A On-Line Appendix

A.1 Identification of the Marginal Utility of Wealth

It is well known in the literature on the estimation of consumption functions that the general form of utility with risk aversion is not identified without quantity and price data, which we do not have. Therefore, we follow the literature and state sufficient conditions to obtain identification of the marginal utility of wealth. We state the conditions below.

Assumption A.1 *The marginal utility of consumption has the following form.*

$$\frac{\partial u_{2i}(c_{nt}, z_{nt}, \varepsilon_{2nt})}{\partial c_t} = u_{2c}(c_{nt}) u_{2z}(z_{nt}) \varepsilon_{2nt}, \tag{63}$$

where $u_{2c}(c_t) > 0$*,* $u_{2z}(z_t) > 0$*, and* $\varepsilon_{2t} > 0$*.*

Assumption A.2 *1)* $E[\log \varepsilon_{2nt} | z_{nt}] = 0$ *for all* n *and* t*. 2)* $E_n[\log(\eta_n) | z_{nt}] = 0$*.*

Assumption A.3 *1)* z_{nt} *has a continuous element* z_{cnt} *with continuous variation on its support* $[\underline{z_c}, \overline{z_c}]$*. 2)* $u_{2z}(\underline{z}_c, .) = 0$*.*

Assumption (A.1) states that the marginal utility of consumption is multiplicatively separable. For example, both the class of constant absolute risk aversion and the class of constant relative risk aversion satisfy this assumption. Assumption A.2(1) formally states that the error is mean independent of z_{nt} with expectation zero. Assumption A.2(2) is the standard normalization needed in a panel data model in order to recover the level of the time component. Finally, Assumption A.3(1) states that at least one variable with continuous variation on its support is required, and Assumption A.3(2) is a boundary condition. Assumption A.3 can be replaced with a parametric assumption on the function $u_{2z}(z_{nt})$.

Lemma A.1 *If* $u_{2c}(c_{nt})$ *is known, and assumptions A.1–A.3 are satisfied. Then* $\eta_n \lambda_t$ *is identified.*

Proof. Without loss of generality, assume that

$$\frac{\partial u_2 i(c_{nt}, z_{nt}, \varepsilon_{2t})}{\partial c_{nt}} = \exp(u_{2c}(c_{nt})) \exp(-u_{2z}(z_{nt})) \exp(-\varepsilon_{2nt}).$$

The above equation satisfies Assumption (A.1). The explicit functional form simplifies the exposition. Equation 4 implies that the Euler for consumption is

$$\exp(u_{2c}(c_{nt})) \exp(-u_{2z}(z_{nt})) \exp(-\varepsilon_{2nt}) = \eta_n \lambda_t. \tag{64}$$

Taking the log and then first derivative of equation (64) and rearranging gives us

$$\Delta u_{2c}(c_{nt}) = \Delta u_{2z}(z_{nt}) + \Delta \log(\lambda_t) + \Delta \varepsilon_{2nt} \tag{65}$$

By assumption A.2(1), then,

$$E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}] = \Delta u_{2z}(z_{nt}) + \log(\lambda_t) \tag{66}$$

Taking the derivative of $E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}]$ with respect to z_{cnt} and z_{cnt-1}, respectively, and integrating back up to z_{cnt} and z_{cnt-1}, respectively, gives

$$\begin{aligned} u_{2zi}(z_{nt}) &= u_{2z}(\underline{z_c}, z_{c'nt}) + \int_{z_c}^{z_{cnt}} \left\{ \frac{\partial E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}]}{\partial z_c} \right\} \mathrm{d}z_c \qquad (67) \\ u_{2zi}(z_{nt-1}) &= u_{2z}(\underline{z_c}, z_{c'nt-1}) + \int_{z_c}^{z_{cnt-1}} \left\{ \frac{\partial E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}]}{\partial z_{c-1}} \right\} \mathrm{d}z_{c-1}, \qquad (68) \end{aligned}$$

which by Assumption A.3(2) and from Chesher's (2007) results is identified. Therefore

$$\begin{aligned} \Delta \log(\lambda_t) &= E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}] - \int_{z_c}^{z_{cnt}} \left\{ \frac{\partial E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}]}{\partial z_c} \right\} \mathrm{d}z_c \\ &\quad + \int_{z_c}^{z_{cnt-1}} \left\{ \frac{\partial E[\Delta u_{2c}(c_{nt})|z_{nt}, z_{nt-1}]}{\partial z_{c-1}} \right\} \mathrm{d}z_{c-1} \end{aligned} \tag{69}$$

and, by Assumption A.2(1),

$$\log(\lambda_1) = E[\Delta u_{2c}(c_{n1})|z_{n1}] - \int_{z_c}^{z_{cn1}} \left\{ \frac{\partial E[\Delta u_{2c}(c_{nt})|z_{n1}]}{\partial z_c} \right\} \mathrm{d}z_c. \tag{70}$$

Hence, λ_t is identified. Finally, by Assumption A.2(2), we have

$$\log(\eta_n) = E_t\{u_{2c}(c_{nt}) - \log(\lambda_t) - u_{2z}(z_{nt})|z_{nt}\}. \tag{71}$$

Using Chesher's (2007) result and the fact that $u_{2c}()$ is assumed known, we use the results from equations (69), (70), and (71). ■

A.2 Estimation of Consumption and Earnings Equations

In the first step, we use the Euler equation for consumption to form the moment condition:

$$E\left[\frac{\partial u_{2i}(c_{nt}, z_{nt}, \varepsilon_{2nt}, \theta_c)}{\partial c_{nt}} - \eta_n \lambda_t \,\middle|\, z_{nt} \right] = 0. \tag{72}$$

Here, we are assuming that the functional form of $u_2()$ is known up to a finite-dimensional parameter vector, θ_c. Recall that we assume that

$$u_{2i}(c_{nt}, z_{nt}, \varepsilon_{2nt}, \theta_c) = \exp(z'_{nt} B_4 + \varepsilon_{2nt}) c_{nt}^{\alpha} / \alpha.$$

Let $\triangle$ denote the first-difference operator. Taking the logarithm of each side of this expression, differencing, and rearranging implies

$$(1-\alpha)^{-1} \triangle \varepsilon_{2nt} = \triangle \ln(c_{nt}) - (1-\alpha)^{-1} \triangle z'_{nt} B_4 + \triangle(1-\alpha)^{-1} \ln(\lambda_t). \tag{73}$$

Let Θ_c denote the $(K+T-1)$-dimensional vector of parameters to be estimated, defined as

$$\Theta_c = \begin{pmatrix} (1-\alpha)^{-1} B_4 \\ \triangle(1-\alpha)^{-1}\ln(\lambda_2) \\ \vdots \\ \triangle(1-\alpha)^{-1}\ln(\lambda_T) \end{pmatrix}.$$

We also define $Y_n = (\triangle \ln(c_{n2}), \ldots, \triangle \ln(c_{nT}))'$ as a vector of endogenous variables and Z_n^c as the exogenous variables:

$$Z_n^c = \begin{bmatrix} \triangle z'_{n2} & D_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \triangle z'_{nT} & 0 & \ldots & D_T \end{bmatrix},$$

where D_t denotes a time dummy for $t \in \{2, \ldots, T\}$. The assumptions in Section 2 imply that the unobserved variable ε_{5nt} is independent of individual-specific characteristics. Therefore $E((1-\alpha)^{-1} \triangle \varepsilon_{2nt} \mid z_{nt}) = 0$. Using equation (73), one can obtain a set of orthogonality conditions,

$$E\left[\left(Y_n - Z_n^c \Theta_c\right) Z_n^c\right] = 0,$$

that can be exploited to estimate Θ_c using an optimal instrumental-variable estimation technique.

We use a traditional fixed-effect estimator to estimate $(1-\alpha)^{-1}\ln(\eta_n)$. Let T_1 be the number of time periods for which the marginal utility of consumption equation is estimated. Let

$$(1-\alpha)^{-1}\ln(\eta_n) \equiv \sum_{t \in T_1} \left[\ln(c_{nt}) - (1-\alpha)^{-1} z'_{nt} B_4 + (1-\alpha)^{-1}\ln(\lambda_t)\right] / T_1. \tag{74}$$

The fixed-effects estimates of $(1-\alpha)^{-1}\ln(\eta_n)$ are obtained as the simple time averages of the estimated residuals of the consumption equation, which correspond to the sample counterparts of $(1-\alpha)^{-1}\ln(\eta_n)$ defined above. In order to form the sample counterpart of (74), we need an estimate of $\{(1-\alpha)^{-1}\ln(\lambda_t)\}_{t=1}^{T_1}$. From the estimate of Θ_c, however, we can only obtain estimates of $\{\triangle(1-\alpha)^{-1}\ln(\lambda_2)\}_{t=2}^{T_1}$. This requires us to make the additional assumption that $E_n[\eta_n \mid Z_{nt}] = 0$, where $E_n[.]$ is the expectation operator over individuals. This assumption enables us to obtain an estimate of $(1-\alpha)^{-1}\ln(\lambda_1)$ as the sample analogue of

$$(1-\alpha)^{-1}\ln(\lambda_1) = -E_n\left[\ln(c_{n1}) - (1-\alpha)^{-1} z'_{n1} B_4\right].$$

We now have estimates of $\{(1-\alpha)^{-1}\ln(\lambda_t)\}_{t=1}^{T_1}$ and $(1-\alpha)^{-1}\ln(\eta_n)$, enabling us to recover α in the third step of our estimation.

Next, we turn our attention to the estimation of the earnings equations. Let $d_{n\tau t} =$

$I_{n\tau t} \times d_{nt}$. Since all the information set in equation (34) is public at period t, we have

(75) $E_t\{d_{n\tau t}d_{n\tau t-1}[\triangle S_{nt} - \triangle b_{0\tau t} - b_\tau \triangle HC_{nt} - \triangle z_{nt}^{p'} B_{\tau 5} - \beta\gamma_\tau \triangle d_{n\tau t+1}] \mid z_{nt}^p, H_{nt}, h_{nt}^*\} = 0,$

where $\triangle HC_{nt} = (\triangle h_{nt}, \triangle h_{nt}^2, \triangle h_{nt-1}, \ldots, \triangle h_{nt-\rho})'$ and $b_\tau = (b_{\tau 1}, b_{\tau 2}, b_{\tau 31}, \ldots, b_{\tau 3\rho})$.

Let $\Theta_{e\tau}$ denote the $(2 + K + \rho + T)$-dimensional vector of parameters to be estimated,

$$\Theta_{e\tau} = \begin{pmatrix} b_\tau \\ B_{\tau 5} \\ \beta\gamma_\tau \\ \triangle b_{0\tau 2} \\ \vdots \\ \triangle b_{0\tau T} \end{pmatrix}.$$

We also define $Y_{n\tau} = (d_{n\tau 2}d_{n\tau 1} \triangle S_{n2}, \ldots, d_{n\tau T}d_{n\tau T-1} \triangle S_{nT})'$ as a vector of endogenous variables and $X_{\tau n}$ as the exogenous variables,

$$X_{n\tau} = \begin{bmatrix} \triangle x'_{\tau 2} & D_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \triangle x'_{\tau T} & 0 & \ldots & D_T \end{bmatrix},$$

where $\triangle x'_{\tau nt} = d_{n\tau t}d_{n\tau t-1}(\triangle h_{nt}, \triangle h_{nt}^2, \triangle h_{nt-1}, \ldots, \triangle h_{nt-\rho}, \triangle z_{nt}^{p'}, \triangle d_{n\tau t+1})$. Letting Z_n be the matrix of conditioning variables,

$$Z_n = \begin{bmatrix} z_{n2}^{p\prime} & H_{n2} & h_{n2} \\ \vdots & \vdots & \vdots \\ z_{nT}^{p\prime} & H_{nT} & h_{n2T} \end{bmatrix},$$

and using equation (75), one can obtain a set of orthogonality conditions:

$$E\left[\left(Y_{n\tau} - X_{n\tau}\Theta_{e\tau}\right) Z_n\right] = 0,$$

which can be exploited to estimate $\Theta_{e\tau}$ using an optimal instrumental-variable technique. The aggregate effect and fixed effect in the earnings equation are estimated in a similar way to those in the consumption equation.

A.3 Estimation of the Final Stage

Note that from the second step, we have estimates of $b_{\tau 1}$, $b_{\tau 2}$, β,γ_τ, and all the other parameters of the production function. In addition, from the first step, we have an estimate of ϕ_{nt},

$$\phi_{nt} = (1 - \alpha)^{-1} \ln(\eta_n \lambda_t).$$

The third step yields estimates of p_{nt}, $p_{1nt}^{(s)}$, $\widetilde{p}_{n\tau t+1}$, $\frac{\partial p_{1nt}^{(s)}}{\partial h_{nt}}$, and $\frac{\partial \widetilde{p}_{n\tau t+1}}{\partial h_{nt}}$. We can form the moment conditions:

$$\begin{aligned} m_{1nt}\left(\Theta u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) \;=\;& \sigma \ln\left[p_{nt}^{(N)} / \left(1 - p_{nt}^{(N)}\right)\right] - B_{0t} - z_{nt}' B_1 + z_{nt}' h_{nt} B_2 \\ & + \theta_0\left(1 - l_{nt}^2\right) + \sum_{s=1}^{\rho} \theta_s h_{nt}(l_{nt-s} + \beta^s) \\ & - \sigma \sum_{s=1}^{\rho} \beta^s \ln\left(\frac{1 - p_{1nt}^{(s)(N)}}{1 - p_{0nt}^{(s)(N)}}\right) \\ & - \exp\left((1-\alpha)\phi_{nt}^{(N)}\right) \sum_{\tau=1}^{M} I_{n\tau t}\left[y_{\tau t}\left(h_{nt}, H_{nt-1}, z_{nt}^p, \theta_e^{(N)}\right)\right. \\ & \left. - \gamma_\tau^{(N)} + \beta \gamma_\tau^{(N)} \widetilde{p}_{n\tau t+1}^{(N)}\right] \end{aligned} \tag{76}$$

and

$$\begin{aligned} m_{2nt}\left(\Theta u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) \;=\;& d_{nt}\left\{\sigma \sum_{s=1}^{\rho} \beta^s \left(1 - p_{1nt}^{(s)(N)}\right)^{-1} \frac{\partial p_{1nt}^{(s)(N)}}{\partial h_{nt}}\right. \\ & - z_{nt}' B_2 - 2\theta_0 l_{nt} - \sum_{s=1}^{\rho} \theta_s (l_{nt-s} + \beta^s) \\ & + \exp\left((1-\alpha)\phi_{nt}^{(N)}\right) \sum_{\tau=1}^{M} d_{n\tau t}\left[b_{\tau 1}^{(N)} + 2 b_{\tau 2}^{(N)} h_{nt}\right. \\ & \left.\left. + \;\beta \gamma_\tau^{(N)} \frac{\partial \widetilde{p}_{n\tau t+1}^{(N)}}{\partial h_{nt}}\right]\right\}, \end{aligned} \tag{77}$$

where $\psi^{(N)} = \left(p_{nt}^{(N)}, p_{0nt}^{(s)(N)}, p_{1nt}^{(s)(N)}, \widetilde{p}_{n\tau t+1}^{(N)}\right)$ are the nonparametric second-step estimates and $\Theta_u = (\sigma, \alpha, \beta, B_{01}, \ldots, B_{0T}, B_1, B_2, \theta_0, \ldots, \theta_\rho)$ are the structural parameters left to be estimated.

There are now two sources of errors in evaluating the sample counterparts of (76) and (77). The first is the forecast errors from replacing the expectations of future variables with their realizations. The second is the approximation error that arises from replacing the true values of the conditional choice probabilities, conditional expectation, and time-invariant individual-specific effects with their estimates. Let us define the 2×1 vector $m_{3nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) \equiv \left[m_{1nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right), m_{2nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)\right]'$ and let T_3 denote the set of periods for which the hours and participation equations are valid. Define the vector

$$m_{3n}^{(N)}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) \equiv \left(m_{3n1}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)', \ldots, m_{3nT_3}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)'\right)'$$

as the vector of the idiosyncratic errors for a given individual over time. Define $\Omega_{nt}^{(N)} \equiv E_t\left[m_{3nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) m_{3nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)'\right]$. The off-diagonal elements of $\Omega_{nt}^{(N)}$ are zero because

$E_t\left[m_{3nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right) m_{3nr}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)'\right] = 0$ for $r \neq t$, $r < t$. The 2×2 conditional heteroscedasticity matrix $\Omega_{nt}^{(N)}$ associated with the individual-specific errors, $m_{3nt}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)$, is evaluated using a nonparametric estimator based on the estimated moments, $m_{3nt}\left(\Theta_{1u}^{(N)}, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)$, derived from an initial consistent estimate of $\Theta_{1u}^{(N)}$. The optimal instrumental-variables estimator for $\Theta_u^{(N)}$ is

(78)

$$\Theta_u^{(N)} \equiv \arg\min_{\Theta u} \frac{\sum_{n=1}^{N} m_{3n}^{(N)}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)\left(\Omega_n^{(N)}\right)^{-1} m_{3n}^{(N)}\left(\Theta_u, \Theta_c^{(N)}, \Theta_e^{(N)}, \psi^{(N)}\right)}{N}.$$

.1 Asymptotic Properties

It is well known in the econometric literature that under certain regularity conditions, pre-estimation does not have any impact on the consistency of the parameters in the subsequent steps of a multistage estimation (Newey, 1984; Newey and McFadden, 1994; Newey, 1994). The asymptotic variance, however, is affected by the pre-estimation. In order to conduct inference in this type of estimation, one has to correct the asymptotic variance for the pre-estimation. The method used for correcting the variance in the final step of estimation depends on whether the pre-estimation parameters are of finite or infinite dimension. Unfortunately, our estimation strategy combines both finite- and infinite-dimensional parameters. Combining results from two sources (Newey, 1984; Newey and McFadden, 1994), however, allows us to derive the corrected asymptotic variance for our estimator.

Following Newey (1984), we can write the sequential-moments conditions for the first- and third-step estimation as a set of joint moment conditions:

$$m_n(\Theta_u, \Theta_c, \Theta_e, \psi) = \begin{bmatrix} \left(Y_n - Z_n\Theta_c\right) Z_n^c \\ \left(Y_{n1} - X_{1n}\Theta_{e1}\right) Z_n \\ (Y_{n2} - X_{Mn}\Theta_{eM}) Z_n \\ m_{3n}(\Theta_u, \Theta_c, \Theta_e, \psi) \end{bmatrix},$$

where $\left(Y_n - Z_n\Theta_c\right) Z_n^c$ is the orthogonality condition from the estimation of the consumption equation, $(Y_{n\tau} - X_{n\tau}\Theta_{e\tau})Z_n$ is the orthogonality condition from the estimation of the earnings equation, and $m_{3n}(\Theta_u, \Theta_c, \Theta_e, \psi)$ is the moment conditions from the third-step estimation. Let $\Theta = (\Theta_u, \Theta_c, \Theta_e)'$, with the true value denoted by Θ_0. Note that each element of ψ is a conditional expectation. Redefine each element as $\psi^j(z^j) = f_{z^j}(z^j)E\left[\widetilde{d}_n^j \mid z^j\right]$, where $\widetilde{d}_{nt}^j = [1, d_{nt}]'$ for the estimation of p_{nt}, $\widetilde{d}_{nt}^j = [d_{knt}^{(r)}, d_{knt}^{(r)} d_{nt}]'$ for the estimation of $p_{knt}^{(r)}$, and $\widetilde{d}_{nt}^j = [d_{n\tau t}, d_{n\tau t} d_{n\tau t+1}]'$ for the estimation of $\widetilde{p}_{n\tau t+1}$. Therefore, $\psi^{j(N)}(z^j) = \frac{1}{N}\sum\limits_{n=1}^{N} \widetilde{d}_n^j J_{\delta_N}(z^j - z_n^j)$. The conditions below ensure that $\psi^{(N)}$ is close enough to ψ_0 for N large enough, in particular that $\sqrt{N}\left\|\psi^{(N)} - \psi_0\right\|^2$ converges to zero.

> **A3:** *There is a version of $\psi_0(z)$ that is continuously differentiable of order κ, greater than the dimension of z and $\psi_{10}(z) = f_z(z)$ is bounded away from 0.*

A4: $\int J(u)\,\mathrm{d}u = 1$ *and for all* $j < \kappa$, $\int J(u)\left(\bigotimes_{s=1}^{j} u\right)\mathrm{d}u = 0$.

A5: *The bandwidth,* δ_N, *satisfies* $N\delta_N^{2\dim(z)}/(\ln(N))^2 \to \infty$ *and* $N\delta_N^{2\kappa} \to 0$.

A6: *There exists a* $\Psi(\omega)$, $\epsilon > 0$, *such that*

$$\|\nabla_\Theta m_n(\omega.\Theta,\psi) - \nabla_\Theta m_n(\omega.\Theta_0,\psi_0)\| \leq \Psi(\omega)\left[\|\Theta-\Theta_0\|^\epsilon + \|\psi-\psi_0\|^\epsilon\right]$$

and $E[\Psi(\omega)] < \infty$.

A7: $\Theta^{(N)} \to \Theta_0$ *with* Θ_0 *in the interior of its parameter space.*

A8: (Boundedness)

(i) Each element of $m_n(\Theta,\psi)$ is bounded almost surely: $E[\|m_n(\Theta,\psi)\|^2] < \infty$;

(ii) $E[Z'_{n'}Z_n] < \infty$, $E[X'_{\tau n}Z_n] < \infty$, $E[\exp((1-\alpha)\phi_{nt})] < \infty$, $E[z_{nt}] < \infty$, $E[y_{\tau t}(h_{nt}, H_{nt-1}, z^p_{nt}, \theta_e)] < \infty$, $\gamma_\tau < \infty$, $E[\nabla_{h_{nt}}\widetilde{p}_{n\tau t+1}] < \infty$, $E[X_{n\tau}] < \infty$ for $\tau = 1,2$;

(iii) $p_{nt}, p^{(r)}_{knt}, \widetilde{p}_{n\tau t+1}, \in (0,1)$, for $k \in \{0,1\}$, $r = 1,\ldots,\rho$, and $\tau = 1,2$;

(iv) $E[\nabla_h f_{z^j}(z^j)] < \infty$ and $E[\nabla_h E[\widetilde{d}^j_n \mid z^j]] < \infty$;

Theorem 1 *Under A1–A8 and* $\Phi(\omega)$, *defined below,*

$$\sqrt{N}\left(\Theta^{(N)} - \Theta_0\right) \Rightarrow N(0,\Sigma(\Theta_0)),$$

where

$$\begin{aligned}\Sigma(\Theta_0) &= E\left[\nabla_\Theta m_n(\omega)\Omega_n^{-1}\nabla_\Theta m_n(\omega)'\right]^{-1} \\ &\quad \times E\left[\nabla_\Theta m_n(\omega)\Omega_n^{-1}\{m_n(\omega)+\Phi(\omega)\}\{m_n(\omega)+\Phi(\omega)\}'\Omega_n^{-1}\nabla_\Theta m_n(\omega)'\right] \\ &\quad \times E\left[\nabla_\Theta m_n(\omega)\Omega_n^{-1}\nabla_\Theta m_n(\omega)'\right]^{-1}.\end{aligned}$$

Assumptions A3–A8 are standard in the semiparametric literature, see Newey and McFadden (1994) for details. One can now use Theorem 1 to calculate the standard for all the parameters in our estimation.

The proof of Theorem 1 will follow from checking the conditions for Theorem 8.12 in Newey and McFadden (1994). We Assume A1–A7 and add the following additional assumption.

Proof of Theorem 1. We first check the various boundedness requirements of Theorem 8.12 in Newey and McFadden (1994). By assumption A8(i), we have that $E[\|m_n(\Theta,\psi)\|^2] < \infty$. It obvious by inspection that $m_n(\Theta,\psi)$ is continuously differentiable in Θ and by A8(ii–iv) that $E[\nabla_\Theta m_n(\Theta,\psi)] < \infty$. Additionally, $\nabla_{\psi\psi}m_n(\Theta_0,\psi_0)$ is also bounded: $E[\|\nabla_{\psi\psi}m_n(\Theta_0,\psi_0)\|] < \infty$.

Second, consider a pointwise Taylor expansion for the j^{th} element of m_n,

$$\begin{aligned}m^j(\omega,\psi) &= m^j(\omega,\psi_0) + \nabla_\psi m^j(\omega,\psi_0)(\psi(z)-\psi_0(z)) \\ &\quad + (\psi(z)-\psi_0(z))'\nabla_{\psi\psi}m^j(\omega,\psi_0)(\psi(z)-\psi_0(z)) + o(\|\psi(z)-\psi_0(z)\|^2),\end{aligned}$$

where the norm over ψ is the sup-norm. Next, note that

$$\begin{aligned}
&\left|m^j(\omega,\psi) - m^j(\omega,\psi_0)\nabla_\psi m^j(\omega,\psi_0)(\psi(z)-\psi_0(z))\right| \\
&\qquad\qquad\qquad\leq \left\|(\psi(z)-\psi_0(z))'\nabla_{\psi\psi}m^j(\omega,\psi_0)(\psi(z)-\psi_0(z))\right\| \\
&\qquad\qquad\qquad\quad + o(\|\psi(z)-\psi_0(z)\|^2) \\
&\qquad\qquad\qquad\leq \|\psi-\psi_0\|^2\left\|\nabla_{\psi\psi}m^j(\omega,\psi_0)\right\| + o(\|\psi-\psi_0\|^2),
\end{aligned}$$

using the triangle inequality and the Cauchy-Schwartz inequality. Therefore, for $\|\psi-\psi_0\|$ small enough,

$$\left|m^j(\omega,\psi) - m^j(\omega,\psi_0) - \nabla_\psi m^j(\omega,\psi_0)(\psi(z)-\psi_0(z))\right| \leq \|\psi-\psi_0\|^2\left\|\nabla_{\psi\psi}m^j(\omega,\psi_0)\right\|.$$

So that

$$\begin{aligned}
\|m(\omega,\psi) - m(\omega,\psi_0) - \nabla_\psi m(\omega,\psi_0)(\psi(z)-\psi_0(z))\| &\leq \|\psi-\psi_0\|^2\|\nabla_{\psi\psi}m(\omega,\psi_0)\| \\
\|m(\omega,\psi) - m(\omega,\psi_0) - \nabla_\psi m(\omega,\psi_0)(\psi(z)-\psi_0(z))\| &\leq \|\psi-\psi_0\|^2\|\nabla_{\psi\psi}m(\omega,\psi_0)\|
\end{aligned}$$

Hence $\Gamma(\omega,\psi-\psi_0) = \nabla_\psi m(\omega,\psi_0)(\psi(z)-\psi_0(z))$ and $\Psi(\omega) = \|\nabla_{\psi\psi}m(\omega,\psi_0)\|$. It follows that both $\Gamma(\omega,\psi-\psi_0)$ and $\Psi(\omega)$ are bounded from the boundedness conditions established above.

Next we establish the form of the influence function. Note that we have

$$\begin{aligned}
\int \Gamma(\omega,\psi)F_0(\mathrm{d}\omega) &= \int f_z(z)E[\nabla_\psi m(\omega,\psi_0)\mid z]\psi(z)\,\mathrm{d}z \\
&= \int \upsilon(z)\psi(z),
\end{aligned}$$

where $\upsilon(z) = f_z(z)E[\nabla_\psi m(\omega,\psi_0)\mid z]$. So, by the arguments on page 2208 of Newey and McFadden (1994), we have the influence function for $m(\omega,\psi^{(N)})$:

$$\begin{aligned}
\Phi(\omega) &= \upsilon(z) - E\left[\upsilon(z)\widetilde{d}\right] \\
&= f_z(z)E\left[\nabla_\psi m(\omega,\psi_0)\mid z\right] - E\left[f_z(z)E[\nabla_\psi m(\omega,\psi_0)\mid z]\widetilde{d}\right].
\end{aligned}$$

Again by the boundedness of $\nabla_\psi m(\omega,\psi_0)$, it follows that $\int\|\upsilon(z)\|\,\mathrm{d}z < \infty$. Finally Assumption A7 guarantees that the Jacobian term converges. ■