Estimating a Model of Excess Demand for Public Housing

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Abstract

The goal of this paper is to estimate a new model that captures excess demand for public housing and to quantify the welfare costs associated with failing to maintain a sufficient supply of public housing communities. We develop a new model that captures excess demand for public housing in equilibrium. We estimate the parameters of the model based on a unique panel data set of low-income households in Pittsburgh. We find that for each family that leaves public housing there are on average 3.85 families that would like to move into the vacated unit. Demolitions of existing units increase the degree of rationing and result in large welfare losses. An unintended consequence of demolitions is that they increase racial segregation in low income housing communities.

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JEL classification: C33, C83, D45, D58, H72, R31

1 Introduction

The market for affordable or low income housing is a prime example of a market that is subject to many distortions that often arise due to government regulations and interventions. Despite the overall importance of providing adequate housing and shelter for low income households, very little is known about the quantitative magnitude of these market distortions and the associated welfare implications. The objective of this paper is to estimate a new model that accounts for rationing in equilibrium and to provide a framework for quantifying the welfare costs associated with policies that fail to maintain an adequate supply of affordable housing.

The Department of Housing and Urban Development (HUD) subsidizes the construction and maintenance of affordable housing communities in cities and metropolitan areas in the U.S.[1] Similar government institutions and programs exist in most European countries. Low income households are eligible for public housing assistance if their income is below a threshold that depends on family status, number of children, and region. Given the current standards for determining eligibility, there is typically a large number of eligible households in each metro area. Supply of public housing units is primarily determined by the current and past political decisions that have allocated funding for local housing authorities. Since rents in public housing are typically a fixed percentage of household income, there is no price mechanism which guarantees that public housing markets clear. Since the demand for public housing often exceeds supply, there is rationing in equilibrium in many local markets.

The federal government has for all practical purposes stopped financing the construction of new housing projects. Existing units are often inadequately maintained because local housing authorities have limited resources. Since the early 1990s, HUD

[1] This paper focuses the market for public housing communities. The other main rental assistance program funded by HUD provides vouchers for household to rent in the private market.

has given financial incentives under HOPE VI and related programs to tear down projects that are considered to be distressed. In some cases demolished units are replaced by mixed income communities that are built with private partners. In other cases, low income households obtain vouchers that they can use to rent apartments in the private markets. Other programs to encourage construction of low income housing emerged as construction of public housing ceased and demolition of public housing began. As detailed in ?), the Low Income Housing Tax Credit (LIHTC) program was created in 1986 as part of the Tax Reform Act of 1986 as an alternative to public housing. They observe that "LIHTC has quickly overtaken all previous place-based subsidized rental programs to become the largest such program in the nation's history." They find, however, that this program has failed to result in new construction that serves the public housing population, for two reasons. One reason is that "... LIHTC actually targets moderate as opposed to low income tenants." They note that ?) finds that only 28 percent of LIHTC residents were in the HUD classification of very low income families whereas 81 percent of residents of traditional public housing developments were in that classification. The other reason Eriksen and Rosenthal (2010) conclude that LIHTC has failed as a substitute for public housing is their finding that LIHTC crowds out 100 percent of unsubsidized rental housing, implying no net increase in rental housing.[2]

It is rather puzzling that recent policies have primarily aimed at reducing the supply of public housing, largely ignoring the fact that there is so much excess demand for living in public housing. If there were strong evidence suggesting large negative spill-over effects (such as higher crime rates and lower educational achievement) associated with living in public housing, then supply reductions could be rationalized as part of paternalistic policy towards the poor. However, ?) who considers the impact

[2]Eriksen and Rosenthal (2010) is the most recent study of crowding out due to LIHTC. See Section 2 of their paper for a detailed discussion of other studies of crowding out by LIHTC.

of demolitions in Chicago finds that there are very few positive effects associated with moving out of the projects using a variety of different outcomes.[3] We know almost nothing about the welfare implications of failing to maintain an adequate supply of affordable housing. Nevertheless, policies that reduce the supply are being adopted in almost all metropolitan areas that have an aging stock of pubic housing.

It is almost impossible to obtain reliable panel data describing the characteristics of applicants on wait-lists for public housing in local markets in the U.S. It is well-known that there exist long wait-lists for public housing in many metropolitan areas. We can typically obtain some aggregate summary statistics that broadly measure the degree of excess demand in these markets. However, these aggregate statistics are not sufficient to estimate a model that captures heterogeneity across agents and cannot be used to construct welfare measures. Local housing authorities are not willing to disclose detailed micro level data on wait-lists because they often contain politically sensitive material about racial sorting and segregation. To our knowledge, there is no empirical research in this area that has ever used household-level wait-list data to study the welfare implications of rationing in public housing markets. As a consequence, we know very little about the welfare implications of building, maintaining, or demolishing public housing units. This is disturbing since currently policy has taken a strong stand on reducing the supply of public housing communities. One key challenge encountered in empirical work is to estimate a model of constrained housing market choices without relying explicitly on household level wait-list data since these data are not broadly available to researchers.

To accomplish this task, we develop an equilibrium model that incorporates supply restrictions that arise from the administrative behavior of the local housing authority. A household can move into public housing if and only if the housing authority offers

[3] Evidence that growing up in a poor neighborhood can have adverse effects on outcomes is presented by **?**).

the household a vacant apartment. The ability of the housing authority to offer apartments to eligible households is largely determined by voluntary exit decisions of households that currently live in housing communities. Exit from public housing is a stochastic event since it is partially determined by idiosyncratic preference and income shocks that are not observed by the administrators. The housing authority's objective is to fill all vacant units. If the potential demand exceeds the available units at any point of time, the housing authority has to ration access to public housing.

Eligible households that have not been offered an apartment in an affordable housing community are placed in our model on a wait list. As households move up on the wait list, their priority increases. Each period a fraction of households on the wait list will receive an offer to move into one of the apartments that have recently become available. If total supply of public housing is fixed and vacancy rates are constant over time, the housing authority adjusts the offer probabilities in equilibrium so that the inflow into public housing equals the voluntary outflow. We define an equilibrium for our model and characterize its properties. We show that there exists a unique equilibrium if there are are no transfers between public housing communities. If transfers are possible we show that equilibrium is also unique as long as the housing authority adopts an equal treatment policy and does not discriminate among current residents.

We then show how to identify and estimate the parameters of the model using data on observed choices, but unobserved wait lists. Since we do not observe the wait list, we do not know which households received offers to move into housing communities. We only observe those offers that were accepted and resulted in a move.[4] The basic insight of our identification approach is that offer probabilities are endogenous and

[4]This type of selection problem is also encountered in labor search and occupational choice models. For a discussion of identification and estimation of labor search model see, among others, **?**) and **?**). **?**) discuss identification in the Roy model.

are constrained to satisfy equilibrium conditions. Hence, offer probabilities can be expressed as functions of the structural parameters of the housing choice models. Moreover, exit is purely voluntary and does not depend on offer probabilities. As a consequence, exit behavior is informative about the structural parameters of the utility function. Imposing the equilibrium conditions then establishes identification of the structural parameters of the model.

We quantify the importance of supply side restrictions and estimate the welfare costs of reducing the supply of public housing using a unique data set from the Housing Authority of the City of Pittsburgh (HACP).[5] We supplement these data with a sample of eligible low income households in the Survey of Income and Program Participation which allows us to follow eligible households outside of public housing.

We find that households that are exceedingly poor and headed by single mothers have strong preferences for public housing. African American households also have stronger public housing preferences than whites. The income coefficient shows that there are strong incentives for households to leave public housing as their income grows larger. These incentives are off-set by the presence of significant moving costs that constrain potential relocations of households. We find that for each family that leaves public housing there are on average 3.85 families that would like to move into the vacated unit. For seniors, the rationing is more pronounced. For each senior that moves out of a housing community there are 13.2 households that would like to move in.

To shed some insights into the welfare effects of reducing the supply of public housing, we consider demolishing some of the existing public housing units. We find that the welfare costs of demolishing even the least desirable units are rather substantial. Moreover, displaced black females are disproportionately disadvantaged, which

[5] **?**) use restricted use data from HUD to study the impact of variations in local housing policies on household behavior.

raises some serious issues related to the distributional impact of these demolition programs. An unintended consequence is that the resulting equilibrium demographic distribution in the remaining public housing communities exhibits some increase in the proportions of female and black residents, and thus an increase in segregation in these already highly segregated communities.

Our empirical estimates are based on Pittsburgh, a market that is characterized relatively moderate rental rates compared to most other metropolitan areas in the U.S. Public housing supply in Pittsburgh is also relatively high compared to large cities such as New York, Boston, or Chicago. Nevertheless, our results suggest that increasing the supply restrictions on housing occupied by the very low income population is problematic, even where there is a substantial supply of moderately priced rental housing. The welfare costs of failing to provide an adequate supply of housing for the very poor are likely to be still larger in cities with tighter housing markets and higher housing rental rates. We, therefore, conclude that it might be time to reconsider existing public housing policies.

The remainder of the paper is organized as follows. Section 2 introduces our data set. Section 3 provides an equilibrium model that treats public housing as a differentiated product that is subject to rationing. Section 4 discusses identification and derives the maximum likelihood estimator for this model. The empirical results are presented in Section 5. Section 6 reports our estimates of the welfare costs of demolitions. We offer some conclusions in Section 7.

2 Data

The U.S. Housing Act of 1937 formed the U.S. Public Housing Program that funds local governments in their ownership and management of buildings to house low-income residents at subsidized rents.[6] Currently, the U.S. Department of Housing and Urban Development funds the efforts of hundreds of city and county housing authorities in the United States. In Pennsylvania alone, there are 92 distinct housing authorities. In 2006, the estimated HUD budget for public housing was $24.604 billion.[7] Within the public housing program, this funding supports administration, building maintenance, and even law enforcement.

The empirical analysis presented in this paper focuses on communities owned and managed by the Housing Authority of the City of Pittsburgh, where approximately 70,000 households were eligible for public housing during the period of study. In 2005 HUD provided the HACP with $83.7 million in grants for public housing, housing vouchers, and other programs. In the same year, HACP received $8.3 million from tenant payments. The public housing stock in the City of Pittsburgh during our study includes about 4,500 habitable units across 34 heterogeneous sites.[8] Only a small number of public housing communities were demolished during the course of our survey.[9] As a consequence the supply of public housing has been approximately fixed during our study period.

There is a great variety of sites, or communities, ranging in size from four units (single family houses converted into several apartment units) to over 600 units in various neighborhoods across the city. Some large communities are high rises, others are low-rise housing spread homogenously over several blocks. These communities

[6] **?**) provides a detailed description of the history and current practices of the various different U.S. Public Housing Programs.

[7] **?**) provides details. Note that this figure does not include housing voucher programs, low-income community development programs, or other none-state owned and managed housing programs.

[8] The number of habitable public housing units varies slightly over time, due to repairs, renovations, and demolition.

[9] Much of the demolition was motivated by the argument that growing up in public housing might be negative for children, although this conjecture is controversial in the literature (**?**). For an analysis of the the impact of public housing demolitions in Chicago see **?**).

are usually designated as either 'family' communities or 'senior' communities, where senior communities target households age 62 or older. There are 34 separate sites. 19 of these sites are family units, 11 are designated for seniors and 4 of them are mixed. There are 16 large communities with more than 100 units, 8 are medium sized, and 10 are small with less than 40 units. Heterogeneity in public housing also arises due to differences in local amenities. The 34 public housing communities in the HACP are located across 19 of Pittsburgh's 32 wards and across 28 census tracts. These public housing communities also vary in terms of neighborhood amenities such as crime, school quality, property values and demographic characteristics.[10]

The HACP data contain records of household entry, exits, and transfers from June 2001 to June 2006 within the 34 public housing communities actively used during this time period. The data set also includes annual updates of each of these households as well as any non-periodic reports that update information about household composition or pre-rent income that is reported to the HACP. These records contain most of the information fields requested of all U.S. housing authorities including age, race, household composition including age and relationship of family members and housemates, earnings, and income adjustment exclusions including disability, medical, and childcare expenses. We also observe the monthly rent being charged to a particular household, the number of bedrooms of the housing unit, whether the community is targeted to seniors, and the address and unit number. There are 7,070 households observed at least once during this time period; there are 2,907 households that move in for the first time, 3,155 households that move out, and 1,244 that transfer from one public housing unit to another.

[10] There is much evidence that suggests that households make residential decisions based on neighborhood characteristics and local public goods. This evidence is based on estimated locational equilibrium models such as?), ?), ?), ?), ?), ?), and ?). ?), ?), ?), ?), ?), ?), ?), and ?) are examples of related empirical approaches which are based on more traditional discrete choice models or hedonic frameworks.

Table 1: Descriptive Statistics of HACP Demographics

	All Units	Family Units	Mixed Units	Senior Units	2 Bedroom Apartments
Age	48.86	40.42	49.06	71.15	34.45
	(20.76)	(16.98)	(20.53)	(11.77)	(13.36)
Percent Female	80.59	84.87	83.85	64.90	84.78
Percent Married	2.66	2.20	2.65	3.93	1.43
Number of Adults	1.16	1.17	1.21	1.06	1.06
	(0.44)	(0.45)	(0.50)	(0.23)	(0.24)
Number of Children	0.95	1.00	1.59	0.00	0.76
	(1.36)	(1.22)	(1.71)	(0.00)	(0.75)
Percent With Children	43.95	53.46	58.31	0.00	57.40
Percent Black	88.53	96.67	97.00	55.59	96.11
Annual Income	9082	8516	9714	9784	6305
	(7776)	(8957)	(6968)	(4602)	(6771)

Standard deviations are given in parenthesis.

Table 1 summarizes key descriptive statistics for the full sample and for four subsamples that are differentiated by community type. Although some families live in senior housing and some seniors live in non-senior housing, age and family composition distributions are bimodal with respect to these two types of communities. In mixed communities, demographic variables look similar to a weighted average of senior and family communities, however there are more cohabiting adults and a higher number of children in mixed housing than in family-only or senior-only housing. The mean age in senior housing is 31 years greater than the mean age in non-senior housing. The majority of households in both senior-only and family-only communities are female, but females are a much larger majority in family-only communities. Blacks households are a very high proportion of residents in family and mixed housing, while senior units have nearly equal proportions of black and white households. Marriage rates are low, 2.20% in family housing and 3.93% in senior housing; there are more cohabiting adults in family housing than in senior housing.[11] There are fewer households in non-senior housing that have children than one might expect (about 53%).[12]

Table 2: Descriptive Statistics of SIPP Compared to Census and HACP

	Census All	SIPP All	SIPP Private	SIPP Public	HACP Public
Age	50.83	52.70	52.72	52.19	48.86
Percent Female	54.6%	59.94%	59.06%	76.56%	80.59%
Percent Married	22.6%	30.79%	32.09%	6.25%	2.66%
Number of Adults	1.450	1.274	1.284	1.094	1.160
Number of Children	0.495	0.617	0.616	0.641	0.950
Percent With Children	24.73%	30.32%	30.27%	31.25%	43.95%
Percent Black	32.64%	28.28%	27.05%	51.56%	88.53%
Annual Income	14079	18979	19391	11184	9082

We only observe households that have lived in public housing at some point during

[11]There is a strong incentive for families to not report the existence of a cohabiting adult or partner, as it would lead to an increase in rent if the cohabiting adult earns an income. As a result, the number of cohabiting adults as well as household income are surely larger than our estimates from the data.

[12]Our sample differs from other studies in that Pittsburgh public housing seems to house a higher percent of black households, female-headed households and households with children; but a much lower percent of married households. For example, Hungerford '96's sample from the 1986-1988 SIPP panel was 52% female, 23% black, 32% married and the mean number of children was 0.21 (?).

the sample period. Once households leave the housing communities, the HACP does not conduct any follow-up surveys. To learn about households that are eligible for public housing, but do not live in one of the housing communities, we turn to the 2001 Survey of Income and Program Participation (SIPP). The SIPP is a survey managed by the U.S. Census Bureau that interviews households every four months for 3 years. Each month, households are asked about their previous four months' family composition, sources of income, and participation in government programs such as public housing and school lunch programs. We create a sample based on the SIPP that contains households that eligible for housing aid.[13]

Table 2 provides some descriptive statistics for our SIPP sample used in this analysis and compares it to Census and HACP data. We find that low-income households that rent in the private market are on average more likely to be married, are less likely to be black, and have substantially higher income than households in public housing. Comparing the SIPP with the HACP sample we find that the SIPP sample is slightly older and as a consequence average income is slightly higher and children are fewer than in the HACP. Comparing the SIPP with the Census, the SIPP contains slightly older heads of household, more female heads of household, more married householders, households with more children, and fewer black households. However, the differences between the SIPP sample and the Census sample of eligible households in Pittsburgh are relatively small.[14]

The 34 communities are classified into broad community types: family large (PH

[13]The SIPP contains only 14 households that participate in public housing in Pittsburgh at some point during the sample period. There are 156 Pittsburgh households eligible for public housing in the first quarter. We, therefore, constructed a sample that also includes households from metropolitan areas with similar characteristics. An appendix that details how we constructed this sample is available from the authors.

[14]An appendix that contains a more detailed description of how our data set was constructed is available from the authors.

Table 3: Transition Matrix

	Private	PH 1	PH 2	PH 3	PH 4	PH 5	PH 6
Private	0	677	144	24	300	59	191
PH 1	855	16264	16	2	75	7	10
PH 2	233	16	5371	3	17	8	7
PH 3	44	2	29	1438	1	0	2
PH 4	572	16	8	1	12156	5	9
PH 5	105	1	0	0	1	2017	29
PH 6	302	0	0	1	47	37	8129
Rows indicate choices in $t-1$ and columns in t.							

1), family medium (PH 2), family small (PH 3), mixed (PH 4), senior large (PH 5), and senior small (PH6). These six types of housing units are fairly homogenous, but seem to attract different types of households. Large, medium, and small low-rise non-senior communities primarily house families with children. However, they also include a significant percent of households without children ranging from 36% to 42%. Although the demographics of senior and family housing differ, there is some overlap. Most senior-dominated communities include a significant percentage of non-senior adults without kids ranging from 13% to 37%. Most family-only communities include some senior households ranging from 0 - 20%, about a third of which are caring for children.

Table 3 shows the transition matrix for the HACP data. We find that locational choices are persistent since most households stay with their past choices. However, the off-diagonal elements of the transition matrix indicate that there is a fair amount of entry into and exit from public housing. Moreover, there are a number of transitions within public housing communities. These transfer are largely voluntary and indicate that households differentiate among the heterogeneous community types.[15]

3 An Equilibrium Model of Housing Markets with Rationing

3.1 The Baseline Model without Transfers

We consider a model with a continuum of low-income households. Each household is eligible for housing aid and can thus, in principle, live in one of the available public housing communities or rent an apartment in the private market. Denote the outside private market option with 0. Let J be the number of different housing communities that are available in the public housing program. Let $d_{jt} \in \{0, 1\}$ denote an indicator variable which equals one if the household chooses alternative j at time t and zero otherwise.[16] Let the vector $d_t = (d_{0t}, ..., d_{Jt})$ characterize choices of a household at t. Since the alternatives are mutually exclusive, we have

$$\sum_{j=0}^{J} d_{jt} \quad = \quad 1 \tag{1}$$

Households differ along a number of characteristics x_t such as income, age, number of kids, number of adults, gender of household head, marital status, and race. We treat these characteristics as exogenous, although it is difficult to endogenize income or family status from a conceptual perspective.[17]

Household preferences are subject to idiosyncratic shocks denoted by ϵ_t. Households face relocation costs if they decide to move. Thus lagged choices, denoted by

[15] In the SIPP sample, we observe 89 transitions from private to public housing and 98 transitions from public to private housing.

[16] In our application, we use quarterly data.

[17] We do not observe labor supply or job market participation in the HACP data. See Jacob and Ludwig (2010) for analysis of the impact of Sectin 8 vouchers on income.

d_{t-1}, are relevant state variables.

Households have preferences defined over all potential elements in the choice set. We model household preferences using a standard random utility specification.

Assumption 1 *Let $u(d_t, x_t, d_{t-1}, \epsilon_t)$ denote the household utility function. We assume that the utility function is additively separable in observed and unobserved states and thus allows the following representation:*

$$u(d_t, x_t, d_{t-1}, \epsilon_t) \; = \; \sum_{j=0}^{J} d_{jt} \left[u_j(x_t, d_{t-1}) + \epsilon_{jt} \right] \tag{2}$$

This specification implicitly treats public housing as a differentiated product.

A key feature of our model is that all potential choices may not be available to a household at any given point of time. A household that is currently renting in the private market may not have access to public housing even if the household meets all eligibility criteria.[18] We, therefore, need to formalize the fact that access to public housing is restricted by a local housing authority.

Assumption 2 *The public housing authority does not evict any households that have lost eligibility.*

This assumption is motivated by policies that are typically used by many local housing authorities. It implies that exit from public housing is purely voluntary. To characterize the voluntary outflow, let P_{jt} denote the fraction of eligible households living in community j at the beginning of period t. The outflow from public housing community j to the private sector, OF_{j0t}, is defined as:

$$OF_{j0t} = P_{jt} \int \; Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq u_j(x_t, d_{t-1}) + \epsilon_{jt}) \; f(x_t | d_{jt-1} = 1) \; dx_t \tag{3}$$

[18] In practice, all eligible households are typically assigned to a waiting list. A household will only receive an offer to move into public housing if it is on top of the waiting list.

where $f(x_t|d_{jt-1} = 1)$ denotes the conditional density function of households with characteristics x_t that live in j at the beginning of period t. As a consequence, the housing authority faces a stream of housing units that become available at each point of time. The authority needs to assign these units to new renters. To model this decision process, we need to model the potential demand for public housing.

Let P_{0t} denote the fraction of eligible households renting in the private market at the beginning of period t. We make the following assumption:

Assumption 3 *All eligible households that are renting in the private market are placed on a wait list for public housing.*

We offer four observations regarding this assumption. First, signing up for the wait list is, for all practical purposes, costless in practice.[19] Second, it is easy to relax the assumption and allow for systematic differences between households on the wait list and eligible households that have not signed up on the wait list. When we discuss the rationing implications, we relax this assumption and consider a case in which a demand signal triggers households to sign up on the wait list. Third, the assumption can be justified by empirical constraints. We do not observe the characteristics of all households on the wait list and neither does the housing authority. We also do not observe the priority ranking of households on the wait-list. Assumption 3 implies that the households that have top priority on the wait-list do not systematically differ from the eligible population.[20] Finally, it is also straight forward to assume that the housing authority has multiple wait lists for households with different family sizes.[21]

Next consider the potential demand for public housing. The probability that a

[19] Of course, it does not matter that all eligible households sign up as long as there are no systematic differences between eligible households and households on the wait-list.

[20] As a consequence, we can solve and estimate the model without observing the conditional distribution of households on the wait list.

[21] We discuss these issues when we estimate the model in Section 5.

households that is currently living in the private sector prefers j at time t is:

$$Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) = Pr(u_j(x_t, d_{t-1}) + \epsilon_{jt} \geq u_0(x_t, d_{t-1}) + \epsilon_{0t}) \tag{4}$$

Let $f(x_t|d_{0t-1} = 1)$ denote the conditional density function of households with characteristics x_t that currently rent in the private market, are eligible for public housing, and thus have been assigned to a wait list. The potential demand for community j is then characterized by the fraction of households on the wait list that prefer j at time t:

$$F_{0jt} = P_{0t} \int Pr(d_{jt} = 1 | x_t, d_{0t-1} = 1) \; f(x_t | d_{it-1} = 1) \; dx_t \tag{5}$$

The most interesting case arises if demand exceeds supply. We therefore make the following assumption:

Assumption 4 *a) The potential demand exceeds the voluntary outflow for each community at each point of time. b) The authority offers the free units to households on the wait list that have the highest priority. The housing authority continues offering units until all available vacant units have been filled with eligible households.*

Assumption 4a is not necessary to obtain a well defined equilibrium, but it holds empirically in almost all large markets in the U.S. It implies that the housing authority can not meet the full demand. Instead it can only offer public housing to a fraction of households that are eligible. Assumption 4b implies that that housing authority follows a first-in-first-out policy. Assumptions 2 through 4 imply that there is a fraction of households denoted by, Π_{0jt}, that will receive offers to move into housing community j at time t. The total inflow into public housing is then given by:

$$IF_{jt} = \Pi_{0jt} \; F_{0jt} \tag{6}$$

To close the model, we need to impose an assumption on the supply of public housing and the vacancy rates.

Assumption 5 *The supply of public housing is constant in each housing community at each point of time.*

We can relax this assumption and allow for exogenous changes in the supply of public housing due to new construction or demolitions. We discuss these issues in detail when we quantify the impact of demolitions in Section 6 of the paper.

Assumption 5 then implies that the outflow must equal the inflow for each housing community at each point of time in equilibrium.[22]

$$IF_{jt} \quad = \quad OF_{jt} \tag{7}$$

An equilibrium for the baseline model can, therefore, be defined as follows:

Definition 1 *Given an initial distribution of household types, an equilibrium for this model consists of a rationing mechanism that determines the fraction of households that receive offers to move into public housing such that*

- *Households choose the preferred housing option among the set of available options.*
- *For each housing community j, the housing authority offers apartments to eligible households on the wait list. Thus a fraction of households on the wait list will receive offers to move into public housing.*
- *The inflow of households equals the outflow of households for each housing community.*

We have J offer probabilities and J market clearing conditions. Moreover, the system of equations which defines the equilibrium is linear in the offer probabilities

[22]The assumption of a constant housing stock is common in many theoretical papers that study housing market equilibrium in urban metropolitan areas. See, for example, **??**), **?**), **?**), and **?**).

and can be solved equation by equation. A unique equilibrium for the economy exists since the potential inflow is at least as large as the voluntary outflow for each community. Hence we have the following result:

Proposition 1 *There exists a unique housing market equilibrium with rationing in the baseline model without transfers.*

As we will see below, uniqueness of equilibrium is essential for identifying the parameters of the model. Next we generalize our model and allow for transfers between public housing units.

3.2 An Extended Model with Transfers

Transfers imply that the demand for public housing must be modified since households may have additional options. The probability that a household that lives in community i at the beginning of the period prefers to move to community j at time t is:

$$Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) = Pr(u_j(x_t, d_{t-1}) + \epsilon_{jt} \geq \max\left[u_i(x_t, d_{t-1}) + \epsilon_{it}, u_0(x_t, d_{t-1}) + \epsilon_{0t}\right]) \quad (8)$$

Note that households only compare options that in the effective choice set, i.e. that are available to them. As before, the potential demand is then characterized by the fraction of households living in community i that prefer j at time t:

$$F_{ijt} \;\; = \;\; P_{it} \int Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) \;\; f(x_t | d_{it-1} = 1) \, dx_t \qquad (9)$$

In contrasts to entry into public housing and exit, there is no stated policy for transfers between public housing units. Nevertheless, we observe a fair number of transfers in practice. A useful modeling approach is then to mimic our assumptions imposed on the (external) wait list to generate a well defined trans policy. Suppose that the housing authority also has an internal mechanism that determines transfer offers. In that case, a fraction of households that is currently living in i are offered the opportunity to transfer to community j.

Assumption 6 *The probability of obtaining an offer from housing community j while living in public housing i is given by Π_{ijt}. Households get at most one offer at each point of time.*

The total realized demand (or inflow) from community i to community j at time t is therefore $\Pi_{ijt}\ F_{ijt}$. Summing over all current housing choices other then j gives the total inflow into housing community j:

$$IF_{jt} \;=\; \sum_{i=0, i\neq j}^{J} \Pi_{ijt}\ F_{ijt} \tag{10}$$

Similarly we can modify the equation that characterizes the total voluntary outflow from community j:

$$OF_{jt} \;=\; OF_{j0t} + \sum_{i=1, i\neq j}^{J} \Pi_{jit}\ F_{jit} \tag{11}$$

where the outflow to the private sector, OF_{j0t}, is defined as:

$$OF_{j0t} = P_{jt}\,\Pi_{jjt} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq u_j(x_t, d_{t-1}) + \epsilon_{jt})\; f(x_t | d_{jt-1} = 1)\; dx_t$$

$$+ \quad P_{jt} \sum_{k=1, k \neq j}^{K} \Pi_{jkt} \int Pr(u_0(x_t, d_{t-1}) + \epsilon_{0t} \geq \max\,[u_j(x_t, d_{t-1}) + \epsilon_{jt}, u_k(x_t, d_{t-1}) + \epsilon_{kt}])$$

$$f(x_t | d_{jt-1} = 1)\; dx_t \qquad (12)$$

In the extended model we have J^2 offer probabilities and J market clearing conditions. Moreover, the system of equations which defines equilibrium is linear in the offer probabilities. An equilibrium for the economy exists if the linear system of market clearing equations has a solution. These solutions (generically) exist, but are not unique, since the number of equations is smaller than the number of unknowns.[23]

The potential for multiplicity in equilibrium arises because we have not sufficiently restricted the ability of the housing authority to allow households to transfer between different units. There are many transfer policies that are consistent with equilibrium in the public housing market. The market clearing conditions alone do not uniquely determine the offer probabilities. To obtain a unique solution to this system of equations, we need to impose additional assumptions. It is plausible that the housing authority does not discriminate based on current residence and uses the same odds ratio for insiders and outsiders. We therefore assume that:

Assumption 7 *The fraction of households that receive an offer to transfer between units in different communities does not depend on current residence:*

$$\Pi_{ijt} \quad = \quad \Pi_{jt} \qquad (13)$$

The odds ratios are the same for household inside and outside of public housing:

$$\Pi_{0jt} \quad = \quad R_{0t}\,\Pi_{jt} \qquad (14)$$

[23] See, for example, the discussion in **?**).

Note that this assumption is plausible since housing authorities are not allowed to discriminate based on income, race, and gender. As a consequence it is hard to believe that they could discriminate based on residency.[24] The parameter R_{0t} measures the relative degree of preferential treatment that is given to outsiders. In practice $R_{0t} >> 1$ and as a consequence households on the wait list get preferential treatment over households that are already in public housing. Substituting Assumption 7 into the definition of equilibrium, we obtain:

$$R_{0t}\, \Pi_{jt}\, F_{0jt} + \sum_{i \neq j} \Pi_{jt}\, F_{ijt} \;\; = \;\; OF_{j0t} + \sum_{i \neq j} \Pi_{it}\, F_{jit} \tag{15}$$

which is a system of J equations in $J + 1$ unknowns. Thus the equilibrium conditions define the offer probabilities up to the factor R_{0t}. We thus have shown the following result:

Proposition 2 *For each value of R_{0t}, there exists a unique housing market equilibrium with rationing.*

In summary, we have developed an equilibrium model of public housing that generates rationing and excess demand in equilibrium. The model explains transfers within public housing since housing communities are heterogeneous.

4 Identification and Estimation

We estimate the model using two different samples. The first sample is a choice based sample that is provided by a local authority. This sample tracks households as long as they stay in public housing. The second sample is a random sample of households that are eligible for housing aid. In this section we introduce a parametrization of our model. We then derive the conditional choice probabilities and develop our

[24]A few transfers in our sample are due to forced relocations or changes in family structure.

maximum likelihood estimator. We then discuss the role that equilibrium conditions play in establishing identification of the model. Finally, we show that our approach works in a Monte Carlo study when the data generating process is known.

4.1 A Parametrization

We assume that the utility associated with community j is given by

$$u_{jt} \;=\; \gamma_j \;+\; \beta \ln(y_{jt}) \;+\; \delta x_t \;+\; mc\, 1\{d_t \neq d_{t-1}\} \;+\; \epsilon_{jt} \quad j = 1, ..., J \tag{16}$$

The utility of the outside option is normalized to be equal to the following expression:

$$u_{0t} \;=\; \ln(y_{0t}) \;+\; mc\, 1\{d_t \neq d_{t-1}\} \;+\; \epsilon_{0t} \tag{17}$$

In the equations above, y_{jt} denotes household net income, mc is a moving cost parameter, and γ_j is a community specific fixed effect.[25] Households that live in public housing typically pay 30% of their income in rent. As a consequence net income is choice specific due to the implicit tax. As income increases, living outside of public housing should become more attractive. We would, therefore, expect that $\beta < 1$. The community specific fixed effects capture observed and unobserved differences among the public housing communities. The specification also accounts for (psychic) moving costs. Idiosyncratic shocks account for factors not observed by the econometrician. Following **?**), we assume that the ϵ's are i.i.d. Type I extreme value distributed.

4.2 Conditional Choice Probabilities

Our main data set is from a local housing authority and follows households as long as they are in public housing. This is, therefore, a choice based sample since we only observe households that have chosen to live in one of the housing communities at

[25]We are implicitly imposing the budget constraint by using net income in the utility function.

time t. A household that lived in community j at the end of the last time period, has potentially three options. First, the household moves back to the private housing market. Second, the household moves to a different housing community. Third, the household stays in its current community j. Given the distributional assumptions on the idiosyncratic shocks, the probability of moving to the private sector is then:

$$\begin{aligned} Pr\{d_{0t} = 1 | d_{jt-1} = 1, x_t\} &= \sum_{k=1, k \neq j}^{J} \Pi_{jkt} \frac{exp(u_0(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t)) + exp(u_k(x_t))} \\ &\quad + \Pi_{jjt} \frac{exp(u_0(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \end{aligned} \tag{18}$$

The probability of moving from community j to community k is given by:

$$Pr\{d_{kt} = 1 | d_{jt-1} = 1, x_t\} = \Pi_{jkt} \frac{exp(u_k(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t)) + exp(u_k(x_t))} \tag{19}$$

and the probability of staying in community j is given by:

$$\begin{aligned} Pr\{d_{jt} = 1 | d_{jt-1} = 1, x_t\} &= \sum_{k=1, k \neq j}^{J} \Pi_{jkt} \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t)) + exp(u_k(x_t))} \\ &\quad + \Pi_{jjt} \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \end{aligned} \tag{20}$$

Finally, we also observe new entrants into public housing. The probability of observing a new household in community j is

$$Pr\{d_{jt} = 1 | d_{0t-1} = 1, x_t\} = \Pi_{0jt} \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \tag{21}$$

The conditional choice probabilities for the choice based sample are thus defined by equations (18), (19), (20) and (21).

Our second sample is a random sample of low income households that tracks households both inside and outside of public housing. In contrast to the choice based sample, this sample does not allow us to identify the exact housing community in which a household lives. As a consequence we only observe a coarser version of the choice set in the random sample. For households that are currently not living in public housing, we have two possible outcomes: 1) the household stays in private housing; 2) the household moves to a public housing unit.

The probability of moving to any of the J public housing communities is given by:

$$Pr\{d_{0t} = 0 | d_{0t-1} = 1, x_t\} \;\; = \;\; \sum_{j=1}^{J} \Pi_{0jt} \, \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \tag{22}$$

Note that (22) is obtained by summing the probabilities in (21) over all possible choices. Similarly, the probability of staying in private housing is defined:

$$Pr\{d_{0t} = 1 | d_{0t-1} = 1, x_t\} \;\; = \;\; 1 \; - \; \sum_{j=1}^{J} \Pi_{0jt} \, \frac{exp(u_j(x_t))}{exp(u_0(x_t)) + exp(u_j(x_t))} \tag{23}$$

Note that we do not observe whether the household obtained an offer and we also do not observe to which housing unit it moved, if it decided to move.

Next consider a household that currently lives in public housing. Again there are two possible outcomes. The household moves back to private housing. Alternatively the household stays in public housing. Consider the first case, in which the household moves back to private housing. Now we do not observe in the random sample in which unit the household lives. However, we can compute relative frequencies based on the choice based sample which assign probabilities to each community type. Let us denote these probabilities by $Pr\{d_{jt-1} = 1 | d_{0t-1} = 0, x_t)$. The choice probability conditional on living in community j is given by equation (18). Summing over all J housing units and properly weighting each conditional choice probability, implies that the probability of moving out of public housing is then:

$$Pr\{d_{0t} = 1 | d_{0t-1} = 0, x_t\} = \sum_{j=1}^{J} Pr\{d_{0t} = 1 | d_{jt-1} = 1, x_t) Pr\{d_{jt-1} = 1 | d_{0t-1} = 0, x_t) \tag{24}$$

Next consider the case in which a household stays in public housing. We cannot distinguish between the case in which a household stays in the same community or moves to a different housing community within public housing. Thus conditional on living in community j, the probability of staying in public housing is the sum of the probabilities in equations (19) and (20), i.e. the probability of staying conditional on living in j at the end of the previous period is

$$Pr\{d_{0t} = 0|d_{jt-1} = 1, x_t) = Pr\{d_{jt} = 1|d_{jt-1} = 1, x_t) + \sum_{k=1,k\neq j}^{J} Pr\{d_{kt} = 1|d_{jt-1} = 1, x_t) \quad (25)$$

Summing over all J housing units and properly weighting each conditional choice probability, implies that the probability of staying in public housing is then:

$$Pr\{d_{0t} = 0|d_{0t-1} = 0, x_t) = \sum_{j=1}^{J} Pr\{d_{0t} = 0|d_{jt-1} = 1, x_t) Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t) \quad (26)$$

The conditional choice probabilities for the random sample are thus defined by equations (22), (23), (24) and (26).

4.3 The Likelihood Function under Enriched Sampling

To compute the likelihood function we need to take into account the fact that we use a random and a choice based sample in estimation. This sampling scheme is also called enriched sampling as discussed in detail by **??**).[26] Let us denote the corresponding sample sizes with N_1 and N_2. Similarly, let T_1 and T_2 denote the length of the two panels. Observations are assumed to be independent across samples ruling out sampling the same household in both data sets. The joint likelihood function of observing the two samples is thus the product of the two likelihood functions

$$L = L_1 \, L_2 \quad (27)$$

The likelihood associated with the random sample L_1 is given by:

$$L_1 = \Pi_{i=1}^{N_1} \Pi_{t=1}^{T_1} l_{1nt} \quad (28)$$

[26]Notice that our sampling scheme satisfies assumptions 9 and 10 in Cosslett (1981) which guarantees a sufficient overlap in the relevant choice sets between the two samples.

where l_{1nt} is given by

$$l_{1nt} = [Pr\{d_{0nt} = 0|d_{0nt-1}, x_{nt}\}]^{1-d_{0nt}}[Pr\{d_{0nt} = 1|d_{0nt-1}, x_{nt},\}]^{d_{0nt}} f(x_{nt}, d_{nt-1}) \quad (29)$$

The likelihood for the choice based sample L_2 is defined:

$$L_2 = \Pi_{i=1}^{N_2}\Pi_{t=1}^{T_2} \frac{Pr\{d_{jnt} = 1|d_{nt-1}, x_{nt}\} f(x_{nt}, d_{nt-1})}{\tilde{Q}_t(J)} \quad (30)$$

where

$$\tilde{Q}_t(J) = \sum_{j=1}^{J} Q_t(j) \quad (31)$$

$Q_t(j)$ is the unconditional probability that choice j is chosen that is defined as:

$$\begin{aligned} Q_t(j) &= \sum_{j=1}^{J} \int Pr\{d_{jnt} = 1|d_{t-1}, x_t\} f(x_t, d_{t-1}) dx_t d_{t-1} \quad (32) \\ &= \sum_{j=1}^{J} \int \sum_{i=0}^{J} Pr\{d_{jnt} = 1|d_{it-1} = 1, x_t\} f(x_t|d_{it-1} = 1) Pr\{d_{it-1} = 1\} dx_t \end{aligned}$$

We assume that $f(x_t, d_{t-1}, \theta)$ is known up to finite vector of parameters θ and treat the the $Q_t(j)$ as unknown. We then define our enriched sampled maximum likelihood estimator (ESMLE) as the argument that maximizes equation (27).[27]

4.4 Imposing the Equilibrium Constraints

One problem associated with the ESML estimator above is that the offer probabilities are not separately identified from the choice specific intercepts. To obtain identification, we use the equilibrium conditions and express the endogenous offer probabilities

[27]If the $Q_t(j)$'s are known, we can define a constrained enriched sampled maximum likelihood estimator (CESMLE) as the argument which maximizes equation (27) subject to the J constraints in equation (32). Finally, one could follow Cosslett (1978,1981) and treat $f(x_t, d_{t-1})$ as unknown and then define Pseudo MLE by concentrating out the weights that characterize the empirical likelihood of the data. These estimators extend the standard choice based estimators discussed in **?**).

as functions of the structural parameters of the choice model. To illustrate the basic ideas, consider first the model without transfers. In that model the structural parameters of the utility function are identified from the exit behavior of households. The conditional exit probability does not depend on the probability of getting an offer to move into public housing. Unattractive housing units will have higher exit rates and lower potential demand than attractive housing communities. Given the voluntary exit rates and potential demand for moving into public housing, the offer probabilities are then uniquely determined by the equilibrium conditions. Solving this linear system of equations, we can express the offer probabilities as functions of the voluntary outflow and the potential demand which only depend on the structural parameters of the utility function. Imposing the equilibrium conditions thus resolves the key identification problem encountered in the model without transfers.

In the model with transfers, the sequential identification argument breaks down since exit probabilities depend on unobserved transfer probabilities. Nevertheless, we can still express the offer probabilities as functions of the structural parameters of the utility function. If a community is attractive, voluntary outflows will be low and potential demand will be high. As a consequence offer probabilities are low. Similarly, if the community is unattractive, voluntary outflows and transfers will be high and the potential inflow will be low. As a consequence, offer probabilities need to be sufficiently large to meet the equilibrium condition. Thus a similar logic for identification applies in the extended model that accounts for transfers.

To provide some additional insights into our approach to identification, we have conducted a Monte Carlo study.[28] We find that our estimator works well under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward

[28]Details are reported in Appendix A.

in samples with 2000 observations. Larger samples help reduce the estimation bias. Imposing the equilibrium conditions works well and establishes identification. The estimates of the offer probabilities that are implied by the equilibrium conditions are accurate.

5 Empirical Results

We implemented our estimator for a number of different model specifications.[29] Table 4 reports the parameter estimates and estimated standard errors for three models that capture the essence of our modeling approach. In column I, we estimate the model with transfers using the full sample.[30] We are thus implicitly assuming that the housing authority has only one wait list. This estimator controls for differences in income, race, age, family status and number of children. In column II, we estimate the model for the subsample of households that are eligible for two bedroom non-senior apartment units. In column III we consider the subsample of senior housing units. These two models thus explicitly assume that there are separate wait lists for different family and apartment sizes. We only control for differences in income, gender and race in these estimators.

[29]In all models, we use the empirical demographic distributions to estimate $f(x_{nt}, d_{nt-1})$. Race (black, white) and age (senior, non-senior) are modeled as a multivariate distribution; sex is a binomial conditional on race-age; number of children is a multinomial conditional on sex and race-age; income is a truncated normal based on number of children, sex, and race-age. We fit a logit model to estimate $Pr\{d_{jt-1} = 1|d_{0t-1} = 0, x_t\}$, which is needed in equations (24), (25), and (26) for the SIPP likelihood. We calibrate R_0 based on the observed ratios of mobility for households inside and outside of public housing.

[30]We have also estimated a version of the model that only used households in the SIPP that live in Pittsburgh. Using the smaller Pittsburgh subsample largely affects the precision of the estimates, but not the magnitude of the point estimates.

Table 4: Parameter Estimates

	I		II		III	
	Full Sample		2 BR Subsample		Senior Subsample	
Income	0.329	(0.028)	0.280	(0.084)	0.380	(0.086)
Moving cost	-3.186	(0.017)	-4.282	(0.065)	-2.355	(0.033)
Black and non-senior	1.222	(0.071)	0.822	(0.178)		
White and senior	0.209	(0.113)				
Black and senior	1.000	(0.101)			0.535	(0.537)
Children	-0.315	(0.123)				
Female	0.053	(0.061)	0.253	(0.205)		
Female and senior	-0.174	(0.094)			-0.179	(0.136)
Female with children	0.426	(0.130)				
	community fixed effects		community fixed effects		community fixed effects	
log likelihood	-688,796		-123,144		-130,184	

Estimated standard errors are given in parenthesis.

Model III allows the fixed effects to differ by race.

We find that blacks have stronger preferences for public housing than whites. This result is largely driven by the fact that black households are overrepresented in public housing in Pittsburgh. We also find that age has an impact. Male seniors have stronger preferences for public housing than female seniors. Females with children also have stronger preferences for public housing than other households. In contrast, fathers or married couples with children have lower valuations for public housing than those without children. We also find that there are significant moving costs that constrain potential relocations of households.

The income coefficient shows that there are strong incentives for households to leave public housing as income increases. This finding is consistent with the fact that there are only a few higher income household in our sample that live in public housing. There are only 52 households in our sample that, at some time during the study, exceed the income eligibility limit of approximately $45,000.[31] Most of these households are headed by a single, black female. We also estimate community specific fixed effects which are not reported in the table above. Our findings suggest that smaller communities are in general more desirable than larger communities.

Estimating a simple discrete choice models that ignore all supply side restrictions, we find that predicted demand exceeds supply by a factor of 7.7 using the full sample and by a factor of 4.3 using the two bedroom sub-sample. Failure to incorporate the supply side restriction in estimation thus leads to a seriously flawed inference and prediction.

Next we analyze the goodness of fit of our model. One measure of goodness of fit is to compare the residency distribution predicted by the model to the actual residency distribution observed in the sample. We find that the predictions that are based on our preferred model are accurate. Our model, thus, matches the unconditional distributions of households among choices well. A more challenging exercise is to predict the composition of the housing communities using our model. We focus on the composition by gender and family status conditional on race. The results are summarized in Table 5. The findings are by and large encouraging. Our model explains the demographic compositions of all communities well.

[31]Note that this limit depends on year and size of household.

Table 5: Actual vs Estimated Composition of Communities

		Private	PH1	PH2	PH3	PH4	PH5	PH6
% Black	Observed	0.24	0.98	0.94	0.90	0.97	0.56	0.55
	Estimated	0.26	0.95	0.92	0.9	0.95	0.51	0.56
% Female	Observed	0.67 / 0.53	0.85 / 0.88	0.89 / 0.75	0.93 / 1.00	0.84 / 0.67	0.63 / 0.53	0.66 / 0.68
	Estimated	0.67 / 0.53	0.82 / 0.67	0.87 / 0.71	0.93 / 0.83	0.84 / 0.64	0.57 / 0.48	0.67 / 0.66
% Have Kids	Observed	0.46 / 0.24	0.55 / 0.64	0.62 / 0.43	0.62 / 0.38	0.58 / 0.1	0 / 0	0 / 0
	Estimated	0.42 / 0.24	0.49 / 0.28	0.57 / 0.36	0.60 / 0.37	0.59 / 0.19	0.06 / 0.02	0.05 / 0.02
Income	Observed	19.3 / 21.0	8.4 / 7.2	12.3 / 12.9	14.1 / 10.3	9.9 / 11.3	9.1 / 8.5	9.3 / 9.8
	Estimated	19.3 / 21.5	8.5 / 6.2	12.3 / 8.1	12.6 / 7.5	9.9 / 8.1	8.3 / 8.0	9.4 / 9.9

Composition Shown by Race black / white.

We compare the observed mobility with the mobility generated under the model. With the model parameters from our preferred model, the predicted number of move-ins during this whole sample is 1796. The actual number is 1581. The predicted move-outs 2273 (actual is 2106). Finally the predicted number of transfers is 374 compared to 349 observed in the data.[32]

6 The Welfare Costs of Demolitions

We are now in a position to estimate the welfare costs associated with demolitions. As we discussed before, the federal government has for all practical purposes stopped building housing projects. To shed some insights into the effects of reducing the supply of public housing, we consider demolishing some of the least attractive public housing units. We analyze how demolitions affect the demand for public housing, the composition of housing communities, and compute standard welfare measures. We consider demolishing communities with a large number of units. These communities have been the target of demolitions in many cities. Our estimates confirm that they have the lowest fixed effect parameter and are thus the least attractive of all communities. The welfare estimates can, therefore, be interpreted as lower bounds for the welfare estimates associated with demolishing more desirable units.

We consider the demolition of public housing community 1 during the third period of a 12-quarter study. We use the estimates based on our preferred model in column II of Table 4. To initialize, the demographic characteristics in the first quarter are the same as those observed in the data. It is well-known that these types of discrete choice models do not yield closed form solutions for compensating variations. We, therefore, follow **??**) and adopt a simulation based approach. An additional complication

[32] Some periods in the HACP data were eliminated. Only quarters overlapping with the SIPP data were included in the estimation.

in our model is that we not only need to simulate draws from distributions of the error terms, but also from the equilibrium offer probabilities. For families of varying demographic characteristics, we compute the median compensating variation for an evicted household earning \$12,000 per year. We find that the estimates range from \$11,656 for a white male with kids to \$116,010 for a black female with kids. White households require lower compensation to leave public housing than black households. Overall, the estimates suggest that there may be significant welfare losses associated with demolishing existing units.[33] The policy experiment shows a decline in overall welfare for low-income blacks. However for some low-income households earning more than \$12,000 a year, there is a small welfare gain.

Compared to the baseline equilibrium, offer probabilities immediately decrease after the eviction because many evicted tenants wish to move back into public housing. Offer probabilities decrease 2.6% for medium communities, 12% for small family communities, 6.3% for mixed family and senior communities, and 16% for mostly senior communities. Over time, the composition of the remaining public housing communities changes. The public housing communities experience an increase of 3% in black households and a 12% decrease in non-black households; there is a 1.3% increase in female-headed households and 2.2% increase in households with children. Average income in the public housing communities decreases 2%. The demolitions of public housing, therefore, lead to an increase in racial and socio-economic segregation.

To better understand the mechanism that drives these welfare costs it is useful to provide a more complete characterization of the rationing process that results in equilibrium. Based on the parameter estimates of our preferred model in column I we estimate the fraction of the population that would like to move into public housing if

[33]Of course, a full cost-benefit analysis would require the inclusion of the cost of maintaining these housing units as well as potential impacts of living in public housing on educational achievements and criminal outcomes.

it was possible. This fraction varies by quarter due to quarterly differences in income and demographic heterogeneity. Table 6 shows the percent willing to move for the 12th quarter (a quarter in the middle of the study).

Table 6: Percent of Households in Community i Who Would Accept an Offer to Move to j

	Would move to:						
Current Residence:	Private	PH1	PH2	PH3	PH4	PH5	PH6
Private		0.006	0.012	0.009	0.008	0.009	0.012
PH1	0.080		0.067	0.054	0.044	0.055	0.071
PH2	0.063	0.020		0.029	0.023	0.029	0.039
PH3	0.075	0.023	0.043		0.028	0.035	0.045
PH4	0.077	0.031	0.056	0.045		0.046	0.059
PH5	0.102	0.022	0.041	0.032	0.026		0.043
PH6	0.085	0.019	0.034	0.027	0.022	0.028	

Comparing the fraction of households willing to move into a housing community with the number of available units in that community, we find that this ratio is equal 3.77 for community 1 which is the least attractive community. For the other three family communities this ratio ranges between 7.10 and 72.71. For senior communities this ratio is equal to 37.79 for communities with a small number of units and 18.17 for communities with a large number of units. If we restrict our attention to the subsample of households that are eligible for two bedroom apartments, the demand-supply ratios are 2.65, 3.90, 15.88, and 4.64 for the four types of housing communities. The fraction of households willing to move into a public housing unit largely depends on the community specific fixed effects and thus reflects the attractiveness of the housing community. However, it also depends on the characteristics of eligible households.

Older households and extremely poor households are more willing to move from the private sector to public housing communities. These households suffer the highest welfare costs from policies that restrict the supply.

7 Conclusions

We developed a new method that can be used to estimate the welfare costs of reducing the supply of public housing. Our estimates are based on an equilibrium model that captures the key supply restrictions. Our empirical analysis of the Pittsburgh metropolitan area, shows that there are significant welfare losses associates with policies that fail to maintain an adequate supply of affordable housing. The welfare effects are likely to be even more pronounced in cities with high housing prices and tight housing markets such as New York or Boston.

We do not dispute that some public housing high rises were in horrible disrepair and contributed to urban blight, and that a lot of people have benefitted from their demolition. Moreover, vouchers may provide a more attractive alternative for some families. Still, public housing appears to be attractive for seniors and very poor households headed by single mothers. Our paper clearly shows that the demand for public housing remains very strong. Our analysis suggests that relieving some of the current rationing by constructing new public housing units may be a good policy. A full cost-benefit analysis of new construction needs to be augmented by estimates of land purchases and construction costs. Nevertheless, it is straight-forward to conduct such a comprehensive analysis based on the framework presented in this paper.

The framework presented in this paper can be extended in a number of fruitful directions. In our model, households maximize current period utility. It is possible to model the dynamic decision problem faced by forward looking households. Households must now forecast if and when they will have access to public housing. The value function that corresponds to this problem depends on current and future offer probabilities. We can then proceed and define demand as before and define a dynamic equilibrium with forward looking households. Characterizing the equilibrium of this model and estimating its parameters is, however, more challenging since the equilibrium conditions are non-linear in the offer probabilities.

It is possible to estimate even richer versions of the model discussed here. We have abstracted from unobserved heterogeneity in tastes for public housing. It is possible that there is stigma associated with living in public housing. **?**) has shown that stigma plays a role in explaining participation in other welfare programs. We can extend our framework and allow for unobserved heterogeneity in tastes for public housing. Such heterogeneity would provide an alternative explanation for the differential flow rates into and out of public housing. Some households may obtain a sufficiently strong negative utility from public housing that they effectively are never interested in the public-sector. Other households might not be affected by stigma and are willing to choose public housing when they receive a sufficiently strong idiosyncratic shock. However, we can still define the equilibrium for this modified model. As long as we can express the offer probabilities as functions of the structural demand parameters, our approach for identification and estimation is valid and can be used to estimate richer specifications of the demand side. Estimating these types of model will allow us to obtain additional insights into the welfare cost of failing to provide an adequate supply of affordable housing in U.S. metropolitan areas.

A A Monte Carlo Study

Since our estimation procedure is non-standard, we conducted a number of Monte Carlo studies to study the properties of the estimators when the true data generating process is known. Below we report the results for one specification that we tested.[34]

Table 7: 95% Confidence Intervals of Estimation Error

Name	Variable	random sample	enriched sample
Fixed Effect PH1	γ_1	[-0.887, 1.763]	[-0.947, 1.763]
Fixed Effect PH2	γ_2	[-.8142, 1.585]	[-1.010, 1.585]
Fixed Effect PH3	γ_3	[-0.806, 1.744]	[-0.850, 1.744]
Beta	β	[-0.191, 0.079]	[-0.191, 0.082]
Offer Prob PH1	π_1	[-0.021 ,0.019]	[-0.020, 0.019]
Offer Prob PH2	π_2	[-0.043 ,0.050]	[-0.046, 0.055]
Offer Prob PH3	π_3	[-0.013 ,0.010]	[-0.013, 0.010]

In our Monte Carlo there is only one observed household characteristic ('income'). We assume that $f(x_t, d_{t-1})$ is log-normally distributed with known mean and variance. We consider a model with three public housing communities with $\gamma_1 = 7.6$, $\gamma_2 = 7.0$ and $\gamma_3 = 0.4$. We set the coefficient of income $\beta = 0.4$. We assign 30 % of the population to private housing, 24, 28, and 18 percent to the three housing communities. This implies that in equilibrium the offer probabilities are $\pi_1 = .11$, $\pi_2 = .24$ and $\pi_3 = 0.05$.

We consider the properties of the estimator above under two sampling designs: random sampling and enriched sampling. For each parameter vector, one hundred

[34]More results for different parametrizations, sample sizes and sampling schemes are available upon request from the authors.

model simulations and estimations are completed, each with sample size 2000. Starting values are initially chosen from a uniform distribution between $(0, 1)$ for β and between $[0, 12]$ for the fixed effects, but any starting values that would lead to unreasonable offer probabilities (probabilities greater than 40%) are rejected. The table above summarizes the performance of the model and reports 95% confidence for the absolute error of parameter estimate and the implied offer probabilities.

In general we find that our estimator works well both under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimate for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2000 observations. In general, larger samples help reduce the estimation bias. Imposing the equilibrium conditions seems to work well, and the estimates of the offer probabilities that are implied by the structural parameters of the model are accurate.