Endogenous Flows of Foreign Direct Investment and International Real Business Cycles*

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Abstract

This paper models flows of foreign direct investment (FDI) in a two country, two sector DSGE framework. The allocation of capital to production capacity abroad is subject to a search-and-matching friction with endogenous capital reallocation. The model is calibrated on observed inflows and outflows of FDI and leads to dynamics of foreign direct investment consistent with the empirical evidence documented in this paper: relative to domestic investment, FDI is more volatile, and inward and outward flows of FDI are positively correlated. This contrasts with a standard International Real Business Cycle model which predicts a negative correlation and low volatility. Moreover, the model generates cross-country aggregate investment correlations consistent with the data.

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1 Introduction

Generally foreign investment is welcomed for bringing new capital to an economy and increasing productivity through the arrival of new technologies. This has also been the main focus of the theoretical and empirical literatures concerned with foreign direct investment. Little attention has been paid, however, to the short and medium run behavior of foreign-controlled firms and, in general, to their importance in understanding the business cycle of open economies. This seems somewhat surprising, as a commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), are large and very volatile. In Canada, for example, foreign-owned firms generate up to one third of employment and control over a fifth of all assets, a share that has been stable over the last four decades.[1]

The bulk of FDI, among developed countries, involves the replication of production capacity abroad, or what is known as horizontal FDI, and in particular for the purpose of serving the host market (Brainard 1993, 1997). What is less well known, and is documented in detail for the case of Canada and other major developed economies in Section 2, is that both net inflows into, and outflows from, a host economy of FDI by foreigners increase during an upturn. Moreover, business cycle fluctuations in net FDI in Canada and net Canadian investment abroad are positively correlated.Thus periods of increased net inflows into an expanding economy are also periods of increased investment abroad by that same economy. The classic international real business cycle model, however, generates a negative correlation between these flows.

The approach taken by this paper, in Section 3, is to model flows of horizontal foreign direct investment in a two-country, two-sector model, in which the allocation of capital to production abroad is subject to a friction of the search and matching type: bringing to fruition a new investment project abroad is costly and time consuming and, once in place, faces an endogenous termination probability. The model therefore provides a theoretical framework with endogenous gross inflows and outflows of foreign direct investment in

[1]These figures are for the manufacturing sector, see Baldwin and Dhaliwal (2001) and Baldwin and Gellatly (2005). The importance of FDI does not limit itself to the case of Canada. For example, the ratio of FDI to domestic investment in the US has risen from 6% in the 1970s to 15% in the 2000s. Lipsey (2000) reports ratios above 10% for many industrialized country over the period 1970 to 1995.

which congestion effects on foreign investment markets impact the response of investment patterns to changes in productivity.

Several considerations motivate the modeling strategy adopted here. First, as argued by Gordon and Bovenberg (1996), due to a lack of knowledge of the domestic economy foreign firms are at a disadvantage in setting up and running a firm. While these authors capture this idea by assuming that output at foreign firms is reduced by some fixed proportion, in the presence of search and matching frictions the cost of investment will be a function of congestion on foreign investment markets. The probabilistic nature of the matching process captures the fact that foreigners incur the cost of more time in setting up a new production facility or acquiring information about a risky investment project, as in Gopinath (2004), and congestion effects are a novel allocative signal .

Quantitatively, the model generates the high cyclical volatility of net FDI flows, and the positive correlation of net foreign direct investment inflows and outflows observed in the data. By contrast, a standard IRBC model with investment adjustment costs predicts a negative correlation, and lower volatilities of FDI flows. As Section 4 elaborates in assessing the quantitative implications of the model, the allocation friction is central to explaining this positive correlation of net inflows and net outflows of FDI. Following a positive technology shock in the host economy, whether in a standard IRBC model with investment adjustment costs or a search in FDI model, flows of net inward FDI increase on impact. In the standard IRBC model, by simple arbitrage, gross flows of FDI from the host economy abroad decrease on impact, generating a counterfactual negative correlation. However, in the proposed model this same drop in the pool of capital goods available for allocation abroad increases the allocation probability for the capital owner in the short run, thus mitigating the drop in new allocations abroad and producing the positive correlation between inward and outward flows observed in the data. This is because, in this context, new allocations are a function of both the pool of capital available and the allocation probability, or congestion on the foreign capital market.

The model is extended in a variety of dimension in order the verify the robustness of the main results. For instance, the the introduction of match specific productivity shocks allows for a notion of endogenous capital reallocation. Endogenous gross repatriation of capital abroad will move inversely with the originating country's business cycle. As a result, the decline in net FDI during an expansion is further limited by a drop in the gross flow of capital repatriated and improves the model's fit with the data.

This paper is related to the growing literature on international real business cycles, dating back to the seminal contribution of Backus, Kehoe and Kydland (1992), and to the transmission channels of international business cycles. This paper considers horizontal FDI, treated in the trade theory literature by papers such as Markusen (1984), Markusen and Venables (2000), and Helpman, Melitz and Yeaple (2004). These Proximity-concentration models of FDI, generally assume a fixed cost to setting up operations abroad, above the the cost of entering the domestic market. As will be discussed below, foreign affiliates pay a cost per investment project initiation. Although allocation of capital to domestic firms will be frictionless, this is only a special case of the search environment when the initiation cost is nil. This setup was explored extensively in a closed economy setting for the allocation of physical capital by Kurmann and Petrosky-Nadeau (2009). Gopinath (2004) models the difficulty in acquiring the information on investment projects in emerging projects as a time consuming search process.

One measure of the international transmission of business cycles, the cross country correlation of aggregate variables, poses a problem for standard IRBC models known of the quantity problem. That is, the ordering of output and consumption cross correlations in the model is opposite to that in the data, (see Backus, Kehoe and Kydland, 1995 or Crucini, 2008). While many papers have made contributions in reducing or solving this problem, few address another quantity problem involving aggregate hours and investment. A result of focusing on net flows of FDI is that the model solves the investment quantity puzzle. That is, contrary to other international real business cycle model, the presence of congestion on foreign investment markets generates a positive cross-country correlation of aggregate investment coherent with the data. To understand this result, consider that aggregate investment has domestic and foreign components. During an expansion in one country, both the domestic and the foreign components increase as investors are drawn to the higher returns. In a standard IRBC setting, both the domestic and foreign components investment in the other country decrease in the meantime such that aggregate investment in the two countries are negatively correlated. However, by getting the correct positive correlation between net flows of FDI in the search based model, the foreign component of investment increases in both countries, raising in the cross-country correlation of aggregate investment.

2 Flows of FDI and Canada - U.S. business cycles

This section reviews evidence on the cyclical characteristics of FDI flows outlined in the introduction. While the Canadian economy is of particular interest for this study because of the large and historically stable share of economic activity originating in the foreign sector, it is increasingly significant for other industrialized economies as they further integrate. The section also documents similar observations on flows for FDI for other major developed countries. Flows of foreign direct investment into Canada, and flows of Canadian direct investment abroad, concerning overwhelmingly the United States, the focus is placed on the similarities and interdependence of both countries.

2.1 Canadian and U.S. business cycles

Despite a large difference in absolute size, in per capita terms the Canadian and U.S. economies are remarkably similar. The evolution of hours worked (indexed), real output, investment and consumption per capita in both countries, over the period 1976-2006, have but for a few episodes followed each other closely.[2] One example is the output per capita gap between the U.S. and Canada appearing during the 1990s, which also shows up as a gap in average hours worked.

While aggregate trends have been similar, Table 1 examines differences in cyclical fluctuations of prominent macroeconomic variables, measured as 2nd moments for Hodrick-Prescott filtered quarterly data over 1976:1 - 2005:4. The Canadian and U.S. economies display approximately the same business cycle characteristics of these variables, although there is evidence of less aggregate consumption smoothing in Canada, seen as the larger relative volatility of consumption, observations also made in Baxter and Crucini (1995), Ambler, Cardia and Zimmermann (2004).

One indicator of business cycle synchronization, the cross-country contemporaneous

[2] A data and technical appendix is available upon request. The time series for the mentioned variables are plotted in Figure 1 of said appendix.

Table 1: Business cycle moments for Canada and the U.S.

1976:1 - 2005:4	Canadian data		U.S. data		Cross-country
variable:	a	b	a	b	correlations
Consumption	0.80	0.87	0.47	0.74	0.60
Hours	0.80	0.83	0.95	0.87	0.63
Investment	3.11	0.69	3.24	0.82	0.45
Output	1.53*		1.42*		0.75

*: standard deviation; a: standard deviation relative to output; b: contemporaneous correlation with output. All moments are Hodrick-Prescott filtered.

correlation of prominent macroeconomic variables, is reported in the last column of Table 1. In their extensive study of international business cycles, Ambler, Cardia and Zimmermann (2004) find much lower, although positive, cross country correlations than those for the Canada - U.S. pair, suggesting a higher than average degree of integration of both economies. While both theoretical and empirical work have often followed trade as a vector of synchronization, the increasingly important channel of flows of foreign direct investment is explored in the next subsection.[3]

2.2 Flows of FDI and foreign controlled firms in Canada

There are essentially two ways in which foreigners can access a domestic economy: (i) by establishing a branch or new business; (ii) through mergers and acquisitions of domestic firms. A commonly used measure of the rate at which foreigners access a domestic economy, flows of foreign direct investment, can further be categorized as either "horizontal" or "vertical". As described by Markusen (2002), horizontal FDI refers to the replication of ca-

[3]Sales by multinational firms have outpaced the expansion of trade in manufactures over the last decades. See Markusen (2004). Kose and Yi (2001) explore and discuss the limitations of the trade approach to solving the quantity puzzles. Ambler, Cardia and Zimmermann (2002) explore the potential of a two country multi-sector model with trade in intermediate goods in addressing the same issue. Other avenues have been explored, such as variable capital utilization in Baxter and Farr (2005), or trade in capital goods in Boileau (1999). Iacoviello and Minetti (2006) explore the implications of imperfect cross-border credit relations for output cross-correlations. See also Schmitt-Grohé (1998) for an evaluation of various mechanisms.

pacity abroad, and vertical FDI to the division of the production process globally in order to exploit the benefits offered by different markets. As Brainard (1997) documents and argues, the majority of FDI between developed countries is horizontal. In addition, the large majority of foreign affiliate sales are destined to the host market. Brainard (1993) estimates that approximately 92% of foreign affiliate production in the United States is destined for the host market. There remains, however, a debate over the principle mode of accessing an economy, although Helpman, Melitz and Yeaple (2004) argue that it occurs mainly through "greenfield" investment.[4]

In order to assess the extent and effect of foreign control over the national economy, in 1962 the Canadian government passed the Corporations Returns Act (CRA), requiring firms doing business in Canada to report financial and ownership data. Of the 40 000 reporting firms in 2004, foreign controlled corporations accounted for 30.7% of total operating revenues and 28.5% of all assets held in Canada,[5] shares that have historically remained stable. The United States play a central role in the foreign control of the Canadian economy, generating 62.6% of the operating revenues of foreign controlled corporations. The closest behind are the United Kingdom and Germany with, respectively, 7% and 6.5% of operating revenues.

As seen by industrial sector, foreign control is most important in oil and gas, manufacturing and mining, and significant in wholesale trade, utilities, and transportation and warehousing.[6] Manufacturing stands out as a sector with a large share of employment and high degree of foreign control, involving nearly one fifth of employment and where just over half of the revenues and assets are under foreign control. In fact, Baldwin and

[4]By "greenfield" investment, one refers to the establishment of a branch or new business. The position taken by Helpman et al. (2004) differs from that of Graham and Krugman (1995) according to whom the evidence is less clear and leans rather towards a larger role for mergers and acquisitions. While this paper will follow Helpman et al. (2004), it worth noting a recent contribution by Nocke and Yeaple (2007). These authors investigate the theoretical determinants of FDI by M&A or greenfield investment.

[5]"The notion of control encompasses both direct and effective control. Direct control is defined as a person, group or corporation holding, directly or indirectly, more than 50% of the voting equity. Effective control implies control through methods other than ownership of the majority voting equity, such as when more than 50% of the directors of a corporation are also directors of another corporation." For additional information , see "Corporations Returns Act, 2004," catalogue no. 61-220. Statistics Canada, vol XI E, p. 3.

[6]See Baldwin and Gellatly(2005).

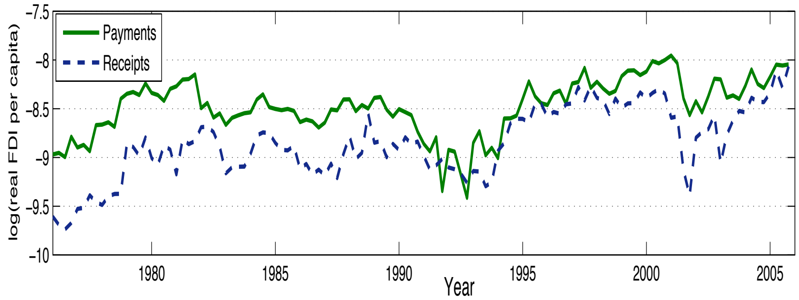

Figure 1: Flows of foreign direct investment, Canadian Balance of Payments

Gellatly (2005) estimate the share of manufacturing employment originating in the foreign sector to be 30% of total sectoral employment. Together, sectors with more than 20% foreign control, in terms of assets, involve 55% of employment. These measures give a sense of the importance of foreign controlled firms for aggregate outcomes.

2.2.1 Flows of foreign direct investment

Flows of foreign direct investment in Canada and flow of Canadian direct investment abroad from the Canadian Balance of Payments are large, historically around 20% of aggregate Canadian investment.[7] The source and destination of these flows is overwhelmingly the U.S., the source of 44% of inflows and destination for 58 % of outflows. Except for a brief period in the early 1990s, payments have always exceeded receipts, leading to a persistent deficit offset only by Canada's historically positive trade balance (see Figure 1).

The business cycle component of net flows of FDI into Canada and flows of Canadian direct investments abroad, along with their cross-correlation are presented in Table 2. Both flows are highly volatile, with H.-P. filtered standard deviations relative to output of 14.19 and 8.8, respectively. By comparison, the relative volatility of aggregate investment is of the order of 3.11. Both net inflows and net outflows, that is net FDI in Canada and net Canadian Direct Investment Abroad move strongly with the Canadian business cycle, with respective contemporaneous correlations with Canadian GDP of 0.36 and 0.40. Moreover, Table 2 also reveals that net inward and net outward flows are positively corre-

[7] It is important to stress that flows of portfolio investment are excluded, keeping only flows of direct investment.

Table 2: The business cycle of foreign direct investment in Canada

1976:1 - 2005:4	Net inflow	Net outflow
Std. dev. rel. to output	14.19	8.8
Contemp.corr. with output	0.36	0.40
Volatility of outflow to inflow	0.62	
Contemporaneous correlation, inflow-outflow	0.27	

2nd moments computed for Hodrick-Prescott filtered data. Source: Statistics Canada

lated. That is, periods of increased net inflows of FDI into Canada are also accompanied by increased net Canadian direct investments abroad. This fact has not received much attention, equilibrium models tending to predict that capital would simultaneously flow to high productivity and out of low productivity countries.

2.3 Evidence for other industrialized nations

This pattern in contemporaneous inflows and outflows of foreign direct investment for a host economy is not unique to the Canada-U.S. pair, as reported in Table 3 using annual data collected by the OECD over the period 1985 to 2007. The contemporaneous correlation between flows of FDI from France to the United Kingdom and in the opposite direction is 0.42, while with the core 15 members of the European Union this correlation is 0.44.

Table 3: The business cycle of foreign direct investment - other countries

	UK	Germany	EU15	Japan
France 1985:2007				
Volatility of outflow to inflow	0.35	0.09	0.32	0.33
Contemp. correlation, inflow-outflow	0.42	0.56	0.67	0.44

2nd moments computed for Hodrick-Prescott filtered data. Source: OECD

3 IRBC with search in FDI

The model develops a framework with *net* inflows and *net* outflows of foreign direct investment in a two country, two sector DSGE model, where *gross* investment flows in both directions evolve endogenously with the business cycle. Each country is populated by domestic and foreign firms and a representative household. For simplicity the model abstracts from trade in consumption goods. Households decide on optimal consumption, an aggregate of goods produced by both types of firms, and allocation of investment goods to firms located at home or abroad. In order to initiate a new investment project abroad, foreign affiliates must disburse a flow cost κ. This cost is paid until the project is brought to fruition, a time consuming task abstracted as a search and matching process with investment goods available for allocation abroad. No such friction applies to changing production capacity at domestic plants located in the home economy.[8] Thus domestic firms rent capital on spot markets while foreign affiliates choose the amount of new projects to initiate. Firms, domestic and foreign, hire labor on competitive domestic markets.

As a matter of notation, the first country is referred to as the "Home" country and the second as the "Foreign" country. Throughout, variables relating to the Foreign economy will be distinguished by an asterisk. For example, k^{fdi} denotes the stock of capital held by foreigners in the "Home" economy while k^{fdi*} denotes the stock of capital held by foreigners in the "Foreign" country, i.e. held by residents of the Home country. We begin by describing the friction to allocating physical capital abroad, domestic and foreign firms, and then examine the problem faced by the representative household in the Home economy, the problem in the Foreign economy being symmetrical.

3.1 Undertaking a foreign direct investment

In order to form a unit of capital abroad, a new project, v, must be initiated at a cost of κ by a foreign affiliate. This cost is reminiscent of Gordon and Bovenberg (1996) who assume that foreign investors, due to a lack of knowledge of the domestic economy, are at a disad-

[8] In fact, the frictionless capital market is a special case of the search environment with $\kappa = 0$. This extreme assumption of no friction to allocating investment goods to domestic firms at home is made for simplicity. As long as allocating investment goods abroad is relatively more costly than the allocation at home, the results go through.

vantage in setting up and running a firm. They capture this idea by assuming that output by foreign firms is reduced by some fixed proportion. Gopinath (2004) assumes that investors in emerging markets must disburse a cost to acquire information on investment projects while the length of the acquisition period is subject to search frictions. Meanwhile, a pool of liquid capital, l, must be made available to be allocated abroad once the right location has been found. This process of matching new projects and liquid capital is abstracted by a constant returns to scale matching technology $m(v,l)$. Denoting $\theta = \frac{l}{v}$ as a relative measure of tightness on international capital markets, the probability for a given project initiation of becoming a productive unit of capital in the current period is given by $\frac{m(v,l)}{v} = m(1,\theta) = \mathcal{P}(\theta)$, with $\partial\mathcal{P}(\theta)/\partial\theta > 0$. The equivalent probability for liquid capital is just $\frac{m(v,l)}{l} = m(1/\theta,1) = \mathcal{Q}(\theta)$, $\partial\mathcal{Q}(\theta)/\partial\theta < 0$.

Once in place, a particular unit of foreign capital faces an probability s of being terminated. When this occurs the unit of capital returns to the pool of liquid capital, net of depreciation, for reallocation. As a result, the total amount of liquid capital available for allocation abroad in the current period is defined as

$$l_t = i_t^{fdi} + (1-\delta)sk_t + u_t, \tag{1}$$

where $u_t = (1-\mathcal{Q}(\theta_{t-1}))l_{t-1}$ is unmatched liquid capital from the previous period carried forward with no net return, and i_t^{fdi} are new investment goods added to the pool of liquid capital.

These assumptions result in the following law of motion for the stock of foreign capital in the Home economy

$$k_{t+1}^{fdi} = (1-\delta)(1-s)k_t^{fdi} + m(v_t, l_t). \tag{2}$$

For ease of comparison with the Balance of Payments, it is useful to rewrite the law of motion as $k_{t+1}^{fdi} = (1-\delta)k_t^{fdi} + m(v_t,l_t) - (1-\delta)sk_t^{fdi}$. The expression $m(v_t,l_t)$ corresponds to gross inflows of foreign direct investment while $(1-\delta)sk_t^{fdi}$ corresponds to gross outflows of foreign direct investment, the difference being net flows of inward FDI. The Home economy's direct investment abroad is likewise decomposed into gross outflows $m(v_t^*, l_t^*)$ and gross inflows $(1-\delta)s^*k_t^{fdi*}$(i.e., returning from the Foreign country).

3.2 Domestic and foreign producers

Domestic and foreign firms produce intermediate goods aggregated into a final homogeneous consumption good by an Armington (1969) aggregator $y_t = G\left(y_t^d, y_t^{fdi}\right)$ $= \left[\phi(y_t^d)^\nu + (1-\phi)(y_t^{fdi})^\nu\right]^{\frac{1}{\nu}}$, with elasticity of substitution $\psi = 1/(1-\nu)$ and relative shares determined by the parameter ϕ. The relative price of the foreign firm's good is then simply $p_t^{fdi} = G_2\left(y_t^d, y_t^{fdi}\right)$ and that of the domestic firm's good $p_t^d = G_1\left(y_t^d, y_t^{fdi}\right)$.

Domestic firms produce with technology $y_t^d = A_t(n_t^d)^{1-\alpha}(k_t^d)^\alpha$, where $0 < \alpha < 1$ and A_t denote total factor productivity in the Home country, hiring both factors of production from households on competitive markets. Optimization yields the following two first order conditions for the demand of domestic labor and capital

$$(n_t^d) \quad : \quad w_t^d = (1-\alpha)\frac{p_t^d y_t^d}{n_t^d}; \tag{3}$$

$$(k_t^d) \quad : \quad r_t^d = \alpha\frac{p_t^d y_t^d}{k_t^d}; \tag{4}$$

where w_t^d and r_t^d are, respectively, the remunerations of labor and capital at domestic firms.

Foreign firms in the Home economy hire domestic labor, n^{fdi}, and make capital adjustment decisions by choosing the number of new projects to initiate, v, with the production technology $y_t^{fdi} = A_t(n_t^{fdi})^{1-\alpha}(k_t^{fdi})^\alpha$. Given allocation process for FDI outline above, foreign firms face the following dynamic program:

$$J(k_t^{fdi}) = \max_{n_t^{fdi}, v_t}\left[p_t^{fdi}y_t^{fdi} - w_t^{fdi}n_t^{fdi} - r_t^{fdi}k_t^{fdi} - \kappa v_t + \beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J(k_{t+1}^{fdi})\right]$$
$$\text{subject to } k_{t+1}^{fdi} = (1-\delta)(1-s)k_t^{fdi} + \mathcal{P}(\theta_t)v_t,$$

where $J(k^{fdi})$ is the value of the foreign firm given its current capital stock, and w_t^{fdi} and r_t^{fdi} are, respectively, the remunerations of labor and capital at foreign firms. The foreign affiliate uses the stochastic discount rate is $\beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}$ as all profits are transferred to the foreign household. Optimization yields the following two first order conditions:

$$(n_t^{fdi}) \quad : \quad w_t^{fdi} = (1-\alpha)\frac{p_t^{fdi}y_t^{fdi}}{n_t^{fdi}}; \tag{5}$$

$$(v_t) \quad : \quad \frac{\kappa}{\mathcal{P}(\theta_t)} = \beta E_t\frac{\lambda_{t+1}^*}{\lambda_t^*}J_{k^{fdi}}(k_{t+1}^{fdi});$$

where $J_{k^{fdi}}(k_{t+1}^{fdi})$ is the marginal value of an additional unit of capital to the firm. While the first condition determining the demand for labor is quite standard, some interpretation of the optimality condition for project initiations is in order. This states that, at the margin, the discounted expected return to an additional unit of capital must be equal to the average cost of setting it up, $\frac{\kappa}{\mathcal{P}(\theta_t)}$, as the average duration to locate a supply of capital is approximately the inverse of the probability $\mathcal{P}(\theta)$. As such, this may be interpreted as a "project creation" condition akin to the job creation condition in labor search and matching models. Differentiating the firm's value function, the marginal value of and additional unit of capital is

$$J_{k^{fdi}}(k_{t+1}^{fdi}) = \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} - r_t^{fdi} + (1-\delta)(1-s)\beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} J_{k^{fdi}}(k_t^{fdi}).$$

In combination with the first order condition for project initiations, this yields the forward looking condition

$$\frac{\kappa}{\mathcal{P}(\theta_t)} = \beta E_t \frac{\lambda_{t+1}^*}{\lambda_t^*} \left\{ \alpha \frac{p_t^{fdi} y_{t+1}^{fdi}}{k_{t+1}^{fdi}} - r_{t+1}^{fdi} + (1-\delta)(1-s) \frac{\kappa}{\mathcal{P}(\theta_{t+1})} \right\} \tag{6}$$

3.3 Domestic households

Households choose a level of aggregate consumption of the final homogeneous good, hours to supply to both domestic and foreign employers, n_t^d and n_t^{fdi} respectively, and have two capital investment options: investing in firms at home, i_t^d, or investing in capacity abroad, i_t^{fdi*}. In addition, there are convex cost to producing new investments goods, domestic and foreign, respectively q_t^d and q_t^{fdi*}.[9] The resulting dynamic program for the representative household is thus

$$\begin{aligned} V(k_t^d, k_t^{fdi*}, u_t^*) = & \max_{c_t, n_t^d, n_t^{fdi}, i_t^d, i_t^{fdi*}} \left[u(c_t, 1-n_t) + \beta E_t V(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) \right] \\ \text{subject to} \quad & w_t^d n_t^d + w_t^{fdi} n_t^{fdi} + r_t^d k_t^d + r_t^{fdi*} k_t^{fdi*} + \Pi_t^* = c_t + q_t^d i_t^d + q_t^{fdi*} i_t^{fdi*} \\ \text{and} \quad & k_{t+1}^{fdi*} = (1-\delta)(1-s^*) k_t^{fdi^*} + \mathcal{Q}(\theta_t^*) l_t^* \end{aligned}$$

[9]It is well known (see Backus, Kehoe and Kydland (1992), Baxter and Crucini (1995)) that without an adjustment cost to the production of new capital goods the volatility of new investment would be much too large in this setting.

where $n_t = n_t^d + n_t^{fdi}$, $\Pi_t^* = p_t^{fdi*} y_t^{fdi*} - w_t^{fdi*} n_t^{fdi*} - r_t^{fdi*} k_t^{fdi*} - \kappa v_t^*$ are profits for firms operating abroad, and $i_t^d = k_{t+1}^d - (1-\delta) k_t^d$. New investment goods destined for foreign direct investment are defined as $i_t^{fdi*} = l_t^* - (1-\delta) s^* k_t^{fdi*} - u_t^*$, where $(1-\delta) s^* k_t^{fdi*}$ is capital recuperated from terminated operations abroad net of depreciation, and u_t^* are units of investment goods not yet allocated. Again, l_t^* is therefore the total amount of investment goods available for allocation to production abroad. Finally, under the assumption that the cost of adjusting physical capital is governed by the function $\Phi(\frac{i_t^j}{k_t^j})$, as in Hayashi (1982), this price of capital goods is given by $q_t^j = \left[\Phi'(\frac{i_t^j}{k_t^j})\right]^{-1}$, for $j = d, fdi^*$, with $\Phi'(\bullet) > 0$ and $\Phi''(\bullet) < 0$, and such that in the steady state $q = 1$.

Denoting the multiplier on the budget constraint λ_t, the optimality conditions are

$$(c_t) \quad : \quad u_c(c_t, 1 - n_t) = \lambda_t \tag{7}$$

$$(n_t^d) \quad : \quad u_{n^d}(c_t, 1 - n_t) = \lambda_t w_t^d \tag{8}$$

$$(n_t^{fdi}) \quad : \quad u_{n^{fdi}}(c_t, 1 - n_t) = \lambda_t w_t^{fdi} \tag{9}$$

$$(i_{t+1}^d) \quad : \quad \lambda_t q_t^d = \beta E_t \lambda_{t+1} \left[r_{t+1}^d + q_{t+1}^d (1-\delta) \right] \tag{10}$$

$$(i_t^{fdi*}) \quad : \quad \lambda_t q_t^{fdi*} = \beta E_t \left[\mathcal{Q}(\theta_t^*) V_{k^{fdi*}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) + (1 - \mathcal{Q}(\theta_t^*)) V_{u^*}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*) \right] \tag{11}$$

The Euler equation for allocation of investment goods to domestic firms, equation (10), has the usual interpretation of equating the opportunity cost of the investment, in terms of current period forgone consumption, to the expected return net of depreciation. The Euler equation governing foreign investment decisions, equation (11), has a similar interpretation. The expected return, however, is an average of the marginal values of matched ($V_{k^{fdi*}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*)$) and unmatched ($V_{u^*}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*)$) capital, weighted by the matching probability $\mathcal{Q}(\theta_t^*)$. The marginal values of allocated and non-allocated investment goods are given by

$$\begin{aligned} V_{u^*}(k_t^d, k_t^{fdi*}, u_t^*) &= \lambda_t q_t^{fdi*}; \\ V_{k^{fdi*}}(k_t^d, k_t^{fdi*}, u_t^*) &= \lambda_t \left[r_t^{fdi*} + q_t^{fdi*}(1-\delta) s^* \right] + (1-\delta)(1-s^*) \beta E_t V_{k^{fdi*}}(k_{t+1}^d, k_{t+1}^{fdi*}, u_{t+1}^*). \end{aligned}$$

Since unmatched liquid capital yields not net return, its marginal value is simply the opportunity cost of funds. The marginal value of matched capital consists of the earnings on the unit, r_t^{fdi*}, and the value of capital separated for reallocation net of depreciation. The last term captures the continuation value if reallocation does not occur.

3.4 Repayment on foreign capital

Each unit of capital allocated abroad generates a surplus for the foreign affiliate and the capital lender. The repayment on capital allocated abroad is determined by Nash bargaining over the total surplus generated by the relationship, defined as $S_t = J(k_t^{fdi}) + \frac{V_{k^{fdi}}(k_t^{d*}, k_t^{fdi}, u_t) - V_u(k_t^{d*}, k_t^{fdi}, u_t)}{\lambda_t^*}$. This results in the following repayment rule:[10]

$$r_t^{fdi} = \eta\alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} + (1-\eta) q_t^{fdi} \delta + \eta(1-\delta)(1-s)\frac{\kappa}{\theta_t} \tag{12}$$

By the first term, the repayment is increasing in the marginal product of capital. The second term captures the loss of value due to physical depreciation, measured by the price of investment goods, the cost of which is split according to the lender's bargaining weight η. The long-term nature of the relationship is captured by the final term. It represents the initiation costs saved by the firm in the continued operation of the unit of capital. By changing the relative threat point of the firm in negotiations, a rise in κ puts upward pressure on the repayment.

The competitive equilibrium is a solution for the quantities $\{y_t, c_t, n_t^d, n_t^{fdi}, k_t^d, k_t^{fdi}, i_t^d, i_t^{fdi}, \lambda_t, \Pi_t, l_t, u_t, v_t, \theta_t\}$, prices $\{p_t^d, p_t^{fdi}, w_t^d, w_t^{fdi}, r_t^d, r_t^{fdi}, q_t^d, q_t^{fdi}\}$ and meeting rates $\{\mathcal{Q}(\theta_t), \mathcal{P}(\theta_t)\}$ in the Home and the Foreign countries that satisfy household, domestic and foreign firm optimization, the rental rate is the solution to the Nash bargaining game, the labor markets clear and the aggregate resource constraint is satisfied.

4 Quantitative results

The model is solved for the rational expectations equilibrium of the log-linear system of equations with the algorithm developed by King and Watson (1998). The quantitative implications are evaluated through impulse responses and unconditional second moments. The results for flows of FDI are discussed first, before looking at aggregate variables and cross-country correlations. In all instances the results are contrasted with those for a standard IRBC with investment adjustment costs.[11] The robustness of the results is examined

[10] The appendix provides details on the derivation of this repayment rule.

[11] The details these models are presented in the appendix. It is important to note that the first corresponds to the search model without allocation frictions, i.e. a model with $\kappa = 0$.

in an extension in which the rate of separation of capital from it's use abroad is endogenized.

4.1 Shocks and calibration

4.1.1 Extraction of a Solow Residual.

The underlying exogenous processes for technology, as in Backus, Kehoe and Kydland (1992), is assumed stationary and to follow a VAR(1) process with possible cross-country spill-overs. Parameter estimates are obtained by extracting Solow residuals for the Canadian and U.S. economies and then estimating the following bivariate VAR(1):

$$\begin{bmatrix} A_t^{can} \\ A_t^{us} \end{bmatrix} = \begin{bmatrix} \rho_c & \rho_{c,us} \\ \rho_{us,c} & \rho_{us} \end{bmatrix} \begin{bmatrix} A_{t-1}^{can} \\ A_{t-1}^{us} \end{bmatrix} + \begin{bmatrix} e_t^{can} \\ e_t^{us} \end{bmatrix}$$

The results of the estimation are presented below for the period 1976:1 - 2005:4. As is usual in this sort of estimation, the persistence parameter is very high. Also, as can be gleaned from the covariance matrix, Canadian and U.S. innovations to the exogenous process for technology are positively correlated. In a subsection below, the sensitivity of the quantitative results to the specification of the exogenous process for technology will be examined.

$$\begin{bmatrix} \rho_c & \rho_{c,us} \\ \rho_{us,c} & \rho_{us} \end{bmatrix} = \begin{bmatrix} 0.9747 & 0.034 \\ -0.0174 & 0.9264 \end{bmatrix} \quad \text{Residuals Covariance matrix :} \begin{bmatrix} 0.079 & 0.021 \\ 0.021 & 0.05 \end{bmatrix}$$

4.1.2 Parametrization and calibration

Time periods are considered to be quarters, and the discount factor is set to $\beta = 0.99$, corresponding to an average annual real yield on a risk-less bond of 4.1%. Preferences are separable in consumption and leisure, and take the form $u(c_t, 1-n_t) = \log(c_t) + \frac{\varpi(1-n_t)^{1-\xi}}{1-\xi}$. The parameter ϖ is fixed such that the average fraction of total hours worked equals $n = 0.2$. Together with $\xi = 4$ this results in a Frisch elasticity of labor supply of 1. Furthermore, the share domestic goods in the production of the final good, ϕ, is set such as to equalize the wage across sectors in the steady state. The share of capital in the production function is set to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$, which corresponds to an annual decline of the productive use of capital of 10%. The elasticity of the investment adjustment cost is 0.025, within a range of values used in different studies (e.g., Baxter and Crucini, 1995, Ambler, Cardia and Zimmermann, 2002, and Baxter and Farr, 2005). Finally, the parameter ν in the Armington aggregator is chosen such that that the steady state equilibrium hours worked in the foreign sector are 1/4 of total hours, which is in the range of employment shares reported earlier, and implies an elasticity of substitution between the foreign and domestic firms' goods of approximately 1.5.

To calibrate parameters relative to foreign direct investment, it is useful to let the theory shed some light on the data. Recall the foreign capital accumulation equation

$$k_{t+1}^{fdi} = (1-\delta)k_t^{fdi} + inflow_t - outflow_t.$$

As the Balance of Payments provides information on foreign direct investment gross inflows and outflows, given a rate of capital depreciation one can compute the implied foreign capital stock in the host economy, using the steady state property $k^{fdi} = [inflow - outflow]/\delta$ to initiate the capital stock. It is then possible, using the time series on outflows, to obtain a time series for the reallocation rate as

$$s_t = \frac{outflow_t}{(1-\delta)k_t^{fdi}},$$

resulting in a mean rate of $s = 0.0602$, which we use to set the value of the exogenous separation rate, and an H.-P. filtered standard deviation relative to output of 1.45 and contemporaneous correlation with output of 0.16. The latter information will be used for the calibration to endogenous capital reallocation.

Next it is assumed that it takes on average a little more than a quarter before liquid capital is allocated and becomes productive, i.e. $\mathcal{Q}(\theta) = 0.75$ in steady state, and we set the household's bargaining weight to $\eta = 0.5$, in the mid-range of possible values.[12] The final parameter left to calibrate is the elasticity of the matching function, which is of the form $m(v_t, l_t) = \chi(v_t)^{\epsilon}(l_t)^{1-\epsilon}$. This parameter only influences the dynamics of the model but does not affect the steady state as the value of the level parameter χ is chosen to match the target meeting rate $\mathcal{Q}(\theta)$. The baseline simulation sets $\epsilon = 0.8$, the reason for which

[12] As it is well know that the results of the search and matching model of equilibrium unemployment are sensitive to the value of this parameter (see Hagedorn and Manovskii, 2008), a series of sensitivity test will be performed below.

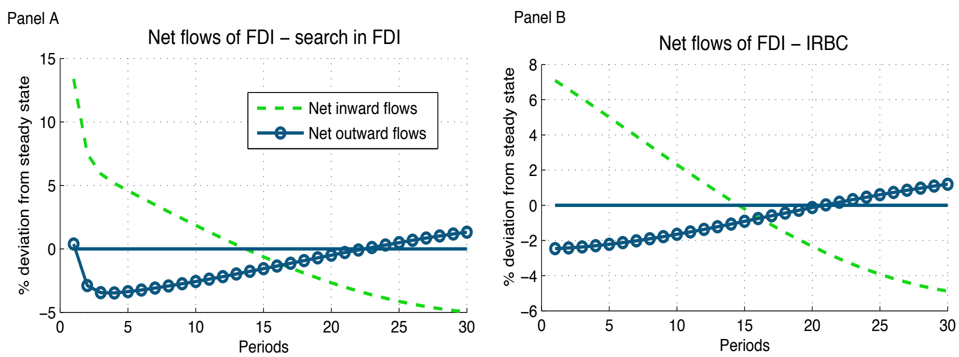

Figure 2: IRFs to a positive "Home" sourced technology shock

will be apparent in the extension to endogenous capital reallocation. A sensitivity analysis of results to variations in these parameters is performed below.

With these calibrations there is sufficient information to endogenously determine the remaining steady state variable and parameters (i.e. θ, κ, σ) such that the system of steady state equations is satisfied. The resulting long-run ratios of interest are the following: the consumption-output ratio equals 76.52% in line with King and Rebelo (1999); the labor share of income amounts to 0.67, which lies in the range reported by Gollin (2002); Furthermore, this calibration implies that the steady state ratio of net FDI to aggregate investment is 23%, that the average initiation cost relative to output equals $v\kappa/y^{fdi} = 1\%$.

4.2 Flows of foreign direct investment

Figure 2 plots the impulse responses to a positive technology shock in the Home economy of net inward and outward foreign direct investment for that expanding economy. The significant difference between the responses of the proposed model (Panel A) and an IRBC model (Panel B), beyond their magnitude, is the behavior of net outward flows (see circled line of panels A and B). In the search model, outward flows drop progressively, whereas in the standard model the drop occurs on impact, generating the positive cyclical correlation of inward and outward flows that is a characteristic of the data.

To detail the response of net outward direct investment flows, it is useful to recall its definition as the difference between gross outflows and gross inflows from the Home to the Foreign economy:

$$Net\ outward : l_t^* \mathcal{Q}(\theta_t^*) - (1-\delta) s^* k_t^{fdi*}$$

The initial response of net outward FDI is determined by that of the gross outflow, $l_t^* \mathcal{Q}(\theta_t^*)$, as k_t^{fdi*} is predetermined. As the opportunity cost of capital abroad increases, households diminish their pool of liquid capital l^*, shifting resources to domestic firms, which leads to a decline in the Home country's pool of capital available for investment abroad, l_t^*. This is the only source of change in net outflows in the model without allocation frictions, and therefore the drop in outward FDI is immediate. When allocation frictions are present, however, the decline in the pool of liquid capital is larger than the initial decline in project initiations at foreign affiliates for two periods after the shock, leading to a short lived increase in the capital allocation probability $\mathcal{Q}(\theta_t^*)$.[13] This reduction in market congestion counters the drop in l^* upon impact, and is seen in the muted initial decline in new allocations abroad. As the easing of congestion in allocation $\mathcal{Q}(\theta_t^*)$ dissipates thereafter, new allocations of FDI decline, reaching their lowest 3 quarters following the shock.

Table 4 presents unconditional second moments for flows of foreign direct investment in the data and generated by the competing models. The standard IRBC model, for the reasons just outlined, generates a strong negative contemporaneous correlation between net inflows and outflows of FDI of -0.59. The proposed model is a substantial improvement, almost perfectly matching the data with a contemporaneous correlation of 0.25 compared to 0.27 in the data. Thus the model is able to replicate the fact that periods of increased net investment abroad are also characterized by increased net inflow of foreign investment.

In the data, the relative volatility of net FDI outflows and inflows is approximately two thirds, and net outflows are as procyclical as net inflows. The standard IRBC model fails on both these counts. The ratio of H.-P. filtered standard deviations is only 0.37, and the correlation of net outflows with the source country's business cycle is strongly negative at -0.55, while the correlation of net inflows is too high at 0.91. On the other hand, the proposed model of search in FDI performs well with respect to the relative volatility of net inflows and outflows, the ratio being 0.67 compared to 0.62 in the data. However, while

[13] Kurmann and Petrosky-Nadeau (2008) show that under relatively weak conditions, if preferences are additive and concave in consumption for example, that congestion on the investment market will be increasing in the expected growth rate of the marginal utility of consumption.

Table 4: 2nd moments for flows of foreign direct investment

1976:1 - 2005:4	Canadian data		Search in FDI		IRBC with FDI	
	a	b	a	b	a	b
Net inflow of FDI	14.19	0.36	10.42	0.83	7.18	0.91
Net outflow FDI	8.8	0.40	6.93	-0.03	2.66	-0.55
	c	d	c	d	c	d
Net outward / Net inward FDI	0.62	0.27	0.67	0.25	0.37	-0.59

a: Standard deviation relative to output; b: Contemporaneous correlation with output; c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr., inward-outward net flows. All moments are Hodrick-Prescott filtered.

the model raises the correlation of net outflows with the domestic business cycle, under the present calibration it is insufficient to be consistent with the data.

4.3 Robustness of results

First, given the lack of direct evidence on the mean allocation rate $\mathcal{Q}(\theta)$, the effects of its variation on the main results, along with the consequence of varying the mean reallocation rate s, are presented in Table 5. In addition, the results are examined with respect to the bargaining weight η, as the work of Hagedorn and Manovskii (2008) has shown that the dynamics of search unemployment models are sensitive to this parameter. Second, we explore the importance of the specification of the exogenous technological process, and third of endogenizing the reallocation probability s.

4.3.1 Sensitivity to search parameters

Beginning with the mean allocation rate, its principle effect is to change the relative standard deviation of net foreign direct investment flows. The change to the correlation between net inward and outward flows of FDI when decreasing the degree of congestion in the allocation of capital abroad (i.e., increasing the mean $\mathcal{Q}(\theta)$) between approximately a year and a half and just over a quarter is virtually not noticeable.

Table 5: Sensitivity to search parameters and calibration

	Baseline	$s = 0.04$	$s = 0.08$	$\mathcal{Q}(\theta) = 0.65$	$\mathcal{Q}(\theta) = 0.85$	$\eta = 0.1$
$\sigma(net\ inflow)/\sigma(Y)$	10.42	9.40	11.46	10.45	10.68	9.75
$corr(inflow, Y)$	0.83	0.86	0.81	0.85	0.82	0.85
$\sigma(net\ outflow)/\sigma(Y)$	6.93	5.70	8.13	6.88	6.99	5.74
$corr(outflow, Y)$	-0.03	-0.09	0.01	0.02	-0.05	-0.2
$\sigma(net\ outflow)/\sigma(net\ inflow)$	0.67	0.61	0.71	0.68	0.65	0.59
$corr(inflow, outflow)$	0.25	0.15	0.31	0.25	0.25	0.03

All moments are Hodrick-Prescott filtered.

As the mean rate s may be affected by the initial foreign capital stock used in its imputation, it is reassuring that the results are very robust to changes in its average value. The volatility of net inward FDI and the relative volatility of net inward and outward flows change little. The main effect is to increase the correlation between inward and outward flow, from 0.15, when $s = 0.04$, to 0.31, when $s = 0.08$.

Reducing the lender's bargaining weight from 0.5 to 0.1 reduces the relative volatility of net inward foreign investment from 10.42 to 9.75. This occurs because, for lower values of the bargaining weight, the expected benefit of a new unit of capital allocated abroad is less elastic to changes in productivity. It also has the effect of reducing both the relative volatility of net inward to outward investment and the correlation between both flows, suggesting that the model is a better fit of the data with a higher bargaining weight, with the exception of the relative volatility of net inward investment.

4.3.2 Sensitivity to the specification of the technological process

An alternative specification of the exogenous process for technology cuts off cross-country spill-overs while fixing identical persistence parameters:

$$\begin{bmatrix} \rho_c & \rho_{c,us} \\ \rho_{us,c} & \rho_{us} \end{bmatrix} = \begin{bmatrix} 0.9747 & 0 \\ 0 & 0.9747 \end{bmatrix} \qquad \text{Covariance matrix}: \begin{bmatrix} 0.079 & 0.02 \\ 0.02 & 0.05 \end{bmatrix}$$

Table 6: Sensitivity to the specification of exogenous process and endogenous separation

	Search in FDI				IRBC with FDI		Search in FDI	
	baseline		no spillovers		no spillovers		End. Sep.	
	a	b	a	b	a	b	a	b
Net inflows of FDI	10.42	0.83	9.40	0.77	5.33	0.89	10.55	0.76
Net outflows of FDI	6.93	-0.03	5.97	0.24	3.36	0.05	8.00	0.03
Net outflows /	c	d	c	d	c	d	c	d
Net inflows FDI	0.67	0.25	0.63	0.33	0.63	-0.31	0.76	0.33

a: Standard deviation relative to output; b: Contemporaneous correlation with output; c: Ratio of outward to inward FDI flow standard deviations; d: contemporaneous corr., inward-outward net flows.

All moments are Hodrick-Prescott filtered.

The results for such a specification are presented in Table 6. Changing the specification of the exogenous process has little effect on the relative volatilities of net inward and outward flows of FDI, but increases the contemporaneous correlation of net investment abroad with the domestic business cycle in all models. In particular, the correlation of net outflows with the domestic business cycle increases to 0.24, compared to 0.4 in the data.

The ratio of volatilities of inward and outward flows hardly changes except in the IRBC model, for which the ratio becomes close to the data. However, the contemporaneous correlation of net inward of outward flows remains strongly negative, at -0.31, while for the proposed model it remains close to the data.

4.3.3 Extension to endogenous capital reallocation

It has so far been assumed that capital allocated to foreign production is reallocated every period at a constant rate s which is independent of economic conditions. It is likely that the opportunity cost of maintaining those units in place depends on the returns from alternative opportunities that evolve over the business cycle. We exam the implications of this for the main results by endogenizing the separation, or reallocation, rate s. As in Mortensen and Pissarides (1994) for the labor market, we assume the existence of match specific random idiosyncratic productivity shocks. Denote match productivity as $a_t > 0$, where a is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$, and follows a log normal distribution $\log(a) \sim N(-\frac{\sigma^2_{\log(a)}}{2}, \sigma^2_{\log(a)})$. The surplus generated by the relationship between a foreign affiliate and the capital lender (i.e. the household) is an increasing function of this shock, $S(a_t)$. Thus there is a reservation strategy for both parties who, once the shock is observed, discontinue the match for realizations of $a_t < \underline{a}_t$, where $\underline{a}_t$ is defined as $S(\underline{a}_t) = 0$ and the separation rate is $s_t = H(\underline{a}_t)$. Using a result of Nash bargaining, the separation threshold is defined by[14]

$$r_t^{fdi} - \underline{a}_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}} - (1-\delta)(1-s_t)\frac{\kappa}{\mathcal{P}(\theta_t)} = 0 \tag{13}$$

In effect, the match is discontinued if the realized marginal product of capital $a_t \alpha \frac{p_t^{fdi} y_t^{fdi}}{k_t^{fdi}}$ plus the search cost saved by maintaining the current unit of capital is inferior to the negotiated repayment. An increase in the average search cost $\frac{\kappa}{\mathcal{P}(\theta_t)}$, for example, by increasing the opportunity cost of exiting the match, lowers the separation probability.

In order to close this extended model, an insurance mechanism funded out of profits from continuing relationships is assumed to insure that, ex-post, the household receives the full ex-ante return to foreign capital, and that the full wage bill and costs of project initiations are covered.[15] Thus aggregate profits returned to the household are

$$\begin{aligned}
\Pi_t &= \int_{\underline{a}_t}^{\infty} \left[a p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t \right] dH(a) \\
&\quad + \int_0^{\underline{a}_t} \left[a p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t \right] dH(a) \\
\Pi_t &= p_t^{fdi} y_t^{fdi} - w_t^{fdi} n_t^{fdi} - r_t^{fdi} k_t^{fdi} - \kappa v_t
\end{aligned} \tag{14}$$

The calibration involves choosing the elasticity of the matching function, ϵ. This parameter is selected such that the relative volatility of the reallocation rate s is close to the data, which is achieved for $\epsilon = 0.8$. The standard deviation of the idiosyncratic productivity shock is equal to $\sigma_a = 0.27$ such that the steady state separation rate equals 0.06.

[14]See appendix for details.

[15]Gordon and Bovenberg (1996) use a similar assumption about the realization of an idiosyncratic productivity shock, and use the law of large numbers to argue that there is no aggregate uncertainty. Here, an insurance funded out of aggregate profits is used to address the issue.

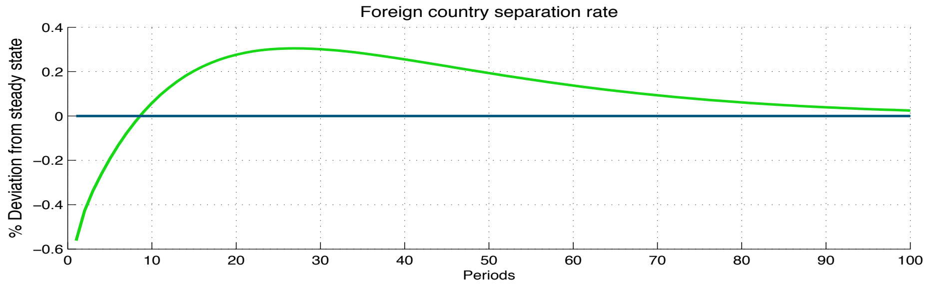

Figure 3: IRF of the separation rate s_t^* to a Home country positive technology shock.

As reported in Table 6, endogenous reallocation of capital already abroad increases the correlation between inflows and outflows even further. Figure 3 shows that a positive innovation in the Home country causes the reallocation rate in the foreign country to drop, reducing the gross flow $(1-\delta)s_t^* k_t^{fdi*}$ on impact. This drop pulls net outward flows from the Home to the Foreign country upward such that, on impact, net outward flows change very little. Thus the key to understanding the response of net foreign direct investment flows are the time-varying congestion, and to a lesser extent reallocation rates, effects that are absent in the standard model of international business cycles.

4.4 Aggregate variables

Figure 4 plots the impulse responses of output, hours and capital, at domestic firms, foreign firms and in the aggregate, to a Home sourced positive technology shock. Panel A presents results for the proposed search model with endogenous reallocation, while Panel B reports results for the IRBC model.

The first observation is that the responses of aggregate variables are quite similar for both models. The differences arise in the response of foreign firms when investing in capacity abroad is subject to a time consuming search process. On impact, hours at foreign firms rise more than at domestic firms in both models. However, the ensuing additional increase in hours at foreign firms is more pronounced in the proposed model, and stems from the different capital stock dynamics: the stock of foreign capital rises more quickly then in the standard case, pushing the labor demand of foreign establishments up further. In both models, hours at foreign firms are more volatile than at domestic firms over the business cycle, which is supported by recent empirical evidence from Europe. Checchi et

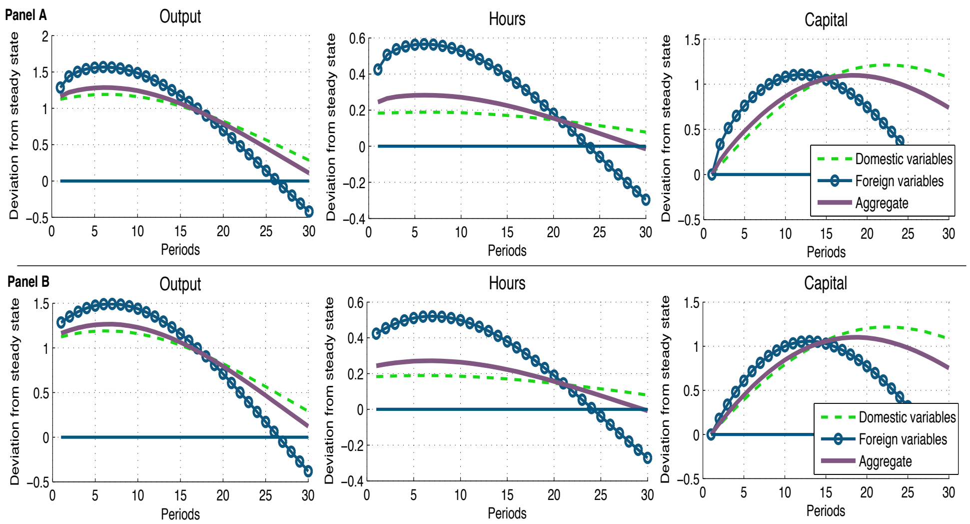

Figure 4: IRFs to a positive "Home" country technology shock

al. (2003) find that foreign controlled firms tend to make larger and more frequent employment adjustments. However, there is no direct evidence of systematic differences in the response of hours to the business cycle.

Table 7 presents the 2nd moments of prominent macroeconomic variables for the three models and the data. Both in terms of standard deviations and correlations with output, all three models are similar in being close to the data, with the well known exception of the volatility of hours. Thus, the ability of the model to generate high volatility in flows of FDI does not come at the expense of creating too much volatility in aggregate investment.

A well know deficiency of standard IRBC models, the quantity problem, concerns the ordering of cross country correlations of consumption, output, investment and hours. The problem of the ordering of consumption and output cross correlations is the most known of the quantity problems, as raised in the work of Backus, Kehoe and Kydland (1995), while the shortcomings related to the cross correlation of hours and investment have been raised in papers such as Ambler, Cardia and Zimmermann (2004). Table 7 shows the performance of the search in FDI model with this respect. All models get the ordering of higher output than consumption cross-correlations right, although the cross-correlation of consumption is lower than in the data, and the three models perform quite well with respect to the labor market. This is due, essentially, to the correlation structure to innovations and the presence of investment adjustment costs. This was first pointed out by Backus, Kehoe and Kydland

Table 7: 2nd moments for prominent macro variables

1976:1 - 2005:4	Canadian data			Search in FDI model						IRBC with FDI		
				Endogenous reall.			Exogenous reall.					
	a	b	c	a	b	c	a	b	c	a	b	c
Consumption	0.80	0.87	0.60	0.55	0.99	0.29	0.55	0.99	0.31	0.56	0.99	0.30
Hours	0.80	0.83	0.63	0.30	0.98	0.45	0.31	0.98	0.45	0.30	0.98	0.44
Investment	3.11	0.69	0.45	3.67	0.96	0.13	4.92	0.93	0.12	4.33	0.96	-0.08
Output	1.53*		0.75	1.41*		0.50	1.41*		0.51	1.38*		0.50

*: standard deviations; a: standard deviation relative to output; b: Contemporaneous correlation with output

c: cross country contemporaneous correlations. All moments are Hodrick-Prescott filtered

(1992), but made more explicit in Baxter and Crucini (1995). However, where aggregate investment is concerned, the cross-correlation is positive in the models with search frictions in foreign direct investment, while an IRBC model with investment adjustment costs generates a negative correlation. The role of foreign direct investment in correcting this correlation is clear when the aggregate investment is seen as the sum of investment in domestic and foreign firms. Driving the point further, raising the size of the foreign sector (as a fraction of total hours) from one quarter to one half, raises the cross-country correlation of aggregate investment from 0.13 to 0.20 in the model with search in FDI, while the correlation is reduced from -0.08 to -0.21 in the standard IRBC model.

5 Conclusion

A commonly used measure of the rate at which foreigners gain control over a domestic economy, flows of foreign direct investment (FDI), represent increasingly important share of aggregate investment in industrialized economies as they further integrate. Given the importance of the foreign sector for aggregate outcomes and the relatively high volatility of direct investment flows, quantitative models of open economies need to be consistent with their dynamics.

As this paper has shown, a combination of frictions in the allocation of physical capital to production abroad, and allowing for the endogenous reallocation of this capital, can replicate the positive correlation between net inflows and outflows of FDI that is a feature of the data. In addition the model can generate the higher volatilities of inward and outward net FDI, while the implication for prominent macroeconomic variables are similar to a standard IRBC model with investment adjustment costs. However, there are important sectoral differences worth mentioning in conclusion. The model implies that, for example, hours worked at foreign establishments are more volatile than hours worked at domestic establishments. An interesting question, and most relevant for economic policy, is whether this is the case in the data. In particular, if one considers the extensive margin of labor adjustments, are jobs at foreign establishments more elastic to the business cycle? If so, this might offer a rationalization for the public's skepticism toward the benefits of increased foreign control of a domestic economy as employment at these firms would be more fragile.

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