

15110 PRINCIPLES OF COMPUTING – EXAM 2- Summer 2015

Name: _____

Section: _____

Andrew Id: _____

| | | |
|--------------------|------|-------|
| Q. 1 | [15] | _____ |
| Q. 2 | [20] | _____ |
| Q. 3 | [20] | _____ |
| Q. 4 | [15] | _____ |
| Q. 5 | [21] | _____ |
| Q. 6 | [9] | _____ |
| TOTAL [100] | | _____ |

Directions: Answer each question neatly in the space provided.
Please read each question carefully. You have **80 minutes** for this exam.
No electronic devices allowed. Good luck!

1. The following question deals with recursion and recursive algorithms [15 points]

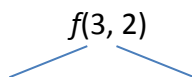
(a) [7 pt] The function f is defined for **non-negative** integers a and b recursively as follows:

$$f(a, b) = \begin{cases} 0 & \text{if } a = 0 \text{ or } b = 0 \\ f(a - 1, b - 1) + 2a - 1 & \text{if } a = b \\ f(a - b, b) + f(b, b) & \text{if } a > b \\ f(a, a) + f(b - a, a) & \text{if } a < b \end{cases}$$

Compute $f(3, 2)$ by drawing a recursion tree showing all of the computation required and then use your tree to compute the answer.

Recursion Tree:

$f(3, 2) =$ _____



State the common name for f or write a very compact non-recursive definition of f :

(b) [5 pts] Complete the recursive function to return a list that consists of the negative numbers of a given list (e.g. negatives([6, 3, -4, 9, -11, -2, 5]) will return [-4, -11, -2])

```
def negatives(lst):
    if _____:
        return _____
    else:
        if _____:
            return _____
        else:
            return _____
```

(c) [3 pts] Write the formula that fMystery function calculates.
(Hint: you can test it with a few numbers)

```
def fMystery(a):
    if a == 0:
        return 0
    else:
        return fMystery(a-1) + 2*a-1
```

2. This problem focuses on representation of data in a computer. [20 points]

The following tables may be helpful in this question:

| | | | | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2^{10} | 2^9 | 2^8 | 2^7 | 2^6 | 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 |
| 1024 | 512 | 256 | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

| | | | | | | | | | | | | | | | | |
|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Bin | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |
| Hex | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |

(a) [2 pts] Compute the decimal value of the byte **10101110** if it is interpreted as an unsigned integer.

(b) [2 pts] Compute the decimal value of the byte **10101110** if it is interpreted as a signed 2's complement integer.

(c) [2 pts] Express the byte **10101101** in hexadecimal.

(d) [2 pts] The ASCII character 'Q' is represented in binary using 7 bits as 1010001. The character is to be sent via satellite using even parity. What eighth bit is sent along with this byte: 1 or 0?

(e) [2 pts] Suppose that the eighth (parity) bit is corrupted during transmission of the eight bits from part (d) and is "flipped" (either from 0 to 1 or 1 to 0). Which of the following is true? Select the appropriate letter and write it here:

- (A) The receiver cannot detect the error.
- (B) The receiver can detect the error but cannot determine which bit is wrong.
- (C) The receiver can detect the error and can correct the bit that is wrong.

(f) [2 pts] In an HTML file for a webpage, the designer used the following font tag that changes the color of the font based on the 6 digit hexadecimal value 876543.

```
<font color="#876543">This is a colorful sentence.</font>
```

The 6 digit hexadecimal value specifies the amount of red, green and blue for the font's color, respectively (in the format RRGGBB). Express the amount of green in the font as an integer between 0 and 255, inclusive. Show your work.

(g) [2 pts] Calculate the sum of the following three **binary** numbers (which represents unsigned integers): 00111, 00101, 01100.

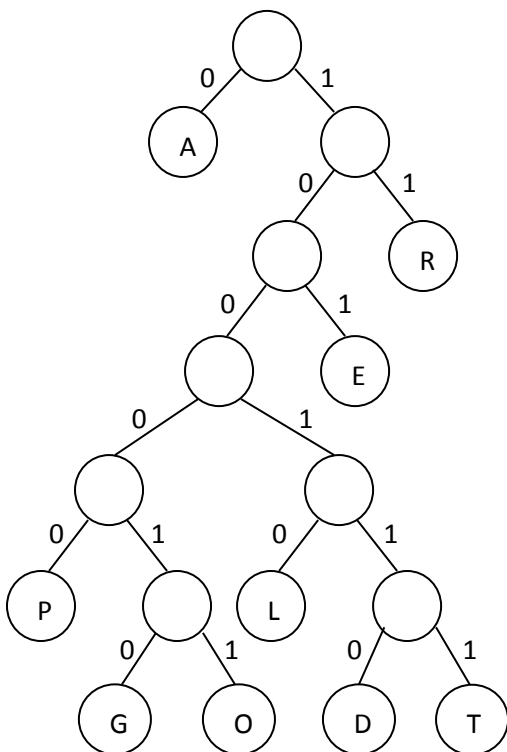
(h) [1 pts] Add the following two **hexadecimal** numbers together: 0x13, 0x98
(Note that "0x" is used before hexadecimal numbers as a convention.)

$$\begin{array}{r} 0x13 \\ + 0x98 \\ \hline 0x_ \end{array}$$

(g) [1 pts] Is MP3 (MPEG3) compression of sound files a *lossless* or *lossy* compression algorithm?

(h) [1 pts] Is JPEG encoding of graphics files a *lossless* or *lossy* compression algorithm?

(i) [3 pts] Based on the following Huffman tree:



What word is represented by the following binary string based on the Huffman tree:

100010111010100111

Suppose we want to encode words made using the nine letters from the tree above using a *fixed-width encoding* with the fewest bits possible for each letter.

How many bits are required to encode each letter?

How many bits are required to (re)encode the English word you decoded above, using a *fixed-width encoding*?

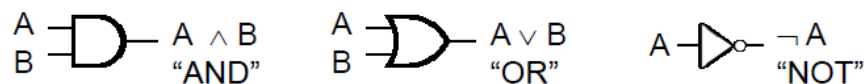
3. The following question involves Boolean logic and digital circuitry.

[20 pts]

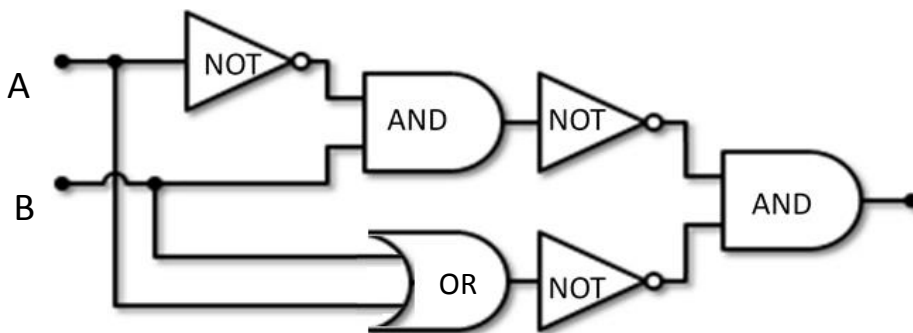
(a) [6 pts] Let $S = (\neg A \vee \neg B) \wedge (\neg A \vee B)$, where A and B are Boolean variables. Fill in the truth table below to compute S.

| A | B | $\neg A \vee \neg B$ | $\neg A \vee B$ | S |
|---|---|----------------------|-----------------|---|
| 0 | 0 | | | |
| 0 | 1 | | | |
| 1 | 0 | | | |
| 1 | 1 | | | |

(b) [4 pts] The Boolean value S from part 3a can be computed by an electronic circuit. Draw this circuit at the gate level of abstraction. Below is a reminder for the diagrams for AND, OR and NOT gates.



(c) [6 pts] Consider the following circuit. Write the Boolean expression that represents the circuit below.



(d) [4 pts] Using the De Morgan's Law, rewrite the equivalent expression $\text{not}(5 \geq y \text{ and } x \neq 4)$

4. This problem focuses on arrays, lists, stacks, and hash tables.

[15 pts]

(a) [2 pts] If you have an array of 7500 elements in computer memory, and you want to insert an element at the beginning of the array, how many elements of the array need to be moved?

(b) [2 pts] If you have a linked list of 7500 elements in computer memory, and you want to insert an element at the beginning of the linked list, how many nodes of the list need to be moved?

(c) [3 pts] A hash table is used to store keys so we can search for them later. Suppose that keys are integers. Consider the following code of a hash function:

| | | |
|--|---|---|
| <pre>def hash(key): return key * 7 % 5</pre> | | Consider the following set of keys: 3, 6, 11, 12. |
| What bucket does each key fall into? | Are there any collisions ? Explain briefly. | How many buckets in total are there in this hash function? |

(d) [5 pts] Show how to compute the following RPN expression using a stack, showing the contents of the stack each time something is pushed or popped. The first three stacks are shown for you. Use as many columns as you need.

8 1 9 + 3 - 13 9 - * 20 - %

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | | | | | | | | | | | |
| 8 | 1 | 9 | | | | | | | | | | | | | | | | | |
| 8 | 8 | 1 | | | | | | | | | | | | | | | | | |
| 8 | 8 | 8 | | | | | | | | | | | | | | | | | |

(e) [3 pts] Suppose that algorithms A and B are two algorithms that can be used to search for a given word in an unsorted collection of 15000 words. Algorithm A operates on a hash table with 500 buckets where the words have been evenly distributed by a hash function, and Algorithm B operates on an array of size 15000 where each array position holds a single word. Which algorithm would be faster in searching for a word that does not exist in the list? Why?

Algorithm _____ would be faster because _____

_____.

5. This question deals with binary trees and graphs.

[21 pts]

(a) [4 pts] Draw the binary search tree that results by inserting the following integers into the tree in the order shown. **Show the tree at each insertion step.** You will draw 8 non-empty trees. 5 1 9 2 6 8 0 7

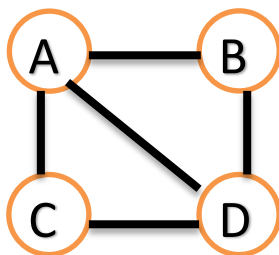
(b) [4 pts] Write a Python list that represents the binary tree from part 5a. Note that this can be done in one of several ways. You can use the representation from the assignments or something else. Write clearly how you intend us to interpret your representation. How do you represent the root, the left subtrees, and the right subtrees?

(c) [2 pts] How many comparisons are needed to find the item 8 in the binary search tree from part 5a, starting from the root and using the binary search algorithm? List all the key values that 8 is compared to until it is found.

(d) [3 pts] If you have a binary search tree with n nodes, what is the minimum numbers of levels in that tree?

(e) [2 pts] Re-order the keys from part 5a (5 1 9 2 6 8 0 7) so that the binary search tree will have **at most 1 child per node**.

(f) [4 pts] Write down the 4 by 4 adjacency matrix for the undirected graph below. Use the Boolean values True and False to indicate whether an edge exists between two nodes. For example, the matrix entry for row A and column C must be True.



(g) [2 pts] Suppose that the graph from part 5f represents a social network where the nodes stand for people and edges stand for friendships. According to this graph, who has the fewest friends (A, B, C, or D)?

6. This question deals with random number generators.

[9 pts]

(a) [5 pts] Recall that the Python `randint(0, n)` function returns a random integer between 0 and n , inclusive. Using the `randint` function with 0 as its first argument, show how to compute the following:

A random integer from the set $\{0, 1, 2, 3\}$

_____ without storing these values in a list first.

A random integer from the set $\{11, 12, 13\}$

_____ without storing these values in a list first.

A random integer from the set $\{-4, -2, 0, 2, 4\}$

_____ without storing these values in a list first.

Write a function to return a random integer which should be square of a number and less than 150 such as $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$.

(b) [4 pts] Consider the following Python functions:

```
def rng1(n):  
    return (n * 2) % 6
```

What is the PRNG period of `rng1`?

Write the sequence of numbers starting at 0 until a repeat.

```
def rng2(n):  
    return (3*n + 1) % 7
```

What is the PRNG period of `rng2`?

Write the sequence of numbers starting at 0 until a repeat.
