Reducing Belief Simpliciter to Degrees of Belief

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- Also, when scientists believe two hypotheses $A$ and $B$ to be true, $A \land B$ *does* seem believable to be true for them (as all other of their logical consequences).
  
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Rational belief comes in a qualitative version—belief \textit{simpliciter}—and in a quantitative one—\textit{degrees} of belief.

Sometimes the concept of qualitative belief is supposed to be eliminable:

- However, even scientists \textit{do} seem to believe in the truth of certain propositions. And they do so without being \textit{certain} of these propositions. (Which rules out: \(X\) is believed\(_P\) iff \(P(X) = 1\).)

- Also, when scientists believe two hypotheses \(A\) and \(B\) to be true, \(A \land B\) \textit{does} seem believable to be true for them (as all other of their logical consequences).
  
  (Which rules out the \textit{Lockean thesis}: \(X\) is believed\(_P\) iff \(P(X) > r\).)

One reason why the concept of belief simpliciter is so valuable is that it occupies a \textit{more elementary} scale of measurement than the concept of quantitative belief does.
So the really interesting question is:

Both qualitative and quantitative belief are concepts of belief. *How exactly do they relate to each other?*
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Plan of the talk:

1. Postulates on Quantitative/Qualitative Belief
2. The Representation Theorem and its Surprising Consequence
3. Applications and Extensions: A To-Do List for the Future
4. Solving a Problem

Postulates on Quantitative/Qualitative Belief

Let $\mathcal{W}$ be a set of possible worlds, and let $\mathfrak{A}$ be an algebra of subsets of $\mathcal{W}$ (propositions) in which an agent is interested at a time.

We assume that $\mathfrak{A}$ is closed under countable unions ($\sigma$-algebra).

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P1 (Probability)

$P : \mathfrak{A} \rightarrow [0,1]$ is a probability measure on $\mathfrak{A}$.

$P(Y|X) = \frac{P(Y \cap X)}{P(X)}$, when $P(X) > 0$.

Read: $P(Y|X)$ is the degree of belief in $Y$ on the supposition of $X$.

$P(Y) = P(Y|\mathcal{W})$ is the degree of belief in $Y$ (unconditionally).

P2 (Countable Additivity)

If $X_1, X_2, \ldots, X_n, \ldots$ are pairwise disjoint members of $\mathfrak{A}$, then

$P(\bigcup_{n \in \mathbb{N}} X_n) = \sum_{n=1}^{\infty} P(X_n)$.
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Let $P$ be an agent’s degree-of-belief function at the time.

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**P2** (Countable Additivity) If $X_1, X_2, \ldots, X_n, \ldots$ are pairwise disjoint members of $\mathcal{A}$, then

$$P(\bigcup_{n \in \mathbb{N}} X_n) = \sum_{n=1}^{\infty} P(X_n).$$
Accordingly, let $\text{Bel}$ express an agent's conditional beliefs.

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It follows: For every $X \in \mathcal{A}$ that is consistent with the agent’s beliefs there is a strongest proposition $B_X$, such that $\text{Bel}(Y|X)$ iff $Y \supseteq B_X$.

In particular, the agent believes $Y$ iff $Y \supseteq B_W$. 
B6 (Expansion) For all \( Y \in \mathcal{A} \) such that \( Y \cap B_W \neq \emptyset \): \( B_Y = Y \cap B_W \).

This postulate is contained in the qualitative theory of belief revision (AGM 1985, Gärdenfors 1988).
Finally, we make quantitative and qualitative belief compatible with each other:

Let $0 \leq r < 1$:

\textbf{BP1'} \, (\textbf{Likeliness}) \, \text{For all } Y \in \mathcal{A} \text{ such that } Y \cap B_W \neq \emptyset \text{ and } P(Y) > 0:\n
\text{For all } Z \in \mathcal{A}, \text{ if } Bel(Z|Y), \text{ then } P(Z|Y) > r.\n
(For $r \geq \frac{1}{2}$, this is one direction of the Lockean thesis; cf. Foley 1993.)
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**Definition**

($P$-Stability) For all $X \in \mathcal{A}$:

$X$ is $P$-stable iff for all $Y \in \mathcal{A}$ with $Y \cap X \neq \emptyset$ and $P(Y) > 0$: $P(X|Y) > r$.

So $P$-stable propositions have stably high probabilities under salient suppositions. (Examples: All $X$ with $P(X) = 1$; $X = \emptyset$; and many more!)
The Representation Theorem and its Surprising Consequence

**Theorem**

Let Bel be a class of ordered pairs of members of a σ-algebra $\mathcal{A}$, and let $P : \mathcal{A} \rightarrow [0, 1]$. Then the following two statements are equivalent:

I. $P$ and Bel satisfy P1, B1–B6, and BP1$^r$.

II. $P$ satisfies P1, and there is a (uniquely determined) $X \in \mathcal{A}$, such that $X$ is a non-empty $P$-stable proposition, and:

   - For all $Y \in \mathcal{A}$ such that $Y \cap X \neq \emptyset$, for all $Z \in \mathcal{A}$:
     
     $$\text{Bel}(Z | Y) \text{ if and only if } Z \supseteq Y \cap X$$

   (and hence, $B_W = X$).

This neither presupposes P2 nor $r \geq \frac{1}{2}$.
With P2 and \( r \geq \frac{1}{2} \) one can prove: The class of \( P \)-stable\(^r \) propositions \( X \) in \( \mathcal{U} \) with \( P(X) < 1 \) is *well-ordered* with respect to the subset relation.
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This implies: If there is a non-empty $P$-stable $X$ in $\mathcal{A}$ with $P(X) < 1$ at all, then there is also a least such $X$. 
With \( P2 \) and \( r \geq \frac{1}{2} \) one can prove: The class of \( P \)-stable propositions \( X \) in \( \mathcal{A} \) with \( P(X) < 1 \) is \textit{well-ordered} with respect to the subset relation.

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With P2 and \( r \geq \frac{1}{2} \) one can prove: The class of \( P \)-stable\(^r \) propositions \( X \) in \( \mathcal{A} \) with \( P(X) < 1 \) is \textit{well-ordered} with respect to the subset relation.

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The next postulate entails, amongst others, that there is a least \( X \) s.t. \( P(X) = 1 \):

\( B \)P2 (Zero Supposition) For all \( Y \in \mathcal{A} \): If \( P(Y) = 0 \) and \( Y \cap B_W \neq \varnothing \), then \( B_Y = \varnothing \).
Finally, we postulate:

BP3 (Maximality)

Among all classes $Bel'$ of ordered pairs of members of $\mathcal{A}$, such that $P$ and $Bel'$ jointly satisfy P1–P2, B1–B6, BP1'$, BP2 (with ‘$Bel'$’ replacing ‘$Bel$’), the class $Bel$ is the largest with respect to the class of beliefs.

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But now $Bel = Bel_P'$ can actually be *defined explicitly* in terms of $P$ and $r \geq \frac{1}{2}$:

**Definition**

Let $P : \mathcal{A} \rightarrow [0, 1]$ be a countably additive probability measure on a $\sigma$-algebra $\mathcal{A}$, such that there exists a least set of probability 1 in $\mathcal{A}$. Let $X_{\text{least}}$ be the least non-empty $P$-stable proposition in $\mathcal{A}$ (which exists).

Then we say for all $Y \in \mathcal{A}$ and $\frac{1}{2} \leq r < 1$:

$Bel_P'(Y)$ (i.e., $Y$ is believed to a cautiousness degree of $r$ as given by $P$) iff $Y \supseteq X_{\text{least}}$. 
One can prove that a similar result holds even when all postulates are generalized to *suppositions that may contradict an agent’s current beliefs*.

That is: Take P1 and P2, add *full* AGM belief revision, make them compatible as before, and voilà: *full* conditional belief is definable explicitly in terms of $P$!
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Semantically, this means that every $P$ determines a *sphere system* of worlds:

And almost all $P$ over finite $W$ have a least $P$-stable set $X_{\text{least}}$ with $P(X_{\text{least}}) < 1$!
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Preface Paradox: What one cannot have (with $X_i \approx \text{‘page } i \text{ is error-free’}$):

$$\text{Bel}(X_1), \ldots, \text{Bel}(X_n), \text{Bel}(\neg X_1 \lor \ldots \lor \neg X_n).$$

What one can have is a different version of Fallibilism:

$$\text{Bel}(X_1), \ldots, \text{Bel}(X_n), P(\neg X_1 \lor \ldots \lor \neg X_n) > 0.$$
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**Conditionalization on Zero Sets:**

$P^*$, with $P^*(Y|X) = P(Y|B_X)$, determines a Popper function.

John Dorling’s (1979) “Duhemian” Example:

$E'$: Observational result for the secular acceleration of the moon.
$T$: Relevant part of Newtonian mechanics.
$H$: Auxiliary hypothesis that tidal friction is negligible.

$P(T|E') = 0.8976$, $P(H|E') = 0.003$. 
while I will insert definite numbers so as to simplify the mathematical working, nothing in my final qualitative interpretation... will depend on the precise numbers...

\[ Bel_P(T|E'), Bel_P(\neg H|E') \text{ (with } r = \frac{3}{4}). \]
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... scientists always conducted their serious scientific debates in terms of finite qualitative subjective probability assignments to scientific hypotheses (Dorling 1979).

$$Bel_P^r(T|E'), Bel_P^r(\neg H|E')$$ (with $$r = \frac{3}{4}$$).
Conditionalization and Qualitative Belief:

- **Standard conditionalization:** If $\text{Bel}_P(H|E)$, then $\text{Bel}'_{P(E)}(H)$. 

- **Jeffrey conditionalization:**
  $$P'(H) = P(H|E) \cdot P'(E) + P(H|\neg E) \cdot P'(\neg E).$$

But for what value $0 < P'(E) < 1$?

Simply let it be high enough so that $\text{Bel}'_{P(E)}(E) \neq H$. 

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- **Jeffrey conditionalization:**
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  P'(H) = P(H|E) \cdot P'(E) + P(H|\neg E) \cdot P'(-E).
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**Indicative Conditionals:**

If two people are arguing ‘If p will q?’ and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q... We can say that they are fixing their degrees of belief in q given p.

(Ramsey 1929)

But when is $X \rightarrow Y$ acceptable *simpliciter*?

$X \rightarrow Y$ is acceptable w.r.t. $P, r$ iff $Bel_P'(Y|X)$.  

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$X \rightarrow Y$ is acceptable w.r.t. $P, r$ iff $Bel^r_P(Y|X)$.

Let $X_1 \rightarrow Y_1, \ldots, X_n \rightarrow Y_n \therefore A \rightarrow B$ be valid iff for all $P, r \geq \frac{1}{2}$, if $X_1 \rightarrow Y_1, \ldots, X_n \rightarrow Y_n$ are acceptable w.r.t. $P$ and $r$, so is $A \rightarrow B$. 
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The resulting logic is exactly E. Adams’ logic of conditionals! E.g.:

\[
\begin{align*}
\frac{X \rightarrow Y, X \rightarrow Z}{X \rightarrow (Y \land Z)} \quad \text{(And)} \\
\frac{(X \land Y) \rightarrow Z, X \rightarrow Y}{X \rightarrow Z} \quad \text{(Cautious Cut)} \\
\frac{X \rightarrow Z, Y \rightarrow Z}{(X \lor Y) \rightarrow Z} \quad \text{(Or)} \\
\frac{X \rightarrow Y, X \rightarrow Z}{(X \land Y) \rightarrow Z} \quad \text{(Cautious M.)}
\end{align*}
\]
Subjunctive Conditionals: For each world $w \in W$, let $Ch_w$ be the chance measure of $w$ (at a fixed time). Then it is plausible that $Ch_w$ and ‘truth of $X \rightarrow Y$ at $w$’ taken together satisfy the analogues of our postulates.

The truth of $X \rightarrow Y$ at $w$ thus entails $Ch_w(Y|X)$ being high, without $Ch_w(Y|X)$ having to be 1.
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Furthermore, if \( P \) satisfies the Principal Principle, then

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More applications: Bayesian statistics, preference aggregation, vagueness, . . . ?

One promising future topic in these areas might thus be: A reunification of logical and probabilistic accounts of inductive reasoning in this or in other ways.
Solving a Problem

A challenge to the theory:

- Intuitively, Expansion/Revision can be problematic:

\[
\begin{align*}
\text{Bel}_P(Y_1 \lor Y_2 \lor \ldots \lor Y_n \mid X), \; \neg \text{Bel}_P(\neg Y_i \mid X) \\
\text{Bel}_P(Y_i \mid Y_i \lor (X \land \neg (Y_1 \lor Y_2 \lor \ldots \lor Y_n)))
\end{align*}
\]
Solving a Problem

A challenge to the theory:

- Intuitively, Expansion/Revision can be problematic:

\[
\begin{align*}
\text{Bel}_P'(Y_1 \lor Y_2 \lor \ldots \lor Y_n \mid X), \neg \text{Bel}_P'(-Y_i \mid X) \\
\text{Bel}_P'(Y_i \mid Y_i \lor (X \land \neg(Y_1 \lor Y_2 \lor \ldots \lor Y_n)))
\end{align*}
\]

- Lottery’s revenge: For the same reason, if both \( P \) and \( \text{Bel} \) represent the same large finite lottery, then \( P(B_W) \) must be very close to 1!
Solving a Problem

A challenge to the theory:

- Intuitively, Expansion/Revision can be problematic:

\[
Bel_P^r(Y_1 \lor Y_2 \lor \ldots \lor Y_n \mid X), \neg Bel_P^r(\neg Y_i \mid X) \\
Bel_P^r(Y_i \mid Y_i \lor (X \land \neg (Y_1 \lor Y_2 \lor \ldots \lor Y_n)))
\]

- Lottery’s revenge: For the same reason, if both \( P \) and \( Bel \) represent the same large finite lottery, then \( P(B_W) \) must be very close to 1!

In both cases, the solution is to make qualitative belief relativized to \textit{partitions} (which are employed by Levi, Skyrms, \ldots anyway):

Possible: \( Bel_P^{r, \{Z_j\}}(Y_1 \lor Y_2 \lor \ldots \lor Y_n \mid X), \neg Bel_P^{r, \{Z_j'\}}(Y_1 \lor Y_2 \lor \ldots \lor Y_n \mid X) \)