Appendix to “Enter at your own risk: HMO participation and enrollment in the Medicare risk market”

Profit-maximization problem

$$\underset{Qp,\ z}{Max}\ \pi = P_p Q_p + P_m Q_m - C_1 Q_m - C_2 Q_p - C_3 Q_m Q_p - C_4 Q_m{}^2 - C_5 Q_p{}^2 - C_6 z Q_m - C_7 z - C_8 z^2$$

subject to:

(1) $Q_m \geq 0$

(2) $Q_p - Q_m \geq 0$

(3) $Q_{mR} - Q_m \geq 0$

To solve this constrained optimization problem, we set up a Lagrangian expression:

$$L = P_p Q_p + P_m Q_m - C_1 Q_m - C_2 Q_p - C_3 Q_p Q_m - C_4 Q_m{}^2 - C_5 Q_p{}^2 - C_6 z Q_m - C_7 z - C_8 z^2 + \lambda Q_m + \rho(Q_p - Q_m) + \sigma(Q_{mR} - Q_m) = 0$$

Taking the first order conditions:

(1)

$$\frac{\partial L}{\partial Q_p} = P_p + Q_p\left[\frac{\partial P_p}{\partial Q_p} + \frac{\partial P_p}{\partial Q_m}\cdot\frac{\partial Q_m}{\partial Q_p}\right] + P_m\frac{\partial Q_m}{\partial Q_p} - C_1\frac{\partial Q_m}{\partial Q_p} - C_2 - C_3\left[Q_m + Q_p\frac{\partial Q_m}{\partial Q_p}\right] - 2C_4\frac{\partial Q_m}{\partial Q_p} - 2C_5 Q_p - C_6 z\frac{\partial Q_m}{\partial Q_p} + \lambda\frac{\partial Q_m}{\partial Q_p} + \rho\left(1 - \frac{\partial Q_m}{\partial Q_p}\right) + \sigma\left(\frac{\partial Q_{mR}}{\partial Q_p} - \frac{\partial Q_m}{\partial Q_p}\right) = 0$$

(2)

$$\frac{\partial L}{\partial z} = Q_p\left[\frac{\partial P_p}{\partial Q_m}\cdot\frac{\partial Q_m}{\partial z}\right] + P_m\frac{\partial Q_m}{\partial z} - C_1\frac{\partial Q_m}{\partial z} - C_3 Q_p\frac{\partial Q_m}{\partial z} - 2C_4 Q_m\frac{\partial Q_m}{\partial z} - C_6\left[z\frac{\partial Q_m}{\partial z} + Q_m\right] - C_7 - 2C_8 z + \lambda\frac{\partial Q_m}{\partial z} - \rho\frac{\partial Q_m}{\partial z} - \sigma\frac{\partial Q_m}{\partial z} = 0$$

(3) $\frac{\partial L}{\partial \lambda} = Q_m \geq 0$

(4) $\frac{\partial L}{\partial \rho} = Q_p - Q_m \geq 0$

(5) $\frac{\partial L}{\partial \sigma} = Q_{mR} - Q_m \geq 0$

(6) $\lambda Q_m = 0$

(7) $\rho(Q_p - Q_m) = 0$

(8) $\sigma(Q_R - Q_m) = 0$

$$\frac{\partial^2 L}{\partial Q_p{}^2} = 2\frac{\partial P_p}{\partial Q_p} + \frac{\partial P_p}{\partial Q_m} \cdot \frac{\partial Q_m}{\partial Q_p} - 2C_5 < 0$$

The second order conditions can be expressed as the following:

$$\frac{\partial^2 L}{\partial z^2} = -2C_4\left(\frac{\partial Q_m}{\partial z}\right)^2 - 2C_6\frac{\partial Q_m}{\partial z} - 2C_8 \quad < 0$$

$$\frac{\partial^2 L}{\partial Q_p \partial z} = \frac{\partial^2 L}{\partial z \partial Q_p} = \frac{\partial P_p}{\partial Q_m} \cdot \frac{\partial Q_m}{\partial z} - C_3\frac{\partial Q_m}{\partial z} - C_6\frac{\partial Q_m}{\partial Q_p} > 0$$

To ensure a maximum, the following criteria must hold:

$$\left(\frac{\partial^2 L}{\partial Q_p{}^2}\right)\left(\frac{\partial^2 L}{\partial z^2}\right) - \left(\frac{\partial^2 L}{\partial Q_p \partial z}\right)^2 > 0 \, and \left(\frac{\partial^2 L}{\partial Q_p{}^2}\right) < 0 \, .$$

Predicted Effects:

Firms are demand-constrained (Case III):

When firms are demand-constrained, an individual firm's Medicare quantity is determined by the constraint. Firms choose the level of quality (z), that corresponds to $Q_m(z, X_m, Q_p) = Q_{mR}(X_m, Q_p)$, such that z is a function of residual market demand, Medicare demand shifters, and private quantity. We proceed by substituting residual market demand for individual firm demand in the profit-maximization problem, taking the first order condition with respect to private quantity, and then determining the predicted effects of the Medicare price and demand shifters on a firm's private and Medicare quantities. Note, here we make the assumption that there are no diseconomies of scope in production.

$$\underset{Q_p}{Max}\,\pi = P_p Q_p + P_m Q_{mR} - C_1 Q_{mR} - C_2 Q_p - C_3 Q_{mR} Q_p - C_4 Q_{mR}^2 - C_5 Q_p^2 - C_6 z Q_{mR} - C_7 z - C_8 z^2$$

First-Order Condition:

$$\frac{\partial \pi}{\partial Q_p} = P_p + Q_p\left[\frac{\partial P_p}{\partial Q_p} + \frac{\partial P_p}{\partial Q_{mR}} \cdot \frac{\partial Q_{mR}}{\partial Q_p}\right] + P_m \frac{\partial Q_{mR}}{\partial Q_p} - C_1 \frac{\partial Q_{mR}}{\partial Q_p} - C_2 - C_3\left[Q_{mR} + Q_p \frac{\partial Q_{mR}}{\partial Q_p}\right]$$
$$- 2C_4 Q_{mR} \frac{\partial Q_{mR}}{\partial Q_p} - 2C_5 Q_p - C_6\left[z \frac{\partial Q_{mR}}{\partial Q_p} + Q_{mR} \frac{\partial z}{\partial Q_{mR}} \cdot \frac{\partial Q_{mR}}{\partial Q_p}\right] - C_7 \frac{\partial z}{\partial Q_{mR}} \cdot \frac{\partial Q_{mR}}{\partial Q_p} - 2C_8 z \frac{\partial z}{\partial Q_{mR}} \cdot \frac{\partial Q_{mR}}{\partial Q_p} = 0$$

Case III (assuming no complementarities):

$$\frac{\partial Q_p}{\partial P_m} = 0 \quad \frac{\partial Q_m}{\partial P_m} = 0$$
$$\frac{\partial Q_p}{\partial X_p} > 0 \quad \frac{\partial Q_m}{\partial X_p} = 0$$
$$\frac{\partial Q_p}{\partial X_m} = 0 \quad \frac{\partial Q_m}{\partial X_m} < 0$$

Firms are unconstrained (Case IV):

Using first-order conditions (1) and (2) from Appendix 1, we form the total differentials and derive the predicted effects below.

Recall:

$$P_P = \frac{-1}{\theta_1} Q_p + \frac{\theta_0}{\theta_1} + \frac{\theta_2}{\theta_1} X_p + \frac{\theta_3}{\theta_1} Q_m$$
$$Q_m = m_1 z + m_2 X_m + m_3 Q_p$$

(1)

$$d\left[\frac{\partial \pi}{\partial Q_p}\right] = \left(\frac{-2}{\theta_1} + \frac{\theta_2}{\theta_1} m_3 - C_3 m_3 - 2C_5\right) dQ_p + \left(\frac{\theta_3}{\theta_1} m_1 - C_3 m_1 - C_6 m_3\right) dz + m_3 dP_m +$$
$$\left(\frac{\theta_3}{\theta_1} m_2 - C_3 m_2\right) dX_m + \left(\frac{\theta_2}{\theta_1}\right) dX_p + (1 - m_3) d\rho - m_3 dC_1 - 1dC_2 - (Q_m + Q_p m_3) dC_3$$
$$-2m_3 dC_4 - 2Q_p dC_5 - z m_3 dC_6 + 0dC_7 + 0dC_8 + \left(\frac{\partial Q_{mR}}{\partial Q_p} - m_3\right) d\sigma + m_3 d\lambda = 0$$

(2)

$$d\left[\frac{\partial \pi}{\partial z}\right] = \left(\frac{\theta_3}{\theta_1} m_1 - C_3 m_1 - C_6 m_3\right) dQ_p - \left(2(C_4 m_1^2 + C_6 m_1 + C_8)\right) dz + m_1 dP_m + 0dX_p$$
$$\left(-2C_4 m_1 m_2 - C_6 m_2\right) dX_m - m_1 dC_1 - 0dC_2 - m_1 Q_p dC_3 - 2m_1 Q_m dC_4 + 0dC_5$$
$$-\left(m_1 z - Q_m\right) dC_6 - 1dC_7 - 2z dC_8 - m_1 d\rho - m_1 d\sigma = 0$$

Case IV (assuming no demand or cost complementarities):

$$\frac{dQ_p}{dP_m} = 0 \qquad \frac{dQ_m}{dP_m} > 0$$
$$\frac{dQ_p}{dX_m} = 0 \qquad \frac{dQ_m}{dX_m} = ?$$
$$\frac{dQ_p}{dX_p} > 0 \qquad \frac{dQ_m}{dX_p} = 0$$

Econometric Estimation

When one cannot assume that the error terms of observations are independent and identically distributed, then specifying the appropriate probability density function to do maximum likelihood estimation may be very complex. One alternative approach is to specify a quasi-likelihood function, which requires the following two things to occur. First, one needs to be able to specify the relationship between the mean and the variance of the dependent variable. And second, the unknown distribution of the dependent variable must be of the linear exponential family, which includes such distributions as binomial, normal, and Poisson (Gourieroux, Monfort, and Trognon, 1984; McCullagh and Nelder, 1983). Once the quasi-likelihood is specified, then the parameter estimates can be found by solving the corresponding quasi-score functions simultaneously.

For estimation of the participation regression, we use generalized estimating equations (GEE), which is the multivariate analogue of quasi-likelihood estimation. We specify the distribution of the dependent variable as binomial. Furthermore, to address the issue of unobserved firm-specific effects, we specify an exchangeable correlation structure (corresponding to the presence of random effects), and this "working correlation matrix" is also incorporated into the maximization problem. Using this method, we are able to obtain consistent parameter estimates. See Liang and Zeger (1986) and Zeger and Liang (1992) for additional discussion of GEE. For the enrollment equations, parameter estimates are calculated using traditional instrumental variables estimation.

The standard errors for both the participation and enrollment regressions are computed using the method described below, which permits within HMO-cluster correlation and allows for the possibility that residuals across HMO clusters are not identically distributed. Here, a cluster includes all observations over the time period of our sample that correspond to a single HMO.

The Huber/White/sandwich estimator of variance is defined as the following:

$$\upsilon = q_c \hat{V} \left(\sum_{k=1}^{M} u_k^{(G)}{}' u_k^{(G)} \right) \hat{V}$$

$$u_k^{(G)} = \sum_{j \in G_k} u_j$$

where:

G_k denotes each cluster, $k = 1,...M$

$\hat{V} = (X'X)^{-1}$

$u_j = (y_j - x_j b)\hat{x}_j$ and is a row vector of scores, where j denotes individual observations in cluster G_k

$q_c = \frac{N-1}{N-k} \cdot \frac{M}{M-1}$, N = number of observations, M = number of clusters, k = number of regressors

Computation of the term in parentheses can be summarized in the following manner. First, the score (u_j) is calculated for each observation within a cluster. Scores are then summed up over the j observations in a cluster, producing a row vector. The outer product is then calculated, which results in a matrix of dimension equal to the number of regressors. One matrix is constructed for each cluster and then these matrices are summed over the clusters in the sample, giving rise to the matrix in parentheses.

By using this method, we have addressed the problem of obtaining consistent standard error estimates given the presence of within-cluster correlation, since we rely only on "between" cluster variation in the computation of this estimator of variance. Additional discussion may be found in the Stata Reference Manual, 1997.