Condition numbers for optimization problems

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Condition

From Oxford English Dictionary:

condition: *the state of something, esp. with regard to its appearance, quality, or working order*

From Webster's Dictionary:

condition: *state of fitness*

Preamble: condition numbers of square matrices

Condition number of $A \in \mathbb{R}^{n\times n}$:

$$\kappa(A) = \|A\| \cdot \|A^{-1}\|.$$

Key parameter for problem

$$Ax = b.$$

- Regularity of the solution:

$$A(x + \delta x) = b + \delta b \Rightarrow \frac{\|\delta x\|}{\|x\|} \leq \kappa(A)\frac{\|\delta b\|}{\|b\|}.$$

- Problem geometry:

$$\kappa(A) = \text{ aspect ratio of } \{Ax : \|x\| \leq 1\}.$$

- Radius of well-posedness (radius of regularity):

$$\kappa(A) = \frac{\|A\|}{\mathrm{dist}(A, \mathrm{Sing})}.$$

Radius Theorem 1

Theorem (Eckart & Young, 1936)

Assume $A \in \mathbb{R}^{n\times n} \setminus \mathrm{Sing}$*. Then*

$$\mathrm{dist}(A, \mathrm{Sing}) = \frac{1}{\|A^{-1}\|} = \max\{\delta : \delta\mathbb{B}_{\mathbb{R}^n} \subseteq A\mathbb{B}_{\mathbb{R}^n}\}.$$

Sing : set of $n \times n$ singular matrices.

Connection with algorithms

Assume A is symmetric and positive definite and let $\bar{x}$ be the solution to $Ax = b$, i.e., the solution to

$$\min_x \left(\frac{1}{2}x^\mathsf{T}Ax - b^\mathsf{T}x\right) \Leftrightarrow \min_x \left(\frac{1}{2}\|x\|_A^2 - b^\mathsf{T}x\right)$$

- After k iterations, steepest-descent method yields x_k such that

$$\frac{\|x_k - \bar{x}\|_A}{\|x_0 - \bar{x}\|_A} \leq \left(\frac{\kappa(A) - 1}{\kappa(A) + 1}\right)^k .$$

- After k iterations, conjugate gradient yields x_k such that

$$\frac{\|x_k - \bar{x}\|_A}{\|x_0 - \bar{x}\|_A} \leq 2\left(\frac{\sqrt{\kappa(A)} - 1}{\sqrt{\kappa(A)} + 1}\right)^k .$$

Theme

- Extend the concept of condition number to linear optimization, conic optimization, and beyond.
- We will emphasize the interplay between condition, problem geometry, radius of regularity, and algorithms.

Condition numbers in optimization

Condition number of an optimization problem of the form

$$\begin{aligned} \min \quad & c^{\mathsf{T}} x \\ & Ax = b \\ & x \geq 0, \end{aligned}$$

or more generally

$$\begin{aligned} \min \quad & c^{\mathsf{T}} x \\ & Ax = b \\ & x \in K, \end{aligned}$$

for some closed convex cone K (e.g., second-order, semidefinite).

Data defining a problem instance

The triple $d := (A, b, c)$.

Condition numbers in optimization

Definition (Renegar)

Assume the instance $d := (A, b, c)$ is given. Define

$$C(d) := \frac{\|d\|}{\inf\{\|\Delta d\| : d + \Delta d \text{ is infeasible or unbounded}\}}.$$

Remarks

- Condition number in terms of a radius of well-posedness.
- This concept can be better understood by concentrating on the primal and dual constraints.

Condition numbers in optimization

Concentrate on the feasibility problems

$$Ax = b, \; x \in K$$

and

$$c - A^{\mathsf{T}} y \in K^*.$$

For convenience, consider the problems in homogenized form

$$Ax = 0, \; x \in K$$

and

$$A^{\mathsf{T}} y \in K^*.$$

For these homogeneous problems the data space is $\mathbb{R}^{m \times n}$.

From equations to constraints

Notice:

Given $A \in \mathbb{R}^{n \times n}$, we have $A \notin \text{Sing} \Leftrightarrow A\mathbb{R}^n = \mathbb{R}^n$.

Equivalently, $A \notin \text{Sing} \Leftrightarrow Ax = b$ has a solution for all $b \in \mathbb{R}^n$.

How do we extend this to constraint systems?

Assume $K \subseteq \mathbb{R}^n$ is a closed convex cone (e.g., $K = \mathbb{R}^n_+$) and $m \leq n$.

Define

$$\mathcal{P} := \{A \in \mathbb{R}^{m \times n} : AK = \mathbb{R}^m\},$$
$$\mathcal{D} := \{A \in \mathbb{R}^{m \times n} : A^{\mathsf{T}}\mathbb{R}^m + K^* = \mathbb{R}^n\}.$$

Well-posed and ill-posed matrices

Notice

- $A \in \mathcal{P} \Leftrightarrow Ax = b, x \in K$ has a solution for all $b \in \mathbb{R}^m$.
- $A \in \mathcal{D} \Leftrightarrow c - A^\mathsf{T} y \in K^*$ has a solution for all $c \in \mathbb{R}^n$.

Furthermore,

- If $m = n$ and $K = \mathbb{R}^n$ then

  $$\mathcal{P} = \mathcal{D} = \text{set of } n \times n \text{ non-singular matrices.}$$

- If $m < n$ then both $\mathcal{P}, \mathcal{D}$ are open and $\mathcal{P} \cap \mathcal{D} = \emptyset$.

Well-posed and ill-posed matrices

Definition

Ill-posed instances $\Sigma := \mathbb{R}^{m \times n} \setminus (\mathcal{P} \cup \mathcal{D})$.

Definition (Renegar)

Condition number of $A \in \mathbb{R}^{m \times n} \setminus \Sigma$

$$C(A) := \frac{\|A\|}{\text{dist}(A, \Sigma)}.$$

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$$C(A) := \frac{\|A\|}{\text{dist}(A, \Sigma)}.$$

Recover former $C(A, b, c) = \max\left\{ C\left(\begin{bmatrix} A & -b \end{bmatrix}\right), C\left(\begin{bmatrix} A \\ -c^{\mathsf{T}} \end{bmatrix}\right)\right\}$.

Radius Theorem 2

Theorem (Renegar, 1995)

(a) *If* $A \in \mathcal{P}$ *then*

$$\mathrm{dist}(A, \Sigma) = \max\{\delta : \delta\mathbb{B}_{\mathbb{R}^m} \subseteq A(\mathbb{B}_{\mathbb{R}^n} \cap K)\}.$$

(b) *If* $A \in \mathcal{D}$ *then*

$$\mathrm{dist}(A, \Sigma) = \max\{\delta : \delta\mathbb{B}_{\mathbb{R}^n} \subseteq A^{\mathsf{T}}\mathbb{B}_{\mathbb{R}^m} + K^*\}.$$

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(b) *If* $A \in \mathcal{D}$ *then*

$$\operatorname{dist}(A, \Sigma) = \max\{\delta : \delta \mathbb{B}_{\mathbb{R}^n} \subseteq A^{\mathsf{T}} \mathbb{B}_{\mathbb{R}^m} + K^*\}.$$

Recall Eckart & Young Radius Theorem:

$$\operatorname{dist}(A, \text{Sing}) = \frac{1}{\|A^{-1}\|} = \max\{\delta : \delta \mathbb{B}_{\mathbb{R}^n} \subseteq A \mathbb{B}_{\mathbb{R}^n}\}.$$

Radius Theorem 3

Suppose we are only allowed to perturb a block of A: Assume $k \leq m$, $\ell \leq n$ and put

$$\Delta := \left\{ \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix} : B \in \mathbb{R}^{k \times \ell} \right\}.$$

Theorem (P. 1998)

Assume $A \in \mathcal{P}$. Then

$$\mathrm{dist}_\Delta(A, \Sigma) = \max\left\{\delta : \delta \mathbb{B}_{\mathbb{R}^k} \times \{0\} \subseteq \{Ax : x \in K, x_{1:\ell} \in \mathbb{B}_{\mathbb{R}^\ell}\}\right\}$$

Renegar's Radius Theorem (a,b) can be recovered.

Good condition implies good geometry

Assume $A \in \mathcal{D}$. Consider the cone $\mathcal{F} := \{y : A^{\mathsf{T}} y \in K^*\}$.

Consequence of Renegar's Theorem: *If $C(A)$ small then $\mathcal{F}$ is thick.*

Theorem (Freund & Vera 1999)

If $A \in \mathcal{D}$ then

$$\tau_{\mathcal{F}} := \max_{\|y\|=1} \{r : \mathbb{B}(y, r) \subseteq \mathcal{F}\} \geq \frac{c_{K^*}}{C(A)}$$

c_{K^*} : *positive constant that depends on the cone K^* only.*

(Similar geometric condition when $A \in \mathcal{P}$.)

Natural question

Does good geometry imply good condition? Not always.

Best-conditioned solutions

Consider special case $K = \mathbb{R}^n_+$. Given $A \in \mathbb{R}^{m\times n}$, write

$$A = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}.$$

Goffin 1980, Cheung & Cucker 2001

Assume $a_i \neq 0,\ i = 1, \ldots, n$. Consider

$$\rho(A) := \max_{\|y\|=1} \min_{j=1,\ldots,n} \frac{a_j^\mathsf{T} y}{\|a_j\|}.$$

Notice

- $A \in \mathcal{D} \Leftrightarrow \rho(A) > 0.$
- $A \in \mathcal{P} \Leftrightarrow \rho(A) < 0.$

Radius Theorem 4

Geometric interpretation

Assume $A \in \mathcal{D}$ and let $\mathcal{F} := \{y : A^\mathsf{T} y \geq 0\}$. In this case

$$\rho(A) = \tau_{\mathcal{F}} = \max_{\|y\|=1} \{r : \mathbb{B}(y, r) \subseteq \mathcal{F}\}.$$

The point $\bar{y}$ where $\rho(A)$ is attained is the "best-conditioned" solution to

$$A^\mathsf{T} y \geq 0.$$

Theorem (Cheung & Cucker, 2001)

Assume $a_i \neq 0,\ i = 1, \ldots, n$. Then

$$|\rho(A)| = \inf \left\{ \max_{i=1,\ldots,n} \frac{\|a_i - \tilde{a}_i\|}{\|a_i\|} : \tilde{A} \in \Sigma \right\}.$$

Good geometry implies good (tweaked) condition

Goffin-Cheung-Cucker condition number

$$\mathscr{C}(A) := \frac{1}{|\rho(A)|}.$$

Renegar's $C(A)$ versus Goffin-Cheung-Cucker's $\mathscr{C}(A)$:

- $|\rho(A)|$: kind of a column-wise *scaled* distance to Σ.
- $\mathscr{C}(A) = C(A)$ if columns of A have all norm one.
- $\mathscr{C}(A) \leq nC(A)$ but $\mathscr{C}(A)$ could be arbitrarily smaller.
- $\mathscr{C}(A)$ filters out poor conditioning due to bad scaling of the columns of A.

Good condition implies good behavior of algorithms

Assume $K \subseteq \mathbb{R}^n$ is a regular cone, $A \in \mathbb{R}^{m \times n}$, and consider the pair of homogeneous problems

$$Ax = 0, \; x \in \mathrm{int}(K) \tag{1}$$

and

$$A^\mathsf{T} y \in \mathrm{int}(K^*). \tag{2}$$

Homogeneous feasibility problem

Determine which of (1) and (2) is feasible and find a solution.

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To find ϵ-solution to conic optimization problem, consider

$$\begin{array}{ccccccc} & \tau\epsilon & - & c^\mathsf{T}x & + & b^\mathsf{T}y & \geq & 0 \\ & \tau c & & & - & A^\mathsf{T}y & \in & K^* \\ - & \tau b & + & Ax & & & = & 0 \\ & & & & & \tau & \geq & 0 \\ & & & & & x & \in & K \end{array}$$

Condition: $C(d)/\epsilon$.

Condition-based analyses of various algorithms

- Interior-point methods (Renegar, Filipowski, Vera, P)
- Ellipsoid method (Freund & Vera)
- Perceptron method (Belloni, Dunagan, Freund, Vempala,...)
- Von Neumann method (Epelman & Freund)
- ...

The above analyses show that a solution to (1) or (2) is found after

$$\mathcal{O}(n^d \log(C(A))),$$

or

$$\mathcal{O}(n^d C(A))$$

iterations, provided $A \notin \Sigma$.

The Perceptron Algorithm

Find a solution to

$$A^{\mathsf{T}} y > 0.$$

(Assume $A \in \mathcal{D}$ and $K = \mathbb{R}^n_+$.)

Perceptron Algorithm (Rosenblatt, 1957)

- $y := 0$
- while $A^{\mathsf{T}} y \not> 0$

  $y := y + \frac{a_j}{\|a_j\|}$, where $a_j^{\mathsf{T}} y \leq 0$

  end while

Theorem (Block-Novikoff 1962)

If $A \in \mathcal{D}$, then the Perceptron Algorithm terminates after at most

$$\mathscr{C}(A)^2 = \frac{1}{\rho(A)^2}$$

iterations.

Some Extensions of the Perceptron Algorithm

Theorem (Dunagan & Vempala 2004)

If $A \in \mathcal{D}$ then a **randomized re-scaled** *version of the Perceptron Algorithm terminates in $\mathcal{O}\left(n \cdot \log\left(\mathscr{C}(A)\right)\right)$ iterations with high probability.*

Theorem (Soheili & P 2011)

If $A \in \mathcal{D}$ then a **smooth** *version of the Perceptron Algorithm terminates in $\mathcal{O}\left(\sqrt{\log(n)} \cdot \mathscr{C}(A)\right)$ iterations while retaining the algorithm's original simplicity.*

Other condition-based analyses

Theorem (Renegar 1995, P & Renegar 2000)

If A is well-posed and K, K^* have barrier functions, then an interior-point algorithm determines which of (1), (2) is feasible and finds such a solution in at most $\mathcal{O}(\sqrt{\vartheta}\log(\vartheta \cdot C(A)))$ iterations.

Here ϑ : parameter of barrier functions for K, K^*.

Theorem (Freund & Vera 1999)

If $A \in \mathcal{D}$ and K^* has a separation oracle, then the ellipsoid method finds a solution to (2) in at most

$$\mathcal{O}(n^2 \log(1/\tau)) = \mathcal{O}(n^2 \log(C(A)/c_{K^*}))$$

iterations.

Does good algorithmic behavior imply good geometry?

Consider the feasibility problem

$$x \in \mathcal{F} \tag{3}$$

where $\mathcal{F}$ is a cone with a separation oracle.

Theorem (Freund & Vera 2009)

Let $\tau \in (0, 1)$ be given. For any separation-based algorithm there exists a cone $\mathcal{F}$ with width τ such that the algorithm needs at least

$$\lfloor \log_2(1/\tau) \rfloor$$

iterations to solve (3).

What about ill-posed instances?

Limitation of previous results

They all assume $A \notin \Sigma$. However, in some interesting cases the feasibility problem is canonically ill-posed.

Example

Homogenization of optimality conditions for linear optimization:

$$\begin{array}{rrrrrrrcl} & & - & c^{\mathsf{T}}x & + & b^{\mathsf{T}}y & \geq & 0 \\ & \tau c & & & - & A^{\mathsf{T}}y & \geq & 0 \\ - & \tau b & + & Ax & & & = & 0 \\ & & & & & \tau & \geq & 0 \\ & & & & & x & \geq & 0 \end{array}$$

Stratified condition numbers

Can we refine or define condition for instances in Σ?

Motivation

- When $K = \mathbb{R}^n$, $\Sigma =$ rank-deficient matrices.
- The set of ill-posed instances Σ can be written as

$$\Sigma = \Sigma_{m-1} \cup \Sigma_{m-2} \cup \cdots \cup \Sigma_1 \cup \Sigma_0$$

  $\Sigma_r =$ matrices with rank at most r.
- Given $A \in \Sigma_i \setminus \Sigma_{i-1}$,

$$\mathrm{dist}_{\Sigma_i}(A, \Sigma_{i-1}) = \sigma_i(A).$$

  $\sigma_i(A)$: i-th (smallest positive) singular value of A.

Stratified condition numbers

Consider again the special case $K = \mathbb{R}^n_+$, and the two associated homogeneous feasibility problems

$$Ax = 0,\ x \geq 0 \ \text{ and } \ A^\mathsf{T} y \geq 0.$$

How can we stratify Σ?

Answer: Use a "canonical" partition $\mathscr{P}(A) = \{B, N\}$ of $\{1, \ldots, n\}$.

Canonical partition

Proposition (Goldman-Tucker)

Assume $A \in \mathbb{R}^{m \times n}$. *Then there exists a unique partition* $B \cup N = \{1, \ldots, n\}$ *such that for some* $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$

$$A_B x_B = 0,\ x_B > 0,\ A_B^\mathsf{T} y = 0,\ A_N^\mathsf{T} y > 0.$$

Observe

- $A \in \mathcal{D} \Leftrightarrow B = \emptyset$
- $A \in \mathcal{P} \Leftrightarrow N = \emptyset$ and $\operatorname{rank}(A) = m$.

Stratified distance to ill-posedness

Assume $A \in \mathbb{R}^{m\times n}$ and $\mathscr{P}(A) = \{B, N\}$. Define

$$L = \ker(A_B^\mathsf{T}) \subseteq \mathbb{R}^m, \text{ and } L^\perp = \text{range}(A_B) \subseteq \mathbb{R}^m.$$

If $N \neq \emptyset$, define

$$\rho_N(A) := \max_{\substack{y\in L\\ \|y\|=1}} \min_{j\in N} \frac{a_j^\mathsf{T} y}{\|a_j\|}.$$

If $B \neq \emptyset$, define

$$\rho_B(A) = \max_{\substack{y\in L^\perp\\ \|y\|=1}} \min_{j\in B} \frac{a_j^\mathsf{T} y}{\|a_j\|}.$$

Observe

- $N \neq \emptyset \Rightarrow \rho_N(A) > 0.$
- $B \neq \emptyset \Rightarrow \rho_B(A) < 0.$

Radius Theorem 5

Theorem (Cheung, Cucker & P., 2008)

For $A \in \mathbb{R}^{m\times n}$

$$\rho_N(A) = \min_{\substack{\mathscr{P}(\tilde{A})\neq\mathscr{P}(A)\\ \tilde{A}_B=A_B}} \max_{j\in N} \frac{\|\tilde{a}_j - a_j\|}{\|a_j\|}$$

and

$$|\rho_B(A)| = \min_{\substack{\mathscr{P}(\tilde{A})\neq\mathscr{P}(A)\\ \tilde{A}_N=A_N\\ \ker(\tilde{A}_B^{\mathsf{T}})\supseteq L}} \max_{j\in B} \frac{\|\tilde{a}_j - a_j\|}{\|a_j\|}.$$

Extended condition number

Definition

$$\bar{\mathscr{C}}(A) := \max\left\{\frac{1}{\rho_N(A)}, \frac{1}{|\rho_B(A)|}\right\}$$

Observations

- $A \notin \Sigma \Rightarrow \mathscr{C}(A) = \bar{\mathscr{C}}(A)$.
- $\bar{\mathscr{C}}(A) < \infty$ for all $A \in \mathbb{R}^{m\times n}$.
- $\rho_N(A)$ relative thickness of $\{y : A_B^\mathsf{T} y = 0,\ A_N^\mathsf{T} y > 0\}$.
- $|\rho_B(A)|$ similar for $\{x : A_B x_B = 0, x_B > 0\}$.

Condition-based complexity for ill-posed problems

Given $A \in \mathbb{R}^{m \times n}$, consider the pair of homogeneous feasibility problems

$$Ax = 0,\ x \geq 0 \ \text{ and } \ A^{\mathsf{T}} y \geq 0.$$

Theorem (Soheili & P., 2010)

Interior-point algorithm that finds $\mathscr{P}(A) = \{B, N\}$ as well as x_B and y such that

$$A_B x_B = 0,\ x_B > 0 \ \textit{ and } \ A_B^{\mathsf{T}} y = 0,\ A_N^{\mathsf{T}} y > 0$$

in at most $\mathcal{O}(\sqrt{n} \cdot \log(n \cdot \bar{\mathscr{C}}(A)))$ iterations.

Related central open problem in optimization

Smale's 9th problem

Is there a polynomial-time algorithm over the real numbers which decides the feasibility of the linear system of inequalities $Ax \geq b$?

Nice Theory. Practical Relevance?

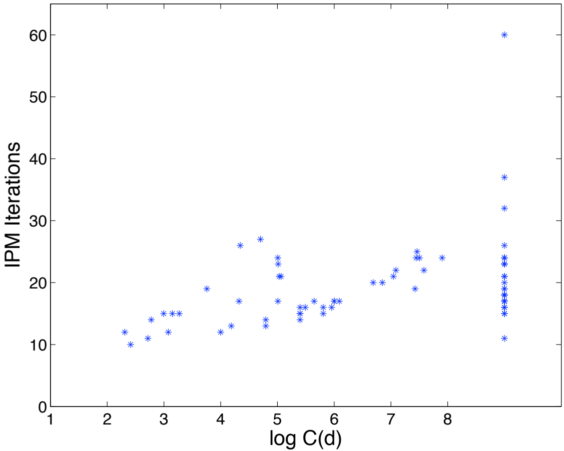
Freund, Ordoñez & Toh, 2007

Empirical study of SDP problems in the SDPLIB suite.

Approach

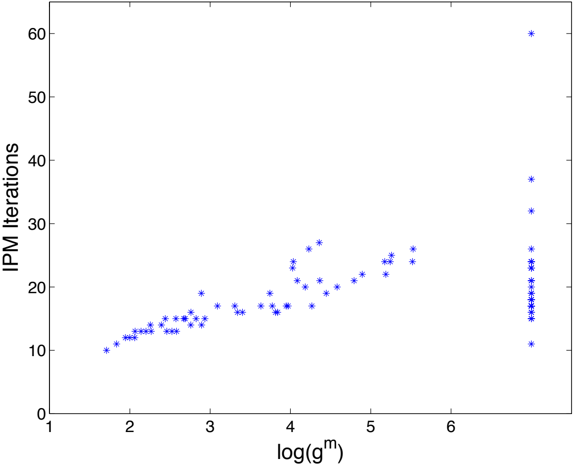
- Estimate condition number $C(A, b, c)$ of each instance (A, b, c)
- Estimate also a certain geometric measure $G(A, b, c)$ (in the spirit of $\tau_{\mathcal{F}}$)
- Run SDPT3 with default settings.
- Determine if there is an empirical relationship between number of iterations and the measures $C(A, b, c)$ and $G(A, b, c)$.

Number of IPM versus $\log(C(A, b, c))$



correlation $= 0.63$

Number of IPM versus $\log(G(A, b, c))$



correlation $= 0.901$

Condition of more general problems

Assume $m \leq n$ and let $\Sigma =$ rank-deficient matrices (i.e., $K = \mathbb{R}^n$).

Regularity of full-rank matrices

Given $A \in \mathbb{R}^{m \times n}$, we have $A \notin \Sigma \Leftrightarrow \delta \mathbb{B}_{\mathbb{R}^m} \subseteq A\mathbb{B}_{\mathbb{R}^m}$ for some $\delta > 0$.

Equivalently, $A \notin \Sigma \Leftrightarrow$ there exists $\delta > 0$ such that for all $\bar{y} \in \mathbb{R}^m$ and all $\bar{x} \in \mathbb{R}^n$

$$\operatorname{dist}(\bar{x}, A^{-1}(\bar{y})) \leq \frac{1}{\delta} \cdot \operatorname{dist}(\bar{y}, A\bar{x}).$$

Metric regularity

Assume $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$, i.e., $x \mapsto F(x) \subseteq \mathbb{R}^m$ for each $x \in \mathbb{R}^n$.

A **generalized equation** is a problem of the form

$$\text{Find } x \text{ such that } b \in F(x).$$

Definition

Assume $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ and $\bar{y} \in F(\bar{x})$. F is *metrically regular* at $\bar{x}$ for $\bar{y}$ if there exists $\kappa > 0$ such that

$$d(x, F^{-1}(y)) \leq \kappa \cdot d(y, F(x)) \tag{4}$$

for all (x, y) in a neighborhood of $(\bar{x}, \bar{y})$. In this case define

$$\operatorname{reg} F(\bar{x} \,|\, \bar{y}) := \inf\{\kappa : (4) \text{ holds}\}.$$

Radius Theorem 6

Theorem (Dontchev, Lewis, Rockafellar, 2003)

Assume $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$, $\bar{y} \in F(\bar{x})$ *and graph*(F) *locally closed at* $(\bar{x}, \bar{y})$. *Then*

$$\inf\{\|B\| : F + B \text{ is not metrically regular}\} = \frac{1}{\operatorname{reg} F(\bar{x} \,|\, \bar{y})}.$$

Interesting connections with fundamental results in analysis: Banach Open Mapping Principle, Lusternik-Graves Theorem, Robinson-Ursescu Theorem.

Summary and conclusions

- Extend concept of condition from linear equations to linear (and conic) optimization
- Similar interplay between condition, geometry, and algorithms
- Similar theorems concerning radius of well-posedness
- Other related work (current & future):
  - "Structured" condition numbers (Doyle, Lewis, P., Packard, Rump, Rohn, Tits,...)
  - Probabilistic analysis of condition numbers (Burgisser, Cucker, Hauser, Spielman, Teng,...
  - Geometric measures (Epelman, Freund, Vera,...)
  - Preconditioning (Epelman)
  - Condition of ill-posed problems