Compressive Sensing
UN Encuentro de Matemáticas, Universidad Nacional
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Course description

It is known that many signals, such as images and audio signals, are compressible. For a signal encoded as a vector in a high dimensional space, compressibility corresponds to having a sparse representation in a suitable orthonormal basis, such as a wavelet or Fourier basis.

Compressive sensing is a modern sampling and data acquisition approach that exploits compressibility when gathering a signal of interest. Advances in this field have shown that an unknown but compressible signal can be collected in a highly efficient manner. Roughly speaking, the number of measurements needed is just a bit higher than the number of entries needed in a sparse representation of the signal. This is achievable via non-adaptive sampling, that is, without any prior knowledge about the specific signal being collected. Compressive sensing is substantially changing the way we gather, process, and analyze large datasets. It has already had important applications in magnetic resonance imaging and seismology among others. Compressive sensing is currently a very active area of research spanning mathematics, statistics, engineering, and computer science.

This three-lecture course will give an introduction to compressed sensing. The first lecture will present an overview of the main concepts, terminology, and results from this field. In particular, we will discuss on the role of ℓ_1-minimization, incoherent projections, and the restricted isometry property for sparse signal recovery. The second lecture will go into some of the techniques from probability and optimization underlying the central results and algorithms in compressive sensing. The third lecture will present extensions of compressive sensing to low-rank matrix recovery. In this context the signals of interest are coded as matrices and the role of sparsity is replaced by low rank. The analogous problem to compressive sampling for sparse vectors is that of reconstructing a low rank matrix from a small subset of its entries.

Instructor

Javier Peña, Carnegie Mellon University.

Main references

For a comprehensive list of references in compressive sensing, please see `http://dsp.rice.edu/cs`

Papers:

- E. Candès & Y. Plan, "A probabilistic and RIPless theory of compressed sensing," *IEEE Trans. Inf. Theory,* vol 57, no. 11, pp. 7235–7254, Nov. 2011.
- E. Candès, J. Romberg, & T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory,* vol 52, no. 2, pp. 489–509, Feb. 2006.
- E. Candès & T. Tao, "Decoding by linear programming," *IEEE Trans. Inf. Theory,* vol 51, no. 12, pp. 4203–4215, Dec. 2005.
- D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory,* vol 52, no. 4, pp. 1289–1306, April 2006.
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- V. Chandrasekaran, S. Sanghavi, P.A. Parrilo and A. Willsky , "Rank-Sparsity Incoherence for Matrix Decomposition," *SIAM Journal on Optimization,* vol. 21, issue 2, pp. 572–596, 2011
- E. J. Cands, X. Li, Y. Ma, and J. Wright, "Robust Principal Component Analysis?," *Journal of ACM,* vol 58, no 3, article no 11, 2011.

Background on probability and optimization:

- R. Adler and J. Taylor, *Random Fields and Geometry,* Springer, 2007
- T. Tao, *Topics in Random Matrix Theory,* American Mathematical Society, 2012
- S. Boyd and L. Vandenberghe, *Convex Optimization,* Cambridge Press, 2004
- M. Grant and S. Boyd, `CVX`*: Matlab Software for Disciplined Convex Programming,* `http://cvxr.com/cvx/`

Popular press:

- R. Baraniuk, "More Is less: Signal processing and the data deluge," *Science,* 331, pp. 717–719, February 2011.
- J. Ellenberg, "Fill in the Blanks: Using Math to Turn Lo-Res Datasets Into Hi-Res Samples," *Wired Magazine,* Feb. 2010.