

4(a)

$$\frac{\frac{\frac{[\_]\_]}{\_}\_I}{\_}\_I}{\_ \vee \_}\_I$$

4(b)

Derivation in text

$$\frac{\frac{\frac{[-\phi]_1 \quad [\phi]_2 \quad \_}{\_}\_E}{\_}\_I}{(\phi \leftrightarrow \_)}_2 \leftrightarrow I \quad \frac{\frac{[\_]\_4 \quad \_}{\_}\_I}{(\phi \leftrightarrow \_)}_3 \leftrightarrow I}{(\phi \leftrightarrow \_) \vee (\phi \leftrightarrow \_)}_1 \vee I$$


4(c)

Derivation in text

$$\frac{\frac{\frac{[-\_]_2 \quad [-\phi]_1}{(\phi \leftrightarrow \_)} \leftrightarrow I 2}{\phi \vee (\phi \leftrightarrow \_)} \vee I}{\phi \vee (\phi \leftrightarrow \_)} \vee E 1$$

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“natural” deduction indeed!



$$\frac{\frac{\frac{[\phi]_5}{\phi \vee \psi} \quad [-(\phi \vee \psi)]_4}{\_}\_I}{\frac{\frac{[\phi \leftrightarrow \psi]_1 \quad \frac{\psi}{(\phi \leftrightarrow \psi)}_5}{\_}\_I}{\frac{[-(\phi \vee \psi)]_4 \quad \phi \vee \psi}{\_}\_I} \quad \frac{\frac{[\phi \leftrightarrow \psi]_2 \quad [\phi]_3}{\psi} \quad [\psi]_3}{\frac{\psi}{(\phi \leftrightarrow \psi) \leftrightarrow \psi}}_2}_1$$

$(\phi \vee \psi) \leftrightarrow (\phi \leftrightarrow \psi) \leftrightarrow \psi$