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Counteroffers and Efficiency in Labor Markets With Asymmetric Information

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Abstract

This paper considers the effect of offer matching on labor market outcomes when the current employer has better information about his worker's productivity than potential employers. Previous research found that when current employers have better information than potential employers, the later use job assignment to infer an employed worker's qualifications. As a result, assignment of workers to jobs is inefficient. I find that when current employers can match outside offers the equilibrium outcome may be efficient despite the asymmetric information. I then analyze the effect of the asymmetric information on investment in human capital made by employers and workers, and find these investment levels to be first best.

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1 Introduction

Labor market competition over employed workers affects firms' practices and compensation policies. It is a common practice in skilled-worker labor markets for competing firms to make offers to employed workers, and for the current employers to make counteroffers. One characteristic of competition for employed workers is that the current employer often has better information regarding the workers' quality than do potential employers. Thus, when outside firms make offers to employed workers they use publicly observable characteristics to infer the worker's skills. Michael Waldman [1984] was the first to observe that outside firms use job assignment to infer employed workers' qualifications, and that employers take this into consideration when they make promotional decisions.

The goal of this paper is to analyze the effect of counteroffers on compensation policies when employers have better information about their workers' productivity than outside firms do. The ability to match outside offers is potentially important as employers have discretion regarding the quality of workers in their firms. By matching offers potential employers make, the current employer decides which workers stay in the firm based on his knowledge of a worker's productivity. In addition, I analyze the effect of the asymmetric information on investment in human capital. I find that despite the asymmetric information, the equilibrium can be efficient.

Previous literature that analyzes the effect of asymmetric information between current and potential employers on firms' practices found that this asymmetric information causes firms to dissipate resources in order to prevent loss of informational rents. In particular, papers that analyze the effect on promotion policies found that insufficient workers are promoted. When outside firms cannot directly observe employed workers' abilities and skills, they use signals such as education, training, past experience, and job assignment to infer an employed worker's qualifications. Waldman [1984] showed

that when employers promote workers to a higher-level job, outside firms infer that the worker is of high quality. In Waldman's model the market is competitive, firms are homogenous ex-ante, and workers are heterogenous with respect to their productivity. Firms try to raid workers assigned to high-level jobs, and make a wage offer equal to the expected marginal revenue product (MRP), conditional on the job assignment. Hence, employers promote workers to high-level jobs only if the increase in productivity exceeds the increase in the wage required to retain a worker in that job. As a result, some workers who would be more productive in high-level jobs than in low-level jobs are not promoted.²

I use a similar framework, but formally analyze competition for employed workers. When employers can match offers, the winner's curse phenomenon arises; that is, outside firms can only raid workers with a wage that exceeds the worker's value. As a result, outside firms will offer a wage equal to the lowest MRP, based on the worker's job assignment and other publicly observed characteristics (e.g., education, past experience). Although the expected marginal product of a worker who's been promoted increases, the lower bound on the worker's productivity doesn't increase; as a result, the increase in wages required to retain the worker is smaller than the increase in productivity efficient matching of workers to jobs causes. Hence, the job assignment that maximizes the output produced also maximizes the employer's profit.

I then extend the model and analyze investment in general and firm-specific human capital. Investment in human capital in the context of asymmetric information is similar to job assignment in the sense that if competing firms observe training (general and firm-specific), such training may reveal information regarding the worker's productivity. Further, if training is observable and general, it will also increase the value of the worker to the

²Bernhardt [1995] builds on Waldman and analyzes a model which includes timing of promotions.

competing firms.³ Similar to promotions, investment in skills that workers and employers make in skills when they have private information conveys information to the market. In my model, unlike promotions, investment may also affect earnings. I first analyze investment in firm-specific skills. I find investment is efficient; in contrast to predictions of the standard theory (symmetric information models) in which workers do not invest in firm-specific skills, workers share investment costs with the employer. Workers invest in firm-specific skills only when it is productive to train a group of high-ability workers (and technology is common knowledge). The logic behind this result is as follows. Outside firms observe that the worker was trained, and infer that the lowest possible MRP of the worker is higher than the lowest MRP of an untrained worker. Therefore, the worker's wage increases if he is trained, although training did not make him more productive in outside firms. In equilibrium, the worker's share in investment cost is equal to the increase in wage, and the employer earns the rents. Since the worker's share in investment costs covers the future increase in wage due to training, the employer trains workers if the increase in output is expected to exceed the cost of training; hence, the equilibrium is efficient.⁴

The analysis of investment in general skills is similar. The difference is that a worker is more productive in outside firms if trained. In equilibrium,

³Acemoglu and Pischke [1998,1999a] examine the effect of the asymmetric information on employers' incentives to invest in workers' general skills on job assignment. In their model, however, training decisions are made before the worker and employer have private information regarding the worker's productivity. The focus of this paper is different in that it explores investment in general and firm-specific skills when employers have private information. Since such decisions are made by an informed employer, these decisions may convey information to competing firms regarding the worker's productivity. Therefore, my analysis applies to firm-specific skills as well as general skills.

⁴The efficiency of investment in human capital is not a result of offer matching. Offer matching lowers outside offers. Outside firms offer the lower bound on the worker's productivity conditional on the information available instead of offering the conditional expected productivity. Training a worker increases the lower bound on his productivity, and therefore offer matching only affects the worker's share in training cost.

the worker's share in investment costs is the increase in wage due to training (which is larger than in the firm-specific skills case). In contrast to the prediction of the standard theory (e.g., Becker [1975]), employers invest in general skills because they capture the rents.⁵ As in the case of investment in firm-specific skills, investment is efficient. The worker's share covers the increase in wages, and the employer invests if the increase in output is expected to exceed training costs.⁶

I then examine workers' investment in human capital prior to entering the labor market. I find that the output level in the economy is the first best level (due to efficient assignment of workers to jobs), and hence, the investment level is also a first best. Whereas wages in each period are not the expected MRP of the worker, the present value of career wages is equal to the expected output produced. This result arises because the market is competitive. The first period wages, which are determined when the information is symmetric, satisfy the condition that the expected profit from hiring a worker is zero; hence, the entry-level wages include the expected profit in the second period, and the expected sum of wages over time is equal to the expected output. Since investment made before entering the labor market maximizes lifetime earnings, it also maximizes the expected output.⁷ The investment level, however, is first best, and is similar to the investment level in symmetric learning case (in which information regarding the worker's productivity is

⁵Acemuglu and Pishcke[1998,1999b] first made the point that employers invest in general skills when there is asymmetric information. In their model employers pay for training, and it is inefficient.

⁶Bernhardt and Scoones [1998] consider investment made before the worker and employer privately learn about the worker's productivity, and hence did not consider the informational content of investment in skills of experienced workers. Chang and Wang[1996] investigate how human capital investment, turnover, and wages are determined under this asymmetric information. In their model, however, outside firms cannot observe whether investment is made.

⁷Bernhardt and Scoones [1998] analyze investment in skills when the current employer has better information about his workers' skills than outside firms. In their model, however, there is no zero profit condition and production is inefficient.

symmetric) because job assignment and output level are first best.

Several other papers are related. Milgrom and Oster [1987] analyze a model of promotion under asymmetric information. They found that minorities who are less visible to the market are less likely to be promoted. Promotions are inefficient in their model because they assume that if a worker is promoted his actual productivity is revealed to the market, and competition for workers in high-level jobs occurs when the information is symmetric. Lazear (1986) and Greenwald (1986) analyze models of asymmetric information. They focus on the effect of the asymmetric information on mobility and do not analyze job assignment or investment in human capital. Lazear (1986) analyzes the effect of stigma of not receiving outside offers. In his model, outside firms receive private signals regarding employed workers' productivity. Information regarding a worker's productivity is revealed in a diffused fashion to the market.⁸ Laing [1994] develops a model of involuntary layoffs and analyzes the properties of the optimal contract. He considers the informational content of layoffs when the employer has better information about the worker's productivity than outside firms. Gibbons and Katz [1991] test empirical implications of asymmetric information, considering the effect of layoffs on future wages.

The paper is organized as follows. In section 2, I describe the model. Section 3 contains equilibrium analysis, and section 4 examines investment in human capital. In section 5 I discuss the robustness of the equilibrium outcome, and section 6 concludes.

⁸See also Waldman [1990] for analysis of optimal contracts in an environment in which information is revealed in a diffused fashion to the market.

2 The model

The economy: In the two-period economy, there is one good and its price is normalized to one. There is no discounting between periods. The identical risk-neutral firms compete over risk-neutral workers when workers are inexperienced, and again when outside firms try to raid employed workers. For notational simplicity, I assume there is one worker, and three firms: the employing firm, which is denoted by e , two outside firms denoted by r . The output of each firm is normalized to equal the worker's labor input. There are two types of jobs in each firm: a high-level and a low-level one, each requiring different skills. Employment contracts pair a wage and a job assignment, and can only be made for one period at a time (spot contracts). Output in the high-level job is unobserved and unverifiable by a third party, and contracts cannot be contingent on output. If a worker quits or is fired, he can not work for that firm again.⁹ The worker's productivity in the low level job, z , is common knowledge. The worker produces θ if assigned to the high-level-job. θ is drawn from a continuous distribution $f(\theta)$ with a lower support below z . This assumption can be made weaker, but the implication of this assumption that the lower support of productivity in the high-level job is weakly lower than the lower support of productivity in the low-level job is important for the efficiency result.¹⁰ I assume the expected productivity $E(\theta)$ is sufficiently lower than z , so talent in high-skill jobs is scarce. Workers display no disutility from effort and have a reservation utility of zero; the worker's utility function is the sum of the first period wage and second period wage.

The assumption that there is no discounting between periods is made to reduce notation. Introducing a discount factor will not change the analysis.

⁹This assumption is only relevant if there are more than two periods. I discuss the multiple period case in Section 4.1.

¹⁰See discussion in section 5.

The assumption that output in low-level jobs is observed publicly, is made for expositional simplicity. The second assumption, that output in high-level job is only observed by the employer, is made to generate asymmetry in information between the employer and outside firms after the first period. The assumption that the unconditional mean productivity is lower than productivity in the low-level job is made for simplicity. As a consequence of this assumption, if a firm has no information about the worker's productivity in the high-level job, it is optimal to assign the worker to the low-level job.

2.1 Timing and information structure of the game

The timing of the game is described in figure 1. At the beginning of the first period, the information is symmetric, all firms and workers know z and $f(\theta)$, and no one knows θ . Outside firms offer the worker a contract, which is a wage and a job assignment pair. The worker decides which offer to accept. Production then takes place. Payments are made at the end of the first period.

At the beginning of the second period, the employer and worker know θ . All firms in the market observe job assignments. If the worker is assigned to the high-level job, outside firms make simultaneous offers, and the employer makes a counteroffer.¹¹ The worker chooses a firm, and production takes place. At the end of the second period payments are made and the worker retires.¹²

¹¹The analysis will not change if the employer offers a job assignment and a wage at the end of the first period and outside firm observe the wage offer and job assignment. The reason is that the employer can match offers after outside firms make offers and the therefore the first offer will not be informative.

¹²In section 5, I generalize the finite bidding process in the basic model to an infinite process, and consider a general version of the two-period model with N periods.

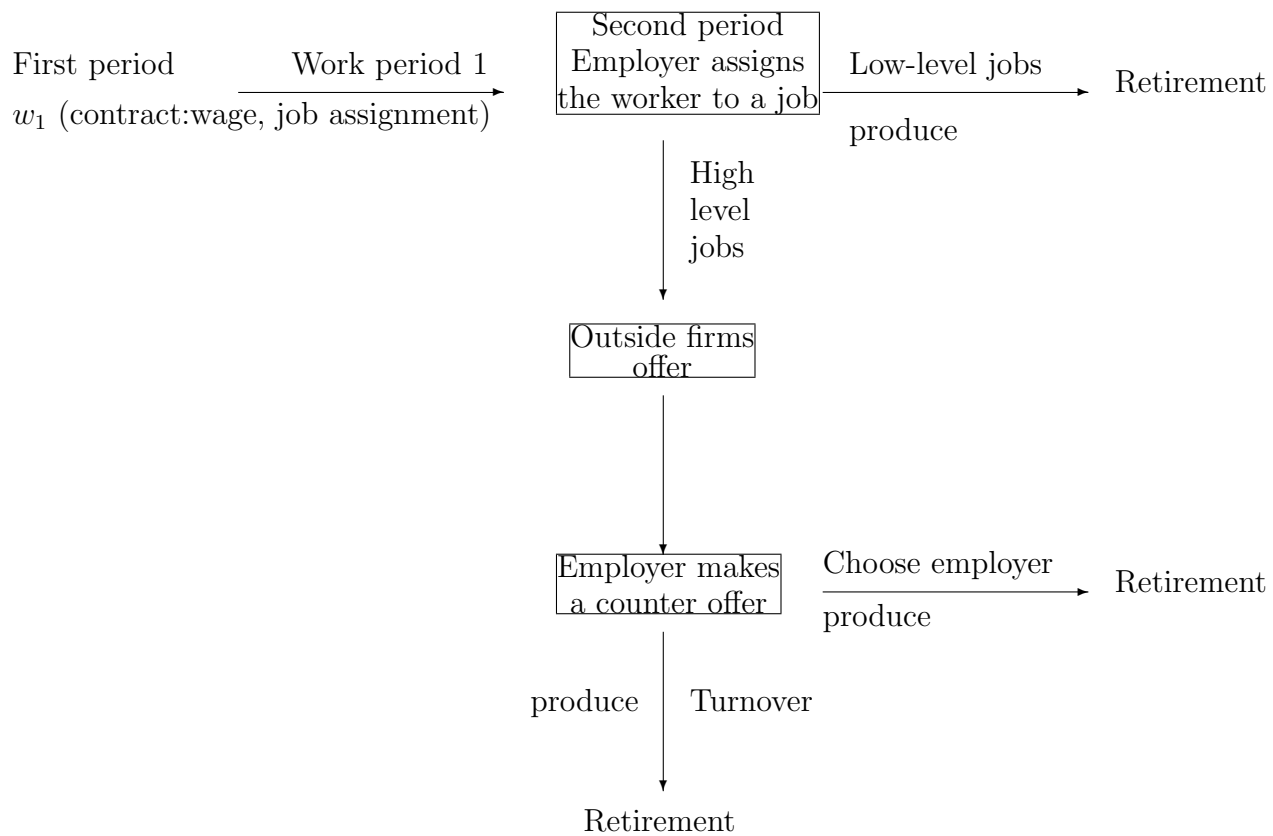


Figure 1: The Two Period Game.

2.2 Strategies

This section describes the strategies of each player.

First period: At the beginning of the first period, the information is symmetric and imperfect: neither party (firms and worker) observes the productivity of the worker in the high-level job. Each firm's strategy is to offer a wage and a job assignment. The worker's strategy is a choice of a firm. I denote strategies by uppercase letters (job assignment and wages) and wage and job assignment realizations by lowercase letters.

Second period: At the beginning of the second period, the employer assigns the worker to a job. The employer knows the worker's ability to perform in both jobs. I denote the employer's job assignment as $J_2^e(\theta) \in \{H, L\}$. J_t^n is a job assignment offered by firm n at time t , H is a high-level job assignment, and L is a low-level job assignment.

After outside firms observe the worker's job assignment, they form beliefs regarding the productivity of the worker based on his job assignment and productivity in the low-level job. Each offer is a job assignment and a wage pair: $[W_2^r(j_2^e), J_2^r(j_2^e)]$, where $r = 1, 2$.

After observing the outside firms' offers the employer makes a counteroffer: $W_2^e(\theta, w_2^1, w_2^2)$.

The worker then observes outside firms' wage offers, and the employer and chooses a firm to work for.

3 Equilibrium Analysis

There are multiple equilibria in the model, which is a typical characteristic of signaling models. In this section, I analyze the unique equilibrium which is robust. I discuss other equilibria and robustness in section 5.

The solution concept I use is perfect Bayesian equilibrium (PBE). An equilibrium consists of ex-ante uniform first period contract offers that clear

the market, a second period job assignment (made by the first period employer), outside firms' wage offers (if the worker is assigned to high-level jobs), the first period employer's wage offer, a worker's choice of a job offer, and outside firms' beliefs about the worker's ability. I start by describing the equilibrium strategies.

Second period Equilibrium strategies

Each worker's equilibrium strategy is to accept the highest offer and accept the current employer's offer if there is a tie. If more than one outside firm makes the highest offer, then the worker chooses an offer randomly.

The employer's strategy is a second period job assignment and a counteroffer. The counteroffer is given by

$$W_2^e = \begin{cases} w_2^M & \text{if } \theta \geq w_2^M, \\ x, \text{ where } 0 \leq x < w_2^M & \text{otherwise} \end{cases} \quad (1)$$

where w_2^M is the highest offer made by an outside firm. I refer to w_2^M as the "market offer."

The employer's second period job assignment is given by

$$J_2^e = \begin{cases} H & \text{if } \theta \geq z, \\ L & \text{otherwise.} \end{cases} \quad (2)$$

The employer assigns the worker to the job in which the worker is the most productive.

Outside firms form beliefs regarding the worker's productivity based on the worker's job assignment. Outside firms offer a worker who is assigned to a high-level job a contract of a low-level job and a wage equal to the MRP in the low-level job:

$$\{W_2^r(j_2^e = H) = z, J_2^r(j_2^e = H) = L\}. \quad (3)$$

The beliefs are the expected MRP in the high-level job conditional on the job assignment and the productivity in that job,

$$E(\theta|H). \quad (4)$$

Outside firms offer the same contract to workers assigned by the employer to low-level jobs, and the beliefs are

$$E(\theta|L). \tag{5}$$

3.1 Equilibrium outcome

Proposition 1: in equilibrium at the second period,

- (a) *The worker stays with the first period employer.*
- (b) *If the worker is assigned to a high-level job in the second period, then outside firms' wage offers are the worker's MRP in the low-level job assignment, $w_2^M = z$.*
- (c) *The second period wage is the MRP in the low-level job, $w_2 = z$, regardless of the second period job assignment.*
- (d) *The worker is assigned efficiently to a job in which he is the most productive:*

$$J_2^e = \begin{cases} H & \text{if } \theta \geq z, \\ L & \text{otherwise} \end{cases}$$

- (e) *The employer's second period informational rent is $\text{MAX}\{0, \theta - z\}$. If the worker is assigned to the low-level job, the employer's rent is zero. If the worker is assigned to the high-level job, the employer's informational rent is $\theta - z$, which is positive.*

Proof: See appendix A. Below is an intuitive discussion.

The current employer's offer (equation 1) maximizes his profit. Since the worker chooses the highest offer and stays with the current employer if there is a tie, the lowest wage that enables the employer to retain the worker is the outside firm's offer. If the market wage exceeds the worker's MRP, however, offering a lower wage than the market wage yields a higher payoff than retaining the worker and paying a wage that exceeds the worker's MRP. This strategy implies that outside firms will only be able to raid an employed worker with a wage that exceeds his MRP.

The offer in equation (3) maximizes the outside firm's profit. To see that, notice that outside firms can only raid an employed worker with a wage higher than the worker's MRP. Therefore, any offer that yields zero expected profit is optimal (because of competition, outside firms offers will not be below z).

Outside firms form beliefs of the worker's MRP in the high-level job. The information available to these firms is the job assignment and the productivity in the low-level job. Interestingly, the expected productivity of a worker does not affect the wage offers outside firms make. This is because it is optimal for an outside firm to offer a wage that is the *lowest* MRP of the worker, provided the worker is assigned to a high-level job in the second period. The information a promotion reveals to outside firms is that the worker is more productive in the high-level job than in the low-level job, that is, $\theta \geq z$. The beliefs should satisfy bayes' law. The beliefs in equation 4 are consistent with the employer's strategy and satisfy bayes' law.

The employer assigns the worker to the job he is most productive. It is key to the efficient job assignment of workers to jobs that the employer can match outside offers. If an outside firm makes the last offer, the wage is the expected MRP conditional on job assignment (and other observable characteristics). Retaining a promoted worker, therefore, requires wage increases. In Waldman [1984] job assignment is inefficient because the increase in wages exceeds the increase in MRP of some workers who are more productive in the high-level job.¹³ Thus, if the employer makes the final the offer, outside firms suffer from the winner's curse. That is, a raided worker's MRP is always lower than the wage required to raid him. As a result, promotion does not increase the market value of an employed worker.¹⁴ Since assign-

¹³In Waldman [1984] workers accumulate firm-specific skills in each period, regardless of their job assignment. Therefore, employers earn rents even if workers are assigned to a low-level job. Such an extension to this model will not change the analysis.

¹⁴As long as there is an arbitrarily small positive probability that the least productive worker in the low level job is more productive in the high level job, the lower bound on the worker's productivity doesn't change when a worker is promoted.

ing workers to jobs according to their comparative advantage is not costly to the employer, and since the employer gains any surplus produced above the market value, profit maximization coincides with output maximization, which implies that workers are assigned to jobs efficiently.

To summarize, the equilibrium is constrained Pareto efficient. I do not compare the equilibrium obtained to the full information case; instead, the benchmark is an allocation that maximizes total welfare, given the information available to all players. In the economy the total surplus is the total output produced.

3.2 First Period Equilibrium Wages

At the beginning of the first period the information is symmetric. Each firm offers a contract of a job assignment and a wage, and the worker chooses a firm.

Proposition 2: in equilibrium, in the first period:

(a) *An ex-ante uniform contract of a low-level job assignment and a wage is offered:*

$$W_1 = z + E_\theta[\text{MAX}\{\theta - z, 0\}] \tag{6}$$

(b) *The market clearing condition is satisfied and each firm earns zero expected profit (over the two periods).*

Proof:

Clearly, offering the worker a lower wage than the wage described in equation (6) is not a profitable deviation, since a firm will not be able to hire a worker with a wage lower than the competitive wage. Offering a higher wage than the competitive wage is not a profitable deviation, since the expected profit the competitive wage yields is zero (over the two periods). It suffices to show two things. First, the expected profit the first period contract combined with the expected profit the second period contract yield zero. Second, in each period the wage is such that the worker's utility exceeds the reservation

utility.

The expected profit a firm earns if it hires a worker (in both periods) is

$$E\Pi^e = (z - w_1) + \int \text{MAX}\{0, \theta - z\}f(\theta)d\theta \quad (7)$$

The first term, $z - w_1$, is the profit in the first period. The second term is the expected profit in the second period. In the second period the worker is assigned to the job in which he is more productive, and hence, the employer's expected profit in the second period is $\int \text{MAX}\{z, \theta\}f(\theta)d\theta - z$. By construction, the first period contract in equation (6) yields zero expected profit. The wages in the first and second period are positive and hence exceed the reservation utility. Therefore, the first period contract satisfies the market clearing condition.

Q.E.D

In the model, workers experience wage decline after the first period. This is due to competition, and is a typical feature of asymmetric learning models. Since employers earn informational rents in the second period, the first period wage includes the future expected information rent, and employers earn zero expected profits.¹⁵ This model, however, is simple and does not include on-the-job training (in which workers accumulate general human capital). Extending the model to include on-the-job training will tend to make age-earning profiles upwards sloping.¹⁶

The first period can be interpreted as a “test” period in which the employee is trained, and in which the employer gains information about whether

¹⁵This is not necessarily true if employers have some monopsony power. For example, in a search model framework, workers may receive one offer each period. In this case the entry level wage is the reservation wage. In the next period if the worker receives an outside offer the analysis remains the same but the age-earnings profiles can be upwards sloping.

¹⁶See Waldman [1984], page 216. Furthermore, the key is that some aspects of productivity are privately observed by the current employer. If some aspects of productivity are publicly observed, then the model is consistent with increasing experience-wage profile as predicted by the standard model.

the inexperienced employee is qualified for the high-level job. In this period the profit, $z - w_1$, is negative and offsets the positive expected informational rent of the second period. Since a firm earns zero expected profit over the two periods, workers receive the entire surplus. Suppose that workers have different initial observable skills z_i which are uncorrelated with productivity in the high-level job.¹⁷ Denote the portion the employer's profit due to the expected informational rent (note that the employer's profit is positive only if the worker is promoted) by $\Delta(z_i)$

$$\Delta(z_i) = \int_{z_i} f(\theta)\theta d\theta - z_i \int_{z_i} f(\theta)d\theta$$

The derivative of $\Delta(z_i)$ is negative.

The informational rent declines because the probability of promotion declines and because the wage in the high-level job increases.

The fact that the wage in the high-level job is increasing in z although the MRP in the high-level job is uncorrelated with z can be a rationale for why firms prefer not to hire "overqualified" workers: "overqualified" worker does not necessarily mean the worker is expected to be more productive, but that he has other skills, that may be irrelevant to the productivity in this job.¹⁸ The reason the wage is larger although the worker is expected to be as productive as other less qualified (in different jobs) workers is that once hired, outside firms expect the worker to be at least as productive in the current job as he is in other jobs. For example, consider two workers who are expected to be equally productive in the job but have other skills that may be uncorrelated with the skills required for the specific jobs. The wage

¹⁷It is straightforward to see that the equilibrium analysis remains the same. That is, workers are promoted efficiently, the second period wage is z_i , and the first period wage is given by equation 6.

¹⁸In this model employers are ex-ante indifferent between workers as they earn zero expected profit. This interpretation is meaningful, however, when market frictions are present, or in situations in which employers seek to fill high-level positions, and use the initial assignment as a "test period", in which they learn about workers qualifications and screen them.

of the overqualified worker will be larger if he is promoted after the first period. Further, since both workers have skills required in other jobs, it is less likely that these workers will be more productive in the job the employer seeks to fill. Put differently, workers who have more qualifications, even if they are irrelevant to the job, may have higher outside options than workers with fewer skills. Therefore, they are paid higher wages even if they are less productive. As a result, if such a worker is expected to be as productive as workers with fewer qualifications, employers earn lower rents.

4 Investment in human capital

Investment in skills employers and workers make when they have private information regarding the worker's skills conveys information to the market. In the previous section I showed that although job assignments convey information regarding a worker productivity and skills, job assignment does not affect wages, and employers promote workers efficiently. Investment in skills, however, can potentially affect earnings. Whereas job assignment did not affect wages, training a group of workers can potentially increase earnings. The difference is that promotion of workers did not change the lower bounds on a worker's productivity since it is profitable to promote a worker if he is more productive in the high-level job. If it is only productive to train some workers (for example, the increase in productivity due to training exceeds the costs of training of only high ability workers), then potentially the lower bound on the productivity of a worker who was trained exceeds the lower bound on the ability of a worker who was not.

I will show next that although trained workers will experience increase in earnings, workers are trained efficiently. The cost of investment are shared by the employer and the worker. I provide analysis for the case in which skills are general and for the case in which skills are firm-specific.

4.1 Firm-Specific Skills

I extend the basic model in the following way. Assume that after learning the worker's productivity in the high-level job, the employer decides whether to train the worker, and proposes a fee (p_w). The worker has to agree, otherwise he is not trained. Training takes place before the beginning of the second period, and outside firms can observe whether the worker was trained. As before, the information available to outside firms is the job assignment. For simplicity, assume training is only productive in the high-level job. If a worker is trained, his productivity increases by $\Delta(\theta)$, and $\Delta(\theta)$ is increasing in θ . The cost of investment $C(\theta)$ depends on the worker's productivity. $\Delta(\theta) - C(\theta)$ is increasing in θ and there exists $\theta_* > 0$ such that $\Delta(\theta_*) - C(\theta_*) = 0$.¹⁹ Outside firms observe whether a worker is trained, but cannot observe the increase in productivity or the cost (as both the costs and the increase in productivity are a function of θ , which is only observed by the worker and the employer). Notice that the wage the employer pays during training has to be such that the worker agrees to participate in training. I interpret the wage reduction a worker is willing to take as the worker's share in the investment cost and denote it by p_w . Assume that outside firms observe p_w .

The efficient investment rule is to invest if the increase in productivity exceeds investment costs, $\Delta(\theta) - C(\theta) > 0$.

Proposition 5:

If the increase in productivity is due to firm-specific skills then:

1. *The wage of a worker in a high-level job that was trained is $\max\{z, \theta_*\}$.*
2. *the employer trains a workers if*

$$\Delta(\theta) \geq C(\theta) \tag{8}$$

3. *Investment cost is shared: the employer's share is $C(\theta) - \max\{\theta_* - z, 0\}$ and the worker's share is $\max\{\theta_* - z, 0\}$.*

¹⁹Otherwise, the productivity of the least productive worker is not greater than the productivity of a worker who is not trained, and investment is clearly efficient.

Proof: The employer's strategy to train workers efficiently is optimal. The profit if the worker is not trained is, $\theta - z$. The profit if the worker is trained is the output produced by a trained worker net of investment cost paid by the employer and the wage:

$\theta + \Delta(\theta) - C(\theta) + \max\{\theta_* - z, 0\} - \max\{\theta_*, z\}$. This equals $\theta + \Delta(\theta) - C(\theta) - z$. If the worker is not trained, the profit is $\theta - z$. Clearly, it is profitable to train workers efficiently ($\theta \geq C(\theta)$). The wage during training (or worker's share of investment costs) maximizes the employer's payoff, as it is the lowest wage the worker will agree to receive and still participate in training.

Workers are indifferent about being trained or not being trained. If trained, a worker's earnings net of investment cost is $\max\{\theta_*, z\} - \max\{\theta_* - z, 0\} = z$. If not trained, the worker earns z as well.

Outside firms' strategy is to offer the MRP of the least productive worker in each group: $\max\{\theta_*, z\}$ if a worker is trained and z .

Q.E.D

The current employer makes the last offer. The results do not change, however, if outside firms observe the employer's offer before making their own offers (any process besides the process in which the employer can not match outside offers will yield the same result). The reason again is that wages will not reflect information beyond that which training provides.²⁰

Wages in the model are the lower bound on the worker's MRP. When the entire increase in productivity is firm specific, the increase in wage is only due to the fact that training signals the worker's ability is above the minimal ability that makes training productive.

In contrast to symmetric information models, investment in firm-specific skills increases earnings. Therefore, workers invest in firm-specific skills. Workers' share in the investment cost is the increase in wage due to training.

²⁰In this model workers are price takers and therefore there is no equilibrium with wage signaling. See Golan [2002] for analysis of bargaining model with asymmetric information in which wage signaling is obtained as an equilibrium outcome.

As a result, the employer captures all the rents and faces only the true cost of investment. Investment therefore maximizes the net benefit of training (increase in output net of cost of training).

4.2 Investment in general skills

The model is similar to the previous model. The only difference is that $\Delta(\theta)$ is increase in general skills. In contrast to symmetric information models, employers earn rents even if investment develops general skills. As a result, employers invest in workers' general human capital.

Proposition 6: If the increase in productivity is due to general skills:

1. The wage of a worker in a high-level job that was trained is $\max\{z, \theta_*\} + \Delta(\max\{z, \theta_*\})$.
2. the employer trains a worker if $\Delta(\theta) \geq C(\theta)$.
3. Investment cost is shared: the employer's share is $C(\theta) - \max\{\theta_* - z, 0\} - \Delta(\max\{z, \theta_*\})$ and the worker's share is $\max\{\theta_* - z, 0\} + \Delta(\max\{z, \theta_*\})$.

Proof: The employer's strategy to train workers efficiently is optimal. Profit from an untrained worker is $\theta - z$. The employer's profit if the worker is trained is: $\theta + \Delta(\theta) - C(\theta) + \max\{\theta_* - z, 0\} + \Delta(\max\{z, \theta_*\}) - \max\{z, \theta_*\} + \Delta(\max\{z, \theta_*\})$. The profit if the worker is not trained is $\theta - z$. It is profitable, therefore, to train workers efficiently (equation 8).

Workers are indifferent about being trained or not being trained. If trained, a worker's earnings net of investment cost is

$$\max\{\theta_*, z\} - \max\{\theta_* - z, 0\} = z$$

If not trained, he earns z as well.

Outside firms' strategy to offer $\max\{\theta_*, z\}$ if a worker is trained and z if not, is optimal. These are the MRP of the least productive worker in each group.

Q.E.D

The difference between investment in general and firm-specific skills is that the increase in a worker's wage is due to both the lower bound on the fundamental ability and the actual increase in productivity. Hence, the increase in wage is larger. The worker share is the increase in wage due to investment and therefore, the employer captures all the rents.

4.3 Workers' investment in human capital

In previous sections I provide "short-term" analysis, in the sense that z is exogenous. I show next that investment workers make before they enter the labor market is first-best.

Investment made before the worker enters the labor market is first-best because output is first-best and due to competition. Although wages in each period are not the expected MRP of a worker, competition for inexperienced workers imposes zero expected profit over the two periods. As a result, lifetime earnings is the expected output produced over the worker's career. Since investment in skills made before they enter the labor market maximize workers' lifetime earnings net of investment costs, it also maximizes the expected output net of investment costs. Output level is first-best due to efficient assignment of workers to job; as a result, investment level is first best.

I extend the basic model and include investment in skills. Before entering the labor market, the worker can invest in skills required in the low-level job. For simplicity assume the worker investment determines the productivity of the worker in the low-level job, z .²¹ $c(z)$ is the cost of acquiring skill level of z (the cost is increasing in z).

An efficient investment in z maximizes the social benefit (SB). The social benefit is the sum of the output produced in the first period (in the low-level job) added to the output produced in the second period (in which the worker

²¹It is straightforward to extend the model to more than one type of investment.

is assigned either to a low or a high-level job) minus investment costs:

$$SB(z) = z + E_{\theta|z}[MAX\{\theta, z\}] - c(z) \quad (9)$$

The worker maximizes the sum of the expected wages net of investment costs. The first period wage, $z + E_{\theta|z}[MAX\{\theta - z, 0\}]$, has two components: the MRP in the low-level job, and the expected value of the employer's informational rent. The second period wage is the MRP in the low-level job z , regardless of the worker's job assignment:

$$W_1(z) + W_2(z) = z + E_{\theta|z}[MAX\{\theta - z, 0\}] + z - c(z) \quad (10)$$

Proposition 7: Worker investment is efficient (investment maximizes the social benefit.) Proof: Immediate. The social benefit and total earnings are equal .

5 Robustness

This section discusses the robustness of the equilibrium outcome and extends the analysis.

5.1 The structure of the game

A key result is that wages are the lower bound on the worker's productivity conditional on information available to outside firms (job assignment). Efficient job assignment is achieved because in the model the lower bound on the worker's productivity doesn't change when job assignment changes, although the expected MRP may change.²²

²²The lower bound doesn't change in the model as a consequence of production technology and distribution assumptions. These assumptions imply that there is a positive probability (can be arbitrarily small) that a worker with productivity $\theta = z$ is efficiently promoted. In section 4, decision to train a worker indicates that his productivity is above a certain threshold. As a consequence, the productivity of the least productive trained worker is above the productivity of the least productive untrained worker; therefore, the equilibrium is efficient only if the worker and employer share investment costs.

There are two job levels to capture the idea that different jobs involve different tasks which may require different skills. The assumption that productivity in the low-level job is constant and publicly observed (therefore, it is uncorrelated with productivity in the high-level job) is for convenience.²³ If outside firms cannot observe productivity in the low-level job, the wage offered to workers in the low-level job is the lower bound on the worker's productivity \underline{z} . The efficient promotion rule is to promote a worker if $\theta \geq z$, and the outside offers to workers in high-level jobs are \underline{z} .²⁴

5.2 Uniqueness

There is a continuum of P.B.E. in the model. The source of multiplicity of equilibria is that outside firms can (correctly) believe that only workers for which the productivity in the high-level job exceeds a threshold level θ_* are promoted. Therefore, outside firms offer a promoted worker a wage of

$$w_2^M = MAX[\theta_*, z] \quad (11)$$

The beliefs are

$$\hat{\theta}(H) = \frac{\int_{max[\theta_*, z]} \theta f(\theta) d\theta}{\int_{max[\theta_*, z]} f(\theta) d\theta}$$

It is profitable then for the employer to promote a worker according to the following rule²⁵:

$$J_2^e = \begin{cases} H & \text{if } \theta \geq MAX[\theta_*, z], \\ L & \text{otherwise} \end{cases} \quad (12)$$

Promoting a worker with $\theta_* > \theta > z$, although efficient, will result in separation of the worker and the first period employer. Therefore, outside firm

²³This assumption creates a model of absolute advantage, in the sense that more able workers are weakly more productive in both jobs.

²⁴Productivity in jobs can be correlated. The assumption required is that there is a small positive probability that the least productive worker is more productive in the high-level job.

²⁵The employer's wage offer strategy is as before, see equation (1).

beliefs satisfy bayes' law. All workers with $z < \theta < \theta_*$ are assigned inefficiently to low-level jobs.

Proposition 3:

In nearby games in which employers make mistakes in promotional decisions with an arbitrarily small probability, the only equilibrium outcome is the one described in proposition 1 (efficient equilibrium).

First, I show that any wage offer that is made by outside firms and exceeds z yields negative expected profit, and therefore, there are no equilibria of the type described above in nearby games. Then I show that offering z is an equilibrium.²⁶ If outside firms offer z in equilibrium, the proof that the equilibrium outcome is as described in proposition 1 is identical to the proof in the original game.

The reason that in nearby games no outside firm offers a wage that exceeds z is that any such offer yields negative expected payoff; offering a wage z and a low-level job contract yields zero expected payoff. More formally, assume there is an exogenous and arbitrarily small probability $\epsilon > 0$ that the employer promotes a worker with ability $\theta < \theta_*$.

Claim 1: Any offer an outside firm makes that satisfies $w_2^r > z$ yields a negative expected payoff.

Proof: The employer's strategy is given by equations 12, and 1. The worker's strategy is similar to the strategy in the original game. The expected payoff of an outside firm is

$$\Pi^r(w_2^r, H) = \begin{cases} \theta - \theta_* & \text{if } w_2^e < w_2^r, \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

If $\theta \geq \theta_*$, the employer will match the offer and the outside firm's expected payoff is zero. If $\theta < \theta_*$, the expected payoff for an outside firm is $(\theta - \theta_*) < 0$.

²⁶Of course, there are no equilibria in which outside firms offer a wage lower than z , as the market is competitive.

Therefore, the expected payoff is:

$$\Pi^l = \epsilon(\theta - \theta_*) < 0$$

Claim 2: An outside firm's offer of a low-level job assignment and a wage equal to the MRP in the low-level job, yields zero profit.

Proof: Immediate. Given the employer's counteroffer strategy (equation 1), the employer will match the offer, and the worker rejects the outside offer.

To summarize, the difference between the payoff in nearby games and the original game is that in the original game the probability that $\theta < \theta_*$ is zero. Since an offer of a low-level job and a wage equals the MRP in the low-level job always yields zero profit, no equilibrium in which the outside firms offer a wage larger than z exists. Since the worker's outside offers are z , regardless of the job assignment, the only optimal job assignment strategy is to assign workers to jobs they are most productive.

5.3 Infinite horizon sequential offers game

The proofs of the previous results hinge on the assumption that the employer makes the final offer. The equilibrium outcome depends, however, on the ability to match offers rather than on the ability to make the final offer. The employer has private information regarding the worker's MRP, and he matches outside offers only if the worker's MRP exceeds the outside offer. As a result, outside firms can only raid a worker with a wage greater than the worker's MRP. This logic holds as long as the employer can match outside offers. The employer, if interested in retaining the worker, offers the lowest wage required to do so. Hence, unless an outside offer exceeds the worker's MRP, the employer's counteroffer is similar to the outside offer. Since outside firms cannot elicit further information observing the employer's counteroffers and successfully raid a worker, they will not match the employer's counteroffer. As a result, the employer makes the final offer in equilibrium, and the

equilibrium outcome is similar to the equilibrium outcome in the finite horizon game in which the employer makes the final offer.

I generalize the basic model and allow each firm to match offers without assuming that there is a final round of offers and counteroffers. The extended model is similar to an ascending auction held by the worker. Apart from modelling the offers and counteroffers as an infinite horizon process, the extended game is similar to the finite horizon game. The employer offers a job assignment and a wage at the end of the first period, and outside firms make an offer after observing the job assignment. The employer then makes a counteroffer, and firms in the market observe the offer and decide whether to raise the previous offer. I assume a firm cannot offer a wage lower than the wage in its previous offer, and if a firm offers a high-level job, it can not offer a low-level job later.²⁷ The offers are observed by all players. If both the employer and outside firms do not raise their previous offers, the worker chooses one of the offers. w_l^M is the highest outside offer a worker receives in round l , where $l = 1, 2, \dots$

I denote the wage offer that the employer makes to its worker in round l by w_l^e . The first round of offers takes place at the beginning of the second period and is denoted by $l = 1$. The employer makes offers in odd rounds, and outside firms make offers in even rounds.

5.4 Equilibrium strategies

The worker's strategy is to accept the highest wage offer. If the highest outside offer and the employer's offer are equal, the worker accepts the employer's offer. This is the equilibrium strategy for any history in any round.

The employer's strategy is a job assignment at the end of the first period, and wage offers in odd rounds. At the end of the first period, the employer

²⁷This assumption is made to simplify the off-equilibrium-path analysis.

assigns the worker to the job in which he is most productive:

$$J_2^e = \begin{cases} H & \text{if } \theta \geq z, \\ L & \text{otherwise} \end{cases}$$

Wage offers are given by $w_1^e = z$, and for $l > 2$

$$w_l^e = \begin{cases} w_{(l-1)}^M & \text{if } \theta > w_{(l-1)}^M \\ w_{(l-2)}^e & \text{otherwise} \end{cases} \quad (14)$$

In every round, outside firms' strategy is to match the highest offer in the history (if it exceeds z). Otherwise, outside firms offer z . I describe formally the "market" offer and beliefs below. are follows. Denote the wage offers' history in round l by h^l . Consider a worker who is assigned to a high-level job. In the first round (of the second period), $l = 2$, the offer is given by the following equations:

If $w_1^e \leq z$, then:

$$\{J_2^M = L, W_2^M = z\}, \text{ beliefs : } \hat{\theta}_2 = E(\theta|h^2) = \frac{\int_z \theta f(\theta) d\theta}{\int_z f(\theta) d\theta}. \quad (15)$$

If $w_1^e > z$ the offer is given by

$$\{J_2^M = H, W_2^M = w_1^e\}, \text{ beliefs : } \hat{\theta}_2 = E(\theta|h^2) = \frac{\int_{w_1^e} \theta f(\theta) d\theta}{\int_{w_1^e} f(\theta) d\theta} \quad (16)$$

In any round, $l > 2$ outside firms offers are given by the following equations:

1. If $w_{l-1}^e \leq w_{l-2}^M$ and $j_{l-2}^M = L$, then

a. If $w_{l-2}^M = z$:

$$\{J_l^M = L, W_l^M = w_{l-2}^M\} \text{ beliefs : } \hat{\theta}_l = E(\theta|h^l) = \frac{\int_z \theta f(\theta) d\theta}{\int_z f(\theta) d\theta} \quad (17)$$

b. If $w_{l-2}^M > z$:

$$\{J_l^M = H, W_l^M = w_{l-2}^M\} \text{ beliefs : } \hat{\theta}_l = E(\theta|h^l) = \frac{\int_{w_{l-2}^M} \theta f(\theta) d\theta}{\int_{w_{l-2}^M} f(\theta) d\theta} \quad (18)$$

2. If $w_{l-1}^e \leq w_{l-2}^M$ and $j_{l-2}^M = H$, then:

$$\{J_l^M = H, W_l^M = w_{l-2}^M\} \text{ beliefs : } \hat{\theta}_l = E(\theta|h^l) = \frac{\int_{w_{l-2}^M} \theta f(\theta) d\theta}{\int_{w_{l-2}^M} f(\theta) d\theta} \quad (19)$$

3. If $w_{l-1}^e > w_{l-2}^M$, then:

$$\{J_l^M = H, W_l^M = w_{l-1}^e\}, \text{ beliefs : } \hat{\theta}_l = E(\theta|h^l) = \frac{\int_{w_{l-1}^e} \theta f(\theta) d\theta}{\int_{w_{l-1}^e} f(\theta) d\theta} \quad (20)$$

Proposition 4:

The above strategies constitute a PBE with the following outcomes:

- (a) *Outside firm offers z to worker assigned to a high level job.*
- (b) *The employer matches outside firm's offer and offers z .*
- (c) *No turnover occurs.*
- (d) *The worker is assigned to the job in which he is most productive.*

Proof: See appendix B.

This extension shows that when an employer learns his workers' productivity privately, outside firms cannot elicit this information by observing the employer's offers. This can be seen in equation 14. The employer's wage offer simply reflects the information available to outside firms. The fact that the employer matches the offer informs outside firms that the worker's MRP is larger than the offer (equations 16, 18, 19, and 20). Thus, any offer that exceeds the employer's offer, and potentially can provide new information to outside firms, yields negative expected payoff due to the winner's curse. Since any offer outside firms make that might extract new information regarding the worker's MRP yields negative expected payoff, outside firms will not match the employer's offer. As a result, the employer makes the final offer.

5.5 Multiple periods

In this section, I show that the equilibrium outcome does not hinge on the assumption that there are only two production periods. There are several ways in which incorporating more periods may affect the equilibrium outcome. If there are more than two production periods, new information about the worker's productivity is not revealed to the market. Three possible sources can reveal information regarding the worker's productivity: promotion, wages, and turnover. I already showed promotion reveals information regarding the worker's productivity, but due to the winner's curse promotion does not affect wages. In the two-period model, information regarding job assignment is revealed to outside firms before they make offers for the second period. Wages are always the highest offer made by outside employers. As long as they are uninformed, employers will not offer wages that reveal information to the market; therefore, wages here will not convey information to the market.

Another reason extending the model to include more than two periods can affect the equilibrium outcome is that it may affect the off-equilibrium path analysis. In the two period model, if outside firms raid an employed worker they learn the worker's productivity. Since this is the last period, however, they cannot benefit from this information. When firms make offers to employed workers for the second period, the offers may differ depending on the number of periods left. If there is more than one period left, especially since they learn about the worker's productivity during this period. Next, I describe the equilibrium outcome and strategies for N periods case and show that the equilibrium outcome does not change. The main point is that employers have better information at the time outside firms make offers, and outside firms can only raid an employed worker with a contract which exceeds the worker's value to the firm.²⁸

²⁸In this paper I only analyze for simplicity spot contracts. If one allows for firms to

Suppose there are $N > 2$ periods, and consider the following outside firm's strategy if it observed a worker who changed employers after the first period.

Outside firm's offer: $W_t^M = \text{MAX}\{w_2, \dots, w_{t-1}, z\}$; that is, each period the firm observes the worker's wage history and offers the highest wage (excluding the first period wage). In the periods proceeding the second period, the profit is positive only if the worker's actual productivity exceeds the wage the firm offered in the second period. As argued previously, however, the employer always matches an offer if the worker's productivity exceeds it. Therefore, for any offer above z in the second period, the firm cannot profitably make a positive profit by offering $w_2 > z$, and the equilibrium outside offers are z . Therefore, firms assign workers to jobs efficiently.

6 Applications and Conclusions

6.1 A note about discrimination

This section extends the basic model and analyzes the effect of offer matching in the presence of asymmetric information on job assignment and wages discrimination.

There are two different groups of workers: a and b (these groups can be race, gender, or class). Both groups have the same distributions of abilities, $f(z, \theta)$. Thus, potential employers have more information on the ability of workers from one group than the workers from the other group. When the worker enters the labor market, potential employers observe past experience, education, and other traditional ability indicators. The employer observes privately θ, z , during the first period. Denote the lower bound on a worker's MRP conditional on publicly observable characteristics of a worker from a

commit to long term contracts, the extension to the N is simple. The only equilibrium contract outside firms can offer is to commit to pay z in all future periods. Due to the winner's curse, no wage above z can be offered.

group k by z_0^k where $k \in a, b$. Workers with the same abilities θ, z from the disadvantaged group, a , have lower z_0 ($z_0^a < z_0^b$). This difference implies that when these workers enter the market, their traditional indicators are less informative and hence, the inference about the MRP they can surely produce is lower than for workers in the advantaged group. As such the expected MRP, conditional on the observable characteristics, is higher in the disadvantaged group than in the other group, whereas the unconditional expected MRP is similar. After a worker works for one period, the employer and the worker realize the ability parameters, and outside firms observe job assignment and z_0^k .

Proposition 8: In equilibrium,

1. *The employer promotes efficiently the same proportion of workers to the high-level job from each group.*
2. *In the second period, workers from the disadvantaged group, a , will earn less on average in both high-level and low-level jobs than workers in group b .*
3. *The average entry wage of workers from the disadvantaged group is higher (conditional and unconditional on the observable characteristics).*

Proof: Proposition 8 follows immediately from the equilibrium properties of the original game.

1. Workers are promoted efficiently. The difference from the original game is that z is unobserved. As a result, the minimum MRP outside firms know a worker can produce is z_0^k instead of z . Hence, in the second period, wages are z_0^k , regardless of the worker's job assignment. Therefore, employers do not need to raise a promoted worker's wage in order to retain the worker. Since the surplus produced is larger when workers are sorted to jobs efficiently, the employer promotes workers from both groups who are more productive in high-level jobs.

2. Wages in the second period are z_0 , regardless of the job assignment. Since on average workers from the disadvantaged group have lower z_0 , the second period average wages in this group are lower.

3. The first period wage is determined before the employer and the worker learn the worker's ability. Since the market is competitive, the expected profit from hiring a worker is zero:

$$\Pi(z_0^k) = \int z f_k(\theta, z) dz - w_1(z_0^k) + \int MAX(z - z_0^k, \theta - z_0^k) f_k(\theta, z) dz d\theta. \quad (21)$$

Imposing zero profit condition on equation 21 yields the first period wages:

$$w_1(z_0^k) = \int z f_k(\theta, z) dz + \int MAX(z - z_0^k, \theta - z_0^k) f_k(\theta, z) dz d\theta. \quad (22)$$

The expected productivity conditional on the observable characteristics is higher in the disadvantaged group by assumption. In addition, the expected MRP in the high-level job conditional on z_0^k is greater in the disadvantaged group. Hence, entry-level wages are greater (the unconditional entry-level wages are greater as well). Whereas the expected productivity in the first period (first term in equation 22) and the expected productivity in the second period are the same in both groups, the average second period wage is lower. Hence, the second term in equation 22 is larger.

Q.E.D

In this model wages are equal to the MRP outside firms know a worker can produce rather than the expected MRP conditional on observable characteristics. In models in which wages are the expected MRP conditional on publicly observable characteristics such as Phelps(1972) and Lundberg and Startz(1983), employers interpret the ability signals of workers from the disadvantaged group less accurately and hence, place less reliance on them. In this model employers are equally informed about workers from both groups after one period. Thus, outside firms can interpret less accurately the signals of one group. Since wages are equal to the highest outside offer a worker receives, workers from the disadvantaged group earn less than workers from the advantaged group. This is despite the fact that the expected MRP is known to be higher.

A major difference from models in which wages are the expected MRP of a worker conditional on his observable characteristics is that if an individual from the disadvantaged group would have had ability signals employers could interpret more accurately (for example, a degree from a good college), he would be paid the same wage as an individual from the advantaged group.

The model predicts that as long as the expected productivity is similar across groups, workers from the disadvantaged group earn higher entry level wages. The model, however, is simple and does not capture other aspects that may cause the observed differences between groups as captured in the empirical literature. For example, differences in employment opportunities for young workers may have permanent effect on career paths. In reality, firms are heterogeneous, and provide different career opportunities. Promotion prospects, quality of training and labor market connections differ across firms. Thus, workers from the disadvantaged group are less likely to be hired by firms with better career prospects, then workers from the advantaged groups are likely to experience better career outcomes.

6.2 Conclusions

Offer matching is a common feature of skilled-workers labor markets. I showed that when employers have better information regarding their workers' productivity, offer matching affects employers' promotion and job assignment decision. Due to the winner's curse, wages are the MRP of the least productive worker (conditional on the information available to outside firms), and the asymmetric information need not cause distortion. I then analyzed investment in human capital, as training programs of employed workers can be used by outside firms to infer workers qualifications. I showed that investment in skills is efficient when investment costs are shared by workers and employers. Due to competition over inexperienced workers, workers' premarket investment maximizes expected productivity. Since the output level is

first-best, investment level is first-best as well.

Several interesting issues are not addresses in this paper. One issue is labor market turnover. No turnover occurs in equilibrium mainly for the reason that each worker is equally productive in all firms. Bernhardt and Scoones [1993] develop a model of job assignment in which workers' productivity is dispersed across firms.²⁹ In their paper, the heterogeneity of worker-firm matches alone doesn't generate turnover. Turnover, however, occurs when outside firms can learn employed workers' productivity by making a costly investment. Once investment is made, the competition is between symmetrically informed employers.³⁰

Other issues that are not explored are outside employers' ability to screen workers, and workers' ability to signal their productivity. The first issue is addressed in Ricart I Costa [1988]. The paper develops a model in which contingent contracts are feasible.³¹ The later issue is explored in Golan [2002] which develops a wage signaling model. The model allows wages to be determined by negotiation between the employer and worker. As a consequence, equilibrium wages of some workers depend on their productivity and therefore reveal information to potential employers.

²⁹Acemuglu and Pische [1998] develop a model of turnover in which workers are heterogeneous with respect to their taste for working in firms.

³⁰Job assignment is inefficient. In their model competition over employed workers takes the form of an auction and therefore, employers can match outside offer. If it is infeasible for outside firms to invest and learn productivity, however, the heterogeneity in firm-work matches alone doesn't cause distortion of job assignment.

³¹As a consequence, wages reflect, to some extent, actual productivity.

Appendix A: Proof of proposition 1

To prove that the strategies and beliefs above constitute a PBE, I show that: 1) each player's behavioral strategy maximizes his expected payoffs given the other player's strategies, 2) outside firms' strategies maximize their expected payoffs given their beliefs and other players strategies and, 3) these beliefs satisfy Bayes' rule and are consistent with the employer's and the worker's equilibrium strategies.

Worker: a worker's strategy, which is to choose the highest offer and stay with the first period employer if there is a tie, maximizes his utility (trivially).

The employer's strategy: First, I show that the wage offer is optimal, and then I show that the job assignment is optimal as well. As denoted earlier, the highest outside offer that a worker receives is w_2^M ; Π_2^e is the employing firm's second period profit. In order to show that the wage offer is optimal, w_2^e should satisfy (for all w_2^M):

$$w_2^e \in \operatorname{argmax} \Pi_2^e, \quad \text{where} \quad \Pi_2^e = \begin{cases} \theta - w_2^M & \text{if } w_2^M \leq w_2^e, \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Next I show that the strategy in (1) maximizes the employer's profit: Consider the two possible cases:

1. $w_2^M > \theta$.
2. $w_2^M \leq \theta$

In the first case each offer that the worker will accept, i.e, $w_2^e \geq w_2^M$, will yield a negative profit (since $w_2^M - \theta < 0$). Hence, offering a wage that is lower than w_2^M and "losing" the worker to another firm is optimal, since it yields zero profit.

In the second case, where $w_2^M \leq \theta$, each offer $w_2^e \geq w_2^M$ will enable the employer to retain the worker; the minimum wage that is required to retain the worker is $w_2^e = w_2^M$. Therefore, the strategy described in equation 1, is optimal.

I show next that assigning the worker to the job he is most productive, is optimal.

Consider a worker with $\theta \geq z$. If the worker is assigned to a high-level job, $j_2^e = H$, then the employer's rent is: $\theta - z$. If the worker is assigned to a low-level job $j_2^e = L$, the employer gains rent of: $z - z = 0$. Hence, assigning such worker to high-level jobs, is optimal.

Consider a worker, $\theta < z$; assigning such worker to a low-level jobs is optimal, since the employer will gain a higher payoff: $\Pi_2^e(L) = z - z = 0 > \Pi_2^e(H) = \theta - z < 0$.

Therefore, assigning the worker, at the end of the first period, to a job he is most productive, is optimal.

Outside firms: I next show that offering a worker that is assigned to a high-level job in the second period, a contract: $\{W_2^r = z, J_2^r = L\}$, is optimal. To prove that, I show that if an outside firm offers a low-level job assignment, then the optimal wage offer is z ; this offer yields zero profit. Then I show, that if the firm offers a high-level job, then for every wage the firm offers, the expected profit is non-positive. Any contract that includes high-level job offer will yield lower payoff to the outside firm than the highest payoff that can be achieved by offering a contract that includes a low-level job assignment offer. Since the market is competitive any wage offer will yield an expected zero profit. Therefore, no outside firm can deviate and offer a wage different from the worker's MRP in low-level job: z . If an outside firm offers a high-level job, the expected profit, $\Pi_2^r(J_2^r = H)$, is always non-positive. Any wage offer should satisfy: $w_2^r \in \text{argmax} \Pi_2^r(J_2^r = H)$, where

$$\Pi_2^r(J_2^r = H) = \begin{cases} \theta - w_2^r & \text{if } w_2^e < w_2^r, \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

There are 2 possible cases: 1. $w_2^r > \theta$, which is a necessary condition to raid a worker in equilibrium. In this case the payoff is negative $\Pi_2^r(J_2^r = H) = \theta - w_2^r < 0$.

2. $w_2^r \leq \theta$. In this case the employer will match the offer, and the expected payoff is zero.

I established that offering a contract $\{z, J_2 = L\}$ yields zero profit, while offering a contract that includes a high-level job assignment yields at most 0 profit. Therefore, offering the contract $\{z, J_2 = L\}$ is optimal.

Beliefs: I proved above that the raiding-firms' strategy is optimal given the other players' equilibrium strategies. Notice that the above equilibrium strategies are optimal for any beliefs about the ability of a worker that is assigned to a high-level job; regardless of the expected MRP of the worker that is promoted, it is always optimal to offer a wage that is equal to the productivity of the least productive worker that the employer promotes. Denote by $\hat{\theta}(H)$, the expected productivity of a worker that is promoted (for any distribution with finite mean). In equilibrium the expected productivity if the worker was assigned to a high-level job in the first period is as described in equation 4. Since a worker is promoted to the job in which he is most productive ($\theta > z$), the beliefs satisfy Bayes' rule; the equilibrium strategies and the beliefs described above constitute a PBE (the strategies are optimal given these beliefs). These strategies imply that the equilibrium outcome is as described in proposition 1.

Q.E.D

Appendix B: Proof of proposition 4

In order to prove that (a)-(d) hold in equilibrium, I prove that the equilibrium strategies constitute a PBE. I show that no player can deviate from the equilibrium strategy and increase by that his expected payoff (given the beliefs), at any node. Then I show that (a)-(d) follow from the equilibrium strategies.

Worker: Since the worker moves only at the terminal nodes, it is straightforward that any deviation from the strategy of choosing the highest wage offer (and choose the employing firm if there is a tie) cannot increase the worker's payoffs.

Employer: I First show that at the end of the first period, the employer cannot make a profitable deviation by assigning the worker to a job in which he is not the most productive. Consider a worker who is more productive in low-level jobs, $z > \theta$. If the employer deviates and assigns the worker to a high-level job, then given the outside firms' strategies and his own strategy, the worker will be raided. Hence, the expected payoff is zero, and the employer cannot deviate profitably.

Clearly, the employer cannot deviate profitably by assigning a worker who is more productive in the high-level job, to a low-level job (since the profit will be zero).

I show next that the employer cannot profitably deviate from his wage offer strategy (equation 14). Consider the case in which $\theta \geq w_{l-1}^M$: if the employer deviates and offer: $w_l^e > w_{l-1}^M$, then in the next round the market will offer a wage: $w_{l+1}^M = w_l^e$, then the employer will not raise the offer in round $l+2$, the market will not raise the offer in round $l+3$, and the employer will stay with the employer. In this case, $\Pi_2^e = \theta - w_l^e$. However, the payoff if the employer offers $w_l^e = w_{l-1}^M$ is higher, since it's the lowest wage that is needed to retain the worker. Hence, any wage offer that is larger is not a profitable deviation.

Clearly, any wage lower than w_{i-1}^M is not a profitable deviation, as the employer cannot retain the worker at a wage lower than the market offer. In this case, the employer will raise the offer in the next round he offers to $w_i^e = w_{i-1}^M$, and receive the same payoff. Hence, any lower offer is not a profitable deviation. Consider the case in which $\theta < w_{i-1}^M$, any offer $w_i^e > w_{i-2}^e$ is not a profitable deviation (cannot offer less by assumption). To see that observe that if $w_i^e \geq w_{i-1}^M$ than the worker will stay with the employer and the profit is $\theta - w_i^e < 0$, it is easy to see that the lower w_i^e the higher the employer's payoff. If $w_i^e < w_{i-1}^M$ than any offer $w_i^e > w_{i-2}^e$ will yield negative payoff (the worker will stay with the employer); the equilibrium strategy wage offer yields zero profit (the worker leaves the firm). Hence, the employer cannot profitably deviate from his wage offer strategies.

Outside Firms: In the first round ($l = 2$) a firm cannot profitably deviate from its contract offer strategy. In case 1 equation 15, it is clear that if $J_2^M = L$ the outside firm cannot offer a wage larger than z and increase the expected payoff. I show next that any contract that include a high-level job offer can yield at most zero profit; hence, it cannot be a profitable deviation. In order to show that, I argue that any wage offer larger than z and a high-level job assignment will yield a negative expected payoff, and an offer of z , yield zero expected payoff.

Consider a contract of $\{J_2^M = H, w_2^M > z\}$. There are two case:

1. $\theta > w_2^M$. In that case the employer will not match the offer (in $l = 3$) and the firm's payoff: $\theta - w_2^M < 0$.

Case 2. $\theta \leq w_2^M$. In this case, the employer will match the offer in the next round: $w_3^e = w_2^M$ and in round 4, the outside firms do not raise the offer. Therefore, the worker stays with the employer and the outside firms' payoff is zero. Therefore, an outside firm cannot deviate profitably. In the second case, equation 16. A firm cannot deviate profitably. The contract in 16 yields zero expected payoff (since in the next round the employer will not raise his offer and then the outside firms will not raise the offer and the

worker stays with the first period employer). Clearly, any offer of a low-level job cannot yield more than zero expected profit. Any wage offer $w_2^M > w_1^e$ and a high-level job offer, cannot yield more than zero expected payoff. The argument is the same as in case 1. If the offer exceeds the worker's MRP in high-level jobs then the worker will accept the firm's offer and the firm's expected payoff will be negative. In all other case the worker will stay with the first period employer and then the firm's expected payoff will be zero. Hence, an outside firm cannot profitably deviate from the strategies in 16.

For any $l > 2$: Any deviation from the strategy in equation 17 cannot increase the expected payoffs. As in the argument for the $l = 2$, no contract that offer a low-level job assignment can yield more than zero expected payoff. Any contract that include a high-level job offer yields zero profit if $\theta \geq w_l^M$ and a negative payoff if $\theta > w_l^M$. In equation 18, the argument is similar. $w_l^M = w_{l-2}^M$ is the lowest offer that outside firms can make. In the case that $\theta \geq w_l^M$ the expected payoff from 18 is zero. Any higher wage offer will yield zero payoff if the wage offered is still lower than the worker's MRP. However, if the wage offer exceeds the worker's MRP then the payoff is negative. In the case where $\theta < w_l^M$ the worker will accept the firms offer. In order to minimize the cost, the firm's needs to offer the lowest possible wage which is w_{l-2}^M . Therefore, a firm cannot profitably deviate from 18. The argument for the contract in 19 is similar. $w_l^M = w_{l-2}^M$ is the lowest offer that outside firms can make. If $\theta \geq w_l^M$ then it yields zero expected payoff. Any offer higher than this wage will yield zero expected payoff if the worker MRP is still larger than the wage offer (the worker will stay with the employer). Any offer that exceeds the worker's MRP will yield negative payoff since the employer will not match the offer and the worker will accept the outside firms' offer. The argument for optimality of equation 20 is as follows: The employer offers a wage that is strictly greater than the outside firms' offer (out of the equilibrium path). Given the firms' believe that the lowest possible MRP of that worker is w_{l-1}^e , the contract in 20 yields zero expected payoff. Any offer

below that wage yields zero expected payoff as well. Any offer that exceeds the employer's previous offer yields zero expected payoff if the offer is still lower than the worker's MRP and a negative payoff if it exceeds the worker's MRP.

Therefore outside firms cannot deviate profitably from their strategy. Note that on the equilibrium path the beliefs satisfy Bayes' law.

$$\hat{\theta}_t = E(\theta|h^t)$$

Since the employer assigns workers efficiently, the expected productivity of a worker in a high-level job, with MRP in low-level jobs of z , is the expected MRP above.

Equations 14, 15, and 17 describe the strategies on the equilibrium path. Clearly, the marker offer is z , the employer's counteroffer is z , and the market will not make an additional offer. Hence, the equilibrium outcome is similar to the equilibrium outcome in the finite horizon game (proposition 1), and is described in proposition 4.

Q.E.D

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