DOMAIN REDUCTION METHOD FOR THREE-DIMENSIONAL EARTHQUAKE MODELING IN LOCALIZED REGIONS. PART II: VERIFICATION AND APPLICATIONS

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November 5, 2002

ABSTRACT

Several examples are used to verify the domain reduction method (DRM), a two-step finite element methodology described in a companion paper for modeling earthquake ground motion in highly heterogeneous three-dimensional localized regions. The first set involves a simple, flat-layered, system. Verification of the DRM for this problem is carried out by comparing the results to those calculated directly by the theoretical Green's function method. Its applicability to more general problems is illustrated by two examples: a basin and a hill with and without a weathered surface layer, and the same stratigraphy. We use a Green's function approach for the first step, which for the examples under consideration needs to be performed only once. For the second step, the domain of computation is reduced in each case to a small neighborhood of the geological feature at hand. The second application considers the ground motion due to a strike-slip double-couple buried 14 km below the free surface in an 80 km $\times$ 80 km $\times$ 30 km region that encloses entirely the Los Angeles basin. This problem is solved first by the finite element method using the single-step traditional approach, in which the ground motion is calculated simultaneously near the seismic source, along the propagation path, and within the region of interest, with a single model that encompasses the entire geological structure, from the source to the region of interest. The DRM is then used to determine anew the ground motion over a much smaller (6 km $\times$ 6 km $\times$ 0.6 km) region contained within the original domain, and results of the two methods within this region of interest are compared.

These examples serve to demonstrate that in many applications the DRM can be signifi-

cantly more efficient than the traditional approach. The DRM can be particularly advantageous if the source is far from the local structure and the local structure is much softer than that of the exterior region; also, if the localized feature exhibits nonlinear behavior, or if for a prescribed source, one wishes to consider a sequence of simulations in which the properties of the local feature, which might include man-made structures, are varied from one simulation to the next.

INTRODUCTION

In a companion paper (Bielak *et al.*, 2002), hereafter referred to as Paper I, we have described a two-step, domain reduction method (DRM) for modeling efficiently source, path, and site effects during earthquakes. A particular case of this method, in which the seismic excitation consisted of a plane wave incident wave and the examples were restricted to two dimensions was presented earlier (e.g., Loukakis, 1988; Loukakis and Bielak, 1995). Alternative two-step methods that share the attractive features of the DRM have been presented also by other authors (e.g., Aydinoğlu, 1993; Zahradník and Moczo, 1996; Moczo *et al.*, 1997). A more detailed bibliography is included in Paper I. Here we assess the DRM by comparing our results with those from established methods, for two particular three-dimensional problems. We also illustrate the applicability to other problems, and discuss extensions and limitations of the method.

The DRM procedure and definition of regions, boundaries, and variables described in Paper I are summarized in Fig.1. In step I (Fig. 1a), one stores the free-field displacement

u_b^0 and u_e^0 within two boundaries Γ and Γ_e for a simpler auxiliary problem without local features ($\Omega^0 \cup \Omega^+$ bounded by Γ^+). These displacements are used for calculating effective seismic forces P_b^{eff} and P_e^{eff} in step II (Fig. 1b). These forces are rigorously equivalent within discretization error to the fault fource P_e. Since they are distributed only in a layer around Ω, which contains the local feature, the domain size for step II can be reduced to a smaller region of interest. In step II, the total wavefield u_i, u_b and the residual wavefield w_e ($w_e = u_e - u_e^0$, where u_e is the total displacement in $\hat{\Omega}^+$) for a complex problem with local features ($\Omega \cup \hat{\Omega}^+$ bounded by $\hat{\Gamma}^+$) are calculated using P_b^{eff} and P_e^{eff}.

In this paper we consider, as a first particular instance, a flat-layered system for the background structure in step I ($\Omega^0 \cup \Omega^+$). Free-field displacements u_b^0 and u_e^0 are calculated by the Green's function method (Hisada, 1994, 1995). Verification of the DRM is carried out by taking the material in Ω to be identical to that of the background structure Ω_0, and by comparing the results to those calculated directly by the Green's function method. Applicability of the DRM is demonstrated by replacing Ω in step II with local structures, including a basin or a hill. Subsequently, we consider an additional problem involving the Los Angeles basin, to illustrate and verify the applicability of the DRM for more realistic situations, which entail a more complex geometry and highly heterogeneous material properties.

FLAT-LAYERED SYSTEM - MODEL VERIFICATION

We consider the two-layer system underlain by an elastic halfspace shown in Fig. 2; its properties are listed in Table 1. These can, of course, be readily scaled. No material damping is considered, for simplicity. The seismic source is a dip-slip double-couple buried at a point 1 km below the free surface. Strike, dip, and rake are 0°, 90°, and 90°, respectively, the seismic moment $M_0 = 6 \times 10^{15}$N·m, and the slip function is shown in Fig. 3. North is aligned with the y axis. The region of interest is a 1-km $\times$ 1-km $\times$ 1-km cube located 10 km east of the epicenter.

Because of the simplicity of the physical setting and of the source, in this example we evaluate the displacements u_b^0 and u_e^0 on Γ and Γ_e in Step I using the 3D Green's functions in the computer code by Hisada (1994, 1995). In Step II the material in Ω is taken to be identical to that of the background material, as shown in Fig. 4. We use an elastic wave propagation finite element simulation code developed for modeling earthquake ground motion in large sedimentary basins (Bao *et al.*, 1998). The wave propagation code is built on top of "Archimedes", an environment for solving unstructured-mesh finite element problems on parallel computers (Bao *et al.*, 1998; "Archimedes", 1998). Archimedes includes 2D and 3D mesh generators, a mesh partitioner, a parceler, and a parallel code generator. We use linear tetrahedral elements. We have added the capability of automatically determining the surfaces Γ and Γ_e once a box that defines the region of interest has been prescribed, and have built-in the necessary operations for evaluating the effective forces P^{eff}. These calculations are also performed in parallel since the contribution to P^{eff} from each element within the layer adjacent to Ω can be evaluated independently of the other elements within that layer. In addition, in this study we have used a lumped mass matrix approach, in which one fourth of the mass of each tetrahedron is assigned to each node. Therefore, the off-diagonal elements of the mass matrix vanish, and consequently, the evaluation of the effective seismic forces involves only a multiplication of the stiffness matrix by a free-field displacement (See (8) in Paper I). We use lumped mass matrices to avoid the need of solving a system of algebraic equations at each time step. With this choice, the only significant algebraic operation at each time step is a matrix-vector multiplication. In addition, since linear elements have nodes only at the vertices, no displacements need to be stored inside the layer between Γ and Γ_e.

The solution from the domain reduction method for points on a fictitious borehole that passes through points B and B' (Fig. 4b) is shown in Fig. 5. Figure 5a depicts the x (east-west) component of the displacement, and Fig. 5b the vertical component, at various depths. (Displacements in the y (north-south) direction are not shown as they essentially vanish, due to symmetry). The complete wavefield, including body waves and surface waves, can be clearly observed in this figure. The corresponding results from the Green's functions evaluations are also shown in Fig. 5, for comparison. Peak values, with their signs, are listed on the right columns for both solutions next to the synthetic seismograms. The agreement between the two sets of waveforms is quite good, with maximum differences in amplitude on the order of five percent. This is consistent with the accuracy we can expect from our finite element approximation, which is tailored to ten points per wavelength, according to the shear wave velocity within each element and a maximum frequency of 1 Hz. The approximate dominant frequency of the surface waves is 0.3 Hz. There are a total of 102,402 elements and 19,143 nodes in the mesh shown in Fig. 4. It took 12 minutes of CPU time on 8 processors of the T3E computer at the Pittsburgh Supercomputing Center to solve the corresponding equation of motion, using a second order central difference method with a time step of 0.01 s.

Notice that the agreement between the finite element solutions and the corresponding Green's functions remains quite close down to the interface Γ, at 700 m. Right below this point, the finite element solution almost vanishes. The same behavior is observed in Fig. 6 for the seismograms on the free surface along AA' (Fig. 4a). The difference between the results from the DRM approach and the Green's functions does not exceed five percent at these locations, and the displacements beyond Γ also essentially vanish. Recall that in the outer region, $\hat{\Omega}^+$, our formulation yields residual displacements; since the material in Ω is the same as that in the background structure for this example, w_e must vanish. The fact that the numerical values of these residual displacements are close to zero provides a useful numerical check. An interesting consequence of the vanishing of w_e for this problem is that, theoretically, the outer boundary $\hat{\Gamma}^+$ must play no role in the solution, regardless of the absorbing boundary conditions one uses there. For the present application the boundary nodes were left unconstrained, thereby implying that the outer boundary is traction-free. The fact that residual displacements in $\hat{\Omega}^+$ are barely visible confirms that for the validation problem the boundary condition on $\hat{\Gamma}^+$ has an insignificant numerical effect. Moreover, since there are no waves leaving the region of interest, Ω, one could modify the material in the exterior region beyond a single-element thick layer surrounding the surface Γ_e, and the results within Ω would not change.

FLAT-LAYERED SYSTEM WITH BASIN AND HILL

Idealized basin. The first example we use to illustrate the applicability of DRM to more complex situations is one that involves a local structure Ω with a sedimentary basin embedded into the same two-layer stratigraphic system we considered in the previous section. The basin is in the shape of a spherical cap, and has a maximum depth of 100 m, and a 150-m radius at its intersection with the free surface, as shown in Fig. 7. It has a uniform shear wave velocity of 125 m/s, P-wave velocity of 250 m/s, and density of 2 gm/cm^3. The seismic source is identical to that for the unperturbed flat-layered system. Thus, there is no need to recalculate the free-field ground motion u_b^0 and u_e^0 on Γ and Γ_e, as we can reuse the seismograms obtained previously. On the other hand, in contrast to the flat-layered system for which the residual displacement vanished outside the region of interest, in this problem the basin generates a scattered wave. Hence, an absorbing boundary must be introduced on $\hat{\Gamma}^+$ to limit the occurrence of spurious reflections. We use a simple dashpot approach (Lysmer and Kuhlemeyer, 1969) for this purpose, which consists of adding viscous dampers at each node on $\hat{\Gamma}^+$. This gives rise to a diagonal damping matrix C with non-zero terms associated only with boundary nodes.

The resulting displacement synthetics along the line BB' (Fig. 7) are shown in Fig. 8, together with the corresponding values for the background (flat-layered) structure. As expected, the basin has the effect of magnifying the amplitude of the free field ground motion. This amplification is confined primarily to points within the basin, where it reaches values of about 50 percent in the x (east-west) direction. The vertical amplification is only of the order of 20 percent.

Figure 9 shows synthetics for locations along AA' (fig. 7) for the x (east-west) component of the displacement. The corresponding results for the vertical component are in Fig. 10. The top panel in each figure depicts the free-field ground motion in the background structure with no valley. There is a discontinuity across the interface Γ because within the region of interest Ω we plot the total displacement, but in the exterior region only the residual displacement is shown. The middle panel shows the corresponding results with the basin present, and the bottom panel the difference between the previous two. The latter is the residual displacement along the line AA' over the entire region of interest. In this case, since the stratigraphy in step I and II is the same, the residual wavefield corresponds identically to the scattered wavefield. The effect of the basin on the ground motion can be seen explicitly from the bottom panel or by comparing the top two panels in each figure. The basin effects in the x (east-west) direction are of the same order in the basin's interior as the free-field motion, especially in the deepest part of the basin. This region is affected most because the prescribed ground motion excites primarily the basin's fundamental mode. Effects for the vertical ground motion are less pronounced. The residual motion is continuous across Γ, as expected, because this fictitious interface has no physical meaning and has been introduced merely for computational convenience. It is clear from Figs. 9b and 9c and Figs. 10b and 10c that the wave motion outside the basin is purely outgoing and that no visible spurious reflections are being generated at the absorbing boundary. The peak amplitude of the scattered wavefield at the edges of the computational domain (x=0, 1000 m) is of the order of ten percent of the peak amplitude of the background free-field motion with no valley.

To summarize the results from the surface response of the basin and its immediate vicinity to the incoming seismic waves, we have plotted in Fig. 11 the distribution of the maximum value of the total surface displacements, over the entire region of interest. Here we have included also the response in the y (north-south) direction. Notice the different scales used for each component; also, the smallest and largest values of these maxima after spatial smoothing are reported in each panel. The effect of the basin on the free surface ground can be clearly seen for the various components of the wavefield. The peak amplification occurs off center, especially for the vertical motion. In addition, there are noticeable backward and forward scattering effects in the vicinity of the basin. The former leads to an increase in the ground motion with respect to that of the free-field motion, while the latter has the opposite effect (Fig. 11a and c).

Idealized hill. To illustrate the applicability of our procedure to the analysis of topographic effects from exposed geological structures, we consider, as a second example, the case of a hill supported on the two-layered system, as shown in Fig. 12. We model the ground motion for two variations of the hill problem. In the first instance, the hill is assumed to be homogeneous with the same properties as the top layer of the background material; in the second, the hill has a weathered surface layer 25-m thick, with the same properties as those of the basin in the previous example. The hill has a square base 350 m $\times$ 350 m, it is 100 m high, and the lateral sides have a slope of 45°. The seismic excitation is the same as before, as is the numerical procedure in almost every detail. The one difference here is that the localized topographical feature is located above the free surface of the layered system, and its lateral and top surfaces are traction-free. It is worth noting that contrary to other methods such as finite differences, no treatment of any kind is required in the finite element method to enforce traction boundary or interface conditions. These are satisfied automatically as a consequence of the variational principle that underlies the finite element formulation.

Seismograms for several locations along the line AA' on the free-surface in Fig. 12 are shown in Fig. 13 for the homogeneous hill. The x (east-west) and z (up-down) components of the displacement are depicted, together with a list of their peak values. A comparison of these results with the corresponding free-field displacements in Figs. 9a and 10a reveals an amplification on the order of 2.5 at the crest of the hill in the maximum amplitude of the east-west component of displacement. For the vertical component this amplification is only about 1.5. This topographic amplification can be clearly observed in Fig. 14, which shows the residual displacement along the same line. In this case, the residual motion is the scattered motion from the hill, since the layered structure beneath the hill is the same as that of the background structure. The residual motion is significantly greater in the east-west than in the up-down direction. The maximum response of the hill occurs late in the excitation, and the wave scattering is significantly stronger than for the basin. The peak value of this scattered motion is of the order of 35 percent of the free-field motion at the edges of the computational domain. This means that the hill's effect on the free-field ground motion is far from negligible.

The distribution of maximum response of the hill's free surface and of its neighboring region is shown in Fig. 15. From this figure it is seen that the prescribed seismic source excites primarily the fundamental mode of the hill. The peak amplitudes of the x (east-west) and z (up-down) components of displacement increase from the base to the top and the maximum peak values occur on the eastward side of the crest for the x-component of displacement, and on the westward side and uphill plane for the vertical component. The displacement in the y (north-south) direction is much smaller than in the previous two cases, but the peak values occur at the foot of the hill and outside of it. Interestingly, in contrast to the basin problem, backward scattering here causes deamplification and forward scattering amplification (Fig. 15a). Even though this example represents an idealized situation, it suggests that interpreting ground motion in the vicinity of a topographic feature as free-field ground motion must be done with caution.

Figures 16, 17, and 18 show the corresponding results for the hill with the weathered layer. The results are qualitatively similar to those of the homogeneous hill, except that the softer layer amplifies significantly the hill response. Compared to the free-field amplitude without the hill, the amplification ratio of the layer east-west displacement component is about 3.6.

An attractive feature of the DRM methodology exhibited by the preceding examples is the relative efficiency of the associated absorbing boundary conditions. We mentioned earlier that by choosing the residual displacements as the unknown field in the exterior region that surrounds the local geological features, the residual ground motion in the exterior region is strictly outgoing and corresponds to the deviation of the actual structure from the background structure. It appears that this perturbation can be small even if the properties of the local feature differ significantly from those of the background structure. In that case, the absorbing boundary is required to dissipate only a small amount of energy compared to that of the free-field motion. This effect is illustrated in Fig. 19, which shows snapshots at various times taken from an animation of the ground motion for the three cases considered thus far: the background structure (left column panels), the basin (middle column panels), and the homogenous hill (right column panels) under the prescribed double-couple excitation. The displaced configuration on the vertical plane of symmetry through the line AA' (see Figs. 4a, 7a, and 12a) is superposed on top of the initial configuration for each system. Visible scattered waves emanate from the two structures and reach the absorbing boundaries. These scattered waves, however, are smaller than the free-field ground motion of the background structure. The reason for this is that some of the input energy is trapped within the structure and is released only gradually. This implies that the amount of energy that the exterior boundary needs to absorb at a given instant when the residual wavefield is chosen as the basic unknown in the outer region can be significantly smaller than that which would need to be absorbed if the total displacements were regarded as the unknowns. Thus, even if the percentage of error is the same for a particular choice of absorbing boundary condition, its performance can be expected to be superior for the domain reduction method.

Figure 19 also serves to illustrate the relative response of the background structures, basin, hill, and their immediate vicinity. It is clear that at any given instant the response of the modified systems differs significantly from that of the background region. Both the basin and the hill exhibit marked spatial variation over short distances compared to that of the free-field ground motion. Even though it is stiffer than the basin in this example, the hill responds more strongly because the basin is confined within the background structure, whereas the hill vibrates freely above the free surface.

LOS ANGELES BASIN - MODEL VERIFICATION

In the preceding examples, because the background structure consisted of a set of horizontal layers overlying an elastic halfspace, we were able to use a theoretical Green's function approach to evaluate the free field motion in Step I of the DRM. If the geometry is complex or the material properties are highly heterogeneous, it becomes necessary to use a purely numerical procedure, such as finite differences or finite elements. To test the DRM in a more realistic situation against the traditional approach, in which the source and the region of interest are incorporated into a single model, in this section we consider an 80 km $\times$ 80 km $\times$ 30 km region that encloses entirely the Los Angeles basin, and use the SCEC Southern California Reference Three-Dimensional Seismic Velocity Model Version 2 (Magistrale et al, 2000) to characterize this region. We use the displacement formulation of the finite element method both for the traditional approach, and for the DRM to determine the ground motion within a small subdomain (region of interest) of the original model, using as seismic source a buried double-couple, which is located well outside the region of interest.

The SCEC velocity model consists of detailed rule-based representations of the major Southern California basins, embedded in a three-dimensional crust over a variable-depth Moho. Outside of the basins, the model crust is based on regional tomographic results. Figure 20 presents a plan view of the shear wave velocity distribution at the free surface of the region considered, together with a vertical cross-section along AA' of the top 15 km of the 30-km deep computational domain across the epicenter of the seismic source . The scale in the plan view has been capped at 350 m/s to highlight the large degree of heterogeneity of the model. This velocity is shown to take values beyond 4000 m/s in the cross-sectional view.

The small red square shown in Fig. 20 represents the region of interest selected for this demonstration example. It is 6 km $\times$ 6km in plan and 0.5 km deep. Due to the complexity, heterogeneity, and refinement of the finite element mesh, we do not sketch the exact location of the two bounding surfaces Γ and Γ_e on which the effective forces are to be applied in Step II (see Fig. 1 for notation). The lateral limits of this localized region along the cross-section AA' are denoted by vertical yellow lines in Fig. 20b. Close-up views of the computational domain to be used in Step II of the DRM are shown in Fig. 21. As in our previous models, this domain extends beyond the region of interest. The displayed shear wave velocity exhibits variations between 184 and 274 m/s at the free surface. Other regions, outside the limits shown in Fig. 21, present shear wave velocities as low as 60 m/s at some locations.

The source is defined as a strike-slip double-couple located at (40 km, 56 km, -14 km), as shown in Fig. 20b, with strike, slip, and rake of 0°, 90°, and 0° , respectively. Its seismic moment $M_0(t)$ is prescribed as:

$$M_0(t) = M_0[1 - (1 + t/T_0)e^{-t/T_0}],$$

with $M_0 = 1 \times 10^{18}\mathrm{N} \cdot \mathrm{m}$, and $T_0 = 0.2$ s. The mesh generated for the simulations is tailored for a maximum frequency of 0.5 Hz; thus, the resulting synthetic ground motions are filtered accordingly.

The verification procedure follows the two steps of the DRM method: (1) large-scale simulation and calculation of the effective forces at the boundaries of the region of interest; and (2) simulation within the reduced domain. To compare the results of the DRM with those of the traditional procedure, which for this particular problem corresponds to Step I of the DRM, the velocity response in Step I is recorded along two lines of free-surface observation points within the region of interest, as shown in Fig. 21a. The cross-section shown in Fig. 21b is taken across the NW-SE diagonal along observation points L. Note that the model used in Step II is only 600m deep. The corresponding synthetics from the traditional approach (Step I) will be compared with those obtained from Step II, in which only the local region is used in the analysis.

For the first step of the calculations, our elastic wave propagation finite element simulation code (Bao *et al.*, 1998) takes as input the original mesh for the entire 80 km $\times$ 80 km $\times$ 30 km model, the DRM limiting surfaces denoted by red lines in the previous figures, the geological and geometric characteristics of original computational domain, and the seismic source. Though no material damping is considered in this model, a simple viscous damping approach is used as in our previous examples to limit the occurrence of spurious reflections at the outer boundaries.

The mesh is partitioned and a communication graph is developed to distribute the computational load among all available parallel processors. The finite elements that intersect the limits of the DRM box are tagged as DRM elements before the beginning of the simulation and each processor stores the number and location of its own tagged DRM elements and nodes. The simulation proceeds as a typical wave propagation analysis, except that in addition to recording responses at locations of interest, the displacement field is recorded at the tagged DRM nodes. Parallel synchronization is essential for the sequential output procedures since each processor outputs its DRM information to a single output buffer.

For Step II, the elements that belong to the outer region, and the seismic source are discarded, as shown in Fig. 21. The calculation proceeds with the reduced mesh and the DRM tagged nodes as multiple seismic sources represented by the effective forces calculated from the displacement field in Step I. This analysis requires much smaller computing and storage capabilities; it may, therefore, be performed either sequentially or in parallel. In either case, the recorded DRM node displacements are assigned to the new mesh nodes with an interpolation scheme. We have developed an automated procedure for which the meshes for Step I and Step II need not be identical. Likewise, the simulation time step may also differ. This represents a clear advantage for code interaction purposes, as it is common to use different numerical calculation procedures, meshes, and software tools for the large-scale ground motion simulation (Step I) and the small-scale ground motion simulation, soil-structure interaction, and building response (Step II). In this particular example, the first and second meshes coincide, and such scheme is not required. Numerous and repeated numerical simulations, such as those required by nonlinear analyses or parametric studies may now be performed with just a fraction of the original computational resources.

The resulting mesh statistics for the present background and local simulations are shown in Table 2. The reduction in required number of mesh nodes and elements is substantial. This fact translates into considerable computing efficiency for further analysis within the local region. For example, Step I required 3 hours on 128 parallel processors of the Cray T3E machine at the Pittsburgh Supercomputing Center for 20,000 time steps of 0.002 s. In addition, approximately 6 Gb of storage space are used to store mesh and data files. By contrast, the second step localized calculation may be performed in a single-processor personal computer with less than 2 percent of the original data storage capabilities. As shown on Table 2, the second step uses only about one percent of the original mesh.

Figure 22 compares the displacement synthetics at the observation points for the traditional approach (Step I for this problem) and the two-step DRM. Clearly, the resulting synthetics consist both of body and surface waves. The two sets of results are essentially identical. Notice that even though for this example the material properties within the localized region are almost uniform in the lateral direction, the spatial variability of the surface ground motion is quite strong. The DRM captures accurately the complex ground motion that is generated as the seismic waves travel from the source through the deep and shallow parts of the geological structure within the extended region.

In dealing with the finite element method, there is one additional point that should be mentioned here. It is well known that the displacement formulation of the finite element method fails for incompressible materials (Poisson's ratio, $\nu = 0.5$) and leads to inaccurate results as ν approaches this limiting value, since the corresponding stiffness matrix becomes nearly singular. For the seismic velocity model of the Los Angeles basin we considered here, there are many locations where ν takes values between 0.4 and 0.44. Independent comparisons of our results with those from finite difference calculations which use an algorithm based on stress-velocity formulation that is insensitive to Poisson's ratio have confirmed the accuracy of our implementation of the traditional finite element methodology (Day, 2002).

DISCUSSION

The initial motivation for developing the DRM came from the desire to deal with problems for which the causative fault is at some distance from the region of interest. By removing temporarily the local geological feature in Step I, we showed via the examples in the previous section that it is possible to greatly simplify the original problem, especially for problems in which some portions of the domain have very low shear wave velocity compared to that of the background geology. This allows one to use coarser meshes for the background system than would be needed with a single-step procedure and, therefore, an increased number of time steps if the spatially discretized equations of motion are solved in Step I by an explicit step-by-step time integration. Only for Step II is a finer mesh, and thus smaller time steps, required in order to represent accurately the ground motion in the presence of a highly contrasting localized geological feature.

There are other problems for which the domain reduction method may be advantageous even if the fault is not far from the region of interest; e.g., for situations in which due to uncertainty of the geometric and material properties of the local feature it may be desirable to repeat the calculations for different combinations of the system parameters, such as in seismic inversion. In that case Step I need only be applied once for a prescribed source. Nonlinear soil behavior extending over a limited region falls in the same category, as the solution must be determined iteratively. Confining the nonlinearity to Step II would then be greatly advantageous. With these applications in mind, we have developed an automated interpolating procedure for which the meshes within the common domains need not be identical for the two steps. Similarly, the time steps in the two simulations may also differ. This allows one to use different codes for the large-scale wave propagation simulation in Step I and for the small-scale ground motion simulation in Step II, for which the corresponding code may include provisions for nonlinear material behavior.

For the DRM procedure to be rigorously valid, as we indicated earlier, the material exterior to Ω (Fig. 1b) must be identical to that of the original problem. However, from the numerical results in the preceding section we saw that for the basin and hill, the residual wavefield in the exterior region is only a fraction of the complete wavefield within the region of interest. This suggests that one might be able to simplify considerably Step II for a general case, yet maintain an acceptable approximation. Nonetheless, we should point out that if in selecting the region $\hat{\Omega}^+$ one leaves out geological features such as a deep layer or a heterogeneity that is present in the original lithology, the domain reduction method will not be rigorously equivalent to a single-step procedure that models the entire region all at once, since any reflections from the heterogeneity or the deep layer due to the residual wavefield will be ignored in Step II. However, provided the background model used to determine the free field motion u_b^0 and u_e^0 is identical to the original one in the domain Ω^+, then the equivalent seismic forces P^{eff} will be exact to within discretization error. The only approximation will be due to the secondary reflections generated by the residual wavefield, and these in many cases will be insignificant.

CONCLUDING REMARKS

The two-step domain reduction method described in Paper I, and illustrated by several three-dimensional examples in this paper provides an efficient and reasonably accurate methodology for modeling earthquake ground motion in complex localized regions with large contrasts in material properties with respect to the background geology. By separating local features with possibly short wavelengths from the background structure, this methodology can make it possible to model earthquake ground motion at higher frequencies and with greater fidelity than has been practical up to now. While this method was originally conceived for cases in which the source is far from the local structure, it can be especially useful for performing repeated analyses in which the source is kept fixed but the properties of the local feature are varied from one simulation to the next. This methodology is equally appropriate if the localized feature exhibits nonlinear behavior or there are engineered structures present within the region of interest. Additional computational savings can be gained if the region of interest is restricted to the local geological feature and its vicinity and excludes heterogeneities or deep layers some distance away. However, errors due to secondary reflections generated by the residual wavefield will occur if the background region contains heterogeneities that are not included in the reduced region. While our numerical results indicate that the outgoing waves are small due to the impedance contrast between the material within the region of interest and the exterior region, the issue of secondary reflections deserves further investigation. We believe that the domain reduction method provides a useful tool towards the assessment of seismic hazard and seismic risk reduction in urban areas within basins with complex topography.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation, under grant CMS-9980063 of the KDI program, and under grant EEC-0121989 of the Engineering Research Centers. The cognizant program directors are Clifford J. Astill and Joy M. Pauschke, respectively. Computing services were provided by the Pittsburgh Supercomputing Center on the Cray T3E. This work was also partly supported by a special project of U.S.-Japan Coorperative Research for Urban Earthquake Disaster Mitigation (No. 11209201), Grant-in-Aid for Scientific Research on Priority Area (Category B), Ministry of Education, Culture, Sports, Science, and Technology of Japan (PI: Tomotaka Iwata), and by Taisei Corporation. We are grateful for this support. We thank Peter Moczo and an anonymous reviewer, whose reviews helped us improve the manuscript. The work was done while one of the authors (C. Y.) was visiting Carnegie Mellon University as a research scholar on leave from Taisei Corporation.

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Figure legends

Figure 1. Summary of two-step domain reduction method (DRM). (a) Step I defines the auxiliary problem over background geological model. Resulting nodal displacements within Γ, Γ_e and the region between them are used to evaluate effective seismic forces P^{eff} required for Step II. (b) Step II, defined over reduced region made up of Ω and $\hat{\Omega}^+$ (a truncated portion of Ω^+). The effective seismic forces P^{eff} are applied within Γ and Γ_e. The unknowns are: the total displacement fields u_i in Ω and u_b on Γ and the residual displacements w_e in $\hat{\Omega}^+$.

Figure 2. Flat-layered system used to verify the domain reduction method (DRM), with seismic source and region of interest (ROI).

Figure 3. Slip function used for seismic source (double couple applied within elastic half-space at 1 km beneath free surface).

Figure 4. Layered system within region of interest. (a) Finite element mesh tailored to shear wave velocity of each layer and the halfspace; (b) Cross-section on vertical plane through AA'. Bold dashed lines show surfaces Γ and Γ_e where effective forces P^{eff} are applied in Step II.

Figure 5. Synthetic seismograms for displacements along downhole line BB' (Fig. 4b). Depth from free surface and shear wave velocity of each material is indicated to left of seismograms. (Other properties are listed in Table I). Scale, in cm, is shown above the origin of the first seismogram. Peak displacements from finite element DRM

simulations and corresponding values from Green's functions calculations are shown to right of seismograms. (a) x- (east-west) component; (b) z- (up-down) component.

Figure 6. Synthetic seismograms for displacements along free-surface (horizontal) line AA'(Fig. 4a). Distance x is measured from origin of x-axis. Other nomenclature as in Fig. 5.

Figure 7. Homogeneous basin embedded in flat-layered system. (a) Finite element mesh; (b) Cross-section through AA'.

Figure 8. Synthetic seismograms for displacements along downhole line BB' (Fig. 7b). Solid lines show response with basin present. Dashed lines correspond to free-field motion (without the valley). Right columns show peak values with and without basin. Traces for points within surface Γ represent total displacement; those for points outside this surface show residual displacements with respect to free-field surface motion of the corresponding points for the flat-layered system. (a) East-west component; (b) Up-down component.

Figure 9. Synthetic seismograms for east-west component of displacement along free-surface horizontal line AA'(Fig. 7a). (a) and (b) show total displacements inside and on Γ and residual displacements outside of it; (c) shows residual displacement at all locations along AA' and thus depicts directly the effect of the basin on surface ground motion.

Figure 10. Same as Fig. 8, but for up-down component of displacement.

Figure 11. Spatial distribution of maximum absolute value of total displacement components of free-surface ground motion within the basin and its vicinity. The point source is located 10 km west (x=-10 km). (a) East-west component; (b) North-south component; (c) Up-down component. Notice different scales in each panel.

Figure 12. Hill on flat-layered system. (a) Finite element mesh; (b) Cross-section through line AA' which traverses the free-surface of the flat-layered system and that of the hill. Two cases of hill are considered in the simulations: one for a homogeneous hill, in which its properties are the same as those of the top surficial layer, and the second in which the hill has a weathered layer with the same properties as those of the basin in Fig. 7.

Figure 13. Synthetic seismograms for displacements of uniform hill along line AA'(Fig. 12a). (a) East-west component; (b) Up-down component. Displacements inside Γ are total; outside they are residual. This is the reason that the seismograms exhibit a discontinuity across Γ_e.

Figure 14. Synthetic seismograms for residual displacements along free-surface line AA' in Fig. 12). (a) East-west component. The peak value of this residual displacement is about twice that of the corresponding value for the flat-layered system; (b) Up-down component. The peak value is about 70% that of the corresponding value for the flat-layered system.

Figure 15. Spatial distribution of maximum value of displacement components of ground motion on free surface of uniform hill and its vicinity. (a) East-west component; (b)

North-south component; (c) Up-down component.

Figure 16. Same as Fig. 13, but for weathered hill.

Figure 17. Same as Fig. 14, but for weathered hill.

Figure 18. Same as Fig. 15, but for weathered hill. Notice the significant increase of peak response due to weathering.

Figure 19. Snapshots of ground displacement on vertical cross-section across AA'(plane of symmetry) at various instants, for the background flat-layered system, the homogeneous basin embedded in the flat-layered system, and the homogeneous hill atop the flat-layered system. Time is measured from the onset of the excitation at the seismic source. Scale is at top left of figure. Displacements in the interior region to Γ are total; those in the exterior are relative to those corresponding to the background layered system (residual field). (a) S-wave arrival; (b) Multi-reflection of S waves and fundamental Rayleigh mode; (c) Fundamental mode Rayleigh mode; (d) First higher Rayleigh mode. Notice radiated residual wavefield from basin and hill, which is concentrated primarily on surface layers. Observe, also, the deformation along boundary of region of interest. Full animation, as well as Figs. 9, 10, 13, 14, 16, and 17, show negligible effect of absorbing boundary conditions on ground motion.

Figure 20. Shear wave velocity model of the Los Angeles Basin. (a) Free surface shear wave velocity distribution, showing a 6km $\times$ 6km region in which the DRM analysis is performed; (b) Cross section AA' shows the shear wave velocity distribution down to 15 km. The blue zones represent softer soils. Notice that the localized region includes

the southeastern portion of the San Fernando Valley. The latitude and longitude of the lower left corner of figure (a) are $33.7275°N$ and $118.9080°W$.

Figure 21. Close-up of the DRM region of analysis with the location of the observation points. (a) Plan view with longitudinal (L) and perpendicular (P) cross sections; (b) 600-m deep cross section along P receiver line. The red line represents the limits of the local region. Not that (b) shows only the top 600 m of the 30-km deep original model. The displayed values of the shear wave velocity present variations between 190 and 316 m/s at the free surface. However, the model used in the simulations includes velocities as low as 60 m/s.

Figure 22. Displacement synthetics (in cm). The figure compares the response at the observation points for the traditional single-step and two-step DRM calculations. The results are identical since the effective force nodes of the first and second step analyses are those from the original mesh; i.e., there are no additional spatial discretization errors between Steps I and II. The readings along LineL (longitudinal) show more pronounced phase differences than those along Line P, consistent with the location of the hypocenter, the magnitude of the shear wave velocity, and the distance between the observation points.