Isogeometric Analysis of Blood Flow: a NURBS-based Approach

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We describe a new approach for constructing patient-specific vascular geometries suitable for isogeometric fluid and fluid-structure interaction analysis of blood flow in arteries. We use solid NURBS (non-uniform rational B-splines) to define vascular geometries as well to perform analysis. It is argued in this paper that this new approach is a viable alternative to the finite element method, which is a standard tool for analysis of vascular systems. Advantages of the new approach are discussed and the technique is demonstrated on a variety of patient-specific arterial models.

1 INTRODUCTION

Patient-specific modeling and simulation-based medical planning has recently become an attractive avenue of research. This research is aimed at providing physicians with tools to construct and analyze combined anatomical/physiological models in order to predict the outcome of alternative treatment plans for an individual patient using techniques from image processing and computational mechanics. The finite element method is considered a standard tool for cardiovascular simulation. In the pioneering paper on the subject (Taylor, Hughes, and Zarins 1998) used real-life geometries to simulate blood flow thus opening the door for designing predictive technologies for vascular modeling and treatment planning.

In the finite element method complex geometrical objects are represented by a mesh of finite elements, typically piece-wise linear tetrahedra or hexahedra. Figure 1 illustrates examples of unstructured tetrahedral and hexahedral mesh of a patient-specific abdominal aorta built using the techniques outlined in (Zhang, Bajaj, and Sohn 2005) and (Zhang and Bajaj 2006). Basis functions, usually of low order, are constructed on the finite element partition, thus generating discrete spaces for approximate solutions of the underlying partial differential equations (see, e.g., (Hughes 2000)).

Isogeomeric Analysis is a new computational technique that improves on and generalizes the standard finite element method. It was first introduced in (Hughes, Cottrell, and Bazilevs 2005), and expanded on in (Cottrell, Reali, Bazilevs, and Hughes 2006). In an effort to instantiate the concept of isogeometric analysis, an analysis framework based on NURBS was built. Mathematical theory of the NURBS-based approach was put forth in (Bazilevs, da Veiga, Cottrell, Hughes, and Sangalli 2006).

In this work we advocate the use of the NURBS-based isogeometric analysis for vascular applications as an alternative to the standard finite element approach. Some of the motivating factors are: 1) NURBS are able to compactly and accurately represent smooth exact geometries, that are natural for arterial systems, but unattainable in the faceted finite-element representation 2) NURBS-based isogeometric analysis is inherently a higher-order technique with approximation properties superior to low-order finite elements. Both factors, as well as some additional ones discussed later in this paper, should render fluid and structural computations more physiologically realistic.

A skeleton-based sweeping method was developed to construct hexahedral solid NURBS meshes for patient-specific models from imaging data. Templates are designed for various branching configurations to decompose the geometry into mapped meshable regions. Piece-wise linear semi-structured hexahedral meshes can also be constructed using this approach. Figure 1 shows examples of semi-structured hexahedral and solid NURBS meshes of a patient-specific abdominal aorta.

The remainder of the paper is organized as fol-

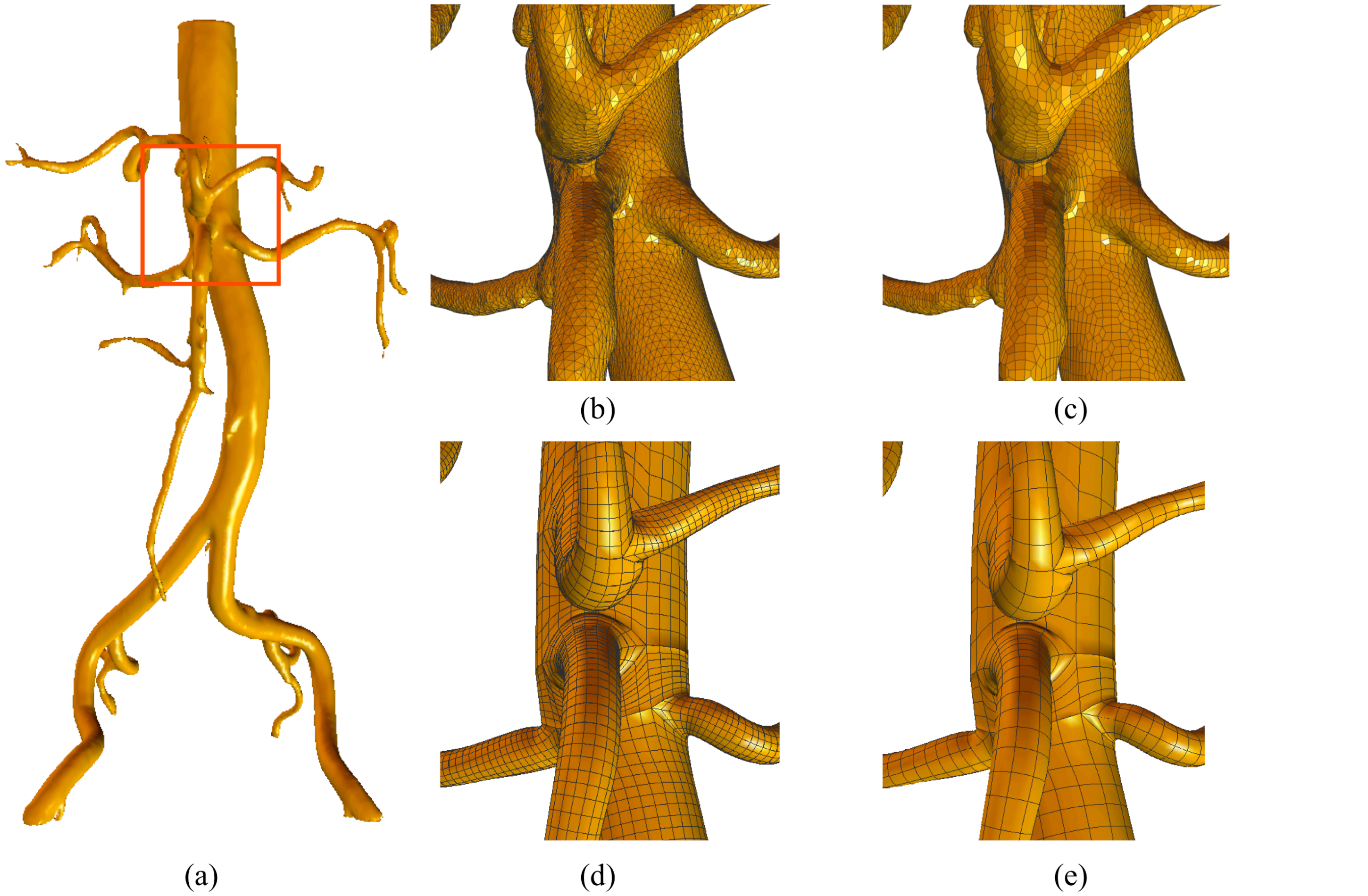

Figure 1: (a) - Geometrical model of a patient-specific abdominal aorta; (b) - Tetrahedral mesh; (c) - Unstructured hexahedral mesh; (d) - Semi-structured hexahedral mesh; (e) - Solid NURBS mesh. All meshes show a zoom on the branching area and are analysis suitable.

lows. In Section 2 we give a brief review of isogeometric analysis based on NURBS. In Section 3 we present a detailed discussion of the NURBS-based arterial cross-section construction and its implications on the analysis procedures from the stand point of accuracy and implementational convenience. In Section 4 we present examples of patient-specific models. In Section 5 we draw conclusions and outline future research directions.

2 ISOGEOMETRIC ANALYSIS USING NURBS

In a NURBS-based isogeometric analysis a physical domain in $\mathbb{R}^3$ is defined as a union of patches. A patch, denoted by Ω, is an image under a NURBS mapping of a parametric domain $(0,1)^3$

$$\Omega = \tag{1}$$

$$\{\mathbf{x} = (x,y,z) \in \mathbb{R}^3 \mid \mathbf{x} = \mathbf{F}(\xi,\eta,\zeta),\ 0 < \xi,\eta,\zeta < 1\},$$

where

$$\mathbf{F}(\xi,\eta,\zeta) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta)\mathbf{C}_{i,j,k}, \tag{2}$$

$$R_{i,j,k}^{p,q,r} = \frac{N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}\sum_{\hat{k}=1}^{l} N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)L_{\hat{k},r}(\zeta)w_{\hat{i},\hat{j},\hat{k}}}. \tag{3}$$

In the above, $R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta)$'s are the rational basis functions, and $\mathbf{C}_{i,j,k}$'s $\in \mathbb{R}^3$ are the control points. In the definition of the rational basis, $N_{i,p}(\xi)$'s, $M_{j,q}(\eta)$'s, and $L_{k,r}(\zeta)$'s, are the univariate B-spline basis functions of polynomial degree p, q, and r; $w_{i,j,k}$'s, strictly positive, are the weights.

In isogeometric analysis the geometry generation step involves construction of a control mesh, which is a piecewise multi-linear interpolation of control points, and the corresponding rational basis functions. The initial mesh encapsulates the 'exact geometry' and, in fact, defines it parametrically.

For the purposes of analysis, the isoparametric concept is invoked (see (Hughes 2000)). The basis for the solution space in the physical domain is defined through a push forward of the rational basis functions defined in (2) (see (Bazilevs, da Veiga, Cottrell, Hughes, and Sangalli 2006) for details). Coefficients of the basis functions, defining the solution fields in question (e.g., displacement, velocity, etc.), are called control variables.

As a consequence of the parametric definition of the 'exact' geometry at the coarsest level of discretization, mesh refinement can be performed automatically without further communication with the original description. This is an enormous benefit. There are NURBS analogues of finite element h- and p-refinement, and there is also a variant of p-refinement, which is termed k-refinement, in which the continuity of functions is systematically increased. This seems to have no analogue in traditional finite element analysis but is a feature shared by some meshless methods. For the details of the refinement algorithms see (Hughes, Cottrell, and Bazilevs 2005).

3 CONSTRUCTION OF THE ARTERIAL CROSS-SECTION

Blood vessels are tubular objects, therefore we choose the sweeping method to construct meshes for isogeometric analysis. A central feature of the approach is the construction of the arterial cross-section template, which is based on the NURBS definition of the circle. A solid NURBS description of an arterial branch is then obtained by extrusion of a circular surface along the vessel path, projection of the control points onto the true surface, and filling the volume radially inward. Arterial systems also engender various branchings and intersections, which are also handled with a template-based approach described in detail in (Zhang, Bazilevs, Goswami, Bajaj, and Hughes 2006). The above procedure generates a multi-patch, tri-variate description of a patient-specific arterial geometry that is also analysis suitable.

In this section we focus on the construction of the cross-section template specific to fluid-structure interaction analysis. We identify the area occupied by the blood, or the fluid region, and the arterial wall, or the solid region. These two subdomains are separated by the luminal surface, or the fluid-solid boundary. Figure 2, which shows a schematic of the NURBS mesh of a circular cross-section, gives an illustration of the above decomposition. NURBS elements are defined as areas enclosed between isoparametric lines. Note that the isoparametric lines correspond to the radial and circumferential directions, and both engender linear parameterization. For computational purposes we isolate the fluid and solid regions by a C^0 line as the solution is not expected to have regularity beyond C^0 at the multi-physics interface. It is important to note that this does not introduce any changes in the geometry of the object or its parameterization.

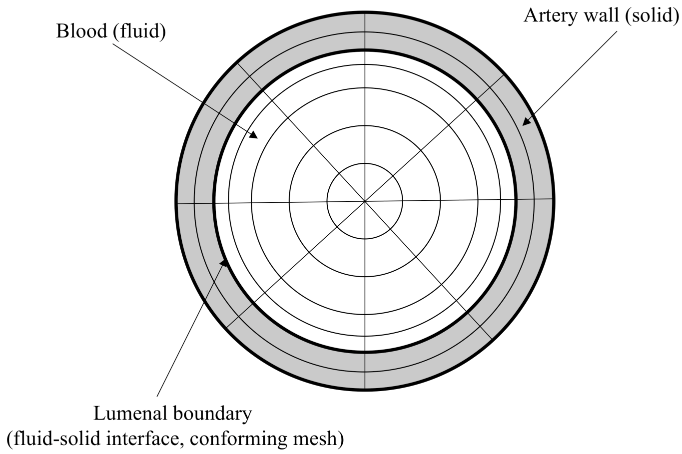

Figure 2: Arterial cross-section template based on a NURBS mesh of a circle. Fluid and solid regions are identified and separated by an interface. For analysis purposes basis functions are made C^0-continuous at the fluid-solid boundary.

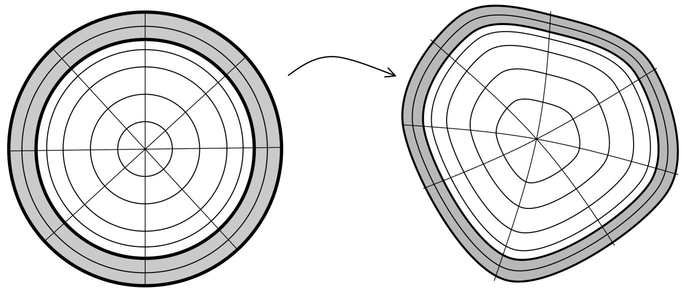

Figure 3: Arterial cross-section template is mapped onto a subject-specific geometry in a way that the topology of the fluid and solid subdomains remains unchanged.

Human arteries are not circular, hence projection of the template onto the true surface is necessary. Only control points that govern the cross-section geometry are involved in the projection process, while the underlying parametric description of the cross-section stays unchanged. The end result of this construction is shown in Figure 3 which shows the mapping of the template cross-section onto the patient-specific geometry. Here the isoparametric lines are somewhat distorted so as to conform to the true geometry, while the topology of the fluid and solid subdomains is preserved along with their interface.

As compared to the standard finite element method, the above approach has significant benefits for analysis, both in terms of accuracy and implementational convenience:

1. In the case of a flow in a straight circular pipe driven by a constant pressure gradient, NURBS basis of quadratic order gives rise to a pointwise exact solution to the incompressible Navier-

Stokes equations. This also has implications on the overall accuracy of the approach.

2. Parametric definition of the NURBS mesh in the fluid region allows one to refine the boundary layer region near the arterial wall. This is crucial for overall accuracy as well as for obtaining accurate wall quantities, such as wall shear stress, which plays an important role in predicting the onset of vascular disease. It is well known that unstructured finite element boundary layer meshes lead to much less accurate solutions for a comparable number of degrees of freedom. In order to circumvent this shortcoming, adaptive boundary layer meshing is required, which is not an easy task, especially for unsteady flows. For recent work in this direction see (Sahni, Muller, Jansen, Shephard, and Taylor 2006).

3. Parametric definition of the NURBS mesh in the solid region allows one to easily define material anisotropy which is present in the arterial wall. See (Holzapfel 2004) for arterial wall material modeling which accounts for anisotropic behavior.

4. Fluid structure interaction applications involve motion of the fluid region. This is typically done by solving an auxiliary linear elastic boundary value problem for mesh movement (see, e.g., (Bazilevs, Calo, Zhang, and Hughes 2006)). Parametric mesh definition in the fluid region allows for a straight forward specification of these elastic mesh parameters. For example, we "stiffen" the mesh in the radial direction so as to preserve boundary layer elements during mesh motion.

4 NUMERICAL EXAMPLES

In this section we present applications of the new methodology to two patient-specific vascular models: a model of the thoracic aorta and a model of the abdominal aorta. Isogeometric analysis is then used to compute blood flow in the models. In all cases, time-dependent, viscous, incompressible Navier-Stokes equations were used as the blood model. The fluid density and dynamic viscosity were chosen to be representative of blood flow. Both models are subjected to a time-periodic inflow boundary condition, which simulates the input from a beating heart. All examples present fluid-structure interaction calculations using the isogeometric approach. The wall is assumed to be nonlinear elastic (see Bazilevs (Bazilevs, Calo, Zhang, and Hughes 2006) for the details of the mathematical formulation). All simulations were run in parallel.

Thoracic aorta model: Data for this model was obtained from CT Angiography imaging data of a healthy male over-30 volunteer. An extra branch, representing a left ventricular assist device (LVAD), was added to the arterial model. Evaluation of LVADs, as well as other electromechanical devices used to support proper blood circulation, is of great interest to the cardiovascular community. The surface geometry, the control mesh, and the solid NURBS model are shown in Figures4a-4c. Figure 4d shows a result of the fluid-structure interaction simulation. Note that the inlet and the three smaller outlet branches were extended for the purposes of analysis.

Abdominal aorta: Data for this model was obtained from 64-slice CT angiography of a healthy male over 55 years of age. The surface geometry, the control mesh, and the solid NURBS model are shown in Figures 5a-5c. Figure 5d shows a result of the fluid-structure simulation. A computational study using a truncated geometrical model of this aorta was performed in (Bazilevs, Calo, Zhang, and Hughes 2006).

5 CONCLUSIONS AND FUTURE WORK

We have developed a NURBS-based modeling and simulation approach for fluid and fluid-structure interaction analysis of blood flow in arteries. Our technique is a viable alternative to the standard finite element method. It possesses a set of attractive features from the view point of accuracy and implementational convenience not engendered by standard finite element discretizations. Rigorous comparison with the standard finite element computations are still necessary in order to strengthen our conjecture.

We have focused on NURBS modeling. Techniques such as A-patches (Bajaj, Chen, and Xu 1995), T-splines (Sederberg, Cardon, Finnigan, North, Zheng, and Lyche 2004), and subdivision (Cirak, Scott, Antonsson, Ortiz, and Schröder 2002) are currently under investigation.

We have successfully applied our method to two patient-specific examples, which involve a thoracic aorta model, and an abdominal aorta model. As part of future work, we would like to apply the techniques described here to modeling and analysis of the human heart.

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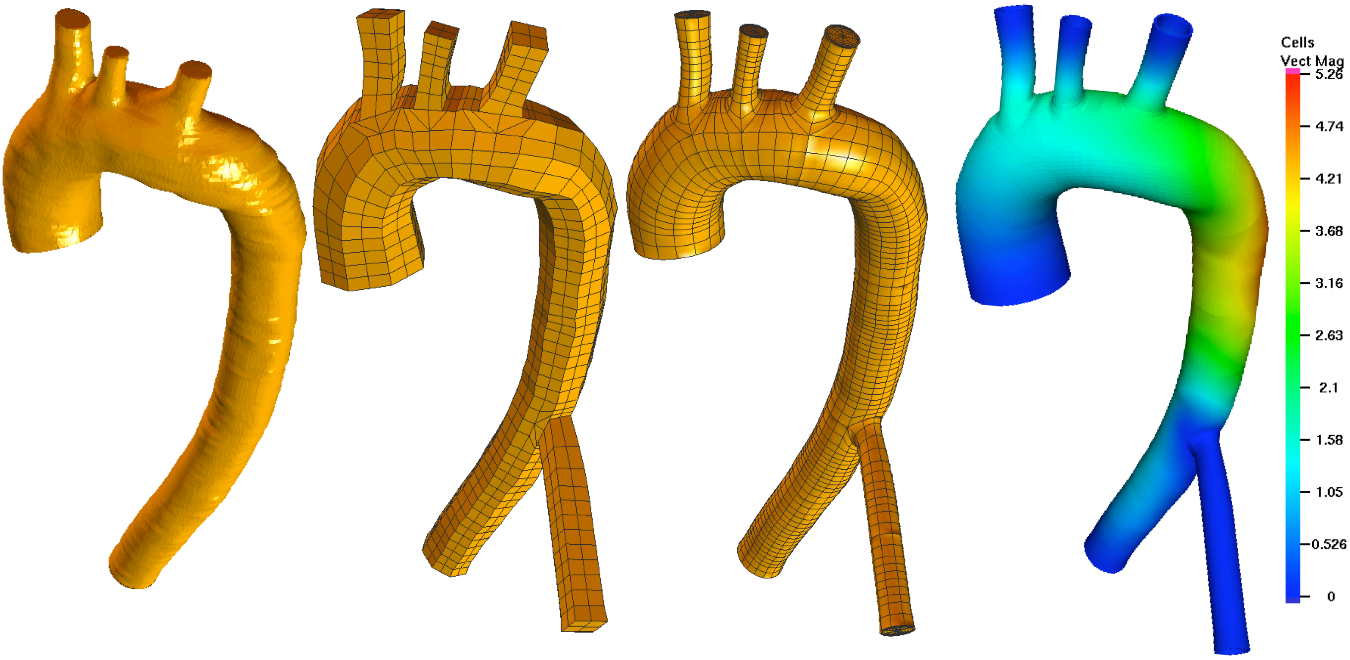

(a) (b) (c) (d)

Figure 4: Thoracic aorta. (a) - surface geometry; (b) - control mesh; (c) - solid NURBS (41,526 elements); (d) - fluid-structure interaction simulation results: contours of the arterial wall velocity (cm/s) during late diastole plotted on the current configuration. Computational model contains an LVAD branch, which is assumed rigid.

(a) (b) (c) (d)

Figure 5: Abdominal aorta. (a) - surface geometry; (b) - control mesh; (c) - solid NURBS mesh (73,314 elements); (d) - fluid-structure interaction simulation results: contours of the arterial wall velocity (cm/s) during late systole plotted on the current configuration. Only major branches are kept in (b-d).

with the data for the thoracic aorta model.

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