on a concurrent line does not preserve betweenness: for example, point  $Q$  is between  $P$  and  $R$  on  $l$ , but its image,  $Q'$ , is not between the images of the other two points,  $P'$  and  $R'$ , on  $m$ .

### An aside: cross ratio

One of the more important properties of a central projection of a line is the following important invariant: preserving 'cross ratio' of distances. Let  $P, Q, R$  and  $S$  be four points on  $l$  with  $P$  and  $Q$  distinct. Then, the cross ratio of the four points is defined as the ratio:

$$(P, Q, R, S) = (PR \times QS) / (PS \times QR),$$

where lengths of the segments are measured along the same direction.



### 11-4

Illustrating the cross-ratio invariance

If  $P', Q', R'$  and  $S'$  are the images of the four points on any line  $m$  under a central projection of  $l$ , then

$$(P', Q', R', S') = (P, Q, R, S).$$

The cross ratio plays an important part in the development of projective geometry; however, we will not consider this any further in this course.

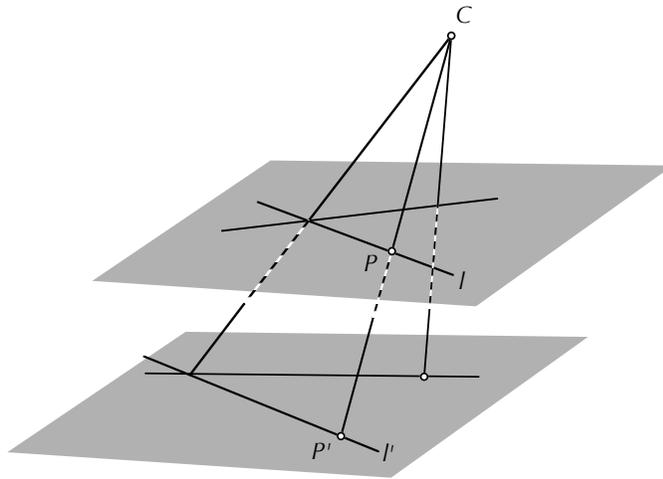
### 11.1.2 Central projection of a plane on a plane

Just as the case of consider a projection of a line on a line through a central projection, we can consider the central projection of a plane onto a plane by mapping every point on one plane to unique point on another by a pencil lines that pass through a point  $C$  not on either plane. Again,  $C$  is called the center of the projection.

#### Case: Parallel planes

Let us again first consider the case when the two planes are parallel (see Figure 11-5).

Consider a line  $l$  in  $p$  and a point  $P$  on  $l$ . The line in the pencil of the projection that passes through  $P$  defines together with  $l$  a plane to which all of the lines in the pencil passing through a point on  $l$  belong. This plane intersects  $p'$  at a line, which is the image of  $l$  under the projection. A central projection of a plane on a parallel plane thus maps lines on lines.



11-5

Central projection of a plane on a parallel plane

We can observe furthermore that this projection uniquely maps between the points in both planes. Therefore, the images of two parallel lines cannot intersect, while the images of two concurrent lines must be concurrent. The projection thus preserves parallelism and concurrence between lines.

Consider a line  $l$  and its image,  $l'$ . As they are on parallel planes, they must be parallel. That is, they are coplanar (because they belong both to the plane defined by them), and their points are associated by a central projection between parallel and coplanar lines. By Property 11-1, the projection multiplies distances by a constant factor. We can show that this factor is constant for the entire projection. We again summarize the findings as follows:

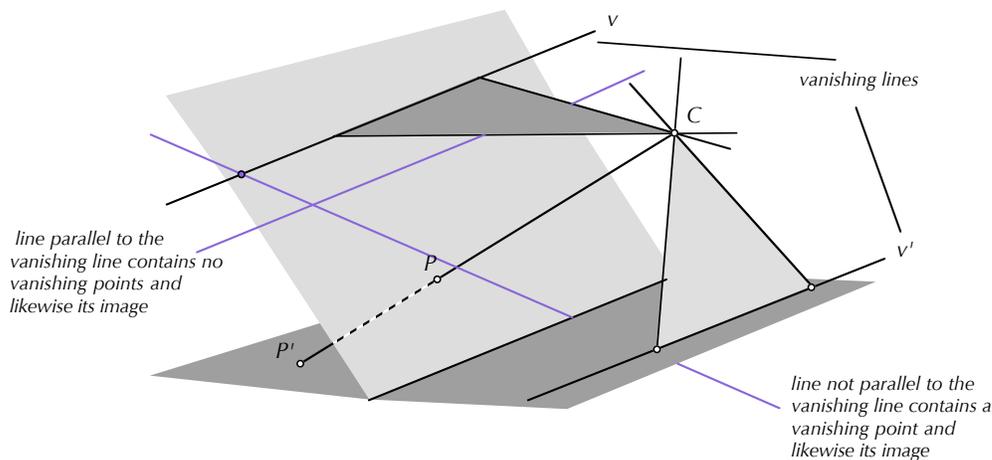
### Property 11-2

A central projection of a plane on a parallel plane uniquely maps between the points in the planes. It maps lines on lines and multiplies distances by a constant factor.

A consequence is that a central projection between parallel planes preserves betweenness between points, and parallelism and concurrence for lines. It also maps conic sections onto conic sections of the same type.

### Case: Concurrent planes

The situation is more intricate for a central projection of a plane on a concurrent plane (see Figure 11-6).



### 11-6

Central projection of a plane on a concurrent plane

The plane through  $C$  parallel to  $p'$  intersects  $p$  at a line,  $v$ . No point on  $v$  can have an image under the projection because the lines in the pencil through these points are parallel to  $p'$ .  $v$  is consequently called a *vanishing line* and every point on it a *vanishing point*. Likewise, the plane through  $C$  parallel to  $p$  intersects  $p'$  at a line,  $v'$ , so that no point on  $v'$  is the image of a point on  $p$ .  $v'$  and the points on it are again called *vanishing line* and *vanishing points*, respectively. Every point on  $p$  that is not on  $v$  has a unique image on  $p'$ , and every point on  $p'$  that is not on  $v'$  is the image of a unique point on  $p$ . The projection thus establishes a unique correspondence or mapping between the points on  $p$  and  $p'$  that are not on  $v$  or  $v'$ .

Consider a line  $l$  on  $p$  that is not the vanishing line. The lines in the pencil through  $l$  form a plane that intersects  $p'$  at a line,  $l'$ , which is the image of  $l$  under the projection. Furthermore,  $l$  and  $l'$  are coplanar, and the projection establishes a central projection of  $l$  on  $l'$  with center  $C$ . Given our investigation of central projections of a line on a line, two cases are of interest:

(a)  $l$  and  $l'$  are parallel.

Then, they have no vanishing points.  $l$  must be parallel to  $v$  and  $l'$  to  $v'$ , and  $l$  is mapped in its entirety on  $l'$ . Furthermore, between-ness and ratio of division are preserved.

(b)  $l$  and  $l'$  intersect.

Then the projection establishes a unique correspondence between the points on the two lines, except for exactly one vanishing point on each line. Between-ness is not preserved for all points. We can summarize this by the following statement:

**Property 11-3**

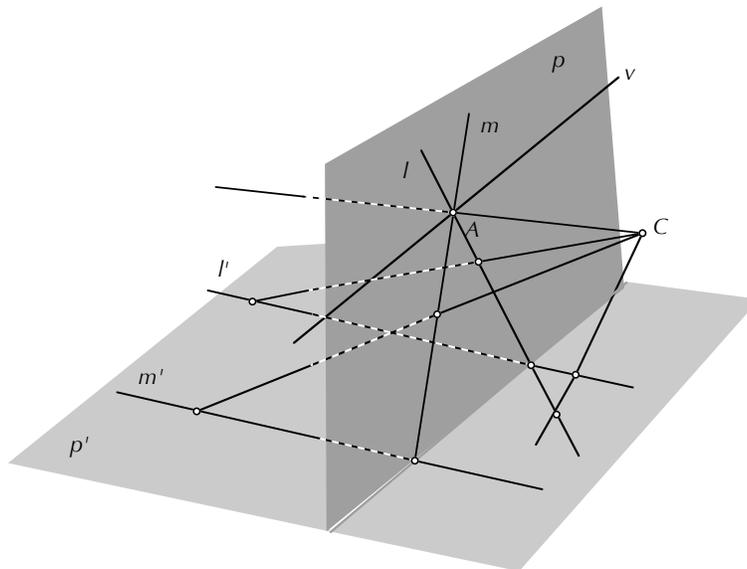
*A central projection of a plane on a concurrent plane establishes a unique mapping between the points on the planes, except for points on the vanishing line in each plane. It preserves collinearity for points that have an image, but not necessarily betweenness, and complete lines are not always mapped onto complete lines.*

**11.1.3 Central projections at vanishing points**

One reason why central projections are important for architectural applications is that they are the basis for an understanding of shadows cast by point light sources. It is therefore worthwhile to look a little deeper into the effects of central projections, especially on points on the vanishing lines, which can be striking.

Any two lines that intersect at the vanishing line are mapped on parallel lines.

To show this, I will employ an argument technique that essentially involves a contradiction. Let us consider two lines,  $l$  and  $m$ , intersecting at point,  $A$ , which thus define a plane,  $p$ . Let us also consider their central projection into a plane,  $p'$ , concurrent with  $p$ . See Figure 11-7.



**11-7**  
Central projections of lines intersecting at a vanishing point

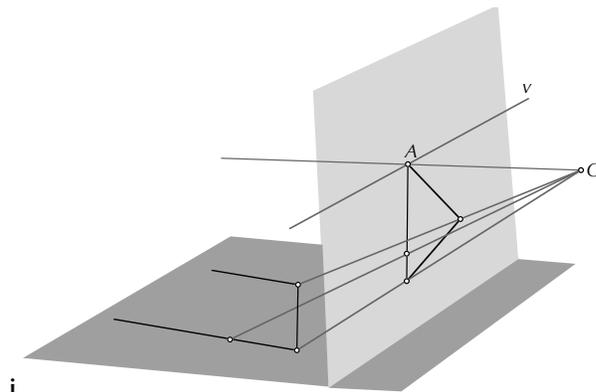
Suppose the center of the projection,  $C$ , establishes a vanishing line,  $v$ , in  $p$  that passes through  $A$ . Clearly,  $A$  has no image. Suppose the images of the two lines,  $l'$  and  $m'$ , meet at a point,  $A'$ , on  $p'$ . Since the projection is uniquely maps all points not on the vanishing lines,  $A'$  must be on the vanishing line in  $p'$  (because otherwise it would be the image of a unique point not on  $v$  where  $l$  and  $m$  intersect). But this is impossible because of the following argument:  $C$  and  $l$  define a plane,  $a$ , that also contains  $l'$ , and  $C$  and  $m$  define a plane,  $b$ , that also contains  $m'$ . Since  $A$  belongs to both planes, they must intersect at the line through  $A$  and  $C$ . This line is parallel to plane  $p'$ , and therefore cannot intersect  $v'$ . Therefore,  $l'$  and  $m'$  cannot intersect at a point of  $v'$  and, therefore, must be parallel.

What is, perhaps, even more striking is that:

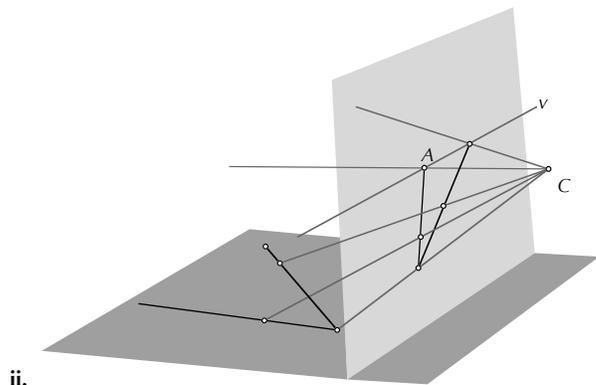
*Two parallel lines are mapped onto intersecting lines that intersect at a vanishing point.*

In order to see this, consider the projection mapping used in the previous case. Let us call it  $f$ . Now consider its inverse, that is, a central projection of  $p'$  on  $p$  with center  $C$ . This projection clearly has the same vanishing lines as  $f$ . By the argument used before, it can be seen that this projection maps two lines,  $k'$  and  $n'$ , on  $p'$  intersecting at  $v'$ , onto two lines,  $k$  and  $n$ , on  $p$  that are parallel. That is,  $f$  maps  $k$  and  $n$  on  $k'$  and  $n'$ . This means that those portions of linear figures that have an image are mapped on a linear figure, but not necessarily onto a similar figure.

Next, consider, for example, the cases illustrated in Figure 11-8.



i.

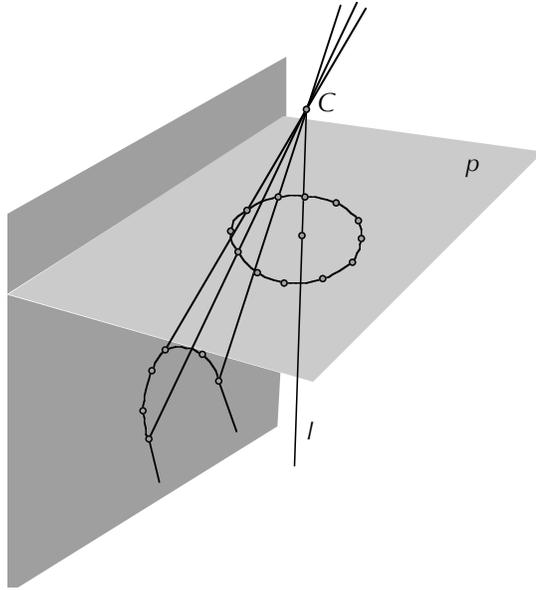


ii.

**11-8**  
Central projection of a triangle on a concurrent plane

The first of these shows the central projection of a triangle with one corner,  $A$ , on the vanishing line on a plane concurrent to that defined by the triangle. The two sides that meet at the vanishing line are mapped exclusive of  $A$  as open segments on parallel rays. The other side is mapped on a segment whose end-points are the images of the corners connected by the side.

The second case shows the central projection of a triangle with one side on the vanishing line. This side has no image, and the other two sides are mapped (exclusive of the corner that is a vanishing point) as half-open segments on a ray whose vertex is the image of the corner not on the vanishing line.



**11-9**  
Central projection of a circle on a plane

Another property of central projections that is important for the modeling of point light sources is their effect on conic sections. Consider the example illustrated in Figure 11-9. It shows the center of a projection,  $C$ , and a plane,  $p$ , with an embedded circle, where  $C$  is on the line,  $l$ , which is normal to  $p$  and passes through the center of the circle. The projection of the circle on a plane  $p'$  parallel to  $l$  can be seen as the intersection of the right circular cone established by the projection lines that pass through the circle with a plane parallel to the axis of the cone; we know that this intersection is a hyperbola. The vanishing line of this projection in  $p$  is the line parallel to  $p'$ ; it passes through two points of the circle, which consequently have no image. This is another example of a bounded set being mapped on an unbounded set by a central projection.

In a practical application,  $C$  may represent the center of a light bulb, the circle the lower rim of a lamp shade and plane  $p'$  a wall partially lit by the bulb and partially shaded by the lamp shade. The hyperbola is the boundary between the lit and shaded portions of the wall. However, note that this boundary only approximates what happens with real light sources. The next section develops this further.

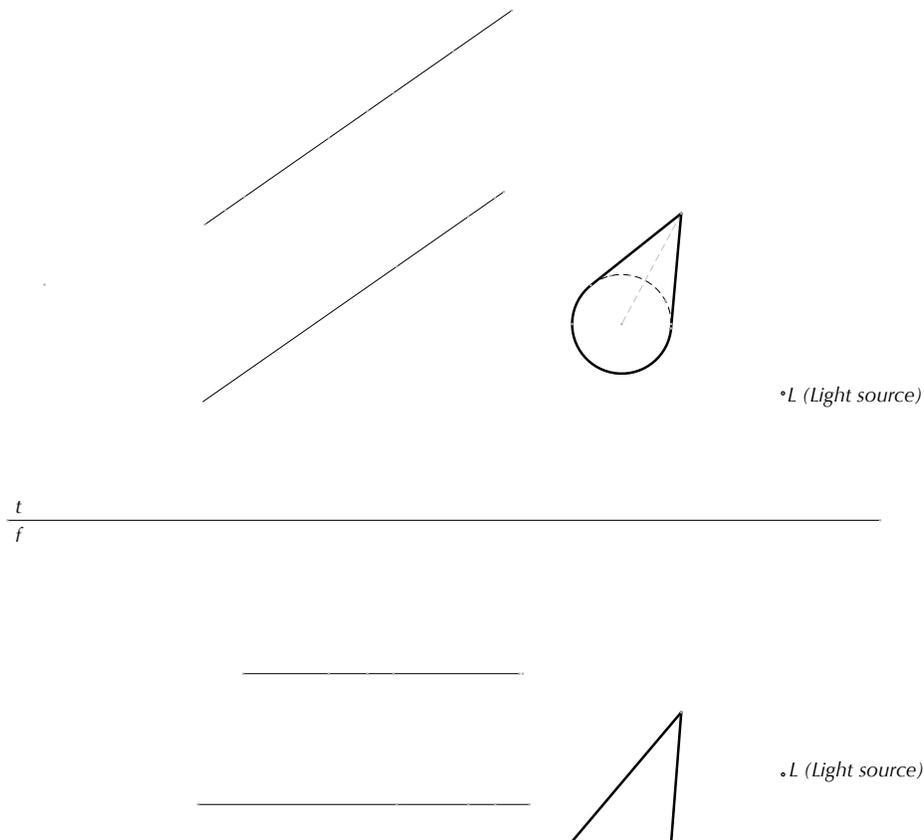
The general effect of a central projection of a plane on a concurrent plane on conic sections is stated in the following:

**Property 11-4**

*A central projection of a plane on a concurrent plane maps conic sections on conic sections, but not necessarily on conic sections of the same type.*

**A worked example**

Suppose you are given the top and front views of a point  $L$  (light source), and an unlimited plane defined by a pair of lines and an inclined cone. We are required to cast the shadow of the cone onto the given plane. See Figure 11-10.

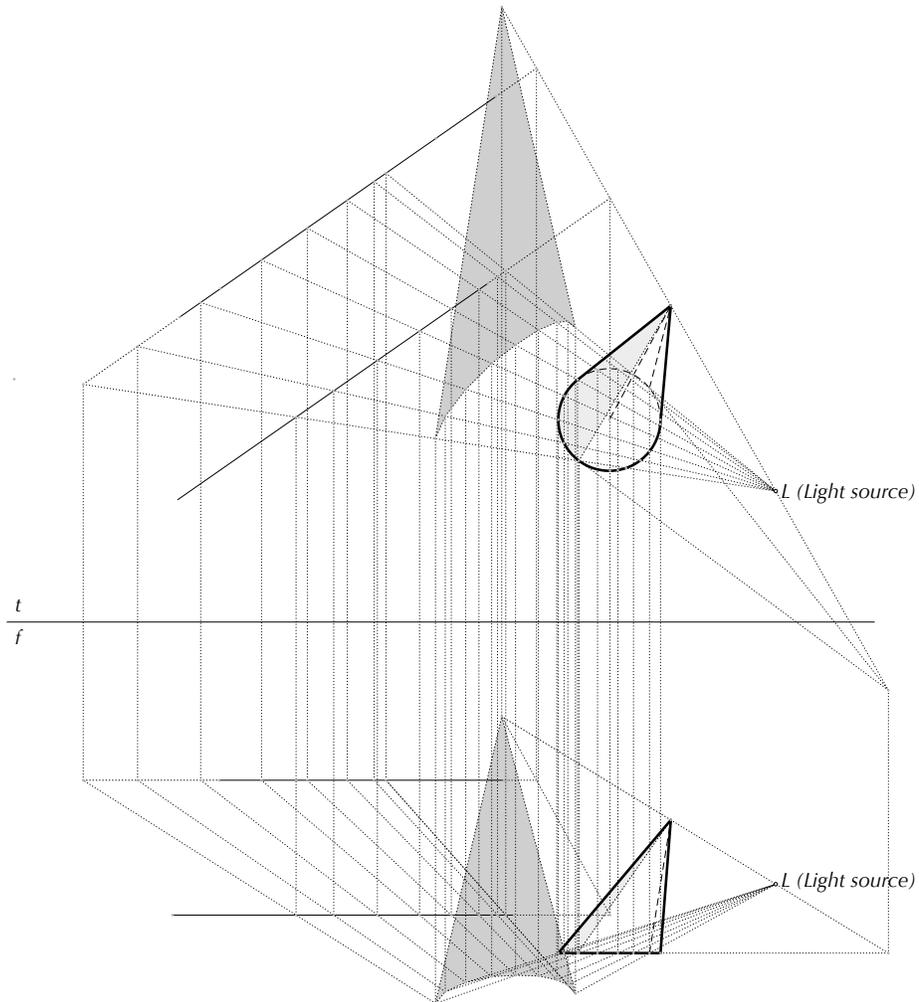


**11-10**

Shadow casting from a single light source (the problem)

For this one needs to find the piercing (shadow) point of each ray of light from the light source through every (shade) point on the prism onto the given plane. This must be done in both views. Join by a line, shadow points corresponding to adjacent (shade) points on

the prism to construct the shadow, and highlight the shadow, in both views. There is the preliminary step of finding the tangent to the cone passing through the light source. See Figure 11-11.

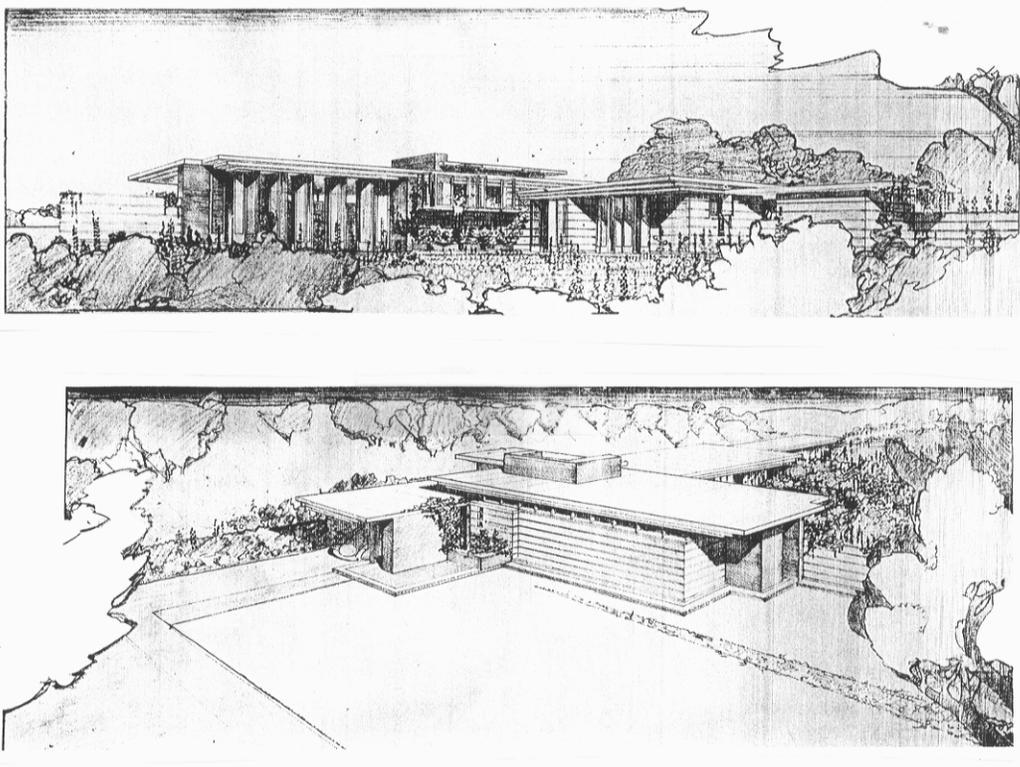


**11-11**  
Shadow casting from a single light source (the solution)

## 11.2 PERSPECTIVE VIEWS

Of all the central projections, perspectives are perhaps the most well known. Perspective, for instance, are useful in architecture for several reasons, the two main ones being: (i) realistic views and (ii) an understanding of how shadows are cast by light sources.

Orthographic views are appropriate representations when exactness is required and geometric problems have to be worked out with precision. However, they do not convey a natural or intuitively obvious view of an object that can be understood by a general audience. For the latter purpose, perspective views are normally used. Figure 11-12 show two examples prepared by Frank Lloyd Wright, one of the great architectural draftsmen of the century.



**11-12**  
Two perspective views of the Jacobs house (Frank Lloyd Wright, 1937)

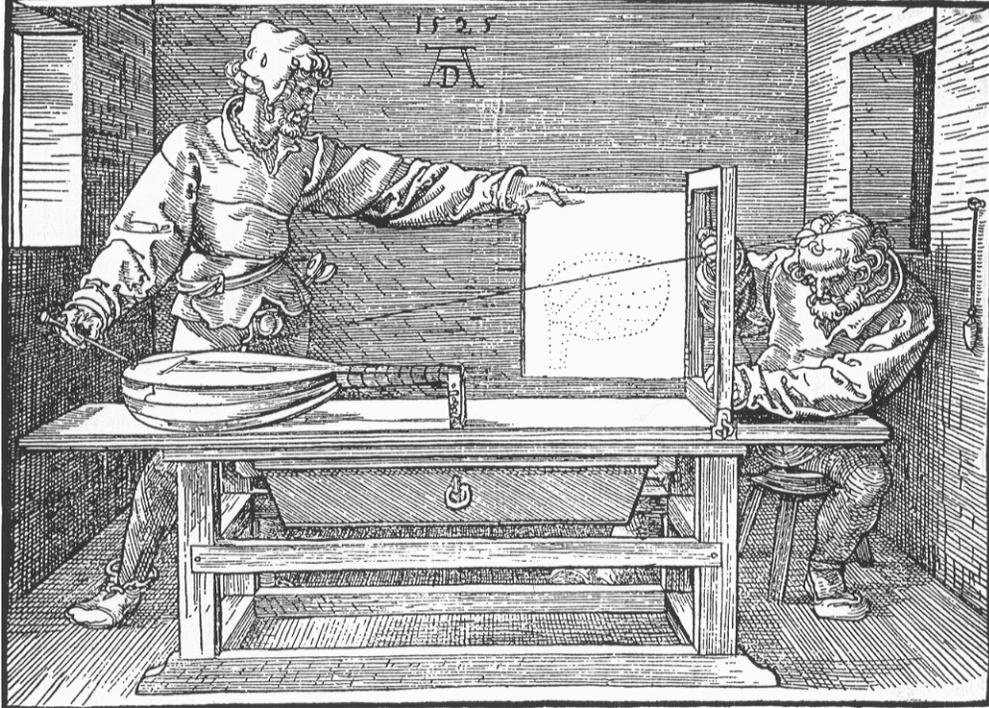
### 11.2.1 Basic terms and constructions

A perspective view of a spatial figure is a composite of a central projection of the figure onto a plane called the *picture plane* (PP) and a distance preserving mapping of the picture plane onto the Euclidean plane (essentially, a sheet of paper). The center of projection is the *station point* (SP). The point where the line normal to PP through SP meets PP is the *center of vision* (CV), and the horizontal line on PP through CV is the *horizon* or *eye line*.

As with orthographic views, two mappings are involved in the construction of a perspective view: one that maps the object onto a spatial picture plane and one that maps the picture plane on a strictly two-dimensional plane, represented in practice by the paper or screen that carries the view.

These principles are clearly demonstrated by the etching produced by the German painter and theoretician Albrecht Dürer shown in Figure 11-13, which depicts a

contraption that allows one to construct a perspective view by following literally the steps involved. Like other artists of the Renaissance, Dürer was keenly interested in the newly discovered perspective drawing techniques.



11-13

Construction of a perspective (Albrecht. Dürer, 1525)

The object to be viewed in perspective is a lute; the station point is given by a hook in a wall; the picture plane is defined by a rectangular frame; and projection lines are represented by a string that is threaded through the hook at one end and can be connected to any point on the lute at the other end by a pointing device; a plumb assures that the string is always stretched tightly between the two ends. While one person attaches the string to a point on the lute, another person determines the coordinates of the point where the string pierces the picture plane (the first mapping); from there, the point can be transferred to a sheet of paper (the second mapping).

Constructing a perspective view directly from a three-dimensional object is, of course, extremely cumbersome. Moreover, the object to be depicted in architectural design normally doesn't even exist in physical form. The techniques developed for the construction of perspective views therefore assume that the object is described through drawings, more specifically, orthographic views. This is straightforward if we are given an orthogonal top and side view of the object, each of which also shows the station point and an edge view of the picture plane. The following is a basic construction to find the perspective view of an arbitrary point on such an object (and thus to construct the perspective view of an object point-wise).

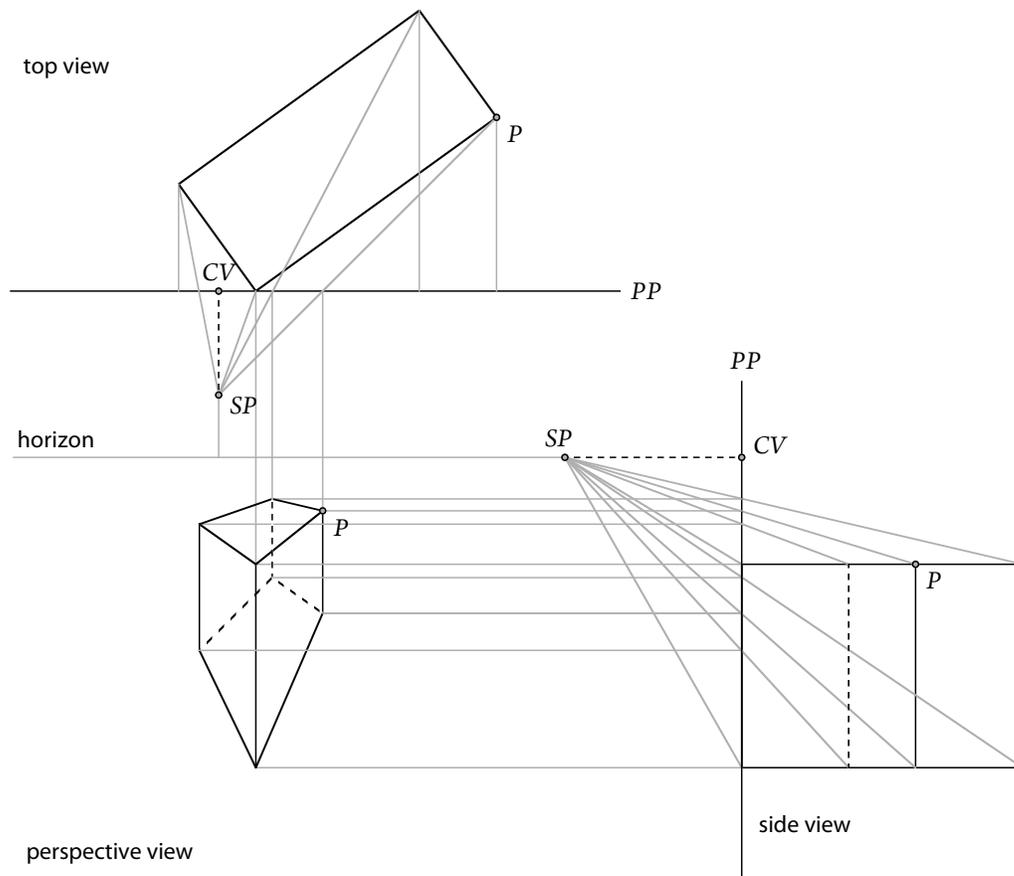
### Construction 11-1

#### Direct Projection Method

Given an orthogonal top and side view of a point  $P$ , each of which also shows a station point  $SP$  and a picture plane  $PP$  in edge view, find the perspective view of  $P$ .

There are three steps.

1. Arrange the given top and side views so that they frame the area of the perspective view as illustrated in Figure 11-14.
2. Draw the projection lines through  $SP$  and  $P$  in both views
3. Determine the points where the projection lines pierce  $PP$  in each view; the point where the perpendiculars through the piercing points intersect is the perspective view of  $P$ .



### 11-14

Direct projection method

This construction is known, for obvious reasons, as the *direct projection* method. The readers should convince themselves that this construction faithfully reflects *Dürer's depiction under the condition that the entire set-up is already given in two-dimensional views*.

### Example

Figure 11-14 illustrates how the above construction can be repeated for each corner of a rectangular prism to produce a perspective view of the entire rectangular prism. (Recall that central projections preserve collinearity of points.)

### 11.2.2 Common constructions

The direction projection method is a very general construction that allows us to piece together a perspective view of an arbitrary object from the views of appropriately selected points from ‘first principles’. Its drawback in practice is that it is generally cumbersome; moreover, the top and side views on which it depends may not be given and may be difficult to construct for a desired station point and picture plane. It is for this reason that simplified procedures have been developed, procedures that depend on a combination of assumptions often met in practice, especially with respect to parallelism between lines defining the object and parallelism between the lines defining an object and the picture plane.

The simplifications that can be used when these assumptions are met are a consequence of the following general properties of central projections:

1. Lines on the picture plane appear in true length.
2. *Parallel lines parallel to the picture plane are mapped onto lines that are parallel to each other and to their originals*; especially, vertical lines are mapped on vertical lines and horizontal lines on horizontal lines (relative to a given ground plane perpendicular to the picture plane).
3. *Parallel lines not parallel to the picture plane are mapped onto lines that converge towards a common point*, which is the vanishing point of the lines under the projection. Moreover, the vanishing point is the point where the line through  $SP$  that is in the same family of parallel lines pierces  $PP$ ; if the given lines are horizontal, this vanishing point is on the horizon.

1 and 2 are direct consequences of Property 11-1; 3 is (a less immediate) consequence of the observations on central projections between lines and planes.

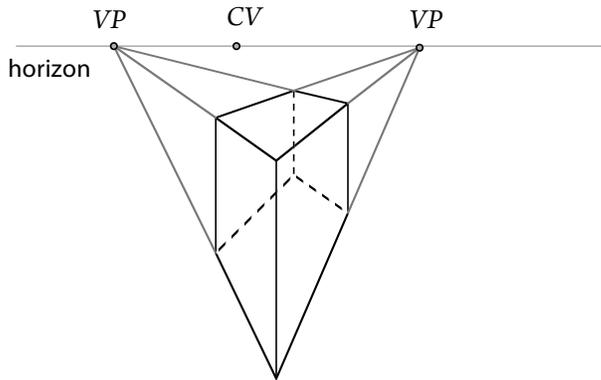
In buildings, horizontal and vertical lines (or more accurately, line segments) dominate due to the laws of gravity. If we select a  $PP$  normal to some horizontal ground plane, we know immediately that vertical lines (that is, lines normal to the ground plane) will remain vertical and parallel horizontal lines (that is, lines parallel to the ground planes) will remain horizontal or meet at a common vanishing point on the horizon on a line in the same family of parallel lines. Furthermore, many buildings are designed on an underlying rectangular grid; that is, the lines defining their most important parts fall into exactly three families: vertical lines and two families of horizontal lines, where the lines in one family are perpendicular to the lines in the other family. Such objects will be called rectangular in the following.

The construction shown in Figure 11-15 depicts such an object in a  $PP$  parallel to the vertical lines on the object, but inclined towards the horizontal lines. Consequently, lines in each of the two families of horizontal parallel lines meet at a distinct vanishing point

$VP$  on the horizon. In general, each family of parallel lines that is not parallel to the  $PP$  meet at a distinct vanishing point. Thus,

**Property 11-5**

*A perspective projection has as many vanishing points as the figure mapped has families of parallel lines not parallel to the  $PP$ .*



**11-15**  
Vanishing points for a family of horizontal parallel lines

**11.2.3 One-point perspectives**

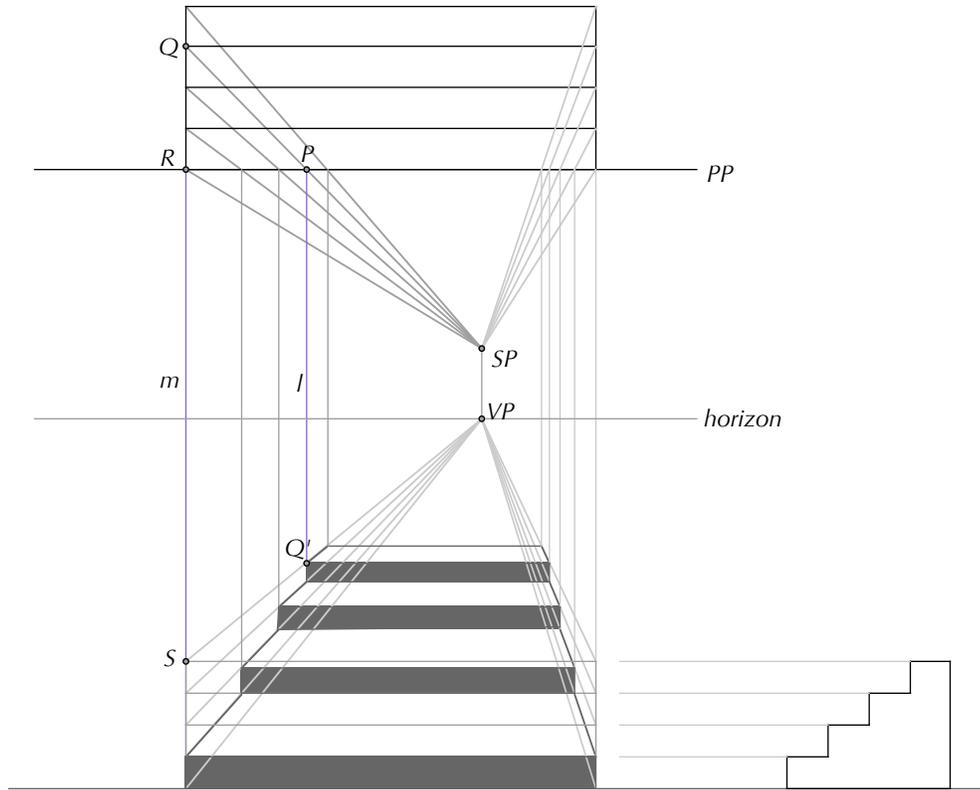
The construction of a perspective view is particularly easy for rectangular objects when the picture plane is parallel to both its vertical lines and one family of horizontal lines. Horizontal lines not parallel to  $PP$  are perpendicular to it, and the view has one vanishing point, which coincides with the center of vision,  $CV$ . The following construction shows how perspective views can be found under these circumstances.

**Construction 11-2**  
*One-Point Perspective*

*Given the top view of a rectangular object and some means of establishing the true heights of horizontal lines above some horizontal ground plane, construct a one-point perspective of the object*

There are four steps.

1. In the top view, select a  $SP$  and a  $PP$  parallel to the vertical lines and one family of horizontal lines. This  $PP$  will appear in edge view parallel to the lines in the selected family. In the illustration of the construction in Figure 11-16, the object to be depicted is a flight of stairs with four steps; the selected  $PP$  is parallel to the front edges of the steps and touches the first step.



#### 11-16

One-point perspective of a flight of four steps

2. Select a horizon parallel to the edge view of  $PP$  and a ground line parallel to the horizon. Project the  $SP$  on the horizon; this point is also the  $VP$  and  $CV$ .
3. The image  $Q'$  of any endpoint  $Q$  of a segment on the object in the perspective is determined as follows:

In the top view, draw the projection line through  $Q$ , and draw the perpendicular  $l$  through the piercing point,  $P$ , with  $PP$  into the perspective view;  $Q'$  will be on this line

In the top view, draw a line through  $Q$  perpendicular to  $PP$ ; this line meets  $PP$  at a point  $R$ . Draw the perpendicular  $m$  through  $R$  in the perspective view.

On  $m$ , mark point  $S$  at the height of  $Q$  from the ground line.  $Q'$  is the point of intersection between the line through  $S$  and  $VP$  and  $l$ .

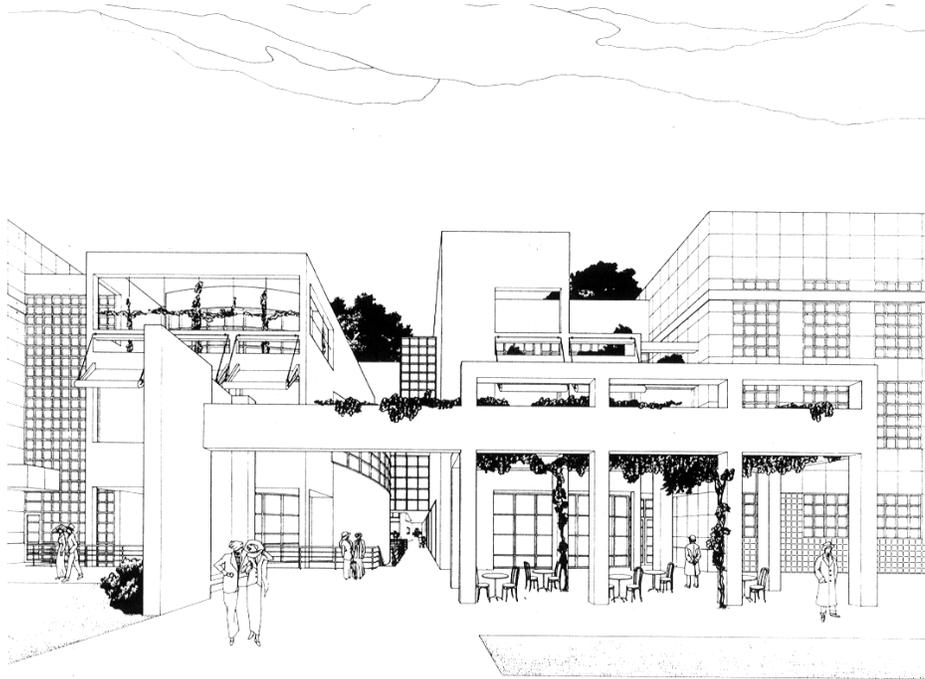
4. The view can be constructed rapidly by finding the views of selected endpoints. A horizontal segment parallel to  $PP$  will be on a horizontal line through the image of one of its endpoints. A horizontal segment perpendicular to  $PP$  will be on a line through one of its endpoints and  $VP$ . Vertical segments appear in point view in the top view. Their images will be on vertical lines through the points where the projection lines through the point view pierce  $PP$ .

A one-point perspective is also known as a *parallel perspective*.

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### Example - Museum for Arts and Crafts by Richard Meier

Figure 11-17 shows a one-point perspective of part of the exterior of the Museum for Arts and Crafts in Frankfurt, Germany, designed by Richard Meier. The center of vision is slightly to the left of the true center-line of the drawing.



**11-17**

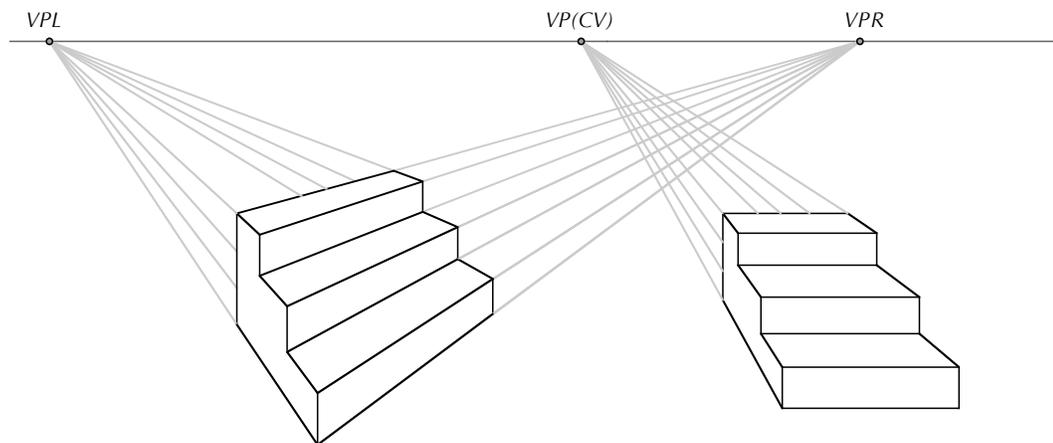
One-point perspective of the exterior of the Museum for Arts and Crafts (*Museum für Kunstgewerbe*) in Frankfurt, Germany, designed by Richard Meier

Observe how a one-point perspective draws the viewer's eye towards the center of vision because it coincides with the vanishing point of the perspective. In the present example, the eye is lead down an open walkway that connects an outdoor café with the main entrance area of the museum. A one-point perspective is appropriate when this type of directional effect is desired, or when a specific center or focus of a composition is to be emphasized. This is not to say that these perspectives haven't been used also for more mundane reasons: they are simply the easiest perspectives to construct.

However, ease of construction ceases being a factor with the spread of computers as drawing tools in architectural offices. Computer programs generate perspectives based on general algebraic methods that work equally well for any selection of a picture plane and for an arbitrary number of vanishing points. But even when they use computers, designers should understand the principles underlying perspective views and the effects that the selection of a picture plane has on the resulting view.

### 11.2.4 Two-point perspectives

Consider the steps example, which we saw in Figure 11-15, both the vertical and horizontal edges of the steps were parallel to the picture. However, if the horizontal edges were inclined, then in the perspective, the vertical edges remain vertical and the horizontal edges converge to two vanishing points. Such a perspective is called a *two-point perspective*. If the figure contained lines that were oblique or inclined lines there would be other vanishing points (see Property 11-5). The two points refer to the number of vanishing points in the principal edges only. That is, in a two-point perspective, the *PP* is parallel to the vertical lines of a rectangular figure, but inclined toward each of the families of parallel horizontal lines. See Figure 11-18.



**11-18**  
1- and 2-point perspectives

The following construction shows how two-point perspective views can be generated.

#### **Construction 11-3** *Two-Point Perspective*

*Given the top view of an object and some means of establishing the true heights of horizontal lines above some horizontal ground plane, usually, a right side elevation, construct a two-point perspective of the object.*

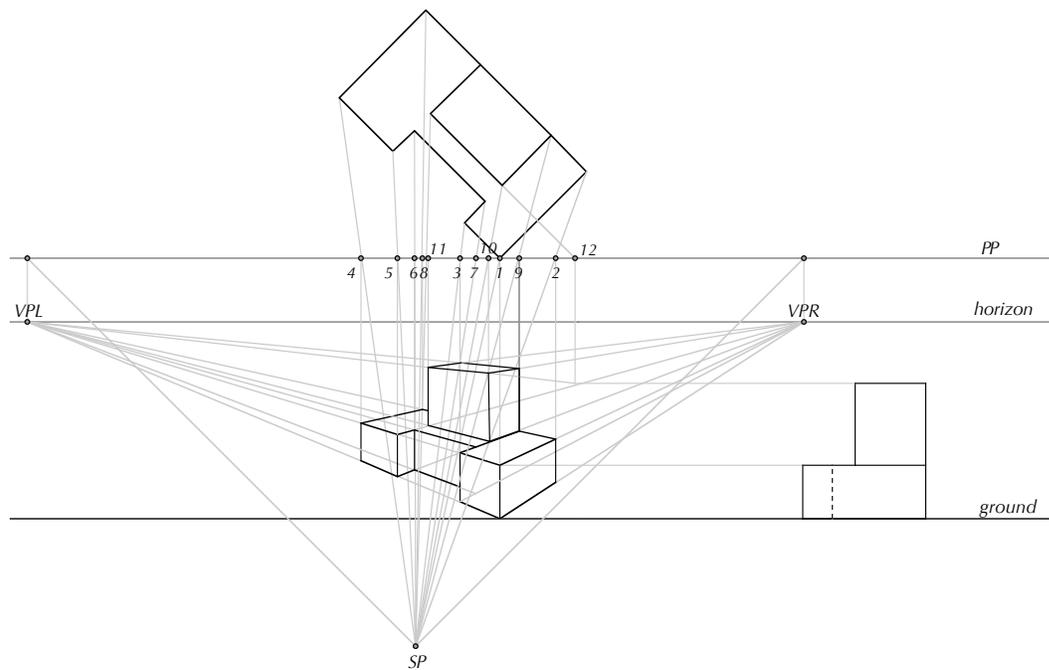
The construction is shown in Figure 11-19.

These are the steps:

1. In the top view, select a *SP* and a *PP* parallel to the vertical lines and inclined to each family of horizontal lines. This *PP* will appear in edge view inclined to the lines in the families.

2. *The next step is to determine the two vanishing points.* For this, select a horizon parallel to the edge view of *PP* and a ground line parallel to the horizon. The two *VPs* are found in the top view by drawing a line parallel to a horizontal line in each family through *SP*. Each of these lines intersects the *PP*. The perpendiculars through the intersection points meet the horizon in the perspective view at the vanishing points.
3. The image of any point—typically, an endpoint of a segment in the object—in the perspective is determined analogously to the parallel perspective method.
4. The view again can be constructed rapidly by finding the views of selected endpoints. A horizontal segment will be on a line through one of its endpoints and the *VP* associated with the family to which the line belongs. Vertical segments appear in point view in the top view. Their images will be on vertical lines through the points where the projection lines through the point view pierce the *PP*. In the example illustrated, all the visible points on the first height level were determined first and the points on the upper height level determined next.

A two-point perspective is also called an *angular* or *office perspective*, and the above construction is also known as *office method*.



**11-19**  
Constructing a two-point perspective

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### Example – Lobby of the J. Paul Getty Museum

Figure 11-20 shows the two-point perspective of the lobby of the museum building in the J. Paul Getty Center, Los Angeles, as designed by Richard Meier.



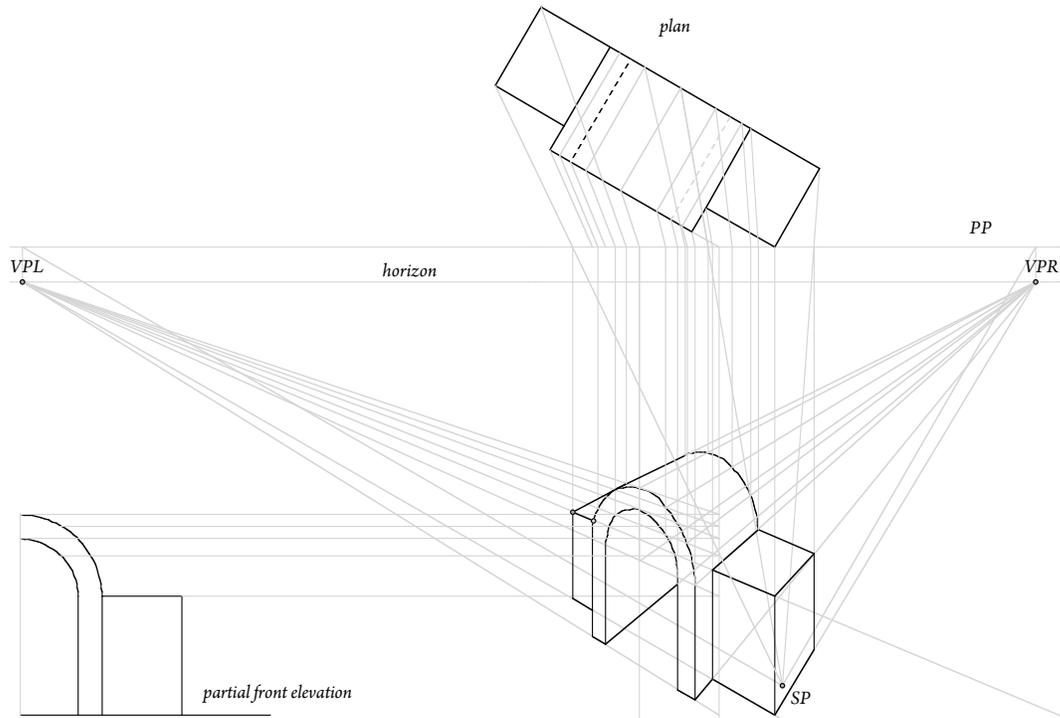
**11-20**

Two-point perspective of the lobby of the museum building in the J. Paul Getty Center, Los Angeles, by Richard Meier (1984)

Readers should compare the effects produced by this technique with those of a one-point perspective: the view has much less of a central focus, and the observer is less forcefully drawn into the picture.

This figure also illustrates that two- or one-point perspectives can have curves or lines not parallel to the families that determine the basic set-up underlying these views. If an object has additional families of horizontal parallel lines, their vanishing point can be determined as indicated in the office method. Other lines and even curves can be found by constructing points on them; this can be done by the same method even if the line is neither horizontal nor vertical: there is always a horizontal and a vertical line parallel to the *PP* that passes through any point on a line or curve, and these lines can be constructed by the methods shown. The only prerequisite is that we are able to determine

the true height of the points under consideration. Especially when dealing with curves, this may require assistance from sections or elevations; but these views serve only to determine the height of selected points above the ground plane and do not have to show the *PP* in edge view, as the direct projection method requires. This is demonstrated in Figure 11-21 for a circular arch.



**11-21**  
**Two-point perspective of an arch**

The figure also demonstrates how perspectives and the views used to construct them are often laid out in practice to save space.

### 11.2.5 Three-point perspectives

One associates with one- and two-point perspectives simplicity and ease of construction. Readers should keep in mind, however, that these techniques are also mathematically correct, that is, result in true central projections of a spatial object on a picture plane, viewed in two dimensions (the only inaccuracies that may occur are caused by drawing or, when computers are used, round-off errors). The simplifications result not from methodological shortcuts, but from the selection of special picture planes that allow, in particular, use of a given top or plan view to draw the picture plane in edge view.

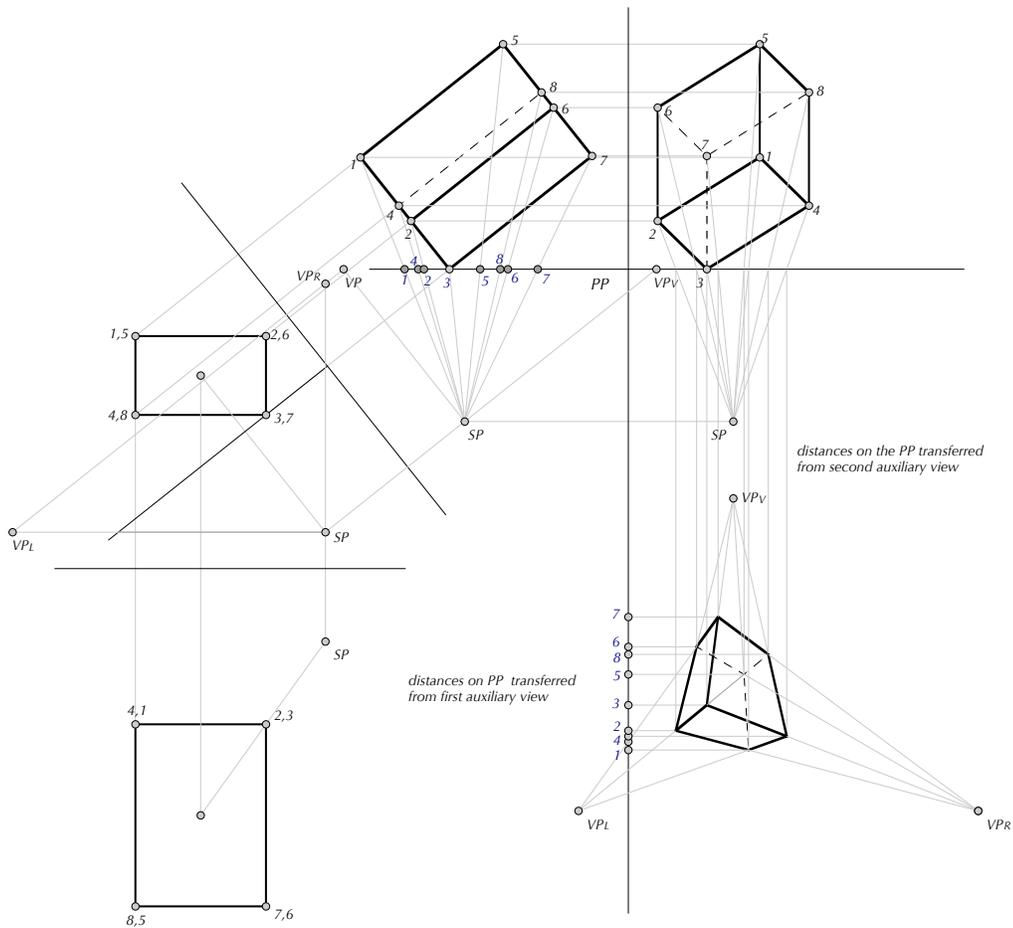
However, the resulting views are not necessarily perceptually correct. If they are used to depict the interior of a low-ceilinged space with a station point at eye level, they may come close to an observer's impression. But when they are used, for example, to show

the exterior of a 30-floor tower with vertical lines perfectly parallel to each other, the result is likely to be less realistic. It is in these cases that a designer may consider using a three-point perspective.

A three-point perspective of a (rectangular) object results from a picture plane not parallel to the vertical or horizontal lines on the object. As a result, none of the normally available horizontal views (floor plan, roof plan) show the *PP* in edge view, a condition on which the previous three constructions crucially depend.

The constructions used so far can nevertheless be used to generate three-point perspectives of objects given in the usual form.

Figure 11-22 illustrates the construction for a rectangular prism, which is shown in plan and elevation with the station located as shown with a view directed towards the center of the prism.



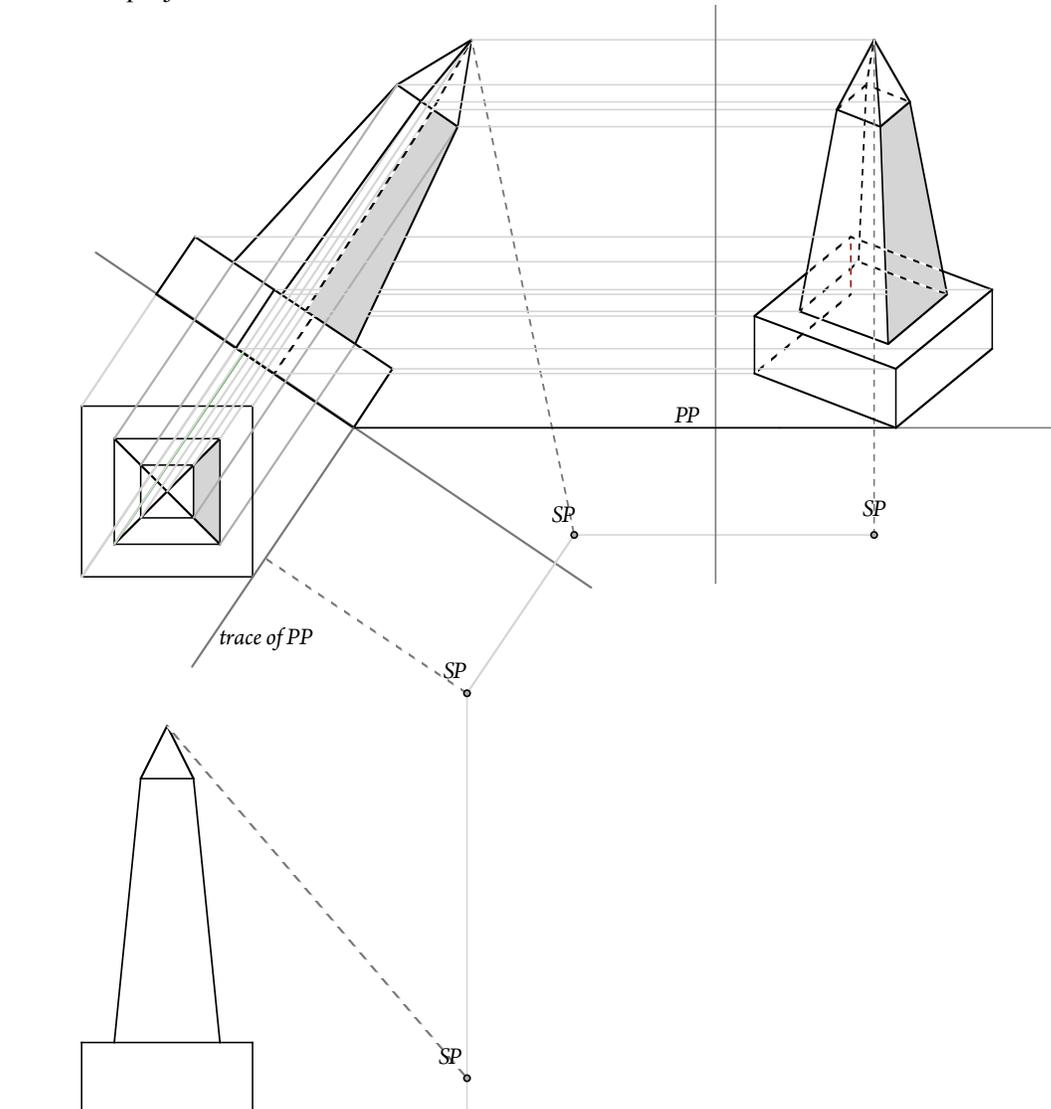
**11-22**  
3-point perspective of a rectangular prism

The vanishing points lie on a horizon in directions parallel to the sides of the prism. Firstly, an auxiliary view showing the *SP* to center in true length is constructed. In this view the vanishing points *VPL* and *VPR* are seen in point view. By choosing a picture plane we can determine the third vanishing point *VPV* along the vertical direction.

Secondly, join  $SP$  to the vertices of the prism in both plan and auxiliary elevation by lines as shown. These meet the  $PP$  in both views.

Finally transfer these distances on the  $PP$  to the vertical and horizontal axes as shown in the right most drawing. Project lines as shown to meet to construct the vertices of the prism, which the perspective can be drawn.

This same construction is demonstrated in Figures 11-23 and 11-24 for a more complex figure, an obelisk. In Figure 11-22 we create an auxiliary side and plan view,  $a$  and  $b$  respectively, that show the  $PP$  for a three-point perspective in edge view as required by the direct projection method.



**11-23**  
Generating a side and plan view for a three-point perspective



objects (for which one would not want to draw a three-point perspective anyway) because one has to find two auxiliary views before starting the construction proper. Methods for constructing three-point perspectives without auxiliaries by generating the vanishing points explicitly can be found in the literature (*AIA Architectural Graphics Standards* gives a method that is relatively easy to follow); but even then the process is time-consuming and not often employed in practice.

But we mentioned already that all of this has changed with the advent of computers and drawing software, for which picture planes in unusual positions pose no problem. As a result, three-point perspectives start to appear more frequently in architectural presentations and publications, and it is worth studying the effects that can be achieved with this technique.