Contest Among Contest Organizers

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This paper studies multiple simultaneous contests where contest organizers elicit solutions to innovation-related problems from a set of agents. Each agent may participate in multiple contests and exert effort to improve her solution at each contest she enters, but the quality of her solution also depends on an output uncertainty. We first investigate whether the profit of an organizer can be improved by discouraging agents from participating in multiple contests. We show, somewhat surprisingly, that organizers benefit from agents' participation in multiple contests when agents' output uncertainty is sufficiently large. A managerial insight from this result is that when organizers elicit major innovation rather than low-novelty solutions that require concentrated effort, encouraging agents to participate in multiple contests may be beneficial to organizers. We further show that the organizer profit is unimodal in the number of contests, and more interestingly, the optimal number of contests increases with agents' output uncertainty. This finding may explain why many organizations in practice run multiple simultaneous contests, and it prescribes a larger number of contests when seeking major innovation.

Key words: Competition, Contest, Crowdsourcing, Innovation, Tournament.

1. Introduction

In the last two decades, innovation contests (also known as tournaments) have emerged as a popular and cost-effective tool for outsourcing innovative solutions to challenging problems. In an innovation contest, a contest organizer elicits solutions to an innovation-related problem from a set of agents, and awards the best solution. With the advancements in information technology and the Internet, the innovation contest industry has evolved into a lucrative business in which billions of dollars are awarded in thousands of contests annually (McKinsey & Company 2009).

Evolution of contests also lead to the rapid growth of contest platforms such as Challenge.gov, designContest, Ennomotive, InnoCentive, Inocrowd, and TopCoder that organize contests for their clients from public or private sector. Take InnoCentive as an example of a contest platform. Since 2001, InnoCentive has organized contests for a large client base including AARP Foundation, Booz

Allen Hamilton, Eli Lilly, NASA, and P&G in subject categories such as business, chemistry, information technology, and life sciences (InnoCentive 2017a). Agents become members of InnoCentive to participate in these free-entry open-innovation contests that are often run simultaneously. InnoCentive members are allowed to (and often do) participate in multiple simultaneous contests.[1] Similarly, since 2001, TopCoder has organized contests on behalf of a diverse group of clients including Best Buy, Comcast, GEICO, HP, and IBM, by encouraging its members to compete in various free-entry software development contests (TopCoder 2017b). During our interviews with TopCoder, we have learned that lower-novelty development challenges that require concentrated effort are often run for shorter durations (e.g., John Hancock - Project Snapshot Bug Hunt contest was run for just 2 days (TopCoder 2017a)), whereas higher-novelty algorithm challenges can be run for longer durations (as long as 3 weeks). In the latter challenges, agents quite often work on multiple contests in parallel.[2] In general, by setting its terms and conditions accordingly (e.g., by setting a tight or loose deadline or by imposing strict or lenient submission rules), each contest platform may encourage or discourage participation in multiple simultaneous contests. Meanwhile, each client organizer determines how to award agents at its own contest.

Many organizations today run multiple simultaneous contests, acting as central planners that determine the awards and terms and conditions of these contests. For instance, in 2016, Elanco (an Eli Lilly affiliate focusing on animal healthcare) has organized five contests that elicit innovative theoretical solutions to animal healthcare problems (Elanco 2017). Some other examples in 2016 are as follows: the US Department of Energy has organized eight contests, Royal Society of Chemistry in the United Kingdom has organized eleven contests, the Gates Foundation has organized fourteen contests (only within Grand Challenges Explorations framework), the US Department of Health and Human Services has organized twenty one contests. Most of these contests run simultaneously, and they provide agents with a selection of problems to work on.

An agent, working on multiple contests, splits her effort over these contests, and this split leads to two important decisions for practitioners. First, a company determines whether to organize multiple contests simultaneously, considering that agents may split their efforts among these contests, potentially worsening the outcome of each contest. Somewhat interestingly, we observe in

[1] Statistical analysis at InnoCentive reveals that in theoretical challenges where agents develop theoretical solutions with no requirement of implementation (e.g., Measuring the Spatial Distribution of Drug Concentrations in Target Tissues (InnoCentive 2017b)), agents work on multiple contests simultaneously. Specifically, among four random days within the past twelve months, 57.4% of agents have opened more than one project room in live theoretical challenges in a day. Note that this number is likely to be a significant underestimation of the actual percentage of agents who have been working on multiple contests because this analysis does not consider that agents can work on some contests offline or allocate one day to one contest and the next day to another contest. We would like to thank InnoCentive marketing department and its director Graham Buchanan for providing this statistic.

[2] We would like to thank TopCoder marketing team for providing this information.

practice that many organizations such as Elanco and the Gates Foundation run multiple simultaneous contests. Second, an organizer of multiple contests or a contest platform can determine whether to discourage agents from participating in multiple contests, which may potentially induce agents to exert more effort at each contest. Indeed, a common assumption in the multiple contests literature is that each agent participates in only one contest (e.g., Azmat and Möller 2009). Yet, in practice, platforms such as InnoCentive allow agents to freely enter multiple contests. In this paper, we aim to guide practitioners about these decisions and to generate insights into the impact of multiple contests. Specifically, we answer the following research questions: (Q1) When should agents be allowed to freely participate in multiple contests? (Q2) How do multiple contests affect each organizer's profit, and when should organizations such as Elanco and the Gates Foundation run multiple simultaneous contests?

To answer these questions, we develop a normative model of innovation contests in which multiple contest organizers elicit solutions from a set of agents. We consider a decentralized case where each organizer chooses an award at its own contest (as in InnoCentive and TopCoder) and a centralized case where a central planner chooses awards at all contests (as in Elanco and the Gates Foundation). After awards are determined, each agent exerts effort to improve her solution, and the quality of her solution also depends on an output uncertainty due to randomness in innovation and evaluation process (e.g., Terwiesch and Xu 2008, Ales et al. 2016a,b). While determining her effort at each contest, each agent considers the cost of her total effort. We factor in two effects identified by economics and new product development literature that determine the shape of an agent's cost function: (i) a "scarcity" effect as it may be increasingly difficult to spare resources (e.g., time or money) for exerting higher total effort (e.g., Albanesi and Sleet 2006), and (ii) an "economies-of-scope" effect as exerting effort at one contest may reduce the cost of effort at another contest (e.g., Willig 1979 and Klette 1996). When each agent works on a single contest, only the scarcity effect is present, so the innovation contest literature (e.g., Mihm and Schlapp 2016, Ales et al. 2016a,b) has often incorporated this effect. When an agent works on multiple contests, however, there may be economies of scope, and we contribute to the innovation contest literature by allowing agents to participate in multiple contests, and by incorporating the economies-of-scope effect into our model. Note that when the economies-of-scope effect dominates, the marginal cost of effort is decreasing; whereas when the scarcity effect dominates, the marginal cost of effort is increasing.

To answer our first research question, we examine an "exclusive" case where each agent can participate in only one contest, and compare it with a "non-exclusive" case where each agent can participate in multiple contests. We show that when agents' output uncertainty is sufficiently large, the organizer profit under a non-exclusive contest is greater than that under an exclusive contest. The intuition is as follows. While an exclusive contest incentivizes agents to exert more effort than a non-exclusive one, a non-exclusive contest attracts a larger number of agents, and hence benefits from a more diverse set of solutions. This diversity effect outweighs the incentive effect when agents face sufficiently large output uncertainty. This result suggests that a non-exclusive contest may be more effective to elicit major innovation from agents, whereas an exclusive contest may provide greater incentives for agents to develop low-novelty solutions that require concentrated effort. For example, as Elanco elicits innovative solutions to animal healthcare problems (e.g., Liver Abscess Formation in Cattle), it may benefit from agents' participation in multiple contests. Similarly, InnoCentive can improve the outcome of theoretical challenges (e.g., Measuring the Spatial Distribution of Drug Concentrations in Target Tissues) by allowing agents' participation in multiple contests. In contrast, TopCoder may improve the outcome of lower-novelty development challenges that require concentrated effort (e.g., finding bugs in software) by discouraging agents from participating in more than one such contest, for example, by imposing tight deadlines.

To answer our second research question, we analyze how the number of contests affects each organizer's profit. An established intuition in economics literature is that more intense competition has an adverse effect on competitors, yet we find that each organizer's profit is unimodal in the number of contests, and hence there is an optimal number of contests. The intuition is as follows. As the number of contests increases, each agent increases her total effort because she can diversify her risk by entering more contests. When the agent's total effort is small, so is the scarcity effect, and hence the economies-of-scope effect is larger, yet as the total effort increases, the scarcity effect starts to dominate. Whenever the economies-of-scope effect is larger (in both decentralized and centralized cases), the organizer profit increases with more contests because an agent's effort at one contest reduces her marginal cost of effort at another contest. Whenever the scarcity effect is larger, the organizer profit decreases with more contests because an agent's effort at one contest raises her marginal cost of effort at another contest. Thus, as the number of contests increases, the organizer profit increases up to an optimal number of contests but decreases afterwards. This finding suggests that when contests exhibit large economies of scope, for example, due to a requirement of common learning or knowledge, contest organizers may benefit from multiple contests. For instance, contest platforms such as InnoCentive may improve the outcome of contests by encouraging agents to participate in multiple contests of the same subject category. Likewise, organizations such as Elanco and the Gates Foundation may benefit from running multiple contests on similar topics. We further show, somewhat interestingly, that the optimal number of contests increases as agents face larger output uncertainty. This prescribes a larger number of contests when organizations seek major innovation rather than low-novelty solutions.

Related Literature. Our paper synthesizes the literature on innovation contests and the small literature on multiple contests.[3]

Early economics literature on contests (e.g., Taylor 1995, Fullerton and McAfee 1999, and Che and Gale 2003) finds that as the number of agents who participate in a contest increases, each agent exerts less effort, and hence this stream of research suggests restricting entry to contests. Yet, these results conflict with innovation contests in practice that often adopt free-entry open innovation in which all agents are allowed to participate. Incorporating the effect of agents' uncertainty, Terwiesch and Xu (2008) and Ales et al. (2016a) show that free-entry open innovation is optimal. Similarly, Boudreau et al. (2011) empirically show that when the effect of agents' uncertainty on the organizer profit (which they call the "order statistics effect") is large, free-entry open innovation is optimal. Meanwhile, Ales et al. (2016b) prove that when agents' uncertainty has a log-concave density function, it is optimal for the organizer to give a single award, i.e., the winner-take-all award scheme is optimal. Using the same framework, Mihm and Schlapp (2016) study how different types of feedback (e.g., public, private, or no feedback) can be used to improve the contest outcome, and Hu and Wang (2017) compare a single-stage contest with a sequential contest in the presence of multiple attributes.[4] We build on the modeling framework of innovation contests established by these studies, and contribute to this literature in two ways. First, we consider multiple contests and the resulting competition among contest organizers for agents' efforts. Second, we consider the economies-of-scope effect that may arise when agents work on multiple contests.

Another stream of related literature analyzes multiple contests where each agent can participate in at most one contest. Azmat and Möller (2009) study two identical contests competing for the participation of a set of identical agents, DiPalantino and Vojnović (2009) examine all-pay contests with heterogeneous awards. Recently, Büyükboyacı (2016) and Hafalır et al. (2016) compare a single contest with two competing exclusive contests. Our contribution to this literature is twofold. First, while this literature overlooks the order statistics effect, we consider this effect as it plays a prominent role in innovation contests in practice (cf. Boudreau et al. 2011). Second, we allow

[3] Besides these two streams of literature, this paper is broadly related to empirical studies on innovation and crowdsourcing (e.g., Bayus 2013, Liu et al. 2014, Mollick and Nanda 2016, and Jiang et al. 2016) and theoretical studies on innovation and new product development (e.g., Dahan and Mendelson 2001, Kavadias and Sommer 2009, and Mihm 2010; for comprehensive surveys, see Krishnan and Ulrich 2001 and Loch and Kavadias 2008). The competition among organizers is also related to the literature on price competition (e.g., Aksoy-Pierson et al. 2013, Federgruen and Hu 2016, and Zhou et al. 2016) as organizers compete with monetary awards, and agents decide on their efforts.

[4] In addition to innovation contests, there are other types of contests and contest-like mechanisms. These include labor or sales contests in which the organizer objective is to maximize the total performance of all agents (e.g., Lazear and Rosen 1981, Kalra and Shi 2001, Moldovanu and Sela 2001, Körpeoğlu and Cho 2017), auction-based mechanisms with deterministic output (e.g., Che and Gale 2003, Siegel 2009), and design contests in which agents select one design approach among a set of approaches (Erat and Krishnan 2012). In recent papers, Nittala and Krishnan (2016) study the design of innovation contests within firms, Bimpikis et al. (2016) study information extraction and disclosure in dynamic contests, and Kim and Lim (2015) study R&D outsourcing via contests in innovation-driven supply chains. For comprehensive surveys, see Konrad (2009) and Vojnović (2015).

agents to participate in multiple contests (i.e., non-exclusivity), and show that the organizer profit under non-exclusivity can be greater than that under exclusivity.

2. The Model

We build our model on the innovation contest literature reviewed in §1. Consider M innovation contests in which M contest organizers ("he") elicit solutions to innovation-related problems from a set of N agents ("she"). Below we describe our model of agents, organizers, and the equilibrium of agents. We discuss our model features and assumptions at the end of the section.

Agents. Each agent $i \in \{1, 2, ..., N\}$ develops a solution to each contest $m \in \{1, 2, ..., M\}$ she participates in, and generates an output $y_{im} \in \mathcal{Y} \subseteq \mathbb{R} \cup \{-\infty, \infty\}$. We can interpret the output y_{im} as the quality of agent i's solution or its monetary value to organizer m. The output y_{im} is determined by two terms: (i) agent i's effort e_{im} at contest m and (ii) agent i's output shock $\widetilde{\xi}_{im}$ at contest m. (As a convention, we use $\tilde{}$ to denote random variables throughout the paper.)

First, each agent i can improve her output by investing effort $e_{im} \in \mathcal{E} \subseteq \mathbb{R}_+$ in contest m. For example, conducting a thorough patent search and literature review, or implementing rigorous quality control systems with high standards improves an agent's output (Terwiesch and Xu 2008). Similarly, a logo designer may exert effort by drawing multiple sketches until she chooses the best one to submit (Ales et al. 2016b). The effort e_{im} leads to a deterministic improvement $r(e_{im})$ of the output, where r is an increasing and concave function of e_{im}.

Second, each agent faces an output shock due to the uncertainty in the innovation process and in the evaluation of solutions. For example, in a graphic-design contest in designContest, an agent may not know the quality of her solution prior to the innovation process. Furthermore, it is difficult for an agent to predict the specific design that an organizer would prefer. Using Lemma 1 of Ales et al. (2016b), we represent both of these uncertainties with a single output shock. The output shock $\widetilde{\xi}_{im}$ ($\in \Xi$) is independent for each agent i and contest m, and follows a cumulative distribution function H and a density function h with $E[\widetilde{\xi}_{im}] = 0$ over support $\Xi = [\underline{s}, \overline{s}]$, where $\underline{s} < \overline{s}$, $\underline{s} \in \mathbb{R} \cup \{-\infty\}$, and $\overline{s} \in \mathbb{R} \cup \{\infty\}$.[5] We assume that h is log-concave, which is satisfied by most commonly used distributions such as uniform and logistic distributions used by Kalra and Shi (2001), Gumbel distribution used by Terwiesch and Xu (2008), and normal distribution. Agent i's output y_{im} at contest m takes the following additive form:

$$y_{im} = y(e_{im}, \widetilde{\xi}_{im}) = r(e_{im}) + \widetilde{\xi}_{im}. \tag{1}$$

[5] Independent and identically distributed shocks are common in the literature. In practice, there may be some contest-specific dependence due to the uncertainty of the evaluation process. In this case, each agent i's output shock for contest m can be modeled as $\widetilde{\xi}_{im} + \widetilde{\epsilon}_m$, where $\widetilde{\epsilon}_m$ is a shock that represents contest-specific fixed effect. Because $\widetilde{\epsilon}_m$ terms would cancel out across agents, they would not affect agents' rankings or our analysis, so they are omitted.

To quantify the impact of a change in the variance of a general distribution H, we introduce the notion of a scale transformation (e.g., Rothschild and Stiglitz 1970) as follows. Two distribution functions H_1 and H_2 differ by a scale transformation if there exists a parameter α such that $H_1(s) = H_2(\alpha s)$ for all $s \in \Xi$. When the output shock $\widetilde{\xi}_{im}$ is transformed by a scale transformation with the parameter α, the transformed random variable $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ has mean 0, and its variance is α^2 times the variance of $\widetilde{\xi}_{im}$. Let $\widetilde{\xi}^N_{(j)m}$ be a random variable that represents the j-th largest output shock among $\{\widetilde{\xi}_{1m}, \widetilde{\xi}_{2m}, ..., \widetilde{\xi}_{Nm}\}$, and let $\mu^N_{(j)} = E[\widetilde{\xi}^N_{(j)m}]$. Noting that $\widetilde{\xi}^N_{(j)m}$ is the $(N-j+1)$-st order statistic among N random variables, its density function is $h^N_{(j)} = \frac{N!}{(N-j)!(j-1)!} H(s)^{N-1}(1 - H(s))^{j-1} h(s)$.

Agent i's utility $U_i = U(e_i, x_i) : \mathbb{R}^{2M}_+ \to \mathbb{R}$ is defined over her vector of efforts $e_i \equiv (e_{i1}, e_{i2}, ..., e_{iM})$ and her vector of awards $x_i \equiv (x_{i1}, x_{i2}, ..., x_{iM})$ that she receives from each contest. Agent i's utility takes the form $U_i = \sum_{m=1}^M x_{im} - \psi(\sum_{m=1}^M e_{im})$, where ψ is a continuously differentiable, increasing, and homogeneous of degree b (> 0) function with $\psi(0) = 0$. We can interpret ψ as the agent's disutility or cost associated with her total effort. Homogeneity of degree b means that $\psi(\gamma e) = \gamma^b \psi(e)$, and it allows us to measure the curvature of ψ. When $b < 1$, ψ is concave; when $b = 1$, ψ is linear; and when $b > 1$, ψ is convex. With this cost function, we consider two effects that determine the curvature of ψ as discussed in §1: (i) a "scarcity" effect because it may be increasingly difficult to spare resources (e.g., money or time) for exerting more effort (e.g., Albanesi and Sleet 2006) and (ii) an "economies-of-scope" effect as exerting effort at one contest may reduce the cost of effort at another contest (e.g., Willig 1979 and Klette 1996). Different contests may exhibit economies of scope due to factors such as a common learning requirement. For example, an agent who conducts a literature review on biology for a contest at InnoCentive may find it easier to conduct literature reviews in the same subject category for other contests. Note that when the scarcity effect dominates, the marginal cost of effort is increasing, leading to a convex ψ; while when the economies-of-scope effect dominates, the marginal cost of effort is decreasing, leading to a concave ψ. We let $g(x) = (\psi'/r')^{-1}(x)$, and assume that $g(x)$ is increasing, $r'(g(x))g'(x)$ is decreasing, and $r''(x)/(r'(x))^2$ is non-increasing in x. These assumptions are satisfied by effort and cost functions commonly used in the literature (e.g., $r(e) = \theta \log(e)$ and $\psi(e) = ce$, where $\theta, c > 0$ used by Terwiesch and Xu 2008 and $r(e) = \theta(e^{1-a} - 1)/(1-a)$ and $\psi(e) = ce^b$, where $a \geq 1$, $b \geq 1$ used by Ales et al. 2016a,b). Similar assumptions are common in the literature reviewed in §1.

Organizers. Each organizer m determines the award A_m that is given to the winner, i.e., the agent with the highest output in contest m. This winner-take-all award scheme is common in practice and literature discussed in §1, and it is optimal in a contest where the output shock density h is log-concave (see Proposition 3 of Ales et al. 2016b). Under the winner-take-all award scheme, if agent i wins contest m, her award is $x_{im} = A_m$; otherwise, $x_{im} = 0$. Each organizer is interested in the highest output in his contest. For example, in a logo-design contest, an organizer is interested in the quality of the best logo because he will use only the best logo. Thus, organizer m's profit Π_m is equal to the highest output in his contest minus the award A_m he gives, i.e., $\Pi_m = \max_i \; y_{im} - A_m$.

The sequence of events is as follows. First, awards $(A_1, A_2, ..., A_M)$ are announced, and then each agent i determines her effort e_{im} at each contest m she participates in, considering the cost of her total effort $\psi(\sum_{m=1}^{M} e_{im})$. Then, each agent i observes her output shock $\widetilde{\xi}_{im}$, and generates an output y_{im} at each contest m. Finally, each organizer m collects all solutions in contest m, and gives the award A_m to the winner.

Equilibrium of agents. We next define and characterize Bayesian Nash equilibrium of the subgame among agents. As is common in the innovation contest literature, we focus on symmetric Nash equilibria, and denote each agent's equilibrium effort at contest m by e_m^*. We solve for a Nash equilibrium by deriving each agent's best-response function stemming from her optimization problem. To do so, we first derive agent i's probability of winning contest m by exerting effort e_{im} given that all other agents exert efforts e_m^* at contest m as follows:

$$P_m(e_{im}, e_m^*) = \int_{s\in\Xi} H(s + r(e_{im}) - r(e_m^*))^{N-1} h(s) ds. \tag{2}$$

Agent i solves the following problem to determine her best-response effort e_{im} at each contest m:

$$\max_{(e_{i1}, e_{i2}, ..., e_{iM})} \sum_{m=1}^{M} A_m P_m(e_{im}, e_m^*) - \psi\left(\sum_{m=1}^{M} e_{im}\right). \tag{3}$$

In a symmetric Nash equilibrium, each agent's effort e_m^* at contest m solves (3). In Lemmas A1 and A2 of Appendix, we present sufficient conditions for the existence of e_m^* that solves (3). For the rest of the paper, we assume that such sufficient conditions are satisfied. Similar sufficient conditions are commonly used in the literature (e.g., Terwiesch and Xu 2008, Ales et al. 2016a,b, Mihm and Schlapp 2016). The first derivative of agent i's problem in (3) with respect to e_{im} is

$$A_m r'(e_{im}) \int_{s\in\Xi} (N-1) h(s + r(e_{im}) - r(e_m^*)) H(s + r(e_{im}) - r(e_m^*))^{N-2} h(s) ds - \psi'\left(\sum_{k=1}^{M} e_{ik}\right). \tag{4}$$

Evaluating (4) at $e_{im} = e_m^*$ for all $i \in \{1, 2, ..., N\}$, we obtain the following characterizing equations:

$$A_m r'(e_m^*) I_N = \psi'\left(\sum_{k=1}^{M} e_k^*\right) \text{ for all } m \in \{1, 2, ..., M\}, \text{ where } I_N \equiv \int_{s\in\Xi} (N-1) H(s)^{N-2} h(s)^2 ds. \tag{5}$$

(5) shows that each agent chooses her effort by optimally balancing the impact of her additional effort on her expected award from each contest (i.e., the marginal benefit of effort) with the impact of her additional effort on her total cost (i.e., the marginal cost of effort). Note that equilibrium efforts at all contests are interlinked via the common cost of total effort.

Decentralized and centralized contests. Throughout the paper, we consider a decentralized case in which independent contest organizers compete for agents' efforts and a centralized case in which multiple contests are managed by a central planner. We use superscripts E and P to denote variables in decentralized and centralized cases, respectively. We drop these superscripts to denote variables that encompass both decentralized and centralized cases.

In the decentralized case, M contest organizers compete for agents' efforts. Given that all other organizers give awards $A_{j\neq m} = A^{*,E}$ and that agents determine their equilibrium efforts based on (5), each organizer m chooses his award A_m to maximize his expected profit

$$\Pi_m(A_m) = r(e_m^*) + E[\widetilde{\xi}_{(1)m}^N] - A_m. \tag{6}$$

We refer to $A^{*,E}$ that solves (6) as the "equilibrium" award.

In the centralized case, all contests are managed by a central planner. The analysis of the centralized case has two objectives. First, as we discuss in §1, many organizations such as Elanco and the Gates Foundation run multiple contests simultaneously, and the analysis of centrally-managed contests provides insights for practitioners in such organizations. Second, we obtain a benchmark for the decentralized case in which independent contest organizers compete for agents' efforts. In the centralized case, the planner's objective is to maximize the expected average profit from contests, which is given by $\overline{\Pi} \equiv (1/M)(E[\sum_{m=1}^M \max_i \ y_{im}] - \sum_{m=1}^M A_m)$. Given the equilibrium effort e_m^* that solves (5), we have $\max_i \ y_{im} = \max_i \ \{r(e_m^*) + \widetilde{\xi}_{im}\} = r(e_m^*) + \max_i \ \widetilde{\xi}_{im} = r(e_m^*) + \widetilde{\xi}_{(1)m}^N$. Thus, the planner's objective is to maximize

$$\overline{\Pi} = \frac{\sum_{m=1}^M r(e_m^*)}{M} + \frac{E\left[\sum_{m=1}^M \widetilde{\xi}_{(1)m}^N\right]}{M} - \frac{\sum_{m=1}^M A_m}{M}. \tag{7}$$

We refer to $A_m^{*,P}$ as the planner's "optimal" award in contest m.

Discussion. Before we proceed to our analysis, we discuss how our model relates to existing models in the literature. We employ the standard model in the innovation contest literature reviewed in §1 with two major novelties. First, we consider multiple contests while the innovation contest literature considers a single contest (i.e., $M = 1$). Second, we allow for a more general cost function of effort than commonly used functional forms in the literature such as a linear cost function (i.e., $\psi(e) = ce$) used by Terwiesch and Xu (2008), a convex cost function (i.e., $\psi(e) = ce^b$, where $b \geq 1$) used by Ales et al. (2016a,b), and a quadratic cost function (i.e., $\psi(e) = ce^2$) used by Mihm and Schlapp (2016).[6] Prior research on contests naturally focuses on the scarcity effect because the economies-of-scope effect disappears when agents work on a single contest. However, when agents work on multiple contests, there may be economies of scope (cf., Willig 1979 and Klette 1996), and our general cost function allows us to analyze cases where the economies-of-scope effect

[6] Following the innovation contest literature reviewed in §1, we assume that agents incur no fixed costs. In this case, agent participation is guaranteed because zero effort results in a non-negative utility for an agent. The inclusion of positive fixed costs renders the analysis intractable but leads to qualitatively similar results as long as fixed costs are not too high to prevent agents from participating in more than one contest.

dominates the scarcity effect and vice versa. In §4.2, we consider a more general cost function that allows us to analyze more complex relationships between these two effects and to give insights into the optimal number of simultaneous contests. Our main difference from the multiple contests literature reviewed in §1 is that while this literature assumes exclusive contests, we allow agents to participate in multiple contests, as it is common in practice. For instance, as discussed in §1, statistical analysis at InnoCentive reveals that in theoretical challenges, in which agents develop innovative theoretical solutions to posted problems, 57.4% of agents work on multiple contests simultaneously. Furthermore, as we show in the next section, the organizer profit can be greater when agents can enter multiple contests compared to when agents can enter only one contest.

3. Exclusive versus Non-Exclusive Contests

In this section, we analyze when agents should be allowed to freely participate in multiple contests. In practice, a contest platform such as InnoCentive or TopCoder can discourage agents' participation in multiple contests, for instance, by setting a tight deadline or by imposing strict submission rules. We refer to the case in which agents are discouraged from participating in multiple contests as the exclusive case and the case in which agents can freely participate in multiple contests as the non-exclusive case. As discussed in §1, the multiple contests literature assumes that each agent can participate in only one contest, i.e., exclusivity.

The following proposition shows that when the output uncertainty is sufficiently large, the average profit of organizers under no exclusivity is larger than that under exclusivity.

Proposition 1. *Suppose that a planner manages contests under exclusivity to find the optimal allocation of agents and awards, and let $\overline{\Pi}^X$ be the average profit organizers in this case.*

(a) *In non-exclusive centralized contests, there exists α_0 such that for any scale transformation $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with the scale parameter $\alpha > \alpha_0$, the average profit of organizers under non-exclusive optimum $\overline{\Pi}^P$ is greater than that under exclusive optimum $\overline{\Pi}^X$.*

(b) *Suppose that $r(e) = \theta\log(e)$ and $\psi(e) = ce^b$ for $\theta, b, c > 0$. In non-exclusive decentralized contests, there exists α_1 such that for any scale transformation $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with the scale parameter $\alpha > \alpha_1$, the average profit of organizers under non-exclusive equilibrium $\overline{\Pi}^E$ is greater than that under exclusive optimum $\overline{\Pi}^X$.*

In Proposition 1, we compare the average profit of organizers under no exclusivity with the average profit of organizers $\overline{\Pi}^X$ under exclusivity when a planner optimally allocates agents to contests and determines the award at each contest.[7] Specifically, Proposition 1 shows that when agents' output

[7] The average profit of organizers $\overline{\Pi}^X$ in this case is an upper bound for the average profit of organizers under an exclusive decentralized case where each agent determines which contest to participate in and determines her effort, while each organizer determines his award. We use this upper bound because in the exclusive decentralized case, generically, a pure-strategy Nash equilibrium of organizers does not exist. Arguments for non-existence are available from authors upon request.

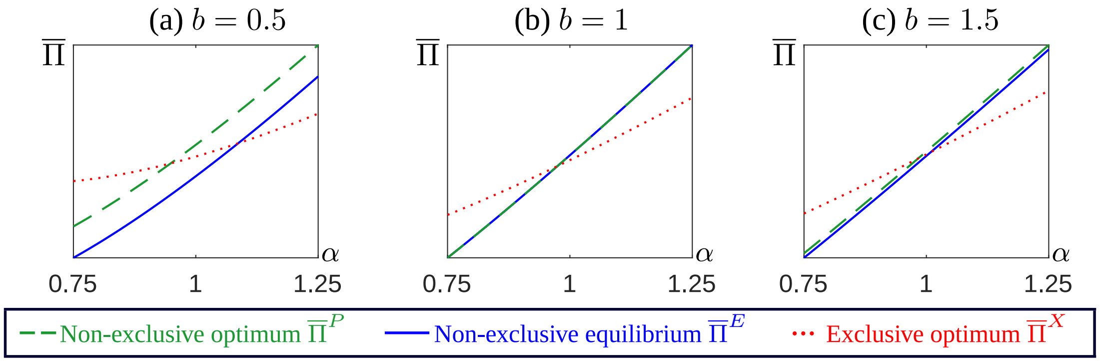


Figure 1 The average profit of organizers $\overline{\Pi}$ as a function of the scale parameter α under different values of the cost function parameter b. Setting: $\widetilde{\xi}_{im} \sim$ Gumbel with mean 0 and scale parameter 1, $M = 5$, $N = 100$, $r(e) = \log(e)$, and $\psi(e) = 0.1e^b$.

uncertainty (i.e., the scale parameter α) is sufficiently large, the average profit of organizers under non-exclusive equilibrium $\overline{\Pi}^E$ and that under non-exclusive optimum $\overline{\Pi}^P$ are greater than that under exclusive optimum $\overline{\Pi}^X$; see Figure 1. Note that as Figure 1 shows, this result holds regardless of the curvature of the cost function (i.e., parameter b). Yet, the exact level of uncertainty that is sufficient for $\overline{\Pi}^E$ and $\overline{\Pi}^P$ to be greater than $\overline{\Pi}^X$ depends on the curvature of the cost function.

The intuition behind Proposition 1 is as follows. The average profit of organizers $\overline{\Pi}$ depends on three terms: the effort term $r(e^*)$, shock term $E[\max_i \widehat{\xi}_{im}]$ $(= E[\max_i \alpha\widetilde{\xi}_{im}])$, and award term A^*. Figure 2 depicts how these terms as well as the organizer profit change with the spread of the output shock measured by the scale parameter α. On one hand, the effort term under exclusivity is higher than that under no exclusivity. This is because the number of agents competing at each contest under exclusivity is smaller than that under no exclusivity. Yet, the difference between effort terms under exclusivity and under no exclusivity stays the same as α increases. On the other hand, the shock term under no exclusivity is greater than that under exclusivity. This is because a non-exclusive contest attracts a larger number of agents, and hence benefits from a more diverse set of solutions. Unlike the effort term, the difference between shock terms under exclusivity and under no exclusivity increases as α increases. Because the award term is the same in all cases, as α increases, the difference in the shock term eventually dominates the difference in the effort term, so the average profit of organizers becomes larger under no exclusivity than that under exclusivity.

Proposition 1 has important implications for practice and the contest literature. First, Proposition 1 suggests that in practice, an organizer may benefit from a non-exclusive contest when he elicits major innovation rather than low-novelty solutions that require concentrated effort (cf. Terwiesch and Xu 2008).[8] For example, InnoCentive may improve the outcome of theoretical challenges by allowing agents to participate in multiple contests because these contests require novel

[8] In the innovation contest literature, agents' uncertainty (also called risk level) is often associated with the novelty of solutions (e.g., Terwiesch and Xu 2008) and with the subjectivity of evaluation criteria (e.g., Ales et al. 2016b).

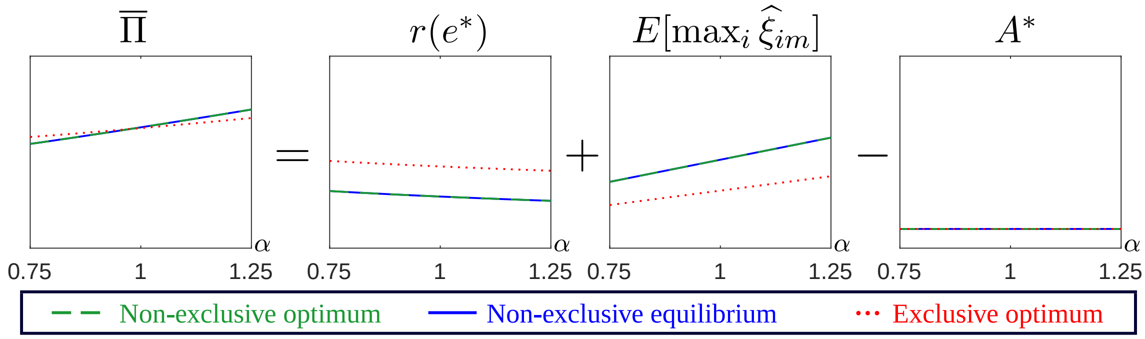


Figure 2 The average profit of organizers $\overline{\Pi}$ and its effort, shock, and award terms under exclusive and non-exclusive cases as a function of the scale parameter α. The setting is the same as Figure 1 with $b = 1$.

solutions to challenging problems. In a similar spirit, Elanco may benefit from agents' participation in multiple contests because these contests elicit innovative solutions to animal healthcare problems. In contrast, TopCoder may improve the outcome of development challenges with low novelty by discouraging agents from entering more than one of these challenges (e.g., via tight deadlines as in John Hancock Project Snapshot Bug Hunt contest). Second, a standard assumption in the economics literature on multiple contests reviewed in §1 is that each agent can participate in only one contest, i.e., exclusivity. Yet, agents' participation in multiple contests is not only common in practice (see discussions in §1 and at the end of §2), but also often beneficial to organizers as Proposition 1 shows. Thus, although exclusivity may be a reasonable assumption for the specific contests this literature studies, relaxing this assumption is crucial while studying multiple innovation contests. Therefore, in the following section, we analyze innovation contests where agents can freely participate in multiple contests.

4. Multiple Simultaneous Contests

In §4.1, we analyze the impact of an additional contest on the organizer profit and compare the centralized and decentralized cases. In §4.2, we consider a more general cost function for agents, and analyze the optimal number of contests.

4.1. Impact of an Additional Contest

In this section, we characterize the agent's equilibrium effort and the award organizers give in centralized and decentralized cases, and we analyze the impact of an increase in the number of contests M on the organizer profit when the scarcity or economies-of-scope effect dominates.

According to these papers, agents face low uncertainty in contests that seek low-novelty solutions with well-defined evaluation criteria, whereas they face high uncertainty in contests that seek novel solutions with more subjective evaluation criteria. Nittala and Krishnan (2016) relate the level of agent's uncertainty to how broadly an organizer defines a problem, which may be associated with the level of novelty an organizer seeks.

Proposition 2. ***(a)*** *In the centralized case, the agent's effort is* $e^{*,P}$ *and the optimal award is* $A_m^{*,P} = A^{*,P}$*, where* $e^{*,P}$ *and* $A^{*,P}$ *satisfy*

$$e^{*,P} = g\left(A^{*,P} I_N M^{1-b}\right), \tag{8}$$

$$r'\left(e^{*,P}\right) g'\left(A^{*,P} I_N M^{1-b}\right) I_N M^{1-b} - 1 = 0. \tag{9}$$

Moreover, the planner objective $\overline{\Pi}^P = \Pi_m^P = r(e^{*,P}) + \mu_{(1)}^N - A^{*,P}$ *is decreasing, constant, or increasing in the number of contests* M *when the cost function parameter* $b > 1$*,* $b = 1$*, or* $b < 1$*, respectively.*
(b) *In the decentralized case, the equilibrium effort* $e^{*,E}$ *and equilibrium award* $A^{*,E}$ *solves*

$$e^{*,E} = g\left(A^{*,E} I_N M^{1-b}\right), \tag{10}$$

$$r'\left(e^{*,E}\right) g'\left(A^{*,E} I_N M^{1-b}\right) I_N M^{1-b} - 1 = \frac{(M-1)(b-1) I_N r'\left(e^{*,E}\right)^2 g'\left(A^{*,E} I_N M^{1-b}\right)}{M^b r''(e^{*,E}) e^{*,E}}. \tag{11}$$

Furthermore, the equilibrium award $A^{*,E}$ *is greater than, equal to, or smaller than the optimal award* $A^{*,P}$ *when* $b > 1$*,* $b = 1$*, or* $b < 1$*, respectively.*

Proposition 2(a) shows that how the planner objective (i.e., the average profit of organizers) $\overline{\Pi}^P$ changes with the number of contests M depends on the curvature of the cost function (i.e., parameter b).[9] Note that because the planner chooses to give equal awards at all contests (as we show in the proof of Proposition 2(a) in Appendix), the planner objective $\overline{\Pi}^P$ is equal to the organizer profit Π_m^P. When the cost of total effort is convex (i.e., $b > 1$), more contests harm each organizer. This is intuitive because an agent's effort at one contest increases the marginal cost of her effort at other contests, and this negative externality induces lower efforts at each contest in equilibrium. When the cost of total effort is linear (i.e., $b = 1$), the agent's problem in (3) becomes decomposable, i.e., each agent treats each contest independently from others. When the cost of total effort is concave (i.e., $b < 1$), interestingly, more contests benefit each organizer. This is because an agent's effort at one contest decreases the marginal cost of her effort at other contests. This positive externality induces higher efforts at each contest in equilibrium. This result shows that organizers may benefit from an increase in the number of contests that exhibit large economies of scope, for example, due to a common learning feature. Proposition 2(a) prescribes that organizations such as Elanco and the Gates Foundation may benefit from running contests on similar topics simultaneously.

Proposition 2(b) shows that compared to the centralized case, each organizer in the decentralized case over-awards agents when the cost function is convex (i.e., $b > 1$), optimally awards agents when the cost function is linear (i.e., $b = 1$), and under-awards agents when the cost function is concave (i.e., $b < 1$). This is because when the cost function is concave, as the number of contests

[9] While how the organizer profit changes with the number of contests M depends on the curvature of the cost function captured by parameter b, as we show in §A.2 of Appendix, the agent utility always increases with M regardless of b.

M increases, organizers can elicit agents' efforts by giving them smaller awards. Internalizing this positive externality, the planner in the centralized case gives larger awards than organizers do in the decentralized case. This finding suggests that it may be beneficial if contest platforms such as InnoCentive encourage larger awards in contests that exhibit common learning features.

To generate further insights about the equilibrium award $A^{*,E}$ and equilibrium organizer profit Π_m^E in the decentralized case, we consider a special form for the effort function r and for the cost function ψ in the following corollary. Note that these special forms are assumed by Terwiesch and Xu (2008) under $b=1$.

COROLLARY 1. *Suppose that* $r(e)=\theta\log(e)$ *and* $\psi(e)=ce^b$ *for* $\theta,b,c>0$.
(a) *The equilibrium effort* $e^{*,E}=\left(\frac{\theta A^{*,E} I_N M^{1-b}}{cb}\right)^{\frac{1}{b}}$, *where the equilibrium award* $A^{*,E}=\frac{\theta(Mb-b+1)}{Mb}$.
(b) *The equilibrium organizer profit* Π_m^E *is decreasing, constant, or increasing in the number of contests* M *when* $b>1$, $b=1$, *or* $b<1$, *respectively.*

Corollary 1(a) shows that the equilibrium award $A^{*,E}$ depends on three factors: the agent productivity θ, number of contests M, and cost function parameter b. Specifically, as Figure 3(a) shows, when b is less than one, in which case the cost function is concave, the equilibrium award $A^{*,E}$ is decreasing in M. This is because the marginal cost of effort is decreasing, so organizers can elicit agents' efforts by giving them smaller awards. In contrast, when b is greater than one, in which case the cost function is convex, $A^{*,E}$ is increasing in M. Finally, when b is equal to one, in which case the cost function is linear, $A^{*,E}$ does not depend on M.

Corollary 1(b) shows that when the cost function is convex (i.e., $b>1$), the organizer profit Π_m^E in the decentralized case decreases with the number of contests M, and when the cost function is linear (i.e., $b=1$), Π_m^E does not change with M. This is intuitive and in line with the centralized case. It is also in line with the common intuition in economics that more intense competition has an adverse effect on competitors. More interestingly, when the cost function is concave (i.e., $b<1$), as Figure 3(b) shows, the organizer profit Π_m^E increases with the number of contests M. This result may occur when different contests exhibit economies of scope due to factors such as a common learning requirement. For instance, an agent who reviews the related literature for a contest in biology or life sciences at InnoCentive may find it easier to review the related literature for other contests in the same subject category. Because organizers benefit from agents participating in such contests, it may be advisable for contest platforms such as InnoCentive to encourage agents to participate in multiple contests of the same subject category.

In this section, we have shown that whenever the economies-of-scope effect outweighs the scarcity effect, increasing the number of contests M improves average and individual profits of organizers. In the following section, we analyze the optimal number of contests.

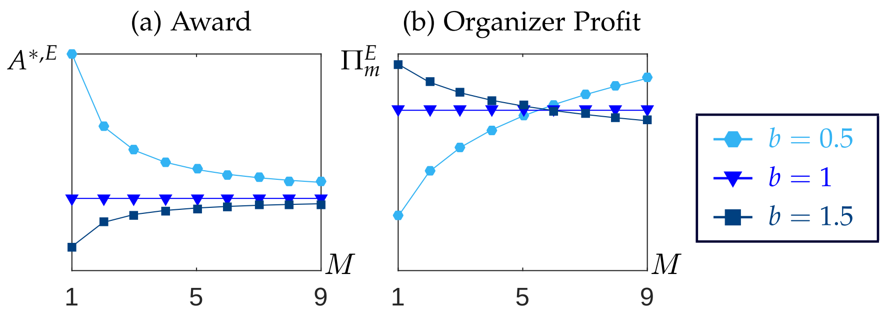


Figure 3 The equilibrium award $A^{*,E}$ and equilibrium organizer profit Π^E_m as a function of the number of contests M. The setting is the same as Figure 1.

4.2. Optimal Number of Contests

In this section, we analyze the optimal number of contests in the centralized case where a planner determines the award at each contest (the insights we generate here can be extended to the decentralized case where organizers compete for agents' efforts). Because whether the scarcity or economies-of-scope effect dominates may depend on agents' efforts, we consider a more general cost function $\psi(E)=a_1\psi_1(E)+a_2\psi_2(E)$, where $E=\sum_{m=1}^{M} e_{im}$ is agent i's total effort, $a_1+a_2=1$, $\psi_1(E)$ is homogenous of degree $b_1>1$, and $\psi_2(E)$ is homogenous of degree $b_2\in(0,1)$. This cost function nicely blends the two effects we consider: when an agent exerts a small effort that requires fewer resources, the economies-of-scope effect (i.e., $\psi_2(E)$) dominates the scarcity effect (i.e., $\psi_1(E)$), but when she increases her effort, the scarcity effect dominates the economies-of-scope effect due to depleting resources. We let $f_1\equiv\psi_1'/r'$ and $f_2\equiv\psi_2'/r'$. As ψ_1 is convex and r is concave, f_1 is increasing; and similar to §2, we assume that f_2 is increasing. Note that this cost function subsumes the cost functions we consider in §2. Specifically, when $a_1=1$ (hence $a_2=0$), $\psi(E)$ is convex; and when $a_2=1$ (hence $a_1=0$), $\psi(E)$ is concave.

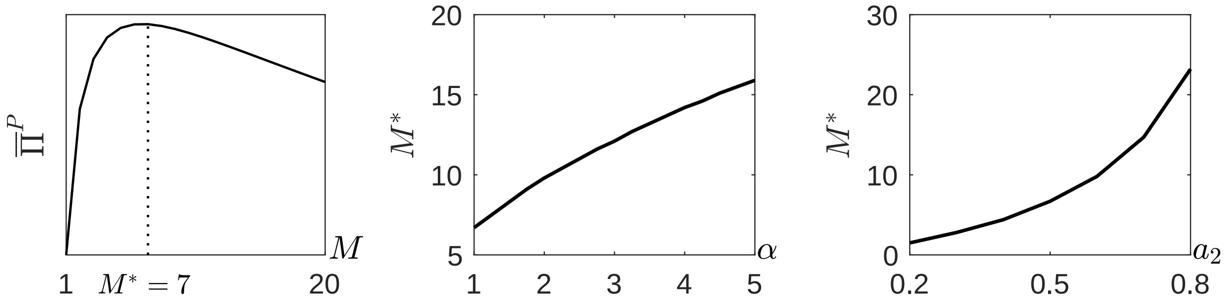
The following proposition presents the main result of this section.

PROPOSITION 3. ***(a)*** *The optimal award $A^{*,P}$ and effort $e^{*,P}$ for the planner's problem solve*

$$a_1M^{b_1-1}f_1(e^{*,P})+a_2M^{b_2-1}f_2(e^{*,P})-A^{*,P}I_N=0, \tag{12}$$

$$a_1M^{b_1-1}f_1'(e^{*,P})+a_2M^{b_2-1}f_2'(e^{*,P})-r'(e^{*,P})I_N=0. \tag{13}$$

Moreover, let $\eta\equiv-a_1(b_1-1)\psi_1'(E^{*,P})+a_2(1-b_2)\psi_2'(E^{*,P})$. The planner objective $\overline{\Pi}^P$ is increasing, constant, or decreasing in the number of contests M when $\eta>0$, $\eta=0$, or $\eta<0$, respectively.
(b) *Suppose that r' is homogenous of degree $-k$, where $k>0$. Then, $\overline{\Pi}^P$ is unimodal in M, i.e., there exists $M^*\in[1,+\infty)\cup\{+\infty\}$ such that $\frac{\partial\overline{\Pi}^P}{\partial M}>0$ for all $M<M^*$ and $\frac{\partial\overline{\Pi}^P}{\partial M}<0$ for all $M>M^*$. Furthermore, under a scale transformation $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with the scale parameter α, M^* is non-decreasing in α.*



(a) $\alpha = 1$, $a_1 = a_2 = 0.5$. (b) $a_1 = a_2 = 0.5$. (c) $\alpha = 1$.

Figure 4 The optimal number of contests M^* and how it changes with the scale parameter α and economies-of-scope weight a_2. Setting: $\widetilde{\xi}_{im} \sim$ Gumbel with mean 0 and scale parameter 1, $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$, $N = 100$, $r(e) = \log(e)$, and $\psi(e) = (1 - a_2)e^{1.5} + a_2 e^{0.5}$.

Proposition 3(a) shows that how the planner objective (i.e., the average profit of organizers) $\overline{\Pi}^P$ changes with the number of contests M depends on the η term, which depends on the relative weight of scarcity and economies-of-scope effects. For a given number of contests M, when the scarcity effect dominates (i.e., $a_1(b_1 - 1)\psi_1'(E^{*,P})$ is large), an additional contest reduces $\overline{\Pi}^P$; but when the economies-of-scope effect dominates (i.e., $a_2(1 - b_2)\psi_2'(E^{*,P})$ is large), an additional contest raises $\overline{\Pi}^P$. This result is in line with our finding in Proposition 2(a).

The major difference of Proposition 3(a) from Proposition 2(a) is that how the planner objective $\overline{\Pi}^P$ changes with the number of contests M depends on the η term, and the η term depends on M. This is due to two reasons: the agent's total effort increases with M and whether the scarcity or economies-of-scope effect dominates depends on the agent's total effort. First, as we discuss in §A.2 of Appendix and in the proof of Proposition 3(a), the agent's total effort increases with the number of contests M because she can diversify risk by participating in more contests. Second, when the total effort is small, the marginal cost arising from the scarcity component of the cost function (i.e., ψ_1') is close to zero, so the economies-of-scope component dominates, and hence $\eta > 0$. On the other hand, when the total effort is large, the marginal cost arising from the economies-of-scope component (i.e., ψ_2') is close to zero, so the scarcity component dominates, and hence $\eta < 0$. Thus, as Figure 4(a) illustrates, when M is small, $\overline{\Pi}^P$ increases with M because the economies-of-scope effect dominates, but as M increases, so does the scarcity effect, and hence $\overline{\Pi}^P$ starts to decrease with M. Therefore, as Proposition 3(b) shows and Figure 4(a) illustrates, the planner objective $\overline{\Pi}^P$ is unimodal in the number of contests M, so there is a unique optimal number of contests M^*.

Proposition 3(b) shows that the optimal number of contests M^* is closely related to the distribution of the output shock $\widetilde{\xi}_{im}$. Specifically, as Figure 4(b) illustrates, when the spread of the output shock $\widetilde{\xi}_{im}$ increases via a scale transformation with the scale parameter $\alpha > 1$, the optimal number of contests M^* increases. The intuition is as follows. As the agent's uncertainty increases, the marginal impact of her effort on the probability of winning decreases, so the total effort $E^{*,P}$ decreases with α. As $E^{*,P}$ decreases, the scarcity component (i.e., $a_1(b_1-1)\psi_1'(E^{*,P})$) decreases, but the economies-of-scope component (i.e., $a_2(1-b_2)\psi_2'(E^{*,P})$) increases. Thus, as agents face larger output uncertainty, additional contests become more beneficial to organizers, and hence the optimal number of contests M^* increases. In practice, this finding suggests that a larger number of simultaneous contests may be more beneficial to organizers when organizers elicit major innovation rather than low-novelty solutions.

Proposition 3 also shows that how the planner objective $\overline{\Pi}^P$ changes with the number of contests M depends on the scarcity weight a_1 and economies-of-scope weight a_2. Specifically, Figure 4(c) demonstrates that as the economies-of-scope weight a_2 increases (and hence a_1 decreases), as expected, the optimal number of contests M^* increases. This result suggests that organizations such as Elanco and the Gates Foundation may benefit from running multiple contests that exhibit large economies of scope (e.g., due to a common learning requirement), supporting the insight we generate in §4.1.

5. Conclusion

We have studied multiple simultaneous contests where contest organizers elicit solutions to innovation-related problems from a set of agents. As each agent splits her effort over a larger number of contests, agents participating in multiple contests may lead to a competition among contest organizers, and possibly a lower profit for each organizer. A plausible remedy in this case could be to discourage agents from entering multiple contests. The goal of this paper is to shed light on the impact of multiple simultaneous contests and to provide insights for practitioners about when to discourage agents' participation in multiple contests and when to run multiple contests.

Our analysis yields the following novel insights. First, when organizers elicit major innovation rather than low-novelty solutions that require concentrated effort, organizers benefit from agents' participation in multiple contests. This finding suggests that organizations such as Elanco and the Gates Foundation may benefit from agents' participation in multiple contests that elicit innovative solutions; whereas TopCoder may benefit from discouraging agents' participation in multiple development challenges that elicit low-novelty solutions and require concentrated effort. Second, the organizer profit may increase with the number of contests when these contests exhibit large economies of scope, for example, due to a common learning requirement. This result shows that organizations such as Elanco and the Gates Foundation may benefit from running multiple contests on similar topics. Finally, we show that there is an optimal number of contests, which increases as organizers seek major innovation rather than low-novelty solutions.

There are several avenues for future research. First, to focus on the impact of competition, our model assumes identical organizers, but it may be an interesting extension to analyze the impact of heterogeneity among organizers. Second, while we consider multiple contests that are run simultaneously, as a future research avenue, one may consider how to dynamically schedule multiple contests to maximize the total or average profits from these contests.

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Appendix

A. Analysis of Agents

In this section, we provide a deeper analysis of agents' equilibrium. In §A.1, we prove the existence of equilibrium for agents. In §A.2, we analyze how the agent's utility changes with the number of contests.

A.1. Existence of Equilibrium

The following two lemmas provide sufficient conditions for the concavity of the agent's utility function and show the existence of the agent's equilibrium.

LEMMA A1. *Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ via a scale transformation with $\alpha > 0$. Ceteris paribus, when α or $r''/(r')^2$ are sufficiently large, there exists $\underline{b} < 1$ such that agent i's utility function $U_i = \sum_{m=1}^{M} A_m P_m(e_{im}, e^*_m) - \psi(\sum_{m=1}^{M} e_{im})$ is concave in agent i's efforts $(e_{i1}, e_{i2}..., e_{iM})$ for all $b > \underline{b}$, where $P_m(e_{im}, e^*_m)$ is as in (2).*

Proof. For notational convenience, we drop e_m^* from $P_m(e_{im}, e_m^*)$. We derive a sufficient condition for U_i to be concave for $M = 2$, but the argument can be generalized. The Hessian matrix of U_i is

$$D^2U_i = \begin{bmatrix} A_1P_1''(e_{i1}) - \psi''(\sum_{m=1}^2 e_{im}) & -\psi''(\sum_{m=1}^2 e_{im}) \\ -\psi''(\sum_{m=1}^2 e_{im}) & A_2P_2''(e_{i2}) - \psi''(\sum_{m=1}^2 e_{im}) \end{bmatrix}. \tag{14}$$

U_i is concave if and only if D^2U_i is negative semi-definite, which is satisfied if and only if for all $m \in \{1,2\}$, $A_mP_m''(e_{im}) - \psi''(\sum_{m=1}^2 e_{im}) \le 0$ and $A_1P_1''(e_{i1})A_2P_2''(e_{i2}) - A_1P_1''(e_{i1})\psi''(\sum_{m=1}^2 e_{im}) - A_2P_2''(e_{i2})\psi''(\sum_{m=1}^2 e_{im}) \ge 0$. A sufficient condition for this is that $A_mP_m''(e_{im}) \le \min\{2\psi''(\sum_{m=1}^2 e_{im}), 0\}$. Under a scale transformation of the output shock to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with α, $P_m(e_{im}) = \int_{s\in\Xi} H\left(s + \frac{r(e_{im})-r(e_m^*)}{\alpha}\right)^{N-1} h(s)ds = E\left[H_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im})-r(e_m^*)}{\alpha}\right)\right]$. The first derivative of $P_m(e_{im})$ is $P_m'(e_{im}) = \frac{r'(e_{im})}{\alpha}E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im})-r(e_m^*)}{\alpha}\right)\right]$. Then, the second derivative of $P_m(e_{im})$ is (where $r_m^* = r(e_m^*)$)

$$P_m''(e_{im}) = \frac{r'(e_{im})^2}{\alpha^2}E\left[\left(h_{(1)}^{N-1}\right)'\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r_m^*}{\alpha}\right)\right] + \frac{r''(e_{im})}{\alpha}E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r_m^*}{\alpha}\right)\right]. \tag{15}$$

When $P_m''(e_{im}) < 0$ for $m \in \{1,2\}$, there exists $\underline{b} < 1$ such that $A_mP_m''(e_{im}) \le \min\{2\psi''(\sum_{m=1}^2 e_{im}), 0\}$ for all m and $b > \underline{b}$. There are two sufficient conditions for $P_m''(e_{im})$ to be negative. First, as α approaches infinity, both expectation terms in (15) converge, and $E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im})-r_m^*}{\alpha}\right)\right]$ converges to a positive constant. Furthermore, because r is increasing and concave, $r'(e_{im})^2/\alpha^2$ (>0) approaches 0 faster than $r''(e_{im})/\alpha$ (<0). Thus, for sufficiently large α, $P_m''(e_{im}) < 0$. Second, $P_m''(e_{im}) < 0$ when $r''/(r')^2$ is sufficiently large. Thus, for sufficiently large $r''/(r')^2$ or α, there exists $\underline{b} < 1$ such that U_i is concave for all $b > \underline{b}$. ■

LEMMA A2. *Suppose that* $\lim_{e\to+\infty}\psi(e) = +\infty$, $\lim_{e\to 0} r'(e) = +\infty$, $\lim_{e\to 0}\frac{\psi'(e)}{r'(e)} = 0$, *and* $\lim_{e\to+\infty}\frac{\psi'(e)}{r'(e)} = +\infty$. *Suppose also that* U_i *is concave in* $(e_{i1}, e_{i2}, ..., e_{iM})$. *There exists a solution to (3) which satisfies (5), and* $(e_1^*, e_2^*, ..., e_M^*)$ *that solves (3) is the symmetric Nash equilibrium.*

Proof. According to Theorem 1.2 of Fudenberg and Tirole (1991), a pure-strategy Nash equilibrium of agents exists if each agent i's set of actions (i.e., the set of feasible efforts at different contests) is a non-empty, convex, and compact subset of the Euclidean space, and her utility U_i is continuous and quasi-concave in her effort e_i. Recall that agent i's utility function $U_i(e_{i1}, e_{i2}, ..., e_{iM}) = \sum_{m=1}^M A_mP_m(e_{im}, e_m^*) - \psi(\sum_{m=1}^M e_{im})$. Because agent i's expected total award $\sum_{m=1}^M A_mP_m(e_{im}, e_m^*)$ is bounded by $\sum_{m=1}^M A_m$, but her cost of effort is unbounded ($\lim_{e\to+\infty}\psi(e) = +\infty$), there exists $\overline{e}$ such that $U_i(e_{i1}, e_{i2}, ..., e_{iM}) < U_i(0, 0, ..., 0)$ when $e_{im} > \overline{e}$ for all $m \in \{1, 2, ..., M\}$ because otherwise, we can find a sequence of vectors $(\overline{e}_{i1k}, \overline{e}_{i2k}, ..., \overline{e}_{iMk})$ such that $\lim_{k\to\infty} U_i(\overline{e}_{i1k}, \overline{e}_{i2k}, ..., \overline{e}_{iMk}) \ge U_i(0, 0, ..., 0)$, which contradicts $\lim_{e\to+\infty}\psi(e) = +\infty$. Thus, without loss of optimality, each agent i's action set can be restricted to $[0, \overline{e}]^M$, which is a non-empty, convex, and compact subset of the Euclidean space. As U_i is concave, so quasi-concave, a pure-strategy Nash equilibrium exists.

We next show that there exists a solution to (3) that satisfies (5). If we evaluate the first-order condition of agent i's problem at $e_{im} = e^*_m$, we obtain (5), which is a sufficient condition for optimality due to concavity of U_i. Therefore, if a solution to (5) exists, then it is a pure-strategy Nash equilibrium. To prove this existence, we convert (5) into M equations, each of which consists of a single variable. From (5), we have $A_m r'(e^*_m) I_N = \psi'(E^*)$ for all $m \in \{1,2,...,M\}$, where $E^* = \sum_{k=1}^M e^*_k$. Since I_N and E^* are the same for all m, we have

$$A_m r'(e^*_m) = A_k r'(e^*_k) \text{ for all } m,k \in \{1,2,...,M\}.$$

From this relationship, we obtain

$$e^*_k = (r')^{-1}\left(\frac{A_m r'(e^*_m)}{A_k}\right). \tag{16}$$

By plugging (16) back into (5), we obtain

$$\Omega_m(e^*_m, A_1, A_2, ..., A_m) \equiv A_m r'(e^*_m) I_N - \psi'\left(\sum_{k=1}^M (r')^{-1}\left(\frac{A_m r'(e^*_m)}{A_k}\right)\right) = 0. \tag{17}$$

If we have a solution to (17) for any $m \in \{1,2,...,M\}$, then we have a solution to (5). Let $\Phi_m(e) = \Omega_m(e, A_1, A_2, ..., A_m)$. Φ_m is continuous in e because r' and and ψ' are continuous. Furthermore, $\lim_{e\to 0}\Phi_m(e) > 0$ because $\lim_{e\to 0} r'(e) = +\infty$ and $\lim_{e\to 0}\frac{\psi'(e)}{r'(e)} = 0$. Also, $\Phi_m(e) < 0$ for sufficiently large e because $\lim_{e\to+\infty}\frac{\psi'(e)}{r'(e)} = +\infty$. Thus, Intermediate Value Theorem dictates that there exists e^*_m such that $\Phi_m(e^*_m) = 0$. Therefore, there exists a solution to (5), and this solution yields the symmetric pure-strategy Nash equilibrium. ■

A.2. Agent Effort and Utility

We analyze how the agent's total effort and utility change with the number of contests M.

LEMMA A3. *Suppose that $r(e) = \theta\log(e)$ and $\psi(e) = ce^b$ for $\theta, b, c > 0$. As the number of contests M increases, the agent's total effort E^* and utility U_i increase under both centralized and decentralized non-exclusive cases.*

Proof. In the centralized case, from Proposition 2(a), we can derive the equilibrium effort as $e^{*,P} = \left(\frac{\theta A^{*,P} I_N M^{1-b}}{cb}\right)^{\frac{1}{b}}$, where the optimal award $A^{*,P} = \frac{\theta}{b}$. Thus, the total effort is

$$E^{*,P} = Me^{*,P} = M^{1+\frac{1-b}{b}}\left(\frac{\theta^2 I_N}{cb^2}\right)^{\frac{1}{b}} = M^{\frac{1}{b}}\left(\frac{\theta^2 I_N}{cb^2}\right)^{\frac{1}{b}}, \tag{18}$$

which is increasing in M. In the decentralized case, by Corollary 1, the equilibrium effort $e^{*,E} = \left(\frac{\theta A^{*,E} I_N M^{1-b}}{cb}\right)^{\frac{1}{b}}$, where the equilibrium award $A^{*,E} = \frac{\theta(Mb-b+1)}{Mb}$. Then, the total effort is

$$E^{*,E} = Me^{*,E} = M\left(\frac{\theta^2 (Mb-b+1) I_N M^{-b}}{cb^2}\right)^{\frac{1}{b}} = (Mb-b+1)^{\frac{1}{b}}\left(\frac{\theta^2 I_N}{cb^2}\right)^{\frac{1}{b}}, \tag{19}$$

which is also increasing in M.

In the centralized case, the agent's utility $U_i = \frac{AM}{N} - \psi(E^*)$ satisfies

$$U_i^P[M] = \frac{\theta M}{Nb} - M\left(\frac{\theta^2 I_N}{b^2}\right) = M\left[\frac{\theta}{Nb} - \frac{\theta^2 I_N}{b^2}\right] \geq 0, \tag{20}$$

because the agent can achieve zero utility by setting effort to zero. Then, the derivative of U_i^P

$$\frac{\partial U_i^P[M]}{\partial M} = \frac{\theta}{Nb} - \frac{\theta^2 I_N}{b^2} \geq 0.$$

In the decentralized case, the agent's utility $U_i = \frac{AM}{N} - \psi(E^*)$ satisfies

$$U_i^E[M] = \frac{\theta\,(Mb - b + 1)}{Nb} - (Mb - b + 1)\left(\frac{\theta^2 I_N}{b^2}\right) = (Mb - b + 1)\left[\frac{\theta}{Nb} - \frac{\theta^2 I_N}{b^2}\right] \geq 0, \tag{21}$$

because the agent can achieve zero utility by setting effort to zero. Then, the derivative of U_i^P

$$\frac{\partial U_i^E[M]}{\partial M} = b\left[\frac{\theta}{Nb} - \frac{\theta^2 I_N}{b^2}\right] \geq 0. \blacksquare$$

Lemma A3 shows that the agent's total effort increases with the number of contests M under both centralized and decentralized non-exclusive cases. The intuition is as follows. Participating in more contests increases the number of draws the agent receives from her output shock, so it leads to risk diversification for her. Thus, the agent finds it beneficial to increase her total effort and to diversify her risk by splitting her total effort over more contests. In fact, as we also show in Lemma A3, this risk diversification leads to a higher utility for the agent as the number of contests M increases, regardless of the curvature of her cost function.

B. Proofs

We start by presenting five lemmas that are used in the proof of Proposition 1, and then we present the proof of Proposition 1. The following two lemmas extend Lemma EC.1 and EC.3 of Ales et al. (2016a) by slightly changing the notation to fit to our paper. Note that Ales et al. (2016a) consider only a single contest, i.e., $M = 1$.

LEMMA A4. *(Adopted from Lemma EC.1 of Ales et al. 2016a) Suppose that density h is log-concave. Then,* $\mu_{(j)}^{N+1} - \mu_{(j)}^{N} < \mu_{(j+1)}^{N+1} - \mu_{(j+1)}^{N}$ *for all* $j \in \{1, 2, ..., N-1\}$.

Proof. Let $\delta_{(j)}^N \equiv \mu_{(j)}^N - \mu_{(j+1)}^N$. We want to show that $\delta_{(j)}^N > \delta_{(j)}^{N+1}$ for all j. From Galton (1902),

$$\delta_{(j)}^N = \binom{N}{j} \int_{\underline{s}}^{\overline{s}} H(s)^{N-j}(1 - H(s))^j ds.$$

Rewriting this equation in terms of density of the j-th highest output shock, $h_{(j)}^N(s)$, and integrating it by parts, we obtain the following relationship:

$$\delta_{(j)}^N = \frac{1}{j}\int_{\underline{s}}^{\overline{s}} h_{(j)}^N(s)\frac{(1 - H(s))}{h(s)} ds = \frac{1}{j} H_{(j)}^N(s)\frac{(1 - H(s))}{h(s)}\Big|_{\underline{s}}^{\overline{s}} - \frac{1}{j}\int_{\underline{s}}^{\overline{s}} H_{(j)}^N(s)\left(\frac{(1 - H(s))}{h(s)}\right)' ds.$$

Using the equation above, we can derive $\delta_{(j)}^{N+1} - \delta_{(j)}^N$ as

$$\delta_{(j)}^{N+1} - \delta_{(j)}^N = \mu_{(j)}^{N+1} - \mu_{(j)}^N - (\mu_{(j+1)}^{N+1} - \mu_{(j+1)}^N) = \int_{\underline{s}}^{\overline{s}} [H_{(j)}^N(s) - H_{(j)}^{N+1}(s)]\left(\frac{(1 - H(s))}{h(s)}\right)' ds < 0,$$

because $H_{(j)}^{N+1}(s) \leq H_{(j)}^N(s)$ for all s (and $H_{(j)}^{N+1}(s) < H_{(j)}^N(s)$ for a measurable subset of Ξ), and log-concavity implies that $\left(\frac{(1-H(s))}{h(s)}\right)' < 0$ for all s. $\blacksquare$

Lemma A5. *(Adopted from Lemma EC.3 of Ales et al. 2016a) Suppose that* $M=1$, $r'(g(x))g'(x)$ *is decreasing in* x, *and that the output shock* $\widetilde{\xi}_{im}$ *is transformed to* $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ *via a scale transformation with* $\alpha>0$. *Then,* $\lim_{\alpha\to+\infty}\frac{A^*}{\alpha}=0$.

Proof. Under a scale transformation of $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$, I_N is converted to $\widehat{I_N}=I_N/\alpha$. Note that when $M=1$, the award $A^*[\alpha]$ under the centralized and decentralized cases both satisfy (9)

$$r'\left(g\left(\frac{A^*[\alpha]I_N}{\alpha}\right)\right)g'\left(\frac{A^*[\alpha]I_N}{\alpha}\right)\frac{I_N}{\alpha}-1=0. \tag{22}$$

Because $r'(g(x))g'(x)$ is decreasing in x, and I_N/α is decreasing in α, in order for A to satisfy (22), $A^*[\alpha]/\alpha$ should be decreasing with α. Since $A^*[\alpha]/\alpha$ is decreasing in α, and $A^*[\alpha]\geq 0$, $A^*[\alpha]/\alpha$ converges. Furthermore, because $\lim_{\alpha\to+\infty}\frac{I_N}{\alpha}=0$, to satisfy (22), we need $\lim_{\alpha\to+\infty}\frac{A^*[\alpha]}{\alpha}=0$. ■

Lemma A6. *For any* k, n_1, *and* n_2 *such that* $n_2>n_1$, *we have* $\mu_{(1)}^{n_2+k}-\mu_{(1)}^{n_2}<\mu_{(1)}^{n_1+k}-\mu_{(1)}^{n_1}$.

Proof. We first show that $\mu_{(1)}^{n_1+2}-\mu_{(1)}^{n_1+1}<\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$. According to Relation 1 on page 44 of David and Nagaraja (2003), $(n+1)\mu_{(1)}^{n_1}=n\mu_{(1)}^{n_1+1}+\mu_{(2)}^{n_1+1}$. From this relationship, we can show that $\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}=\frac{1}{n_1+1}(\mu_{(1)}^{n_1+1}-\mu_{(2)}^{n_1+1})$. Lemma A4 shows that $(\mu_{(1)}^{n_1+1}-\mu_{(2)}^{n_1+1})$ is strictly positive and decreasing in n_1. Because $\frac{1}{n_1+1}$ is also strictly decreasing in n_1, we have $\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$ strictly decreasing in n_1. We use induction for the rest of the proof. From above, we have $\mu_{(1)}^{n_1+j+2}-\mu_{(1)}^{n_1+j+1}<\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$. Suppose that for $k>1$, we have $\mu_{(1)}^{n_1+k}-\mu_{(1)}^{n_1+k-1}<\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$. Then, because $\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$ strictly decreasing in n_1, $\mu_{(1)}^{n_1+k+1}-\mu_{(1)}^{n_1+k}<\mu_{(1)}^{n_1+k}-\mu_{(1)}^{n_1+k-1}<\mu_{(1)}^{n_1+1}-\mu_{(1)}^{n_1}$. Thus, we can rewrite this relationship as $\mu_{(1)}^{n_1+1+k}-\mu_{(1)}^{n_1+1}<\mu_{(1)}^{n_1+k}-\mu_{(1)}^{n_1}$. Using an induction as above, we can show that for any $n_2>n_1$, we have $\mu_{(1)}^{n_2+k}-\mu_{(1)}^{n_2}<\mu_{(1)}^{n_1+k}-\mu_{(1)}^{n_1}$. ■

Lemma A7. *Let* $N_1=\lfloor N/2\rfloor$ *and* $N_2=\lceil N/2\rceil$. $\mu_{(1)}^{N_1}+\mu_{(1)}^{N_2}>\mu_{(1)}^{N_1'}+\mu_{(1)}^{N_2'}$ *for any* $N_1'\in\{1,2,..,N\}\setminus\{N_1,N_2\}$ *and* $N_2'=N-N_1'$.

Proof. Suppose without loss of generality that $N_2'>N_1'$ (note that $N_1'=N_2'$ is not possible as $N_1'\notin\{N_1,N_2\}$). Then, we have $N_2'>N_2\geq N_1>N_1'$, and $k\equiv N_2'-N_2=N_1-N_1'>0$. In this case, because $N_1'<N_2$, by letting $n_1=N_1'$ and $n_2=N_2$, Lemma A6 dictates that $\mu_{(1)}^{N_1'+k}-\mu_{(1)}^{N_1'}>\mu_{(1)}^{N_2+k}-\mu_{(1)}^{N_2}$. Thus, $\mu_{(1)}^{N_1}+\mu_{(1)}^{N_2}>\mu_{(1)}^{N_1'}+\mu_{(1)}^{N_2'}$. ■

Lemma A8. *Suppose that the are two exclusive contests. There exists* α_e *such that for any scale transformation* $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ *of the output shock* $\widetilde{\xi}_{im}$ *with scale parameter* $\alpha>\alpha_e$, *it is optimal for the planner to distribute the number of agents as evenly as possible between contests, where one contest has* $\lfloor N/2\rfloor$ *agents and the other one has* $\lceil N/2\rceil$ *agents.*

Proof. Suppose to the contrary that it is not optimal for the planner to distribute agents as proposed. Suppose that, instead, it is optimal for the planner to assigns N_1 agents to the first contest and N_2 agents to the other contest. Let A_1 and A_2 be the optimal award in the first and second contest, respectively. Note that we have $N_1 \notin \{\lfloor N/2 \rfloor, \lceil N/2 \rceil\}$ and $N_2 = N - N_1$. In this case, because agents are split between contests, each contest is an individual contest with different agents, so the equilibrium effort can be derived from (5) under $M=1$. Thus, $e_m^* = \left(\frac{\psi'}{r'}\right)^{-1}(A_m I_{N_m})$. Then, the planner objective can be written as $\overline{\Pi} = \frac{1}{2}\sum_{m=1}^{2}\left(r(e_m^*) + \mu_{(1)}^{N_m} - A_m\right)$. Consider the alternative case in which the planner assigns $\lfloor N/2 \rfloor$ agents to the first contest and $\lceil N/2 \rceil$ agents to the second contest, and sets the award at each contest so that agents' efforts are the same at each contest. Specifically, the award in the first contest is $\frac{A_1 I_{N_1}}{I_{\lfloor N/2 \rfloor}}$ and the award in the second contest is $\frac{A_2 I_{N_2}}{I_{\lceil N/2 \rceil}}$. Also, consider a scale transformation $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with scale parameter $\alpha > 0$. After the transformation, $E[\widehat{\xi}_{im}] = \alpha E[\widetilde{\xi}_{im}]$ and $\widehat{I_N} = I_N/\alpha$ for all N. After the transformation, when the number of agents at each contest changes from N_1 and N_2 to $\lfloor N/2 \rfloor$ and $\lceil N/2 \rceil$, respectively, and the awards are set as discussed above, the change in the planner objective can be written as

$$\Delta\overline{\Pi} \equiv \frac{\alpha}{2}\left(\mu_{(1)}^{\lfloor N/2 \rfloor} + \mu_{(1)}^{\lceil N/2 \rceil} - \sum_{m=1}^{2}\mu_{(1)}^{N_m} + \frac{A_1}{\alpha}\left(1 - \frac{I_{N_1}}{I_{\lfloor N/2 \rfloor}}\right) + \frac{A_2}{\alpha}\left(1 - \frac{I_{N_2}}{I_{\lceil N/2 \rceil}}\right)\right). \tag{23}$$

As Lemma A7 shows, $\left(\mu_{(1)}^{\lfloor N/2 \rfloor} + \mu_{(1)}^{\lceil N/2 \rceil} - \sum_{m=1}^{2}\mu_{(1)}^{N_m}\right) > 0$, and as Lemma A5

shows, when $r'(g(x))g'(x)$ is decreasing in x, $\lim_{\alpha\to\infty} A_m^*/\alpha = 0$ for each m. Thus, for sufficiently large α, we have $\Delta\overline{\Pi} > 0$, which contradicts the optimality of N_1 and N_2. Because there is a finite number of combinations for N_1 and N_2, there exists α_0 such that for any $\alpha > \alpha_0$, $\Delta\overline{\Pi} > 0$ for any N_1 and N_2. Thus, when $\alpha > \alpha_0$, it is optimal for the planner to have one contest with $\lfloor N/2 \rfloor$ agents and the other contest with $\lceil N/2 \rceil$ agents. ■

Proof of Proposition 1. We prove the result for two contests but the result can be generalized to any number of contests $M > 2$. Let $e^{*,N,M}[A] = \left(\frac{\psi'}{r'}\right)^{-1}\left(\frac{AI_N}{M^{b-1}}\right)$. Under the exclusive case, suppose that a planner optimally chooses the set of agents and awards at each contest so as to maximize $\overline{\Pi}$ in (7). Under exclusivity, let $N_1^{*,X}$ and $N_2^{*,X}$ be the optimal number of agents and $A_1^{*,X}$ and $A_2^{*,X}$ be the optimal award at contest 1 and 2, respectively.

(a) We first compare the planner objective under exclusivity with that under no exclusivity. Under exclusivity, the equilibrium effort is $e_1^{*,X} \equiv e^{*,N_1^{*,X},1}[A_1^*]$ in contest 1 and $e_2^{*,X} \equiv e^{*,N_2^{*,X},2}[A_2^*]$ in contest 2. Without loss of generality, suppose that $e_1^{*,X} \le e_2^{*,X}$. After incorporating the optimal solution, the planner's objective function under exclusivity becomes

$$\overline{\Pi}^X = \frac{1}{2}\sum_{m=1}^{2}(r(e_m^{*,X}) + \mu_{(1)}^{N_m^{*,X}} - A_m^{*,X}). \tag{24}$$

We next show that when the output shock is sufficiently spread out, the planner objective is higher under no exclusivity. Under no exclusivity, consider the case where the planner offers an award A at each contest so that the equilibrium effort at each contest is $e_m^{*,P} = (e_1^{*,X} + e_2^{*,X})/2$. Note that by (5), we know that A satisfies $AI_N r'\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right) = \psi'(e_1^{*,X}+e_1^{*,X}) = 2^{b-1}\psi'\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right)$. Thus, award A satisfies the following upper bound:

$$A = \frac{2^{b-1}}{I_N}\frac{\psi'}{r'}\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right) \leq \frac{2^{b-1}}{I_N}\frac{\psi'}{r'}(e_2^{*,X}) = \frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{I_N}. \tag{25}$$

Then, the planner objective under no exclusivity can be written as

$$\overline{\Pi}^P = r\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right) + \mu_{(1)}^N - A \geq \frac{1}{2}\sum_{m=1}^{2} r(e_m^{*,X}) + \mu_{(1)}^N - \frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{I_N}. \tag{26}$$

Suppose that output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. The difference between the planner objective under no exclusivity and that under exclusivity satisfies

$$\overline{\Pi}^P - \overline{\Pi}^X \geq \alpha\left(\mu_{(1)}^N - \frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}} + \frac{1}{2}\sum_{m=1}^{2}\frac{A_m^{*,X}}{\alpha} - \frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{\alpha I_N}\right). \tag{27}$$

As Lemma A5 shows, when $r'(g(x))g'(x)$ is decreasing in x, $\lim_{\alpha\to\infty} A_m^{*,X}/\alpha = 0$ for each m. Furthermore, because $N > N_m^{*,X}$ for sufficiently large α (as shown in Lemma A8), by definition of a mean of an order statistic, $\mu_{(1)}^N > \frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}}$. Thus, for sufficiently large α, $\overline{\Pi}^P - \overline{\Pi}^X > 0$.

(b) We next compare the average profit of organizers in equilibrium under no exclusivity with that under exclusivity. Suppose that $r(e) = \theta\log(e)$ and $\psi(e) = ce^b$ for $\theta, b, c > 0$. By Corollary 1, under no exclusivity, the equilibrium effort $e^{*,E} = \left(\frac{\theta A^{*,E}I_N M^{1-b}}{cb}\right)^{\frac{1}{b}}$, where the equilibrium award $A^{*,E} = \frac{\theta(Mb-b+1)}{Mb}$. Then, the average profit of organizers in equilibrium under no exclusivity is

$$\overline{\Pi}^E = r(e^{*,E}) + \mu_{(1)}^N - A^{*,E} = \frac{\theta}{b}\log\left(\frac{\theta^2 I_N M^{1-b}(Mb-b+1)}{cb^2 M}\right) + \mu_{(1)}^N - \frac{\theta(Mb-b+1)}{Mb}. \tag{28}$$

For contest m, the equilibrium effort under exclusivity is $e_m^{*,X} = \left(\frac{\theta A_m^{*,X} I_{N_m^{*,X}}}{cb}\right)^{\frac{1}{b}}$, where the optimal award $A_m^{*,X} = \frac{\theta}{b}$. Then, the average profit of organizers under exclusivity is

$$\overline{\Pi}^X = \frac{1}{2}\sum_{m=1}^{2}\left(r(e_m^{*,X}) + \mu_{(1)}^{N_m^{*,X}} - A_m^{*,X}\right) = \frac{1}{2}\sum_{m=1}^{2}\left(\frac{\theta}{b}\log\left(\frac{\theta^2 I_{N_m^{*,X}}}{cb^2}\right) + \mu_{(1)}^{N_m^{*,X}} - \frac{\theta}{b}\right). \tag{29}$$

Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. The difference between the average profit of organizers under no exclusivity and that under exclusivity

$$\overline{\Pi}^E - \overline{\Pi}^X = \frac{\theta}{b}\log\left(\frac{I_N\,(Mb-b+1)}{M^b(I_{N_1^{*,X}}I_{N_2^{*,X}})^{1/2}}\right) + \alpha\left(\mu_{(1)}^N - \frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}}\right) - \frac{\theta(M-1)(b-1)}{bM}. \tag{30}$$

As $N > N_m^{*,X}$ for sufficiently large α (see Lemma A8), by definition of an order statistic, $\mu_{(1)}^N > \frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}}$. Thus, for sufficiently large α, $\overline{\Pi}^E - \overline{\Pi}^X > 0$, so there exists α_1 such that for any scale transformation $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with $\alpha > \alpha_1$, the average profit of organizers under non-exclusive equilibrium $\overline{\Pi}^E$ is greater than that under exclusive optimum $\overline{\Pi}^X$. ■

Proof of Proposition 2. **(a)** We first prove that it is optimal for the planner to give equal awards at all contests. Then, we prove the results in Proposition 2(a).

Suppose to the contrary that it is optimal for the planner to set different awards for different contests. Specifically, there exist two contests in which the planner sets a higher award for one of these contests. Without loss of generality, we label the contest with the highest award as contest 1 and the contest with the lowest award as contest 2. Then, in the optimal award scheme $(A_1^*, A_2^*, ..., A_M^*)$, $A_1^* = \max\{A_1^*, A_2^*, ..., A_M^*\} > A_2^* = \min\{A_1^*, A_2^*, ..., A_M^*\}$. Because r' is decreasing, we have $e_1^* > e_2^*$ from (16). Consider a perturbation with an alternative set of awards $(A_1, A_2, ..., A_M)$ such that $e_1 = e_1^* - \epsilon$ and $e_2 = e_2^* + \epsilon$ (with $\epsilon \in [0, (e_1^* - e_2^*)/3]$), and $e_j = e_j^*$ for all $j > 2$. Because the total effort $E = \sum_{k=1}^M e_k = \sum_{k=1}^M e_k^* = E^*$, we can deduce from (5) that $A_j = A_j^*$ for all $j > 2$, and that $A_1 = \frac{r'(e_1^*)A_1^*}{r'(e_1)}$ and $A_2 = \frac{r'(e_2^*)A_2^*}{r'(e_2)}$. Then, the change in the planner objective $\overline{\Pi}$ after the perturbation can be written as (noting that $e_1 = e_1^* - \epsilon$ and $e_2 = e_2^* + \epsilon$)

$$\Delta \equiv \frac{1}{M}\sum_{i=1}^{2}(r(e_i) - r(e_i^*) - A_i + A_i^*) = \frac{1}{M}\sum_{i=1}^{2}\left(r(e_i) - r(e_i^*) - \frac{r'(e_i^*)A_i^*}{r'(e_i)} + A_i^*\right)$$

$$= \frac{1}{M}\left[r(e_2^* + \epsilon) - r(e_2^*) - (r(e_1^*) - r(e_1^* - \epsilon)) - A_1^*\left(\frac{r'(e_1^*) - r'(e_1^* - \epsilon)}{r'(e_1^* - \epsilon)}\right) + A_2^*\left(\frac{r'(e_2^* + \epsilon) - r'(e_2^*)}{r'(e_2^* + \epsilon)}\right)\right].$$

Taking the limit $\lim_{\epsilon \to 0} \frac{M\Delta}{\epsilon}$, and noting from (5) that $r'(e_1^*)A_1^* = r'(e_2^*)A_2^*$, we obtain

$$\delta \equiv r'(e_2^*) - r'(e_1^*) - A_1^*\left(\frac{r''(e_1^*)}{r'(e_1^*)}\right) + A_2^*\left(\frac{r''(e_2^*)}{r'(e_2^*)}\right) = r'(e_2^*) - r'(e_1^*) - r'(e_2^*)A_2^*\left(\frac{r''(e_1^*)}{r'(e_1^*)^2} - \frac{r''(e_2^*)}{r'(e_2^*)^2}\right).$$

Because r' is decreasing and $e_1^* > e_2^*$, $r'(e_2^*) - r'(e_1^*) > 0$. Because a sufficient condition for $\delta > 0$ is that $\frac{r''}{(r')^2}$ is non-increasing, the perturbation improves the planner objective. This contradicts the optimality of the award scheme $(A_1^*, A_2^*, ..., A_M^*)$. Thus, the planner should set $A_1^* = A_2^* = \cdots = A_M^*$. Because it is optimal for the planner to set equal awards for all contests, without loss of optimality, the planner's problem can be rewritten as

$$\max_A \ r(e^*) + \mu_{(1)}^N - A, \text{ where } e^* = g(AI_N M^{1-b}). \tag{31}$$

We next prove that the agent's effort $e^{*,P}$ and optimal award $A^{*,P}$ is a solution to the planner's problem. Because ψ is homogeneous of degree b, ψ' is homogeneous of degree $(b-1)$. When $r'(g(x))g'(x)$ is decreasing in x, the planner's problem is concave in A, so the following first-order condition of the planner's problem (31) evaluated at $A = A^{*,P}$ and $e^* = e^{*,P} = g\left(A^{*,P} I_N M^{1-b}\right)$ is sufficient for optimality:

$$r'(e^{*,P})\frac{\partial e^*}{\partial A}\bigg|_{e^*=e^{*,P}, A=A^{*,P}} - 1 = r'(e^{*,P})g'\left(A^{*,P} I_N M^{1-b}\right) I_N M^{1-b} - 1 = 0. \tag{32}$$

We next show that the planner objective $\overline{\Pi}^P = \Pi_m^P = r(e^{*,P}) + \mu_{(1)}^N - A^{*,P}$, and $\overline{\Pi}^P$ is decreasing, constant, or increasing in the number of contests M when the cost function parameter $b > 1$, $b = 1$, or $b < 1$, respectively. Applying Envelope Theorem to $\overline{\Pi}^P \equiv \max_A r(e^*) + \mu_{(1)}^N - A$, we obtain

$$\frac{\partial \overline{\Pi}^P}{\partial M} = r'(e^{*,P})\frac{\partial e^*}{\partial M} = (1-b)r'(e^{*,P})g'\left(A^{*,P} I_N M^{1-b}\right) A^{*,P} I_N M^{-b}. \tag{33}$$

As ψ'/r' is increasing, $g'>0$; and as r increasing, $r'>0$. Thus, from (43), $\overline{\Pi}^P$ is decreasing, constant, or increasing in M when $b>1$, $b=1$ or $b<1$, respectively. Note that plugging the planner's optimal solution $e^{*,P}$ and $A^{*,P}$ into its objective, we can obtain $\overline{\Pi}^P=\Pi_m^P=r(e^{*,P})+\mu_{(1)}^N-A^{*,P}$.

(b) We find the symmetric equilibrium in the decentralized case, and then compare the equilibrium award with the optimal award. The organizer m's first-order condition stemming from (6) is

$$\frac{\partial \Pi_m}{\partial A_m}=r'(e_m^*)\frac{\partial e_m^*}{\partial A_m}-1=0. \tag{34}$$

Applying Implicit Function Theorem to (17), we obtain $\frac{\partial e_m^*}{\partial A_j}=-\frac{\partial \Omega_m}{\partial A_j}/\frac{\partial \Omega_m}{\partial e_m^*}$. When $m\neq j$,

$$\frac{\partial \Omega_m}{\partial A_j}=\psi''\left(\sum_{k=1}^{M}(r')^{-1}\left(\frac{A_m r'(e_m^*)}{A_k}\right)\right)\frac{A_m r'(e_m^*)}{r''\left((r')^{-1}\left(\frac{A_m r'(e_m^*)}{A_j}\right)\right)A_j^2}=\psi''\left(E^*\right)\frac{A_m r'(e_m^*)}{r''(e_j^*)A_j^2}, \tag{35}$$

where the second equality follows from (16). When $m=j$, we have

$$\begin{aligned}\frac{\partial \Omega_j}{\partial A_j}&=r'(e_j^*)I_N-\psi''\left(\sum_{k=1}^{M}(r')^{-1}\left(\frac{A_j r'(e_j^*)}{A_k}\right)\right)\left[\sum_{k\neq j}\frac{r'(e_j^*)}{r''\left((r')^{-1}\left(\frac{A_j r'(e_j^*)}{A_k}\right)\right)A_k}\right]\\&=r'(e_j^*)I_N-\psi''\left(E^*\right)\left[\sum_{k\neq j}\frac{r'(e_j^*)}{r''(e_k^*)A_k}\right].\end{aligned} \tag{36}$$

Furthermore, we have the following derivative:

$$\begin{aligned}\frac{\partial \Omega_m}{\partial e_m^*}&=A_m r''(e_m^*)I_N-\psi''\left(\sum_{k=1}^{M}(r')^{-1}\left(\frac{A_m r'(e_m^*)}{A_k}\right)\right)\sum_{k=1}^{M}\frac{A_m r''(e_m^*)}{r''\left((r')^{-1}\left(\frac{A_m r'(e_m^*)}{A_k}\right)\right)A_k}\\&=A_m r''(e_m^*)I_N-\psi''\left(E^*\right)\sum_{k=1}^{M}\frac{A_m r''(e_m^*)}{r''(e_k^*)A_k}.\end{aligned} \tag{37}$$

Using (36) and (37), we can obtain $\frac{\partial e_m^*}{\partial A_m}=-\frac{\partial \Omega_m}{\partial A_m}/\frac{\partial \Omega_m}{\partial e_m^*}$. Evaluating $\frac{\partial e_m^*}{\partial A_m}$ at symmetric equilibrium $A_m=A_k=A^{*,E}$ and $e_m^*=e_k^*=e^{*,E}$, we can rewrite (34) as

$$\begin{aligned}&-r'(e^{*,E})\frac{r'(e^{*,E})I_N-\psi''\left(Me^{*,E}\right)\left[(M-1)\frac{r'(e^{*,E})}{r''(e^{*,E})A^{*,E}}\right]}{A^{*,E}r''(e^{*,E})I_N-\psi''\left(Me^{*,E}\right)M}-1\\&=-\frac{r'(e^{*,E})^2 I_N}{A^{*,E}r''(e^{*,E})I_N-\psi''\left(Me^{*,E}\right)M}+\frac{r'(e^{*,E})^2}{r''(e^{*,E})A^{*,E}}\frac{\psi''\left(Me^{*,E}\right)(M-1)}{A^{*,E}r''(e^{*,E})I_N-\psi''\left(Me^{*,E}\right)M}-1=0.\end{aligned} \tag{38}$$

Evaluating (17) at symmetric equilibrium and letting $g=(\psi'/r')^{-1}$ yields $e^{*,E}=g\left(A^{*,E}I_N M^{1-b}\right)$. Using $A^{*,E}I_N=M^{b-1}\frac{\psi'(e^{*,E})}{r'(e^{*,E})}$, and $\psi''\left(Me^{*,E}\right)=M^{b-2}\psi''\left(e^{*,E}\right)$, we can rewrite (38) as

$$\frac{r'(e^{*,E})^3 I_N M^{1-b}}{\psi''\left(e^{*,E}\right)r'(e^{*,E})-\psi'(e^{*,E})r''(e^{*,E})}-1=\frac{(M-1)\psi''\left(e^{*,E}\right)I_N r'(e^{*,E})^2}{M^b r''(e^{*,E})\psi'(e^{*,E})}\frac{r'(e^{*,E})^2}{\psi''\left(e^{*,E}\right)r'(e^{*,E})-\psi'(e^{*,E})r''(e^{*,E})}.$$

Since $g'(x)=\frac{1}{(\psi'/r')'(g(x))}=\frac{r'(g(x))^2}{\psi''(g(x))r'(g(x))-\psi'(g(x))r''(g(x))}$ and $\psi''\left(e^{*,E}\right)=\frac{(b-1)\psi'\left(e^{*,E}\right)}{e^{*,E}}$, we get

$$r'\left(e^{*,E}\right)g'\left(A^{*,E}I_N M^{1-b}\right)I_N M^{1-b}-1=\frac{(M-1)(b-1)I_N r'\left(e^{*,E}\right)^2 g'\left(A^{*,E}I_N M^{1-b}\right)}{M^b r''(e^{*,E})e^{*,E}}.$$

We next show that the equilibrium award $A^{*,E}$ is greater than, equal to, or smaller than the optimal award $A^{*,P}$ when $b>1$, $b=1$, or $b<1$, respectively. Note that the left-hand side of (11) is the same as the left-hand side of (9) if we replace superscript ^{*,P} with ^{*,E}. The right-hand side of (9) is zero, and the right-hand side of (11) is less than, equal to, or greater than 0 when $b>1$, $b=1$, or $b<1$, respectively. Because $r'(g(x))g'(x)$ is decreasing in x, we have $r'(e^*)g'(AI_N M^{1-b})$ decreasing in A. Thus, the equilibrium award $A^{*,E}$ is greater than, equal to, or smaller than the optimal award $A^{*,P}$ when $b>1$, $b=1$, or $b<1$, respectively.■

Proof of Corollary 1. **(a)** Suppose that $r(e)=\theta\log(e)$ and $\psi(e)=ce^b$. Then $r'(e)=\frac{\theta}{e}$, $\psi'(e)=cbe^{b-1}$, and hence $g(x)=\left(\frac{\theta x}{cb}\right)^{\frac{1}{b}}$ and $g'(x)=\frac{\theta}{cb2}\left(\frac{\theta x}{cb}\right)^{\frac{1-b}{b}}$. By (10), the equilibrium effort can be written as $e^{*,E}=\left(\frac{\theta A^{*,E}I_N M^{1-b}}{cb}\right)^{\frac{1}{b}}$. Incorporating these in (11), after simplifications, we obtain

$$\frac{\theta}{A^{*,E}b}-1=-\frac{\theta(M-1)(b-1)}{bMA^{*,E}},$$

which yields the following equilibrium award:

$$A^{*,E}=\frac{\theta}{b}+\frac{\theta(M-1)(b-1)}{Mb}=\frac{\theta}{b}\frac{M+(M-1)(b-1)}{M}=\frac{\theta}{b}\frac{(Mb-b+1)}{M}.$$

Thus, the organizer profit in equilibrium $\Pi_m^E=r(e^{*,E})+\mu_{(1)}^N-A^{*,E}$ can be written as

$$\Pi_m^E=\frac{\theta}{b}\log\left(\frac{\theta^2 I_N}{cb^2}\right)+\frac{\theta(1-b)}{b}\log(M)+\frac{\theta}{b}\log\left(\frac{Mb-b+1}{M}\right)+\mu_{(1)}^N-\frac{\theta}{b}\frac{Mb-b+1}{M}.$$

(b) The derivative of the organizer's problem with respect to the number of contests M is

$$\begin{aligned}\frac{\partial\Pi_m^E}{\partial M}&=\frac{\theta(1-b)}{bM}+\frac{\theta}{b}\frac{M}{(Mb-b+1)}\frac{(b-1)}{M^2}-\frac{\theta(b-1)}{bM^2}=\left(1-\frac{1}{Mb-b+1}+\frac{1}{M}\right)\frac{\theta(1-b)}{bM}\\&=\left(\frac{M^2b-bM+M-M+Mb-b+1}{M^2b-bM+M}\right)\frac{\theta(1-b)}{bM}=\frac{1}{j}\left(\frac{(M^2-1)b+1}{b(M-1)+1}\right)\frac{\theta(1-b)}{bM^2}.\end{aligned}$$

Thus, Π_m^E is decreasing, constant, and increasing in M when $b>1$, $b=1$, and $b<1$, respectively. ■

Proof of Proposition 3. We can show that it is optimal for the planner to give equal awards at all contests, as in Proposition 2(a). Note that the argument in Proposition 2(a) does not depend on ψ, so it applies here. Then, without loss of optimality, the planner's problem can be written as

$$\max_A\ r(e^*[A])+\mu_{(1)}^N-A, \tag{39}$$

where the equilibrium effort $e^*[A]$ under award A satisfies the first-order condition (using (5))

$$a_1\psi_1'(Me^*[A])+a_2\psi_2'(Me^*[A])-r'(e^*[A])AI_N=0. \tag{40}$$

(a) We first characterize $e^{*,P}$ and $A^{*,P}$. Letting $f_j=\psi_j'/r'$, $j\in\{1,2\}$, and using the homogeneity of ψ_j', we can write $\Psi(e[A],M)\equiv a_1M^{b_1-1}f_1(e^*[A])+a_2M^{b_2-1}f_2(e^*[A])-AI_N=0$. From the planner's problem in (39), we obtain the first-order condition $r'(e^*[A])\frac{\partial e^*[A]}{\partial A}\Big|_{A=A^{*,P}}-1=0$. By Implicit Function Theorem,

$$\frac{\partial e^*[A]}{\partial A}\bigg|_{A=A^{*,P}}=-\frac{\frac{\partial\Psi(e^*[A^{*,P}],M)}{\partial A}}{\frac{\partial\Psi(e^*[A^{*,P}],M)}{\partial e^*[A]}}=\frac{I_N}{a_1M^{b_1-1}f_1'(e^*[A^{*,P}])+a_2M^{b_2-1}f_2'(e^*[A^{*,P}])}.$$

Thus, we have the characterizing equations for $e^{*,P}=e^*[A^{*,P}]$ and $A^{*,P}$ as follows:

$$a_1M^{b_1-1}f_1(e^{*,P})+a_2M^{b_2-1}f_2(e^{*,P})-A^{*,P}I_N=0, \tag{41}$$

$$a_1 M^{b_1-1} f_1'(e^{*,P}) + a_2 M^{b_2-1} f_2'(e^{*,P}) - r'(e^{*,P}) I_N = 0. \tag{42}$$

Next, we apply Envelope Theorem to $\overline{\Pi}^P \equiv \max_A \; r(e^*) + \mu_{(1)}^N - A$, and we obtain

$$\frac{\partial \overline{\Pi}^P}{\partial M} = r'(e^*[A]) \frac{\partial e^*[A]}{\partial M}\bigg|_{A=A^{*,P}}. \tag{43}$$

We use Implicit Function Theorem to obtain $\frac{\partial e^*[A]}{\partial M}\Big|_{A=A^{*,P}}$, which has the same sign as $\frac{\partial \overline{\Pi}^P}{\partial M}$

$$\frac{\partial e^*[A]}{\partial M}\bigg|_{A=A^{*,P}} = -\frac{\frac{\partial \Psi(e^*[A^{*,P}],M)}{\partial M}}{\frac{\partial \Psi(e^*[A^{*,P}],M)}{\partial e^*[A]}} = -\frac{a_1(b_1-1)M^{b_1-2} f_1(e^*[A^{*,P}]) + a_2(b_2-1)M^{b_2-2} f_2(e^*[A^{*,P}])}{a_1 M^{b_1-1} f_1'(e^*[A^{*,P}]) + a_2 M^{b_2-1} f_2'(e^*[A^{*,P}])}.$$

Since $f_j' = (\psi_j'/r')' > 0$, we have $\frac{\partial \Psi(e^*[A^{*,P}],M)}{\partial e^*[A]} > 0$. Thus, the sign of $\frac{\partial \overline{\Pi}^P}{\partial M}$ is the same as

$$\eta \equiv -\frac{\partial \Psi(e^*[A^{*,P}],M)}{\partial M} r'(e^{*,P}) M = -a_1(b_1-1)\psi_1'(E^{*,P}) + a_2(1-b_2)\psi_2'(E^{*,P}),$$

where $E^{*,P} = M e^*[A^{*,P}]$. Then, $\overline{\Pi}^P$ is increasing, constant, or decreasing in M when $\eta > 0$, $\eta = 0$, or $\eta < 0$, respectively.

(b) Suppose that r' is homogenous of degree $-k$. Then, f_j is homogenous of degree $k + b_j - 1 > 0$ for $j \in \{1,2\}$. We assume that $2k + b_j > 2$, which requires that $k > 0.5$ (this is equivalent to $r'(g(x))g'(x)$ being decreasing in §2). These assumptions, for instance, are satisfied when $r(e) = \theta \log(e)$ as in Terwiesch and Xu (2008). Note that f_j'/r' is homogenous of degree $2k + b_j - 2 > 0$. Then, letting $E^{*,P} = M e^{*,P}$ and multiplying both sides of (42) by $M^{2k-1}/r'(e^{*,P})$ yields

$$\phi(E^{*,P}, M) \equiv a_1 \frac{f_1'}{r'}(E^{*,P}) + a_2 \frac{f_2'}{r'}(E^{*,P}) - M^{2k-1} I_N = 0. \tag{44}$$

By Implicit Function Theorem,

$$\frac{\partial E^{*,P}}{\partial M} = -\frac{\frac{\partial \phi(E^{*,P},M)}{\partial M}}{\frac{\partial \phi(E^{*,P},M)}{\partial E^{*,P}}} = \frac{(2k-1)M^{2k-2} I_N}{a_1 \left(\frac{f_1'}{r'}\right)' (E^{*,P}) + a_2 \left(\frac{f_2'}{r'}\right)' (E^{*,P})} > 0,$$

because $2k + b_j - 2 > 0$ and $k > 0.5$ implies $\left(\frac{f_j'}{r'}\right)' > 0$ and $2k - 1 > 0$. Furthermore,

$$\frac{\partial \eta}{\partial M} = [-a_1(b_1-1)\psi_1''(E^{*,P}) + a_2(1-b_2)\psi_2''(E^{*,P})] \frac{\partial E^{*,P}}{\partial M} < 0,$$

because $\psi_1'' > 0$, $\psi_2'' < 0$, and $\frac{\partial E^{*,P}}{\partial M} > 0$. Thus, there exists $M^* \in [1,\infty) \cup \{+\infty\}$ such that $\eta > 0$ and hence $\frac{\partial \overline{\Pi}^P}{\partial M} > 0$ for all $M < M^*$ and $\eta < 0$ and hence $\frac{\partial \overline{\Pi}^P}{\partial M} < 0$ for all $M > M^*$; i.e., $\overline{\Pi}^P$ is unimodal in M. Note that this definition allows for $M^* = 1$, in which case, $\frac{\partial \overline{\Pi}^P}{\partial M} < 0$ for all $M > M^* = 1$, and it allows for $M^* = +\infty$, in which case, $\frac{\partial \overline{\Pi}^P}{\partial M} > 0$ for all $M < M^* = +\infty$.

We next show that under a scale transformation $\widehat{\xi}_{im} = \alpha \widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with the scale parameter α, M^* is non-decreasing in α. Let M^* be defined as above. Then $\eta > 0$ and hence $\frac{\partial \overline{\Pi}^P}{\partial M} > 0$ for all $M < M^*$ and $\eta < 0$ and hence $\frac{\partial \overline{\Pi}^P}{\partial M} < 0$ for all $M > M^*$. Consider a scale transformation $\widehat{\xi}_{im} = \alpha \widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with the scale parameter α. After the scale transformation, $\widehat{I_N} = I_N/\alpha$, so (44) becomes

$$a_1 \frac{f_1'}{r'}(E^{*,P}) + a_2 \frac{f_2'}{r'}(E^{*,P}) - M^{2k-1} \widehat{I_N} = 0.$$

As $\left(\frac{f_j'}{r'}\right)' > 0$, and $\widehat{I_N}$ decreases with α, from above, $E^{*,P}$ decreases with α. Then,

$$\frac{\partial \eta}{\partial \alpha} = [-a_1(b_1-1)\psi_1''(E^{*,P}) + a_2(1-b_2)\psi_2''(E^{*,P})]\frac{\partial E^{*,P}}{\partial \alpha} > 0,$$

because $\psi_1'' > 0$, $\psi_2'' < 0$, and $\frac{\partial E^{*,P}}{\partial \alpha} < 0$. Thus, $\eta > 0$ and hence $\frac{\partial \overline{\Pi}^P}{\partial M} > 0$ for all $M < M^*$. Therefore, M^* is non-decreasing in α.