

Cognitive inconsistencies and non-symmetric friendship ^{*}

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Abstract

Non-reciprocated relationships, such as all workers knowing the president of the company but only a few of the workers being known by the president, and non-symmetric relationships, such as workers thinking that they know the president and thinking that the president does not know them, are endemic to most social situations. While such inconsistencies may be expected in relationships such as giving advice and lending money, they are rarely expected to occur in seemingly symmetric relationships such as friendship. Nevertheless, they do. We suggest that research in this area has been hampered by the confused language used for describing 'symmetries' and 'non-symmetries'. We present a framework for thinking about these relations that clearly distinguishes cognitive inconsistencies and non-symmetric and non-reciprocated relations. Then, we employ this framework and structural theory to suggest that owing to cognitive inconsistencies, any interaction-based relationship, including friendship, can potentially be non-symmetric. We examine a series of hypotheses concerning interaction and interaction-based behaviors that derive from this theory using friendship relations. We find that we are able to predict both who is friends with whom, non-symmetry in friendship, and non-reciprocities in the expectation for and recall of friendship.

0. Introduction

The study of friendships has been a focus of researchers for several decades (Moreno 1934; Newcomb 1961; Bell 1981; Fischer 1982; Hallinan and Williams

^{*} This work was supported in part by a grant provided by the NSF under grant No. IST-8607303.

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1987; Wellman 1988; Krackhardt 1992). Part of this work has uncovered structural consequences of friendships, but a good portion of this research has explored the more basic question of what constitutes a friendship link. Throughout this research a common thread has been the assumption that friendships are inherently symmetric. This assumption, however, is suspect. Ego biases (Kumbasar 1993, 1994) and other factors may affect the degree of symmetry in these networks. Further, research in this area has been hampered by the lack of a consistent vocabulary for describing non-symmetries. The use of an inconsistent vocabulary has resulted in researchers ignoring fundamental cognitive differences in the basis for non-symmetry in a dyadic relation such as friendship. In this paper, we present a framework for discussing cognitive inconsistencies and non-symmetry with respect to relationships. We then demonstrate that there is a systematic cognitive basis for the existence of non-symmetry in friendship.

1. A framework for discussing cognitive inconsistency and dyadic non-symmetry

In discussing the relations between individuals, it is important to distinguish between the tie at the sociometric level and the tie at the full ordered pairwise/paired comparison (Krackhardt style data) level. At the sociometric level we are concerned with ego's self report of his or her relations with others. At the sociometric level the relation is of the form R_{ij} such that R_{ij} represents Person i sends Relation R to Person j . In contrast, when full ordered pairwise/paired comparisons are used we are concerned with each person's report on the relations between all other pairs of individuals, whether or not ego is actually one of the members of the pair. Thus, at the Krackhardt style data level the relation is of the form R_{ijk} such that $R[ijk]$ represents that Person k perceives that Person i sends Relation R to Person j (Krackhardt 1987a).

In either type of data inconsistencies can arise. Further, procedures for compressing Krackhardt style cognitive-social network data into sociometric data may induce certain types of inconsistencies. In the case of sociometric data, inconsistencies exist just in the case $R_{ij} \neq R_{ji}$. Whether these inconsistencies result from non-symmetries or non-reciprocities at the cognitive level depends on how the data was collected, as will be seen. However, for Krackhardt style data, there are many different types of inconsistencies. In Krackhardt style data inconsistencies, specifically cognitive inconsistencies, occur for a pair of individuals i and j , where R_{iji} does not equal any of the following: R_{ijj} ; R_{jii} ; R_{jij} .

These various types of inconsistencies have been confused. This confusion exists at two levels. First, words that describe different types of inconsistency are often used interchangeably: e.g. asymmetry¹ and non-reciprocity. For example, Holland

¹ In contrast, and in keeping with graph theory, we reserve the term asymmetry to refer to a property of a matrix. A matrix is asymmetric in the case where for every (i, j) dyad either $(R_{ij} = 1 \text{ and } R_{ji} = 0)$ or $(R_{ij} = R_{ji} = 0)$.

and Leinhardt (1979: 66) note that the central question they are trying to answer is whether there is "... anything in sociometric data besides... a tendency for choices to be reciprocated." They answer this question by looking at the u-m, a, n distribution and note that (1979: 72) "from the triad census t we can obtain ... the number of mutual, m, asymmetric, a, and null, n, dyads in the di-graph." Second, theoretical discussions often move back and forth between the sociometric and Krackhardt style data level without denoting the level at which the argument holds.

Before proceeding, it is worth separating out the different types of inconsistencies that can arise at the Krackhardt style data level. We are concerned only about those inconsistencies in which the perceiver is directly involved. Consequently, there are only four 'relational primitives' for any (i, j) pair. In terms of friendship, these primitives are:

- (1) R_{iji} : whether i perceives that self (i) considers other (j) a friend;
- (2) R_{ijj} : whether j perceives that other (i) considers self (j) a friend;
- (3) R_{jii} : whether i perceives that other (j) considers self (i) a friend;
- (4) R_{jjj} : whether j perceives that self (j) considers other (i) a friend.

For any (i, j) dyad, then, there are the following 16 outcomes (where, e.g., $R_{ijk} = 1$ implies that k perceives that i considers j a friend; $R_{ijk} = 0$ implies that k perceives that i does not consider j a friend):

	R_{jji}	R_{ijj}	R_{jii}	R_{jjj}
A	0	0	0	0
B	0	0	0	1
C	0	0	1	0
D	0	0	1	1
E	0	1	0	0
F	0	1	0	1
G	0	1	1	0
H	0	1	1	1
I	1	0	0	0
J	1	0	0	1
K	1	0	1	0
L	1	0	1	1
M	1	1	0	0
N	1	1	0	1
O	1	1	1	0
P	1	1	1	1

Of these outcomes, only the A and P outcomes are completely consistent; the remaining 14 possible outcomes show some sign of inconsistency or disagreement about whether i and j are friends. Inconsistencies occur any time any two of the

relational primitives above disagree. There are six possible disagreements (four primitives taken two at a time), each representing one of four possible types of disagreements.

Primitive inequality	Type of disagreement
$R_{iji} \neq R_{ijj}$	Non-confirmation
$R_{iji} \neq R_{jii}$	Non-symmetry
$R_{iji} \neq R_{jij}$	Non-reciprocity
$R_{ijj} \neq R_{jii}$	Reflected non-reciprocity
$R_{ijj} \neq R_{jij}$	Non-symmetry
$R_{jii} \neq R_{jij}$	Non-confirmation

Non-confirmations reflect a lack of agreement between the two parties about the existence of a tie from i to j (or vice versa). Non-symmetries stem from one perceiver's inconsistency in his or her belief that the tie is reciprocated (resulting in a non-symmetric matrix within his or her cognitive map of the structure). Non-reciprocities (or reflected non-reciprocities) represent a difference between the two parties each sending (or receiving) a relation to (or from) the other.

Depending on how sociometric data are collected, then one of these inconsistencies may underlie that data. For example, if the sociometric network is the result of asking each individual to state for each other individual whom i has relation R to, then resultant inconsistencies in the sociometric data are due to $R_{iji} \neq R_{jij}$. That is, inconsistencies in the sociometric data are due to non-reciprocities in the corresponding locations in the Krackhardt style data.

Taking account of these different bases for inconsistencies at the cognitive level, i.e. in Krackhardt style data, can actually help clarify various discussions in the literature. We argue that Heider would be most concerned about non-symmetries, Davis would be most concerned about non-reciprocities, and non-confirmations are often the methodological concerns of researchers who wish to uncover actual relations from one party to the next (e.g. Krackhardt 1990).

Let us consider these first two claims in more detail. Heider's formulation of inconsistency is not based on actual balance, but rather based on people's perceptions of the reciprocated liking: i.e. on non-symmetry. That is, according to a strict interpretation of Heider's theory, it is not the case that if Person 1 likes Person 2, and Person 2 likes Person 1 that balance exists; rather, balance exists if Person 1 thinks that Person 1 likes Person 2 and Person 1 thinks that Person 2 likes Person 1. In this case, an inconsistency (or lack of balance) exists in the case where $R_{iji} \neq R_{jii}$. This type of inconsistency is actually a case of non-symmetry in Krackhardt style data. On the other hand, Davis's idea of balance is more tied to the traditional notion of reciprocity: Person 1 likes Person 2, and Person 2 likes Person 1. In this case, the person doing the 'liking' decides whether the relationship exists. That is, the relationship tie from Person 1 to Person 2 is determined by Person 1's perception, whereas the relationship tie from Person 2 to Person 1 is determined

by Person 2's perception. In Davis's view, an inconsistency occurs when $R_{iji} \neq R_{jij}$. This is actually a case of non-reciprocity in Krackhardt style data.

The implications and source of the inconsistencies in Krackhardt style data are different. For example, a Heiderian inconsistency or imbalance is observed in outcome J (1 0 0 1) above: Person i sees self as considering Person j a friend, but does not see Person j reciprocating that friendship; similarly, Person j sees self as considering Person i a friend, but does not see Person i reciprocating that friendship. This outcome is not inconsistent from the Davis perspective, however, since both parties see themselves as friends of the other.

As we noted earlier, it is often the goal of a research agenda to translate these primitives using a set of combining rules into a two-dimensional relational matrix (Krackhardt 1987a). For example, one rule might be that $R_{ij} = 1$ iff $R_{iji} = 1$ (this is the one used by Davis and most researchers in their studies). Another might be that $R_{ij} = 1$ iff $R_{iji} = 1$ and $R_{ijj} = 1$ (Krackhardt 1990; Krackhardt and Kilduff 1990). These rules may be encoded in terms of the set of mutually exclusive outcomes of the four primitives given above. For example, the former rule above may be rewritten as

$R_{ij} = 1$ iff Outcome is from set (I J K L M N O P), else 0,

whereas, the latter can be written as

$R_{ij} = 1$ iff Outcome is from set (M N O P), else 0.

If we restrict ourselves to binary outcomes of $R_{ij} = 0$ or 1, we can enumerate all possible rules from this process by counting all possible subsets of the outcome set above. This number is equal to $\binom{16}{1} + \binom{16}{2} + \binom{16}{3} \cdots + \binom{16}{15}$, for a total of 65 534 rules that could be used to reduce the primitives to a R_{ij} matrix.

Which of the 65 thousand rules should be used is determined by the theory one is exploring. Frequently, researchers employ rules without being clear why they have chosen that particular rule. Sometimes rules are chosen because only one or two of the four primitives is available in the data collected. For example, in the Davis data sets only R_{iji} and R_{jij} are available. Thus for Davis, $R_{ij} = 1$ iff $R_{iji} = 1 \wedge R_{jij} = 1$ iff $R_{jij} = 1$. In this case, non-symmetry at the sociometric level occurs just in case non-reciprocity exists at the Krackhardt style data level. It is important to note that in the study of friendship most researchers collect sociometric data in a fashion similar to Davis (Krackhardt 1987a). That is, most collect data of the form R_{iji} and R_{jij} . Thus, for most researchers the non-symmetries observed at the sociometric level are actually non-reciprocities at the Krackhardt style data level. In writing up their results, these researchers typically use either or both terms, non-symmetry (or asymmetry) and non-reciprocity. In contrast, in the current study, we have data on all four primitives and will set forth several different rules testing different models of inconsistency.

2. Revisiting the nature of friendship

A common thread in the research on friendship has been the assumption that friendships were inherently symmetric in nature. Such assumptions were based on

an appeal to common sense and experience (e.g. Bell 1981), on empirical observations (e.g. Newcomb 1961), and on theory (e.g. Heider 1958; Davis 1968). These latter two scholars demonstrate the pervasiveness of this idea, as they come to the same conclusion from two very different perspectives. Davis, the sociologist, notes that friendship entails time spent together in the same proximity; since time and proximity are physically constrained to be symmetric, then the friendship tends toward symmetry (he further predicts transitivity with the same logic). Heider, the psychologist, makes the same prediction from a very different base. He argues that lack of symmetry in 'liking' produces imbalance and discomfort, and therefore "... it tends to become symmetrical; i.e., a balanced state exists if both (p L o) and (o L p) are true" (p. 205). Indeed, it is not infrequently that data are collected that presume symmetry; i.e. the researchers collect data in a way that does not permit non-symmetric relations to be recorded (e.g. Freeman et al. 1988).

The problem with this tendency toward symmetry is that too many non-symmetries are noted in the real world. A review of 1000 sociometric matrices by Davis and Leinhardt (1972) forced them to alter their model to include a preponderance of non-symmetric ties (these are actually non-reciprocated at the Krackhardt style data level.) Hallinan (1978) found a sizable number of non-symmetric/non-reciprocated friendship links among school children. Moreover, she found many of these links persisted, despite her best efforts to explain them away as temporary aberrations.

To reconcile these discrepancies, several scholars have appealed to psychology, suggesting that while non-symmetry may exist in the actual world (sociometric level), symmetry is the dominant model in the mind (Krackhardt style data level). Perhaps one of the most convincing lines of work that point to inherent symmetry in friendship relations was started by DeSoto (1960) and extended by Freeman (1992). DeSoto asked subjects to 'learn' a set of relationships among a group of four hypothetical people. He demonstrated that subjects took about 50% longer to 'learn' an observed interaction that was non-symmetric (and transitive) than one that was symmetric (and transitive). Freeman (1992) replicated these studies precisely and discovered that subjects made errors in their learning trials by filling in relationships that would make them symmetric.

In his perhaps most important extension of this work, Freeman (1992) argues that people have a strong tendency to view the structure of friendships around them as being symmetric and transitive, yielding an ultrametric structure. While he notes that 'real' friendship structures are not often ultrametric, the persistence in the literature is due in large part to the fact that people, including scientists, cognitively alter their perception of the social environment to minimize non-symmetries (and intransitivities). In other words, according to Freeman, individuals are cognitively forcing R_{ij} to be equal to R_{ji} . That is, individuals insist on seeing symmetries even when they do not exist.

While this work is compelling in empirical support and in logic, we point to two issues that are left unresolved. First, what the DeSoto/Freeman studies have definitively shown is that it is easier to learn friendship structures that

are symmetric and transitive.² That does not mean that people cannot learn non-symmetric ones, nor that they refuse to admit that they exist. In fact, in both the Freeman and DeSoto studies, people did learn the non-symmetric relations – it just took longer. In the real world of social living, people have hours, weeks, years to get to learn the structures in which they are embedded. Despite the DeSoto/Freeman findings, it is reasonable to expect that they will in fact learn and perceive non-symmetric friendship relations.

Second, the Freeman and DeSoto experiments were conducted in the laboratory. There were no other cues of history or complexity to assist the subjects in seeing anything but what is cognitively simpler, that is a symmetric and transitive order. It could be argued that the real world is much messier, and non-symmetries might be apparent to the individual perceiver if given an opportunity to report it.

We propose that non-symmetric and non-reciprocated friendships are an important and omnipresent part of the social world (sociometric level), even in the minds of people who are experiencing it (Krackhardt style data level). We propose that Carley's constructural theory (1990, 1991, forthcoming) can be used to predict when non-symmetries at the sociometric level will be present. Using this model, we derive predictions regarding the existence, initiation, and recall of friendship relations. We then test these predictions using cognitive social structure data collected by Krackhardt and Kilduff (1990) on friendship in a distribution firm referred to as PACDIS.

We suggest that friendship is a perceived relationship occurring as a byproduct of individual interaction and communication. Individuals can differ in their perception of whom they interact with and in their perceptions of who interacts with them; i.e. R_{iji} may not equal R_{jij} , and R_{iji} may not equal R_{ijj} . Friendship can similarly be perceived as either symmetric or non-symmetric. Indeed the English language is fraught with terms that indicate that people recognize non-symmetries in affective relationships: e.g. obsequious and unrequited love. This contrasts with the view of friendship as an exchange relationship in which, ultimately, friendships will tend toward symmetry. For example, Hallinan (1978: 194) proposed a four-stage exchange model of friendship formation in which ultimately all friendships are symmetric:

- (1) individual i decides to seek out individual j as a friend;
- (2) individual i behaves in a way that offers friendship to j ;
- (3) individual j recognizes the offer of friendship from i ;
- (4) individual j accepts the offer and reciprocates with friendship.

Hallinan notes that during Stages 2 and 3, the friendship between i and j is non-symmetric, i.e. i extends friendship to j but j does not extend friendship (yet) to i . We note that this non-symmetry at the sociometric level is due to non-reciprocity at the Krackhardt style data level (i.e. $R_{iji} = 1$ and $R_{jij} = 0$). While

² It should be noted that DeSoto and Freeman both demonstrated that "influence" relations were more easily learned when they were non-symmetric.

Hallinan's model posits that non-symmetries exist, it also suggests that the non-symmetry is only a temporary condition, and that in the near future the non-symmetry will disappear: either j will accept the friendship offer of i or i will withdraw the offer.

Hallinan's (1978: 207) data supported her contention that, at least among sixth graders, non-symmetric friendships were less stable than symmetric ones. In later work with her colleagues, she found some support for the tendency for friendships to be reciprocated (Hallinan and Williams 1987; Hallinan and Kubitschek 1988). Of more interest to us here, however, is the less-emphasized fact that there were a substantial number of non-symmetric friendships observed. For example, Hallinan and Williams (1987) found that, on average, 53% of the friendships in their sample were not reciprocated. Further, while non-symmetric relations may have been less stable than symmetric ones in Hallinan's study (1978), they were hardly negligible, lasting virtually half as long as the symmetric friendships. Hallinan's major explanation for the existence of these non-symmetries was that they are likely to occur when a lower status person makes a friendship overture to a higher status person (1978: 194). While this may explain why the non-symmetry is initiated, it does not explain its substantial duration.

In her later work, Hallinan has argued that status non-symmetry can result in stable friendship non-symmetries: "... There is also a tendency toward non-symmetry in friendship choices. Those who are more popular are typically the group members who are held in higher esteem by their peers because they possess some respected characteristic, talent, skill or resource. The result is the emergence of a status hierarchy in friendship relationships that is characterized by a tendency toward non-symmetry in social networks." (Hallinan and Kubitschek 1988: 83). Indeed, they found that 38% of the A-B friendships in their triad study were not reciprocated, but their variable POPULARITY did not significantly explain the results as predicted.

In contrast to this exchange perspective, we take a constructural perspective – a sociocognitive perspective in which friendship is a perceived relationship derived by the individual from his or her cognitive structure. The constructural perspective (Carley 1990, 1991, forthcoming; Kaufer and Carley 1993) provides an integrated and dynamic view of cognition, interaction, friendship, and social structure in which social structure through interaction affects cognition (and friendship), and cognition motivates interaction and hence alters social structure (and friendship). This perspective combines the view that individual's cognitive structures evolve in response to their position in the social structure (House 1977; Stryker 1977; Cartwright 1979; Boyd and Richerson 1989; Friedkin 1990a, 1990b; Morgan and Schwalbe 1990) with the view that evaluation of the cognitive structure provides the basis for action and interaction (and friendship) (Emerson 1962; Mead 1962; Garfinkel 1968; Granovetter 1973; Collins 1986; Marsden 1988; Turner 1988). The constructural perspective treats the social network and the distribution of resources as dynamic entities which are continually adjusted (or constructed) as individuals interact and exchange information. Under this perspective, individuals evaluate and determine their position in the social network by determining for

