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Author(s): Frederic Schick
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DUTCH BOOKIES AND MONEY PUMPS*

CONCLUSIVE arguments in philosophy are rare, so any such argument we find we prize. If it is not only conclusive but clever, all the better for it. The Dutch book arguments of the theory of probability seem to fill the bill. The idea behind them was first presented by Frank Ramsey[1] and by Bruno deFinetti.[2] It was fully developed in three independent papers in 1955.[3] The arguments offer us something worth having; they offer a rationale for the whole theory. They have now become a settled part of the literature.

I want to show that they all fail. They seem to do the job only because of an assumption they all take for granted, an assumption that may often be false. Where this assumption is dropped, the arguments no longer work. Very similar arguments have been offered in support of some basic principles of preference, and these fail too, and in the same way. The moral is not that our theories of probability and of preference are in any trouble, but only that they are not as easy to justify as is believed.

I

A Dutch book argument has this form. There are several possible bets—say that there are three. Suppose I am willing to pay x for the first, y for the second, and z for the third. The sum of these prices I am willing to pay is $x + y + z$, but the bets at issue are such that, come what may, I will win *less* than this sum if I place all the bets at these prices. I can therefore be made a sap of by any bookie who sells me the three bets together. What puts me in this fix are the probabilities that I set, for it is these (along with the stakes) that determine what I will pay. My probabilities are thus jointly improper: they can be said to be *incoherent*.

* I am indebted to Isaac Levi and to Teddy Seidenfeld for some helpful comments on an earlier draft.

[1] "Truth and Probability," in his *The Foundations of Mathematics* (London: Routledge & Kegan Paul, 1931); reprinted in Henry E. Kyburg, Jr. and Howard Smokler, eds. *Studies in Subjective Probability* (New York: Wiley, 1964).

[2] "Sul significato soggestivo della probabilità," *Fundamenta Mathematicae,* XVII (1931): 298–329, and "La Prévision: Ses lois logiques, ses sources subjectives," *Annales de l'Institut Henri Poincaré,* VII (1937): 1–68. An English translation of the latter appears in Kyburg and Smokler, *op. cit.*

[3] Abner Shimony, "Coherence and the Axioms of Confirmation," *Journal of Symbolic Logic,* XX, 1 (March 1955): 1–28. R. Sherman Lehman, "On Confirmation and Rational Betting," *ibid.,* XX, 3 (September 1955): 251–262. John G. Kemeny, "Fair Bets and Inductive Probabilities," *ibid.,* XX, 3 (September 1955): 263–273.

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Consider a specific case involving the mutually exclusive propositions h and k. Suppose I set the probability $p(h)$ on h, $p(k)$ on k, and $p(h \vee k)$ on $h \vee k$. This means that, where the stake S is small, I am willing to pay $p(h)S$ for the bet that would get me S if h and otherwise get me nothing, and am willing to pay $p(k)S$ for the bet that would get me S if k and otherwise get me nothing. Also that I am willing to pay $-p(h \vee k)S$ for the bet that would get me $-S$ if $h \vee k$ and otherwise nothing. (Being willing to pay $-x$ means demanding a payment of no less than x, and a gain of $-x$ is a loss of x.)[4]

What if I bought all these bets together? There are three possibilities: either h is true, or k, or neither. If h is true, I would win the first bet, getting S, and also the third, getting $-S$. If k, I would win the second bet, getting S, and also the third, getting $-S$. In both these cases, my total gains would be zero. If neither h nor k, I would win no bet and again would gain zero. So the three bets would yield me nothing, come what may. The sum of the prices I am willing to pay is $(p(h) + p(k) - p(h \vee k))S$—where $p(h \vee k) < p(h) + p(k)$, this is *more* than zero. Therefore, where $p(h \vee k) < p(h) + p(k)$, I would win less, come what may, than I now am willing to pay. Put $-S$ for S throughout above, and the same is true where $p(h \vee k) > p(h) + p(k)$. So I can be played for a fool by any sharp bookie unless neither inequality holds. I can be played for a fool unless $p(h \vee k) = p(h) + p(k)$ for all mutually exclusive h and k. We thus arrive at the addition principle of probability theory. (An analogous argument yields the multiplication principle.)

This analysis has a 'therefore' in it. How did it get in there? It entered via the unspoken assumption that I am willing to pay

$$(p(h) + p(k) - p(h \vee k))S$$

for the three bets together, that the value I set on them together is the sum of the values I set on them singly. This, however, is not always true—it isn't always true *of me*. Nor is it always true of others. The value people set on three items together (three bets, arrangements, propositions, whatever) need not be the sum of the values of these same three items. It may be greater or less than this sum. In the typical case, it is less.

The unspoken assumption is that of value additivity. We could put it another way too. We could say that the assumption is that the bets

[4] It will be convenient to think of the payments and gains and losses in terms of money, but the general analysis speaks of utilities only. For the general thesis, put 'utility' for every instance of 'value' below. (Other changes must then be made too, but all of them are marginal.)

are value-wise independent, that the value the agent sets on bet A is the same whether or not he thinks bet B is in effect, and the same also whether or not he thinks both B and C are in effect. The assumption of independence is that, where I know my bet portfolio, the values I set on new possible bets are not affected by this.[5]

Such independence can't be taken for granted. Where the stakes are large and I know I am committed to B, the highest price I would pay for A is sure to be less than it otherwise would be, for the thought of losing both bets would make me more averse to the risk. In the situations considered here the stakes are small, but the point still holds: the values of the bets need not be independent. Suppose that the probabilities I set do not conform to the principles; say that

$$\mathrm{p}(h \vee k) \neq \mathrm{p}(h) + \mathrm{p}(k)$$

though h and k are mutually exclusive. I will then see that the bets together could yield me less than the sum of their costs. Perhaps I know there are bookies around who are also wise to this. I would be a predictable loser if I here set independent values—if my values were additive. Since I don't want to lose, the values I set are *not* independent.

The lurking presence of bookies in fact is not essential to this. (It is not essential to the argument we started with either.) Suppose that there are no bookies. No bets could then be placed, so I could not lose. But the value I set on a bet is what I would give to put it into effect, the most I would give *if* there were takers. There may not be any takers. Still, I don't want my values to be such that, if there were, I would play the fool. So I don't set such values. That is, my values are not independent.

The values a person sets are typically not independent (or additive). Where his probabilities don't conform to the principles, prudence indeed requires that his values *not* be independent. This may be worth noting, but it does not take us far. We can never assume that a person's probabilities go against the principles. So we cannot argue that his values had better not be independent. The converse of this holds too: since we cannot assume that the agent's values are independent, we cannot argue that his probabilities must conform to the principles.

[5] Additivity and independence here come to the same because we are speaking of the *incremental* value of A given B, of the value the agent sets on A & B in excess of that which he sets on B. Let this be written $\mathrm{v}(A/B)$; then $\mathrm{v}(A/B) = \mathrm{v}(A \,\&\, B) - \mathrm{v}(B)$. The incremental value of A given B is independent of B where $\mathrm{v}(A/B) = \mathrm{v}(A)$. This implies additivity, $\mathrm{v}(A \,\&\, B) = \mathrm{v}(A) + \mathrm{v}(B)$, and vice versa.

Strictly, a blanket assumption of independence is not needed for the argument. A weaker assumption would serve. Let us return to the addition principle. We might suppose only that, for every pair of propositions, there is at least one betting situation of the relevant sort in which the agent's values are independent. In order for him to avoid being used, the probabilities of the propositions involved would have to satisfy the principle in these situations. This could then be generalized. The phrase 'in these situations' is redundant; the probabilities that a person sets can't vary with the circumstances he might now be in. The relations between his probabilities thus cannot vary either, and so the principle would hold throughout.

What about this weaker assumption? In a relevant situation (for the addition principle), the agent is betting, say, on h, on k, and on $\sim(h \vee k)$ together. Are there always some bets of this sort on which he sets independent values, and value-independent bets of this sort for every other proposition pair? I see no grounds for thinking there are. We have not made any progress. The weaker assumption has no better standing than the more comprehensive one.

Suppose that the assumption of independence (or additivity) is dropped—that even the weaker assumption is dropped. What does the argument presented then prove? That is, what follows from only the fact that, were I to place certain bets together, I would win less, come what may, than the sum of what I would pay for these bets singly? What follows is not that the probabilities that determine the values I set on these bets are improper, but only that *either* they are improper *or* the values I set on the bets are not independent, that I value the conjunction of the bets at less than the sum of their separate values. Where I am aware that my probabilities are Dutch-bookable, I must *either* change these probabilities *or* make sure that the bets are not value-wise independent—more precisely, that their values are *sub*-additive for me (are *sufficiently* sub-additive). Since Dutch book arguments all go beyond this point and conclude that the probabilities must be changed, all these arguments need the assumption we have just rejected. Without that assumption, none of these arguments work.[6]

A terminological note. The above takes Dutch-bookability to have to do with sets of several bets. This is the race-track usage; philo-

[6] Some readers may hope to replace the assumption with the thesis that a person's probabilities are invariant over the values he might have. They may then want to argue that, since our values *might* be independent, our probabilities had better be coherent, for otherwise we could in that case be had. But could one not equally argue that, since our probabilities might be incoherent, our values had better not be independent?

sophers have generalized it. They speak of Dutch-bookability also in the one-bet case, where a person is willing to pay more for some single bet than he could win. The avoidance of Dutch-bookability in the single-bet case requires that $0 \leq p(h) \leq 1$ and also that $p(h) = 1$ where h is logically true. My point about independence does not reflect on the arguments that establish this. It bears only on the arguments dealing with several-bet Dutch-bookability. These however, are central to any analysis of the Ramsey sort; so it suffices here to show that these arguments fail.

II

Look now at a closely related kind of situations. In this, a set of arrangements is offered, not together, but in sequence. As in a Dutch book case, I am willing to pay x for the first, y for the second, and z for the third, and these are in fact the prices charged. The sum of these prices I am willing to pay is $x + y + z$, but the arrangements are such that my benefit if I accept all the offers must be less than that sum. Again I have left myself open to a shabby abuse.

This is the premise of the familiar "money pump" arguments, the first of which appeared in 1955 in a paper by Donald Davidson, J. C. C. McKinsey, and Patrick Suppes.[7] (The authors attribute the idea to Norman Dalkey.) Suppose that you prefer B to A, C to B, and A to C, these three propositions being pairwise exclusive. You believe that A holds. Let your bookie-exploiter now enter and offer to undo A and to set up B instead, for a small consideration—a dollar will do. You give him a dollar and now expect B. He then offers to cancel B and to guarantee C, for another dollar. You know he can do it, so you give him the dollar. He then offers to cancel C and to guarantee A, again for a dollar, etc. The suggestion is that he will bleed you dry and that it serves you right for having cyclical preferences. (Your going bankrupt is not essential. You will already be looking foolish after the very first cycle, for you will have paid good money to get to be where you were at the start; you will have paid a positive price for a zero benefit.)

Here the conclusion proposed is that your preferences are improper: cyclical preferences are incoherent. A similar argument is available to support the transitivity of coherent preference and thus (with acyclicity) to require strict partial orderings. Still another such argument goes to support the transitivity of indifference.[8]

[7] "Outlines of a Formal Theory of Value, I," *Philosophy of Science*, XXII, 2 (April 1955): 140–160.

[8] Yet another supports conditionalization; see Paul Teller, "Conditionalization, Observation, and Change of Preference," in W. L. Harper and C. A. Hooker, eds., *Foundations of Probability Theory, Statistical Inference, and Statistical*

The trouble with these arguments is the same as the trouble with the Dutch book sort. Either an implausible assumption is being made or the arguments fail. The assumption the arguments need is that the agent is willing to pay for any sequence of arrangements the sum of what he would pay for the arrangements singly, that the value of the sequence to him is the sum of the separate values he sets. The arrangement offers being sequential, the willingness assumed extends over time: the agent is assumed to be willing over time to pay the full sum of the separate values.

This is an assumption of diachronic additivity. It implies that, where several arrangements have been made, one after the other, the value the agent sets on the next is the same as it would have been otherwise. So an equivalent assumption is this: that the arrangements are value-wise independent, that if the agent knew of the arrangements he had already accepted, this would not affect the value he set on the arrangement just offered him. Again, the additivity/independence assumption cannot be taken for granted. Indeed, in typical cases it is false, and for the obvious reasons: the gradual depletion of the agent's funds, his awareness of being exploited, and the like.

Where we must either place or not place a set of bets together, their total value will be weighed; this was the point of the preceding section. No sensible person will pay more for the bets jointly than he can win. But is there not a difference in the diachronic case? Here at no occasion must you make any deals all together. The arrangements are offered you one by one, and each time your friendly pumper addresses a preference you have. If the price is not too high, how can you decline his offer? Why should the zero value to you of any whole cycle keep you from taking a step that would yield you a benefit? No doubt an arrangement will be worth less to you the less money you have; for the less you have, the less you will pay. Still, if you pay any money at all, time after time, you are still being pumped.

Does a person with cyclical preferences have no grounds for declining offers? Let him look back and see the arrangements he has already paid for. He may then come to see which way the wind is blowing, that if he accepts the current offer, he will then get another, and then another, and still another, every cycle bringing him back to where he was at the start, only poorer. Seeing what is in store for him, he may well reject the offer and thus stop the pump. Of course,

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Theories of Science, vol. I, (Boston: Reidel, 1976). See also Bas C. van Fraassen, "Belief and the Will," this JOURNAL, LXXXI, 5 (May 1984): 235–256.

he *might* accept the offer, he might fall for the short-run gain. He need not fall for it, however; this is what has to be stressed. He need not act as if he wore blinders.

Again, the agent prefers C to B, B to A, and A to C. This much remains fixed. It does not follow that the values he sets on the arrangements he is offered are all positive. In the absence of special information, he sets a positive value on the pumper's canceling X in favor of some preferred outcome Y—this for all X and Y. But where he has made certain arrangements already and now looks back, he may get the drift. He may see he is being pumped and refuse to pay for any further deals. His values would then be different. He would set a *zero* value on any new arrangement.

The basic point is as above: the independence of the agent's values cannot be assumed. Suppose that we do not assume it. What would a money-pump argument prove? Let the total benefit from some sequence of arrangements be less than the sum of what you would pay for these arrangements singly, this being due to your preferences (or indifferences) over the outcomes. What follows from that alone? Only that *either* your preferences (or indifferences) are incoherent *or* the values you set on the offered arrangements are not independent. Without the assumption, the arguments do not establish that coherent preferences (or indifferences) are acyclical or transitive. They establish only that *either* this *or* the arrangements offered are not value-wise independent.

III

A general term may be useful. Let me say that a set of a person's dispositions is an *exploitable* set where, given value independence, it could be enlisted by other people to guarantee these others a benefit at his expense. The separate dispositions involved can be said to be *jointly* exploitable. I have not argued that nothing is wrong with jointly exploitable dispositions. My point has been only that their being exploitable does not reveal any fault in them. Some such dispositions are jointly improper, but not all. Probabilities that violate the usual principles of probability are exploitable (Dutch-bookable), and such probabilities are incoherent. They are incoherent not because they are exploitable but because they violate these principles, the principles of probability being criteria of coherence. Intransitive preferences are also both exploitable (money-pumpable) and incoherent. They are incoherent because they violate the principle of preference transitivity, not because they can be exploited. Intransitive indifferences are exploitable too. But I find them *not* incoherent, for I don't subscribe to any principle of indifference transitivity.[9]

What qualifies a principle as a criterion of coherence? This is a question to which I have no answer. Or perhaps the question is rather what counts as a principle of probability, etc. Again, I have left this question dark. I have argued only that certain arguments shed no light on it.

FREDERIC SCHICK

Rutgers University

BOOK REVIEWS

Motivated Irrationality. DAVID PEARS. New York: Oxford, 1984. viii, 259 p. $18.95.

David Pears offers us a complex and subtle exploration of issues centering on weakness of will, self-deception, and practical reasoning. The views he discusses—including the Freudian approach to self-deception, Aristotle's theory of practical reasoning, and Donald Davidson's account of weakness of will—are criticized cautiously. Pears aims at improving, not rejecting.*

The book's most significant contribution will be, I believe, its application of *attribution theory* in explaining human irrationality. Consider an example in which "A girl has a lot of evidence that her lover is unfaithful, but she does not believe it" (44). We inherit explanations of such irrationality [construed as the "incorrect processing of information" (14)] from the Freudian dichotomy of incompetence or willfulness. But recent social (cognitive) psychology offers us an intermediate account in which reason itself may have "bad habits or perversions" (9).

One development of this story would have the girl being the victim of our natural tendency to overweigh *salient* or *vivid* evidence. The flowers at the door, for example, are given greater weight than the lover's abrupt partings. A second possibility is that the girl has made the "fundamental attribution error"—that of explaining another's behavior in terms of "an obvious disposition" ("He called because he cares about me") rather than a more distant disposition or mere

[9] The merits of this appear in my *Having Reasons* (Princeton, N.J.: University Press, 1984), chaps. 2 and 3.

* I am grateful to Mr. Pears for comments on an earlier draft.

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